# Seismic torsional vibration in elevated tanks

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**Abstract.** Some elevated water tanks have failed due to torsional vibrations in past earthquakes. The overall axisymmetric structural geometry and mass distribution of such structures may leave only a small accidental eccentricity between centre of stiffness and centre of mass. Such a small accidental eccentricity is not expected to cause a torsional failure. This paper studies the possibility of amplified torsional behaviour of elevated water tanks due to such small accidental eccentricity in the elastic as well as inelastic range; using two simple idealized systems with two coupled lateral-torsional degrees of freedom. The systems are capable of retaining the characteristics of two extreme categories of water tanks namely, a) tanks on staging with less number of columns and panels and b) tanks on staging with large number of columns and panels. The study shows that the presence of a small eccentricity may lead to large displacement of the staging edge in the elastic range, if the torsional-to-lateral time period ratio ( $\tau$ ) of the elevated tanks lies within a critical range of  $0.7 \le \tau \le 1.25$ . Inelastic behaviour study reveals that such excessive displacement in some of the reinforced concrete staging elements may cause unsymmetric yielding. This may lead to progressive strength deterioration through successive yielding in same elements under cyclic loading during earthquakes. Such localized strength drop progressively develop large strength eccentricity resulting in large localized inelastic displacement and ductility demand, leading to failure. So, elevated water tanks should have  $\tau$  outside the said critical range to avoid amplified torsional response. The tanks supported on staging with less number of columns and panels are found to have greater torsional vulnerability. Tanks located near faults seem to have torsional vulnerability for large  $\tau$ .

**Key words:** lateral-torsional coupling; elevated water tanks; stagings; reinforced concrete; frame-type; inelastic; strength deterioration.

### 1. Introduction

Torsional failure of some reinforced concrete as well as steel elevated water tanks has occurred in past earthquakes (Steinbrugge 1965, Steinbrugge and Moran 1954, Jain *et al.* 1994). The latest failure of this kind was the torsional failure of a reinforced concrete elevated water tank during 1993 Killari, India, earthquake (Jain *et al.* 1994). In this case, the tank container vertically collapsed burying the six supporting columns directly underneath the bottom slab of the container. This vertical collapse and the evidence of a circumferential displacement of about 0.5 metre suggest that

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torsional vibrations may have been the primary cause of failure. But, elevated water tanks with their broadly axisymmetric geometry and mass distribution, do not appear to be prone to torsion since, centre of stiffness (CS) and centre of mass (CM) tend to coincide. However, small eccentricity between CS and CM may arise due to accidental reasons. For instance, a geometrical imperfection or nonuniformity in construction in the columns could slightly move the CS. Sloshing of the water mass also may cause a shift in the CM at a given instant of time. Asymmetric placements of ladders, stairs and pipelines may also give rise to a small eccentricity.

The lateral and torsional motions of unsymmetric structures are coupled. Literature on this topic (e.g., Skinner *et al.* 1965, Shepherd and Donald 1967, Penzien 1969, Hoerner 1971, Kan and Chopra 1977, Tso and Dempsey 1980, Kan and Chopra 1981, Chopra and Hejal 1988), mainly focusing on the buildings, clearly shows that dynamic amplification of the rotational response becomes significant as the uncoupled lateral and torsional frequencies of the structure get close to each other. The above studies also conclude that this amplification effect is most pronounced when the eccentricity is small. Evidence of small eccentric structures exhibiting considerable torsional response during earthquakes due to closely-spaced torsional and lateral natural periods is also recorded in the literature (e.g., Lin and Papageorgiou 1989).

Numerous studies on the nonlinear behaviour of eccentric systems under lateral-torsional coupling have been reported in the literature (Rutenberg 1992, Chandler *et al.* 1996). Most of them study the response under recorded earthquake ground motions or associated spectra, and of systems without strength deterioration but under cyclic loading. The amplification in torsional response due to lateral-torsional coupling is reported to be absent in small eccentricity yielding systems with elastoplastic characteristics for resisting elements (Tso and Sadek 1985). This is attributed to the considerable de-tuning of the uncoupled lateral and torsional natural periods on yielding. In fact, two earlier studies (Irvine and Kountouris 1980, Tso and Sadek 1984) concluded that the peak ductility demand in small-eccentricity systems is similar to that in the corresponding symmetric systems. Also, it is reported that including the stiffness degrading characteristics in the load-resisting element does not appreciably change the peak ductility demand in comparison with that obtained without including the same (Tso and Sadek 1985).

The study presented in this paper discusses the possibility of amplification of the displacement response of the lateral load-resisting elements of the frame stagings of elevated water tanks with small eccentricity, due to tuning between torsional and lateral natural period. In fact, the displacement of the load-resisting elements is more important than overall rotation of the structure from the view point of design. On the contrary, the most of the above-mentioned existing literature focused only on the rotational response of the asymmetric systems. It is observed here, that in addition to the various parameters identified in the past, the ratio of torsional stiffness to lateral stiffness also affects the torsional response. Inelastic torsional behaviour is studied to observe the effect of strength deterioration of the reinforced concrete members under cyclic loading. The excessive elastic displacement at one edge of the staging may cause localized yielding and considerable strength drop in successive inelastic excursion. The large strength eccentricity from such progressive effect of torsion may create a very high localized displcement and ductility demand. It is observed elsewhere (Dutta 1995a) that elevated water tanks supported on usual axisymmetric frame-type staging (Fig. 1a) are likely to have tuning between torsional and lateral natural periods and hence, are torsionally vulnerable. Four alternate configurations (Figs. 1b, 1c, 1d and 1e) are suggested to avoid this problem.

# 2 Modeling of elevated water tanks

# 2.1. Idealized systems

Generally, elevated water tanks have two types of modes of lateral vibration namely, the impulsive and the sloshing modes of vibration. Similarly, impulsive and sloshing modes of vibration also exist under torsional motion. The natural periods of the sloshing modes of vibration of elevated water tanks are usually quite large in comparison with both the impulsive natural periods and the natural periods contained in the earthquake ground motions. This is evident from a few example tank problems solved in the literature (Jain and Sameer 1993). The impulsive mode of vibration strongly dominates the dynamic behaviour of elevated water tanks due to the participation of the whole structural mass and a major part of the water mass. Hence, it is considered that the coupling between impulsive modes of vibration in translation and torsion will primarily generate the torsional vibration in elevated water tanks during earthquakes. Accordingly, only impulsive modes of vibration, in translation and torsion, are considered in this study. So, the structure is modeled as a single-storey system with two degrees of freedom namely, the lateral translation and the rotation of the CM in horizontal plane (Fig. 1a). For circular cylindrical elevated water tanks, translational motion may produce hydrodynamic pressures. Torsional motion of the staging may produce

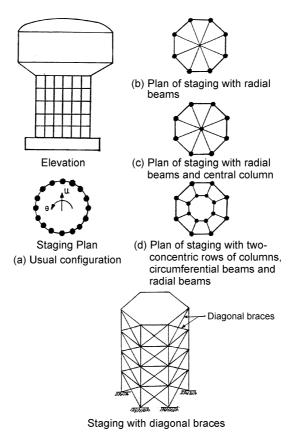


Fig. 1 Usual and a few alternate tank staging configurations

translational motion normal to the direction of excitation. However, any possible effect of these is neglected in the present study.

Elevated water tanks have resisting elements (columns) of circular frame staging placed near the perimeter of the tank. The lateral stiffness of these elements are represented by the load-resisting elements near the edges of the idealized systems as shown in the plan in Fig. 2b and 2d, respectively. The load-resisting elements are often referred as elements for simplicity in the rest of the study. These elements in the idealized systems are assumed to have only in-plane stiffness as indicated in the figure while no out-of-plane stiffness. So, the idealized models contain a rigid floor diaphragm, representing the comparatively rigid container of the tank, supported by four and two lateral load-resisting elements, and referred as four-element and two-element systems, respectively.

The plans of a staging with many columns and the corresponding idealized four-element system are shown in Fig. 2a and 2b, respectively. Four separate groups of columns are identified and marked in the figure. The two groups of columns marked by unshaded ellipses have the circumferential beams approximately spanning along the direction of ground motion (shown by arrows in the figure). These columns mainly contribute lateral stiffness in the direction of ground motion, and torsional stiffness. These can be adequately represented by two parallel load-resisting elements along the direction of ground motion, a distance D (i.e., diameter of the staging) apart. The two groups of columns marked by shaded ellipses mainly contribute to the stiffness of the staging in the direction perpendicular to that of ground motion. So, these columns do not contribute lateral stiffness in the direction of ground motion; however, these columns do contribute torsional stiffness. In the four-element idealized system, these groups of columns are represented by the two elements located at a distance D from each other along the direction perpendicular to that of ground motion (Fig. 2b). In stagings with four columns, the orientation of circumferential beams is such that all four columns equally participate in resisting the lateral force as well as torsional moment approximately with same stiffness, say, k (Fig. 2c). Such stagings are represented by the twoelement system shown in Fig. 2d wherein the two load-resisting elements (each with stiffness 2k) are at a distance D apart where D represents the diameter of the actual staging i.e., the distance between two diametrically opposite columns. These two elements equally participate in resisting lateral force as well as torsional moment. Now these idealized models are compared with actual elevated water tanks regarding the ratio of torsional and lateral stiffness and the ratio of torsional and lateral strengths as described below. These are two important parameters which may regulate lateral-torsional coupled behavior in elastic as well as post-elastic range.

A closed-form expression was derived for lateral stiffness,  $(K_x)$  of frame staging shown in Fig. 1a by the method outlined by Sameer and Jain (1992) and assuming equal heights for all panels. The

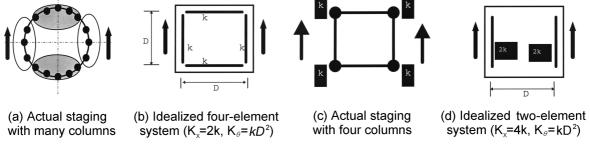


Fig. 2 Plans of stagings and corresponding idealized systems

closed-form expression for torsional stiffness ( $K_{\theta}$ ) for the same was also derived (Dutta 1995a). Subsequently, the expression for ratio of torsional and lateral stiffnesses was obtained as follows:

$$\frac{K_{\theta}}{K_{x}} = \frac{D^{2}}{4} \left[ \frac{0.0025 N_{p} (4N_{p}^{2} - 1) + N_{p} + 2(N_{p} - 1)K_{r}}{N_{p} + \frac{(N_{p} - 1)K_{r}}{\cos^{2}\left(\frac{\pi}{N_{r}}\right)}} \right]$$
(1a)

where  $N_p$ ,  $N_c$  and  $K_r$  are number of panels, number of columns and the ratio of flexural stiffness of columns and beams, respectively. The variation of ratio  $K_{\theta}/(K_xD^2)$  and the variation of  $\tau$  for such tanks are reported elsewhere (Dutta *et al.* 1996a). The above expression shows that for stagings with less number of columns and panels, the nondimensionalized stiffness ratio  $K_{\theta}/(K_xD^2)$  approaches to 0.25. For instance, this ratio is 0.255 when  $N_p$ ,  $N_c$  and  $K_r$ , all are 4. For stagings with considerably large number of columns and panels, the ratio approaches 0.5 (e.g., if  $N_p$ ,  $N_c$  and  $K_r$  are 8, 20 and 4, respectively, the ratio becomes 0.47). If D is the distance between two extreme end elements of the idealized systems in either direction, the ratio of torsional stiffness  $K_{\theta}$  and lateral stiffness  $K_x$  may be expressed in non-dimensional form as

$$\frac{K_{\theta}}{K_{x}D^{2}} = \begin{cases}
0.25 & \text{for a two-element system} \\
0.50 & \text{for a four-element system} 
\end{cases}$$
(1b)

Hence, the four-element systems reflect the stiffness ratio of the extreme category of stagings with large number of columns, and panels. On the otherhand, the two-element systems have stiffness ratio closer to that of stagings with four columns and small number of panels. An increase in the number of columns considerably increases the stiffness ratio, while an increase in number of panels increases the stiffness ratio only marginally (Dutta 1995a).

The ratio  $S_{\theta}/S_x$  of torsional strength,  $S_{\theta}$ , and lateral strength,  $S_x$ , of the idealized systems should also closely match with that of the frame stagings of elevated water tanks for fairly accurate prediction of their inelastic coupled lateral-torsional behaviour. The range of variation of  $S_{\theta}/S_x$  ratio for the circular frame-stagings of elevated water tanks are studied elsewhere (Dutta *et al.* 1996b). The analytical expressions of  $S_{\theta}$ ,  $S_x$  and  $S_{\theta}/S_x$  are derived in the same study. It is observed that  $S_{\theta}/S_x$  becomes D/2 for stagings with lesser number of columns and panels (say,  $N_c$ =4 and  $N_p$ =4) while for stagings with large number of columns and panels, this ratio becomes D where D represents the diameter of the staging. Hence, the two-element and four-element systems having  $S_{\theta}/S_x$  ratio D/2 and D, respectively (refer Fig. 2) are adequate representative of frame stagings with lesser number of columns and panels, and those with large number of columns and panels, respectively.

A staging with large number of columns and panels has higher degree of indeterminacy than that with four columns and small number of panels. Likewise, the idealized four-element system has higher indeterminacy than the idealized two-element system. However, these idealized systems do not represent the exact number of indeterminacy of the elevated water tanks they are modeling. But, behaviour of a staging with four columns with a direction of excitation perpendicular to one of the braces as shown in Fig. 2(c) may be adequately featured by two-element model. Let the translation and rotation of staging be u and  $\theta$ , under coupled lateral-torsional motion. In this case, two columns

on one side will have resultant displacement of same magnitude  $\sqrt{[u+D\theta/(2\sqrt{2})]^2+D^2\theta^2/8}$  (though the direction of resultant displacement will be different) and will yield together. Similarly, two columns on the other side will have displacement  $\sqrt{[u-D\theta/(2\sqrt{2})]^2+D^2\theta^2/8}$  and should yield together. This possibility of simultaneous yielding of two columns on each side, together, is represented through the concentrated modeling of stiffness in two-element model. The actual rigorous modeling of staging structure is avoided as the objective of the present work is to qualitatively recognize the magnification of inelastic torsional displacement due to strength deterioration, triggered by accidental eccentricity and closely spaced torsional and lateral natural periods. Apart from severe complexity involved, rigorous modeling also may not yield a very accurate result due to uncertainty involved in other parameters e.g., actual magnitude and direction of eccentricity, and parameters involved in hysteresis behaviour. The present models with  $K_\theta/(K_xD^2)$  and  $S_\theta/S_x$  ratio conforming to that of real tanks is expected to give an acceptable prediction of staging edge displacement in elastic range and at least qualitatively good prediction of the same in inelastic range. Such idealized one storey models are also extensively used to study the seismic behaviour of asymmetric buildings (Chandler *et al.* 1996).

The effect of torsion in two-element system in the post-yield range is expected to be larger than that in a four-element system because of a lower  $K_{\theta}/(K_xD^2)$  and  $S_{\theta}/S_x$  ratios of the former. Hence, a detailed study is conducted on idealized two-element systems under uni-directional ground motion, followed by a limited study on four-element systems under both uni-directional and bi-directional ground motions.

In this study, small eccentricity in the elastic range is introduced by increasing the stiffness of one lateral load-resisting element and decreasing the stiffness of another element along the direction of ground motion, by an equal amount. The element with lesser lateral stiffness will hereinafter be called a *flexible element* and the other *stiff element*.

### 2.2. System properties

In the current study, systems with  $T_x$ =0.5 sec, 1.0 sec and 2.0 sec, representing the typical natural periods in acceleration-sensitive, velocity-sensitive and displacement-sensitive regions, respectively, of the usual design response spectrum, are considered. These values of lateral natural periods also represent the realistic lateral natural periods of elevated water tanks. Earlier studies have used  $T_x/T_\theta$  as the influencing parameter for studying torsional response. However, in the present study,  $\tau = T_\theta/T_x$  is used. Systems with large  $\tau$  are torsionally flexible, and those with small  $\tau$  are torsionally stiff. The natural period ratio  $\tau$  for elevated water tanks generally takes values between 0.4 and 1.5 (Dutta 1995a). So, in the present study,  $\tau$  is varied from 0.25 to 2.0.  $\tau$  can be expressed as  $[r/D]/\sqrt{[K_\theta/(K_xD^2)]}$  where r, the radius of gyration of the mass, may independently change due to the change in container diameter or depth of water in the container etc. even if D and  $K_\theta/(K_xD^2)$  are fixed for a staging.

Normalised eccentricity is considered in two ways, namely (a) constant e/r, and (b) constant e/D. The radius of gyration r of the mass is a measure of its spread in plan. Hence, a constant e/r implies that eccentricity is a fixed percentage of the plan dimension of the mass. On the other hand, a constant value of e/D implies that eccentricity is a fixed percentage of the distance D between the extreme lateral force-resisting elements. Since r is dependent on the geometric distribution of mass in plan, it is not easy to physically visualize the meaning of constant e/r. However, one can easily reflect on the meaning of constant e/D, since D is the physical distance between lateral force

resisting elements i.e., the diameter of the tank staging. In the current study, small eccentricity cases of both e/r = 0.05, and e/D = 0.05 are considered. A large eccentricity case of e/D = 0.2 is also considered to observe the basic trends in the behaviour of large-eccentricity systems.

A constant value of damping is considered throughout the study. For harmonic single-frequency ground motion, 2% of critical damping is considered in each mode of the corresponding uncoupled symmetric system. This is adopted to facilitate decoupling of modes and to arrive at an approximate, but closed-form solution. The damping matrix so obtained is

$$[C] = \begin{bmatrix} 2m\omega_x \zeta & 0\\ 0 & 2mr^2\omega_\theta \zeta \end{bmatrix}$$
 (2)

where m is the mass of the system; r is the radius of gyration of the mass about CM;  $\omega_x$  and  $\omega_\theta$  are uncoupled lateral and torsional frequencies; and  $\zeta$  is the modal damping ratio taken as 0.02. While analysing the system for synthetic spectrum-consistent ground motions and for ground motion represented by idealized spectra, 2% of the critical damping is considered in each mode of the torsionally-coupled system.

### 2.3. Equations of motion

The equations of motion of the lateral-torsionally coupled systems in the nonlinear range can be represented as

$$\begin{bmatrix} m & 0 \\ 0 & mr^2 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} C \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{\theta} \end{Bmatrix} + \{ p \} = - \begin{bmatrix} m & 0 \\ 0 & mr^2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_g(t) \\ 0 \end{Bmatrix}$$
 (3)

where u,  $\dot{u}$  and  $\ddot{u}$  are the lateral displacement, velocity and acceleration of CM with respect to ground, respectively;  $\theta$ ,  $\dot{\theta}$  and  $\dot{\theta}$ , are the rotational deformation, velocity and acceleration of CM with respect to ground, respectively; [C] is the damping matrix;  $\{p\}$  is the stiffness-related resisting force vector, and  $\ddot{u}_g(t)$  is the translational ground acceleration at any time instant t. [C] is chosen such that the damping in each mode of the initial linear elastic system is 2% of critical damping. The damping matrix so obtained is kept constant throughout the analysis. The stiffness of individual element changes as it undergoes yielding, which introduces nonlinearity in  $\{p\}$  in the inelastic range.

The nonlinear equations of motion (Eq. 3) are numerically solved in the time domain by Newmark's  $\gamma$ - $\beta$  method using the iterative Modified Newton-Raphson technique. The Newmark's parameters are chosen as  $\gamma$ =0.5 and  $\beta$ =0.25. For systems with lateral natural period,  $T_x$ =1 sec and 2 sec, the time step for integration is taken as 0.01 sec, while for systems with  $T_x$ =0.5 sec, it is taken as 0.005 sec. These time steps were found to be sufficiently small from sample convergence studies conducted in each case.

# 3. Ground motions used

Three sets of ground motions are used in this study, namely harmonic single-frequency synthetic ground motions, spectrum-consistent synthetic ground motion, and ground motions with idealized

spectra.

Harmonic single-frequency synthetic ground motions are sinusoidal pulses of different frequencies. These ground motions are characterized in terms of the ratio  $\beta$  of ground motion frequency  $\omega$  to uncoupled lateral frequency of the structure  $\omega_x$ . In the present study,  $\beta = \omega/\omega_x$  is varied from 0.5 to 1.5. These ground motions have similarity with the nature of the ground motion generated near a fault (more specifically, the motion generated due to the movement between two sides of a fault parallel to the direction of the fault). For the example of a near-fault motion, one may refer to the accelerogram data of Port Hueneme earthquake, March 18, 1957.

A ground motion of 20.48 sec duration and consistent with a spectrum similar to the one given in Indian seismic code (IS: 1893-1984) for 2% of critical damping, is generated by a procedure detailed in literature (Khan 1987). The response spectrum regenerated from the synthetic ground motion time history has a maximum departure of around 10% from the target spectrum in the acceleration sensitive region (i.e., for natural periods upto 0.5 second), as shown in Fig. 3a. The generated time history is shown in Fig. 3b. This ground motion is referred as spectrum-consistent synthetic ground motion in the rest of the study.

A general consensus on overall seismic behaviour may be gathered considering two extreme spectra encompassing most ground motions, namely (a) the flat acceleration spectrum, i.e., the spectral acceleration (SA) is constant, and (b) the hyperbolic acceleration spectrum. Further, two variations in the hyperbolic spectrum, namely  $SA \propto 1/T$  and  $SA \propto 1/T^{2/3}$ , are considered which represent the actual and design spectra in the large period region. The responses under these three idealized spectra are obtained. Since, the responses under these three spectra are very similar, the response under the hyperbolic spectrum given by  $SA \propto 1/T^{2/3}$  is only presented for the sake of brevity.

The result of inelastic analysis exhibiting the effect of strength deterioration is presented only under spectrum-consistent synthetic ground motion to conclude about the broad reflections of the trends in behaviour. While studying the response under four-element system under bi-directional ground motion, another uncorrelated synthetic ground motion of the same duration of 20.48 sec is used along the other axis of symmetry of the system. It has a similar response spectrum and same

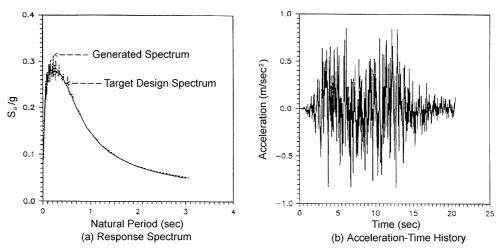


Fig. 3 Spectrum-consistent synthetic ground motion

peak ground acceleration as that of the synthetic ground motion. The fault-parallel and fault-normal pulses generated near strike-slip faults are also employed in the possible simulated idealized forms to understand the behaviour of elevated tanks located near faults. Such simulated fault parallel ground motion has a net residual slip; while the fault-normal motion has no residual slip but there is a half-cycle displacement pulse, indicating momentary opening and closing of the earth in slip region.

# 4. Elastic response to harmonic ground motions

### 4.1. Analytical derivation

The harmonic single-frequency ground motion is taken as

$$\ddot{u}_g(t) = a_g \sin \omega t \tag{4}$$

The equations of motion for idealized systems, Eq. (3), are solved analytically. The lateral displacement of CM of the damped system is

$$u(t) = -\frac{a_g}{\omega_r^2} \frac{(QR - PS)\cos\omega t + (PR + QS)\sin\omega t}{R^2 + S^2}$$
 (5)

the rotation of CM is

$$\theta(t) = -\frac{a_g}{\omega_r^2} \frac{\left(\frac{e}{r}\right) (R\sin\omega t - S\cos\omega t) \left(\frac{1}{r}\right)}{R^2 + S^2}$$
(6a)

and the contribution of rotation to the element displacement is

$$\frac{D}{2}\theta(t) = -\frac{a_g}{\omega_r^2} \frac{\left(\frac{e}{r}\right) (R\sin\omega t - S\cos\omega t) \left(\frac{D}{2r}\right)}{R^2 + S^2}$$
 (6b)

where

$$P = \frac{1}{\tau^2} - \beta^2$$

$$Q = \frac{2\beta\zeta}{\tau}$$

$$R = (1 - \beta^2)P - \frac{4\beta^2\zeta^2}{\tau} - \left(\frac{e}{r}\right)^2$$

$$S = 2\beta\zeta \left\{ P + \frac{1 - \beta^2}{\tau} \right\}$$
(7)

and

$$\beta = \frac{\alpha}{\alpha}$$

Compare a four-element system (Fig. 2b) and a two-element system (Fig. 2d) having same lateral

time period,  $T_x$ , the time period ratio,  $\tau$ , normalized eccentricity, damping and distance D between the load-resisting elements. Since,  $\tau = [r/D]/\sqrt{[K_{\theta}/(K_xD^2)]}$ , between the above two systems, the four-element system will have r as large as  $\sqrt{2}$  times the r of the two-element system because of larger  $K_{\theta}/(K_x D^2)$  ratio (values of this ratio for the above two systems can be obtained from Fig. 2). As discussed in section 2.1, the number of columns and panels of a staging regulates the influencing parameters related to the resistance of the staging frame to coupled lateral-torsional movements and hence, becomes the criteria for choosing idealized models. The discussion is section 2.1 shows that physically, the idealized four-element and two-element systems will represent two extreme categories of elevated water tanks, one supported on staging with large number of columns and panels, and the other supported on staging with less number (e.g., 4) of columns and panels, respectively. If both of them have same  $T_x$ , normalized eccentricity, damping, staging diameter D and  $\tau$ , the container radius (which primarily regulates the value of r) of the first tank will approximately be  $\sqrt{2}$  times that of the second one. Because of a smaller D/r ratio of the first tank, the contribution of torsional response to the element displacement,  $(D/2)\theta(t)$ , will be smaller as seen from Eq. (6b). So, this shows that an additional nondimensionalized parameter  $K_{\theta}/(K_{x}D^{2})$  (which is representative of number of columns and panels in the staging) is needed to be specified with usual parameters like  $T_x$ ,  $\tau$ , normalized eccentricity and modal damping to freeze D/r ratio and hence, to find out  $(D/2)\theta(t)$  as well as element displacement as a whole. This is particularly important for elevated water tanks as they may have wide variation of D/r ratio due to large variation in ratio of staging radius and container radius. This discussion shows that between two elevated water tanks having same  $T_x$ ,  $\tau$ , normalized eccentricity and modal damping, the one supported on staging with lesser columns and panels will exhibit a higher element displacement due to torsional coupling.

It may be worth-mentioning here that  $r\theta(t)$  as observed from expression of  $\theta(t)$  (given by Eq. 6a) is independent of D/r ratio and hence,  $K_{\theta}/(K_xD^2)$  ratio. The previous studies (e.g., Kan and Chopra 1977, Tso and Dempsey 1980) have considered directly or indirectly  $r\theta(t)$  as a measure of torsional effect instead of increase in displacement of individual element. Hence, the effect of  $K_{\theta}/(K_xD^2)$  ratio was not recognized. Moreover, the maximum element displacement being a function of both translation and rotation need not be maximum when  $\theta(t)$  will be maximum.

The flexible element displacement for four-element system is

$$\Delta_{f}(t) = -\frac{a_{g}}{\omega_{x}^{2}} \frac{\left(QR - PS - \frac{eS}{r\tau\sqrt{2}}\right)\cos\omega t + \left(PR + QS + \frac{eR}{r\tau\sqrt{2}}\right)\sin\omega t}{R^{2} + S^{2}}$$
(8)

and the stiff element displacement for four-element system is

$$\Delta_{s}(t) = -\frac{a_{g}}{\omega_{s}^{2}} \frac{\left(QR - PS + \frac{eS}{r\tau\sqrt{2}}\right)\cos\omega t + \left(PR + QS - \frac{eR}{r\tau\sqrt{2}}\right)\sin\omega t}{R^{2} + S^{2}}$$
(9)

The displacement of CM of the corresponding symmetric system is

$$u_0(t) = -\frac{a_g(1 - \beta^2)\sin\omega t - (2\beta\zeta)\cos\omega t}{\omega_x^2 - (1 - \beta^2)^2 + (2\beta\zeta)^2}$$
(10)

Hence, the amplitude of element displacements,  $\Delta_f(t)$  and  $\Delta_s(t)$ , normalised with respect to the amplitude of  $u_0(t)$  is

$$\Delta_{fn} = \frac{\sqrt{\left(QR - PS - \frac{eS}{r\tau\sqrt{2}}\right)^2 + \left(PR + QS + \frac{eR}{r\tau\sqrt{2}}\right)^2}}{R^2 + S^2} \sqrt{\left(1 - \beta^2\right)^2 + \left(2\beta\zeta\right)^2}$$
(11)

and

$$\Delta_{sn} = \frac{\sqrt{\left(QR + -PS + \frac{eS}{r\tau\sqrt{2}}\right)^2 + \left(PR + QS - \frac{eR}{r\tau\sqrt{2}}\right)^2}}{R^2 + S^2} \sqrt{\left(1 - \beta^2\right)^2 + \left(2\beta\zeta\right)^2}$$
(12)

### 4.2. Results and discussion

The absolute displacement u of CM, the absolute rotation  $\theta$  about CM, and the absolute displacements  $\Delta_f$  and  $\Delta_s$  of the flexible and stiff elements, respectively, increase with increase in lateral natural period  $T_x$  as seen from Eqs. (5), (6), (8) and (9). However, the normalised maximum displacements  $\Delta_{fn}$  and  $\Delta_{sn}$  of flexible and stiff elements, given by Eqs. (11) and (12), do not depend on lateral natural period  $T_x$  except through the frequency ratio  $\beta(=\omega/\omega_x)$ .

The normalised maximum flexible-element displacement  $\Delta_{fn}$  of the four-element system with e/r = 0.05 is shown in Fig. 4a. In general, the peak in  $\Delta_{fn}$  increases with decrease in  $\beta$ , except around  $\beta = 1$ . A similar trend is noticed in the normalised stiff element displacement  $\Delta_{sn}$  of the same system in Fig. 4b. Fig. 4 shows that peak displacements of either flexible or stiff element in many cases exceeds the displacement of reference symmetric system by more than 50% and even may be as high as more than twice that of symmetric system. Likewise, the rotational response, the peaks approximately occur when  $\beta$  is close to  $1/\tau$ , i.e.,  $\omega \approx \omega_{\theta}$ . Moreover, the peaks are high when  $\beta$  is less i.e., the duration of the pulse is much longer than the lateral time period of the system.

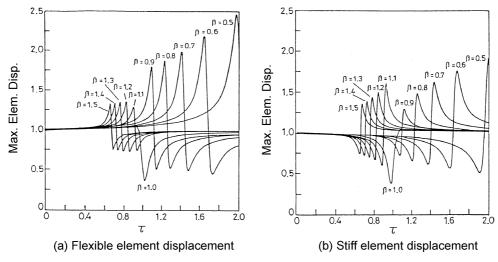


Fig. 4 Maximum normalised element displacement of four-element system for single-frequency harmonic ground motion (e/r=0.05)

For  $\tau > 1$ , the flexible element undergoes larger displacement than the stiff element; while for  $\tau < 1$ , the stiff element undergoes larger displacement than the flexible element. The general increase in maximum element displacement with increase in  $\tau$ , (even as high as more than twice that of the reference symmetric system) is owing to the increase in torsional flexibility of the system. The increase in element displacement is not high for systems with lower value of  $\tau$ .

Since, the harmonic pulses closely represent the nature of near-fault motion, the trends of the results are indicative of the behaviour of elevated tanks located near faults during earthquake. A near-fault motion may have pulse of large duration such that frequency of pulse,  $\omega \approx \omega_{\theta}$  of tanks. This may cause a very high increase in element displacement in tanks located near faults due to torsional coupling. This effect will be minimum for tanks with lower value of  $\tau$ .

# 5 Elastic response to ground motions with idealized spectra

## 5.1. Analysis

This section presents the formulation for modal displacements of flexible and stiff element of idealized systems under ground motion consistent with idealized spectra (Fig. 6). From the governing equations of motion, Eq. (3), the participation factor  $P_i$  for mode i of the two degree of freedom system is

$$P_1 = \frac{\phi_{11}}{\phi_{11}^2 + \phi_{21}^2 r^2} \text{ and } P_2 = \frac{\phi_{12}}{\phi_{12}^2 + \phi_{22}^2 r^2}$$
 (13)

where  $\phi_{1i}$  and  $\phi_{2i}$  are the mode shape vector elements corresponding to lateral and rotational degrees of freedom, respectively, in mode *i*. The normalised displacement of the flexible element in mode *i* is

$$\Delta_{fni} = \begin{cases} \left(\phi_{1i} - \frac{D}{2}\phi_{2i}\right)P_i\left(\frac{\omega_x}{\omega_i}\right)^2 & \text{for Spectral Acceleration (SA)=constant} \\ \left(\phi_{1i} - \frac{D}{2}\phi_{2i}\right)P_i\left(\frac{\omega_x}{\omega_i}\right)^{4/3} & \text{for SA} \propto 1/T^{2/3} \\ \left(\phi_{1i} - \frac{D}{2}\phi_{2i}\right)P_i\left(\frac{\omega_x}{\omega_i}\right) & \text{for SA} \propto 1/T \end{cases}$$

$$(14)$$

Similarly, the normalised displacement of the stiff element in mode i is

$$\Delta_{sni} = \begin{cases} \left(\phi_{1i} + \frac{D}{2}\phi_{2i}\right)P_i\left(\frac{\omega_x}{\omega_i}\right)^2 & \text{for SA=constant} \\ \left(\phi_{1i} + \frac{D}{2}\phi_{2i}\right)P_i\left(\frac{\omega_x}{\omega_i}\right)^{4/3} & \text{for SA} \approx 1/T^{2/3} \\ \left(\phi_{1i} + \frac{D}{2}\phi_{2i}\right)P_i\left(\frac{\omega_x}{\omega_i}\right) & \text{for SA} \approx 1/T \end{cases}$$
(15)

From Eqs. (14) and (15), the normalised maximum element displacement is a function of  $\omega_r/\omega_0$ 

 $\{\phi\}_1$ ,  $\{\phi\}_2$  and which are independent of  $T_x$  (Dutta 1995a). So, the normalised maximum element displacement in all the three cases of idealized ground motions spectra are independent of  $T_x$ . Modal analysis is conducted to obtain the response of the coupled system for the idealized flat and hyperbolic spectra. The modal responses are combined by the CQC method (e.g., Gupta 1990) using a modal damping of 2% of critical damping.

# 5.2. Results and discussion

The maximum displacement of the load-resisting elements, normalized with respect to the displacement of a similar symmetric system with same  $T_x$ , for four-element system with e/D=0.05and e/D = 0.20, and that for two-element system with e/D = 0.05 under the hyperbolic spectrum SA  $\propto 1/T^{2/3}$  is shown in Fig. 5. The curves in Fig. 5 are independent of  $T_x$ . For small-eccentricity systems, maximum normalised element displacement has a minimum near  $\tau=1$ , accompanied by a maximum each on either side within a critical range of  $0.7 < \tau < 1.25$  (Fig. 5). At  $\tau = 1$ , the rotation of CM is maximum but the lateral movement of CM becomes minimum resulting in a minimum value of element displacement (Dutta 1995a). The further details in this regard is available elsewhere (Dutta 1995a). Also, a comparison of responses of four-element and two-element systems with e/D = 0.05 in Fig. 5 confirms that lateral-torsional coupling is stronger in two-element systems than in four-element systems. In small-eccentricity systems, the maximum element displacements exceed those of the corresponding symmetric systems by upto about 15% and 30% for four-element and two-element systems, respectively. In large-eccentricity systems with four-element (e/D=0.20), this exceedance is by upto about 30% for the hyperbolic spectra (Fig. 5) while by upto about 40% for flat spectrum (these results are not presented in the limited scope of the present paper). Thus, an increase in eccentricity from 5% to 20% does not significantly increase the effect of torsional coupling on maximum element displacement. The responses under hyperbolic spectrum, as presented in Fig. 5, is very much similar with those under flat spectrum as well as under spectrumconsistent synthetic ground motion. The later two cases also include two sharp peaks in normalized element displacement within a critical range of  $0.7 < \tau < 1.25$ . So, the responses under flat spectrum and under spectrum-consistent synthetic ground motion are not included for the sake of conciseness.

There is a general concern in the literature about torsionally very flexible systems (which have

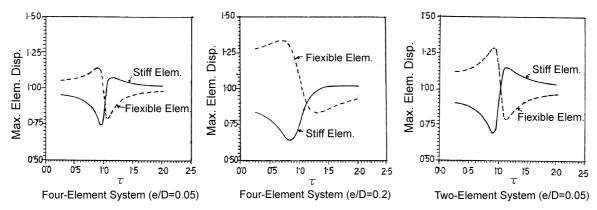


Fig. 5 Maximum normalised element displacement of eccentric systems under ground motion with hyperbolic spectrum

large  $\tau$ ). The systems with large  $\tau$  even exhibit high torsional response under near-fault motion as observed from the response under harmonic motion. This issue is also examined for response under ground motion with flat as well as hyperbolic spectrum considering systems with  $\tau$  upto 50. However, the increase in maximum normalised element displacement is observed to be very small. This observation can be explained as follows. The participation factor  $P_i$ , given by Eq. (13), reduces considerably with increase in  $\tau$ , i.e., with an increase in r for the predominantly torsional mode. This implies that if the elevated tank is located away from the fault, lateral shaking is not expected to excite considerable torsional response if rotational moment of inertia is very high, unless very high rotational kinetic energy is supplied. If the ground motion contains significant torsional component, the system with large  $\tau$  may be relatively easily excited in torsion. While torsional ground motion is possible in long-plan buildings owing to spatial variation of earthquake ground motion, in elevated water tanks, which usually have relatively small plan dimension, it is very unlikely.

# 6. Inelastic response under spectrum-consistent synthetic grounnd motion

# 6.1. Hysteresis model for strength deterioration

A number of sophisticated hysteresis models are available in the literature (e.g., Dutta 1995b) for reinforced concrete members, e.g., three parameter model (Park et al. 1987) and Roufaiel-Meyer model (Chung et al. 1990). These models incorporate stiffness degradation and pinching characteristics of reinforced concrete members in addition to the strength deterioration characteristics under cyclic loading. But the sophistication of these models can be fully utilized only through the calibration of the parameters through experimental data. However, the objective of the present study is to qualitatively examine the possibility of amplification of ductility demand due to unsymmetric yielding and subsequent progressive strength deterioration only in small-eccentricity systems. Hence, the isolated effect of strength deterioration to amplify the ductility demand is studied conveniently through a simple idealized strength deteriorating model employed for each of the load-resisting elements; instead of employing one of the rigorous models incorporating all other characteristics with separate calibration study of its parameters. In this model, the number of yield excursions regulates the extent of strength deterioration. But, for simplicity, the strength deterioration is considered as a regime-independent phenomenon in this model, i.e., the amount of plastic strain accumulated does not control the extent of strength decay.

The hysteresis rules employed in this simplified strength deteriorating hysteresis model are:

- 1. The backbone curve is elastic-perfectly plastic.
- 2. Each yielding on either side, i.e., positive side or negative side, causes a deterioration in the yield force by a definite fraction  $\delta$  of the original (undeteriorated) yield force. This deterioration is effective only at the next yielding, on either the positive side or the negative side.
- 3. If a yielding is followed by a small amount of unloading such that the current force after unloading is still higher than the new deteriorated strength after the last yielding, then a further loading will cause immediate yielding at the current force level itself.

Strength deterioration in reinforced concrete members largely depends on their detailing. A properly detailed member may exhibit a very small deterioration in strength, while a poorly detailed member exhibits a very large drop in strength under cyclic loading. However, most of the normally

detailed reinforced concrete structures exhibit considerable strength deterioration. Experimental studies are available on the load-deformation behaviour of reinforced concrete members (e.g., Brown and Jirsa 1971, Wight and Sozen 1975, Scribner and Wight 1980, Ehsani and Wight 1985, Saatcioglu and Ozcebe 1989) which provide load-deformation curves for different reinforced concrete members with different detailing schemes. For the present study, these curves are carefully examined. The total drop in strength is divided by the total number of yield excursions to obtain the average amount of strength deterioration in each yield excursion. From a number of such curves available in the literature (Brown and Jirsa 1971, Wight and Sozen 1975, Scribner and Wight 1980, Ehsani and Wight 1985, Saatcioglu and Ozcebe 1989), in most cases, the average rate of deterioration  $\delta$  in a single yield excursion is found to be around 5% of the initial yield strength for ordinarily detailed reinforced concrete specimen. However, it is found to take values upto 10%. Hence, the values of  $\delta$  used in this study are 0.0, 0.02, 0.05, 0.08, and 0.1. However, in case of bidirectional ground motion, only cases of  $\delta$  = 0.05 and 0.1 are considered.

# 6.2. Ductility reduction factor $R_{\mu}$

The extent of inelasticity in a structure under a specified loading depends on the ratio of the elastic strength demand and the actual lateral strength. The response reduction factor, R, which is the ratio of the maximum lateral strength experienced by the structure if it were to remain elastic and the design lateral force for a system, is one way of indicating the expected extent of inelasticity under a specified loading. Building structures possess large redundancies, and hence, are assigned a large value of R (e.g., NEHRP provisions specify R as high as 8 for certain building systems). However, the elevated water tank structures do not enjoy such a high degree of redundancy, and are assigned a significantly lower value of R (e.g., R=2.5 in NEHRP Recommended Provisions 1991). Since factors of safety are involved in the process of design, all structures including elevated water tanks will always possess significant overstrength over and above the design lateral force (e.g., Jain and Navin 1995). This implies that yielding will take place not at the design value of lateral force but at a higher value. A factor called the ductility reduction factor  $R_{\mu}$  is defined in the literature as the ratio of the maximum lateral force that will be experienced by the structure if it were to remain elastic and the yield lateral force. So, the ductility reduction factor  $R_{\mu}$  will be less than the response reduction factor R. This implies that  $R_{\mu}$  for elevated water tanks will be less than 2.5. In the current study, two cases of  $R_{\mu}$ =1 and  $R_{\mu}$ =2 are considered, the former being the elastic case for the symmetric system.

# 6.3. Effect of strength deterioration

While presenting results, the maximum element displacement of the eccentric system is normalized with the maximum element displacement of the corresponding symmetric system of the same lateral natural period  $T_x$ .

#### 6.3.1. Study of two-element systems

Two-element systems, with both stiffness-eccentric strength-symmetric and stiffness-symmetric strength-eccentric systems, with lateral natural period  $T_x$  of 0.5 sec, 1 sec and 2 sec have been analyzed. The variation in the normalized maximum element displacement with  $\tau(=T_\theta/T_x)$  is

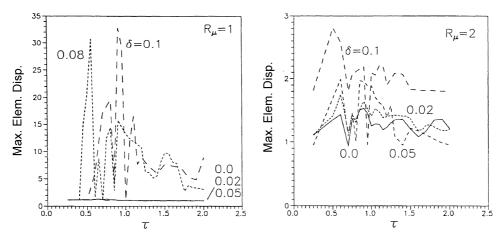


Fig. 6 Maximum normalized element displacement of stiffness-eccentric two-element system ( $T_x$ =0.5 sec, e/D=0.05)

studied. Sample response of stiffness-eccentric strength-symmetric system with lateral natural period  $T_x$ =0.5 sec and e/D=0.05 is presented in Fig. 6. These results represent the generalized trends in effect of strength deterioration observed from all other cases studied. So, the results of all other cases, available elsewhere (Dutta 1995a), are not included in the present paper.

For small-eccentricity systems studied in the present paper, the normalized element displacements presented in all the figures are almost same as the ratio of the maximum element ductility demand of these systems normalized with respect to the maximum element ductility demand of the corresponding symmetric systems.

Figs. 6 shows that the effect of eccentricity on maximum element displacement, and hence on ductility demand, generally increases with increasing rate of strength deterioration,  $\delta$ . For a high rate of strength deterioration of  $\delta$ =0.1 (or 0.08), the eccentric systems show a very high normalized element displacement even for  $R_u$ =1 as it enters the inelastic range through unsymmetric yielding. The displacement of the symmetric system is small due to an elastic range behaviour when  $R_{\mu}=1$ . As compared to such elastic range displacement, the localized inelastic range displacement of the eccentric systems are found to be very high. The maximum element displacement and ductility demand of eccentric systems are, in many instances, more than twice the element displacement and ductility demand of the reference symmetric systems. A large element displacement is also observed for  $R_u=2$  even when  $\delta=0.05$ . Since, a rate of strength deterioration of 0.05 is not unexpected in reinforced concrete members, elevated water tanks can have large displacement and ductility demand in load-resisting elements due to small accidental eccentricity. A small accidental eccentricity causes early yielding of one of the elements. A higher rate of strength deterioration further lowers the strength of that element under repeated loading. This leads to a larger strength eccentricity resulting in a further increase in element displacement, and hence, in the ductility demand. The effect may be even larger if both stiffness-eccentricity and strength-eccentricity occur together.

### 6.3.2. Study of four-element systems

Idealized four-element systems with small stiffness eccentricity and strength symmetry are studied

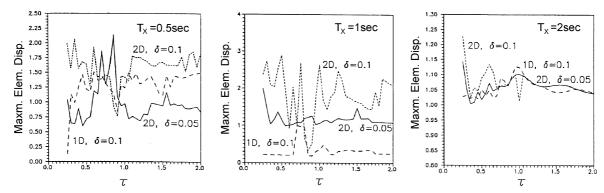


Fig. 7 Maximum normalised element displacement of stiffness-eccentric four-element system under bidirectional and uni-directional ground motion (e/D=0.05)

to investigate their inelastic behaviour. The variations in normalized maximum element displacement with  $\tau$  under bi-directional synthetic ground motion for two values of rate of strength deterioration,  $\delta$ =0.05 and 0.1, and for  $R_{\mu}$ =2, are shown in Fig. 7. The responses of the four-element systems under uni-directional ground motion with  $\delta$ =0.1, are also presented to study the effect of bi-directional ground motion. The curves are marked 1D and 2D to indicate the responses under uni-directional and bi-directional ground motions, respectively. Three values of lateral natural periods,  $T_x$ =0.5 sec, 1.0 sec and 2.0 sec, are considered.

As expected, the four-element systems show a greater effect of torsional coupling under bidirectional motion than under uni-directional ground motion. Four-element system have two additional elements with symmetric characteristics oriented along the perpendicular direction of ground motion in addition to the two elements with unsymmetric characteristics oriented along the direction of ground motion. Under uni-directional ground motion, these additional elements may remain elastic. So, even if the stiffness in the direction of ground motion becomes zero because of the yielding of both the elements, the torsional resistance generally may not reduce below 50% of the original torsional stiffness.

However, the situation may be different, if a four-element system is subjected to bi-directional ground motions. In this case, the additional elements are also expected to exhibit considerable post-yield range response during ground shaking owing to the ground motion parallel to their orientation. So, under bi-directional ground motion, the torsional resistance may become zero during ground shaking depending on the correlation between the ground acceleration in two mutually perpendicular directions. Hence, as observed in an earlier study using an elasto-plastic behaviour (Correnza and Hutchinson 1994), the present study also shows that this drop in the torsional stiffness may increase torsional response. Another reason of increasing response under bi-directional ground motion may be the introduction of unsymmetry in strength in the additional elements due to their unsymmetrical yielding and the subsequent deterioration in strength under the combined action of the lateral motion in the direction of the additional elements (i.e., along the direction of symmetry) and the torsional motion.

Further, if the curves of  $\delta = 0.1$  and 0.05 in Fig. 7 are compared with the corresponding curves for two-element systems for  $R_{\mu}=2$  in Fig. 6, a four-element system exhibits a lesser effect of torsion than two-element systems. This is so because a two-element system becomes a mechanism under coupled lateral-torsional motion if any one of the elements undergoes yielding, while a four-element

system has greater redundancy, torsional stiffness and strength. This implies that the frame stagings with large number of panels and columns represented by four-element systems are less vulnerable to the effect of torsion even in the post-yield range.

Under bi-directional ground motion, four-element systems with  $T_x$ = 0.5 sec and 1 sec, indicate high element displacement and ductility demand, upto twice or more than that of the corresponding symmetric system; this has serious implications. Such a high displacement and ductility demand caused by torsion may not be acceptable as far as the detailing of lateral load resisting elements is concerned. This clearly indicates that even elevated water tanks supported on staging with many columns and panels are highly vulnerable to the effect of torsional coupling in a real event of earthquake involving bi-directional ground motion.

Comparison of the element displacement curves in Fig. 7 for systems with  $\delta$ =0.1 and 0.05 under bi-directional ground motions shows that the effect of torsional coupling increases with rate of strength deterioration in four-element systems also.

### 7. Inelastic behaviour of elevated tanks located near tectonic faults

The inelastic torsional behaviour of large-eccentricity systems with e/r = 0.5 (appropriate for buildings) under fault-normal ground motion was reported to be significant (Goel and Chopra 1990). Inelastic behaviour of steel planar building frames under fault-parallel and fault-normal ground motions has also been reported to be very critical (Murty and Hall 1994). A similar study on elevated water tanks supported on frame stagings is expected to provide crucial inputs in their design. Elasto-plastic behaviour is assumed for each lateral load-resisting elements. Hence, the same hysteresis model with  $\delta = 0.0$  is used for the study.

However, for small eccentricity systems (e/D=0.05), the torsional coupling effect increases the maximum element displacement and hence, ductility by only a small amount (around 20-25%) as

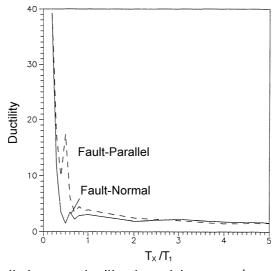


Fig. 8 Variation of element displacement ductility demand in symmetric systems under fault-parallel and fault-normal ground motions with different pulse durations

compared to a similar symmetric system.

The variation of element displacement ductility demand of a symmetric system for  $R_{\mu}$ =2 with  $T_x/T_1$  varying from 0.2 to 5 is shown in Fig. 8. This figure shows that when the pulse duration  $T_1$  is large, i.e.,  $T_x/T_1$  is small, the symmetric system itself produces a very high ductility demand. Ductility demand stabilizes at a value of around 2, when the pulse duration becomes less than the lateral natural period (i.e.,  $T_x/T_1 \ge 1$ ). So, a pulse of large duration may produce a large ductility demand in a system due to increased amplitude in lateral translation irrespective of symmetry or small eccentricity in it. Since, a pulse duration of around 2 sec is not unexpected, elevated water tanks having lateral natural period less than 2 sec may encounter a large ductility demand if situated near a tectonic fault. Large displacement may not be acceptable in elevated water tanks even from the operational point of view e.g., fracture of the connected pipelines. Moreover, such structures having large masses concentrated at considerable heights may undergo collapse owing to secondary P- $\Delta$  effects.

### 8. Conclusions

In the context of torsional failure of elevated tanks in earthquakes, present paper aims to study the torsional behaviour of water tanks supported on concrete frame-type staging (Fig. 1a), using simple idealized models. The salient conclusions of the study are listed below.

1. The torsional response due to small eccentricity arising due to accidental reasons are usually not considered by the designers. The elevated tanks with  $\tau$  within the range of  $0.7 < \tau < 1.25$  may exhibit peaks in displacement of load-resisting elements at one edge of the staging in the elastic range due to even a small accidental eccentricity. The present study shows that an accidental eccentricity of 5% may cause an elastic-range displacement demand as high as about 1.5 to even more than 2 times that of the symmetric systems under near-fault motion. Hence, the elastic range force demand will also exhibit an increase of similar order. In the event of an earthquake consistent with design spectrum, such displacement demand and force demand may be as high as 1.4 times that of the symmetric system. So, due to such a small eccentricity, the staging will not remain elastic in the event of even a moderate level earthquake, though it is designed to remain so.

Such increase in elastic range displacement at one edge of the staging may cause unsymmetric yielding in the staging. It may in turn generate a very high localized displacement and ductility demand in staging load-resisting elements due to asymmetrically progressive localized damage caused by strength-deteriorating characteristics of reinforced concrete members under cyclic loading. The effect becomes more severe with increase in rate of strength deterioration of concrete. Such effect of torsional coupling is found to be present not only in systems designed to behave inelastically ( $R_{\mu}$ =2) but also in systems marginally designed to behave elastically without any overstrength ( $R_{\mu}$ =1). Such a high displacement and ductility demand in staging load-resting elements will clearly give rise to a high curvature ductility demand in individual reinforced concrete members.

2. To limit the inelastic displacement by controlling progressive damage, one may think of using reinforced concrete members with a very low value of  $\delta$ ; as the present study shows that systems having load-resisting elements with higher  $\delta$  show very high displacement/ductility demand, after the yielding is triggered by accidental small eccentricity if  $0.7 < \tau < 1.25$ . It is observed elsewhere (Brown and Jirsa 1971, Wight and Sozen 1975, Scribner and Wight 1980, Ehsani and Wight 1985,

Saatcioglu and Ozcebe 1989) that  $\delta$  is dependent on shear span to depth ratio, longitudinal reinforcement, and spacing, quantity and configuration of transverse reinforcement. But in absence of a well accepted standardized quantitative guidelines relating  $\delta$  with above parameters, it is practically difficult to eliminate progressive, asymmetrically localized torsional damage. Such detailing of reinforcement, even if standardized, would be effective if adequate quality control can be assured.

An alternate feasible way to avoid torsional failure may be to ensure elastic behaviour of staging. So, to stop unexpected asymmetrically localized yielding, the peaks in the elastic range element displacement should preferably be avoided. To achieve this, the elevated tanks should be configured to have  $\tau$  outside the critical range of  $0.7 < \tau < 1.25$  detuning the torsional and lateral natural periods.

- 3. Between two similar elevated tanks, (i.e., with same lateral time period,  $T_x$ , the time period ratio,  $\tau$ , normalized eccentricity, and damping), the tank supported on staging with less number of columns and panels (represented by two-element systems) are found to be torsionally more severely vulnerable compared to the tank supported on staging with large number of columns and panels in elastic as well as inelastic range. Such behaviour may be attributed to the lesser torsional-to-lateral stiffness ratio and lesser torsional-to-lateral strength ratio of the former as compared to the later. So, staging with small number of columns and panels should be avoided as far as practicable.
- 4. As inferred from the response under harmonic pulses, the elevated tanks with higher  $\tau$  may exhibit considerable increase in element displacement if located near faults. Hence, the elevated tanks near faults should be configured with lesser  $\tau$ . Such tanks may also demand very high displacement and ductility capacity in the inelastic range under a long duration pulse characteristics of near fault motion even due to a purely translational behaviour.

The range of variation of natural period ratio  $\tau$  of the elevated tanks with usual staging configuration (Fig. 1a) is studied elsewhere (Dutta 1995a) for usual range of variation of the influencing parameters namely, the relative stiffness of columns and beams, number of columns and number of panels. Most of such tanks are likely to have  $\tau$  within the critical range of  $0.7 < \tau < 1.25$  under tank empty, tank full or inter mediate water-depth conditions. Thus, the problem of amplified torsional response exists in many of these tanks. The alternate staging configurations with radial beams (Fig. 1b), with radial beams and a central column (Fig. 1c), and two circular rows of columns (Fig. 1d), increase  $\tau$ . The radial beams and central column do not contribute to torsional stiffness while they increase lateral stiffness by increasing framing action. In the configuration of Fig. 1d, the inner circular row of columns contributes less to torsional stiffness because of smaller radius while it adds substantially to lateral stiffness. Configuration of Fig. 1e can be used to reduce  $\tau$  since the diagonal braces induce truss action, which creates relatively smaller increase in lateral stiffness than in torsional stiffness. These alternate configurations may be considered in order to keep  $\tau$  outside the critical range, and therefore the problem of large torsional response can be avoided.

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