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Towards improved models of shear strength degradation in reinforced concrete members

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Abstract. Existing models for the shear strength degradation of reinforced concrete members present varied conceptual approaches to interpreting test data. The relative superiority of one approach over the others is difficult to determine, particularly given the sparseness of ideal test data. Nevertheless, existing models are compared using a suite of test data that were used for the development of one such model, and significant differences emerge. Rather than relying purely on column test data, the body of knowledge concerning degradation of concrete as a material is considered. Confined concrete relations are examined to infer details of the degradation process, and to establish a framework for developing phenomenologically-based models for shear strength degradation in reinforced concrete members. The possibility of linking column shear strength degradation with material degradation phenomena is explored with a simple model. The model is applied to the results of 7 column tests, and it is found that such a link is sustainable. It is expected that models founded on material degradation phenomena will be more reliable and more broadly applicable than the current generation of empirical shear strength degradation models.

Key words: reinforced concrete; shear strength degradation; earthquake engineering; columns; plastic hinges; displacement capacity; ductility capacity; bridge piers.

1. Introduction

Field and laboratory evidence indicate that a class of reinforced concrete members is vulnerable to shear failures that develop after the member's flexural strength is attained. It appears that the shear failures are associated with degradation of the concrete under repeated and reversed cyclic loading, such as may occur in response to seismic ground shaking. The shear failures often preclude the member from reaching the displacement capacity that otherwise could be obtained if flexural failures governed, and thus play a significant role in establishing expectations of seismic performance of existing buildings and bridges.

The notion that shear strength degrades with increasing cyclic demands was put forward after the 1971 San Fernando Earthquake in a publication of the Applied Technology Council (1983), but little data were available to quantify the degradation process. In subsequent years, several models were developed and calibrated to the results of experimental tests on columns (Watanabe and Ichinose 1991, Aschheim and Moehle 1992, Priestley *et al.* 1994). These models were quite useful in that they provided a basis for evaluating the performance of existing construction short of using current design criteria, since it became possible to avoid the unnecessary retrofit of existing

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construction. However, distinct conceptual and numeric differences among the models highlight uncertainty about the nature of the degradation process and appropriate mechanical models. Given these uncertainties, one should be judicious in using any of these models to evaluate a member that deviates from those represented in the data set used in establishing the model. Lacking anything better, in some circumstances it may be prudent to apply all three models to better gauge the uncertainty that surrounds the result of any one model.

The broader topic, shear strength of reinforced concrete members, has been the subject of an enormous amount of research, particularly since 1950 (Collins 1991). The objective of this paper is not to provide an answer to the question of shear strength degradation, but rather, to outline a basic approach that can be used to establish a phenomenologically-based mechanical model. The underlying premise is that the degradation process can be modeled and that models founded on material degradation phenomena have the potential to be more robust than those based on column test data alone. The development of such a model is illustrated with an example model, whose simplicity belies the importance of the result — that a link to material degradation phenomena can be sustained.

2. Modeling and test data interpretation

Experimental test data suggest that various mechanisms of shear resistance may be mobilized in a reinforced concrete member, and that their relative contribution to the shear resistance of a cross section changes as loading progresses (Wight 1973). In analyzing experimental test data, it is often very difficult to identify the relative prominence of these mechanisms. In practice, it has become common to assign a portion of the measured shear strength to a term associated with the contribution of the transverse steel, V_s, and to assign the remaining strength to a term associated with the concrete, V_c. Effects of axial load may be considered within the V_c term, or may be assigned a separate term.

The basic task in formulating an empirical model is to identify a mechanical model that can be used to calculate the observed data. Failing this, one seeks to calculate a conservative estimate of the observed data, so as to avoid unsafe predictions of strength or deformability. The premise is that the data set can be used to validate a model that has broader applicability.

The modeler-analyst has only a few constraints to contend with in developing a model. For example, various formulas may be used to represent the steel and concrete contributions, but an untenable situation results if the contribution of one component is so large as to require the other to be negative in order to obtain the experimentally-observed shear strength. But to truly understand the underlying phenomena requires careful analysis of the test data, and, moreover, careful design of the test program itself. In most test programs, parameters such as axial load, longitudinal reinforcement, transverse reinforcement, and column height are varied independently, leading to coupled changes in the values of the parameters and the shear demand. Tests in which the parameter values were varied in such a way that the shear demand were held constant would be particularly revealing.

3. Existing shear strength degradation models

Three models that quantify shear strength degradation were developed in the 1990s. These models

(Watanabe and Ichinose 1991, Aschheim and Moehle 1992, Priestley *et al.* 1994) illustrate that different concepts may be used to represent empirical observations of column shear strength degradation. Common to all models is a relationship between displacement ductility or plastic rotation capacity and the intensity of shear. Since plastic flexural hinges are expected, shear demands depend on the length of the member. Where the shear demand is high, the ductility or rotation capacity is small. Column deformability increases as the shear required to develop flexural hinging is reduced. Where the shear demand is sufficiently small then shear does not significantly affect the deformability of the column, and ordinary methods to determine flexural deformation limits apply.

The Watanabe and Ichinose (1991) model superposes the strengths obtained by arching and truss mechanisms. Axial load is not considered to have an effect on shear strength. The shear strength in the plastic hinge zone is decreased by (1) reducing the concrete strength as a function of the plastic rotation in the plastic hinge zone and (2) limiting the angle of the truss, to limit the contribution of the truss mechanism. The latter is paradoxical, because inelastic response is typically associated with the more steeply-inclined struts; in this model the steeply inclined struts are located away from the plastic hinge zone.

The model developed at UC Berkeley (Aschheim and Moehle 1992) considers the shear strength to be the sum of a stationary value attributed to the transverse steel, V_s, and a degrading value attributed to the concrete, V_c. The V_c term considers the shear that may be sustained to specified displacement ductility limits to increase linearly with axial load.

The model developed at UC San Diego (Priestley *et al.* 1994) has steel and concrete contributions that are similar to those of the Berkeley model, but considers axial load effects differently. In the San Diego model, the axial load is resisted by a diagonal strut between the ends of the member, and its contribution to shear strength is the transverse component of this strut. Thus, the axial load contribution to shear strength is a function of the column aspect ratio (shear span divided by member depth). As axial load increases to the limit of purely concentric loading, the transverse component of the strut reduces to zero.

Efforts to identify whether the Berkeley or San Diego approach is more appropriate for modeling the effect of axial load on shear strength are hampered by a lack of suitable data. Differences between the models would be most visible for relatively high axial load ratios ($P/(A_g f_c')$), but for the data sets used to develop the models, no specimens with displacement ductility capacities in excess of unity have axial load ratios above 20% for rectangular columns and 40% for circular columns.

Different sets of experimental test data were used to develop each model, although some specimens were common to the data sets used in the Berkeley and San Diego models. If the models are intended for general use, they should be capable of representing available test data with a suitable degree of conservatism. The models were applied to the test data that were used in the development of the Berkeley model. Using each model, the shear demand that the member should be capable of sustaining was calculated for the experimentally-determined displacement capacity. With this approach, a model is considered to be unconservative if it overestimates the shear strength of the test columns, since columns that are exposed to the higher, calculated demands will tend to fail at smaller displacements.

This data set is described briefly by Aschheim *et al.* (1997). To tabulate the data in terms of displacement ductility capacity required some assumptions be made to determine plastic hinge rotations for use in the Watanabe and Ichinose model; these are also described in Aschheim *et al.* (1997).

Figs. 1, 2, and 3 plot the ratio of calculated shear strength, V_{model}, to the peak shear strength, V_{exp}, reported for the test columns. The data are sorted in order of increasing displacement ductility capacity, and the symbol at the top of each bar identifies the test specimen. The relative contributions of arch, truss, transverse reinforcement, concrete, and axial load terms, as they pertain to the various models, are identified separately in the plots. Asterisks identify those specimens that are considered in greater detail later in the paper. Figs. 1 and 2 illustrate the variability that may result when models are applied to data sets different from those used in model development. For this data set, the Watanabe and Ichinose model appears to be fairly conservative and the San Diego model is occasionally fairly unconservative. Fig. 3 is provided to complete the presentation but does not test the Berkeley model since this dataset was used to establish the model.

The variability of Figs. 1, 2, and 3 illustrates a problem inherent in pure empiricism. The models for shear strength degradation are simply hypotheses postulated by the researchers that can be used to calculate the trends observed in the data sets used by the researchers. The underlying mechanism of shear strength degradation has not been addressed directly. An approach that explicitly considers degradation as it develops at the material level may be more robust for representing shear strength degradation in columns.

Early in the development of the Berkeley model it was observed that the V_c term was elevated where a greater amount of transverse steel was present (e.g., Aschheim and Moehle 1992b). This observation implies that the transverse reinforcement enhances the contribution of the concrete, a notion that is parallel to the idea that confinement enhances the behavior of concrete in compression.

4. Degradation of confined concrete

It is generally accepted that the strength and strain capacity of concrete is enhanced in the

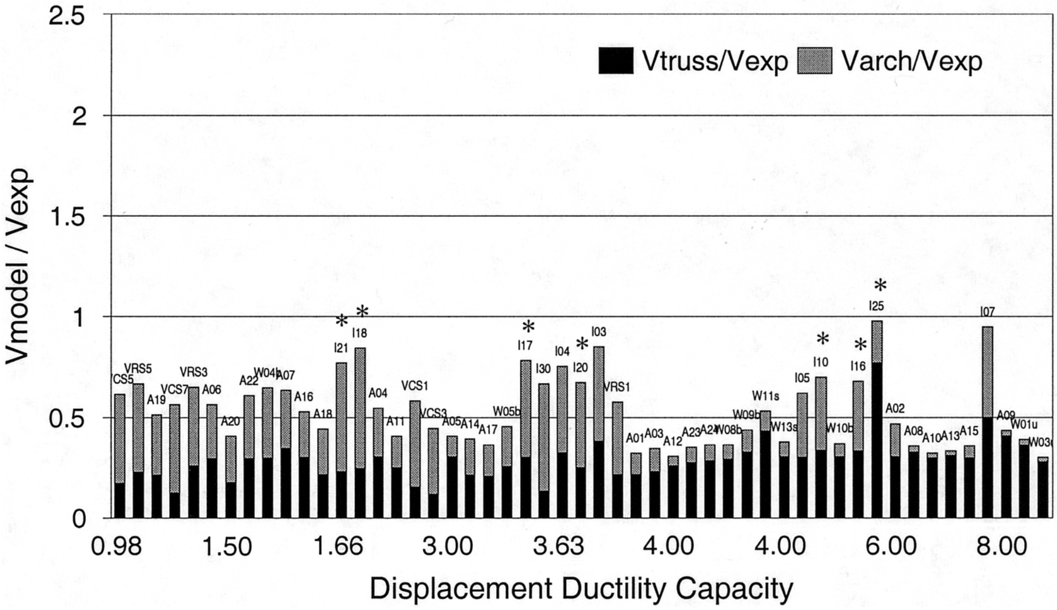


Fig. 1 Watanabe and Ichinose model

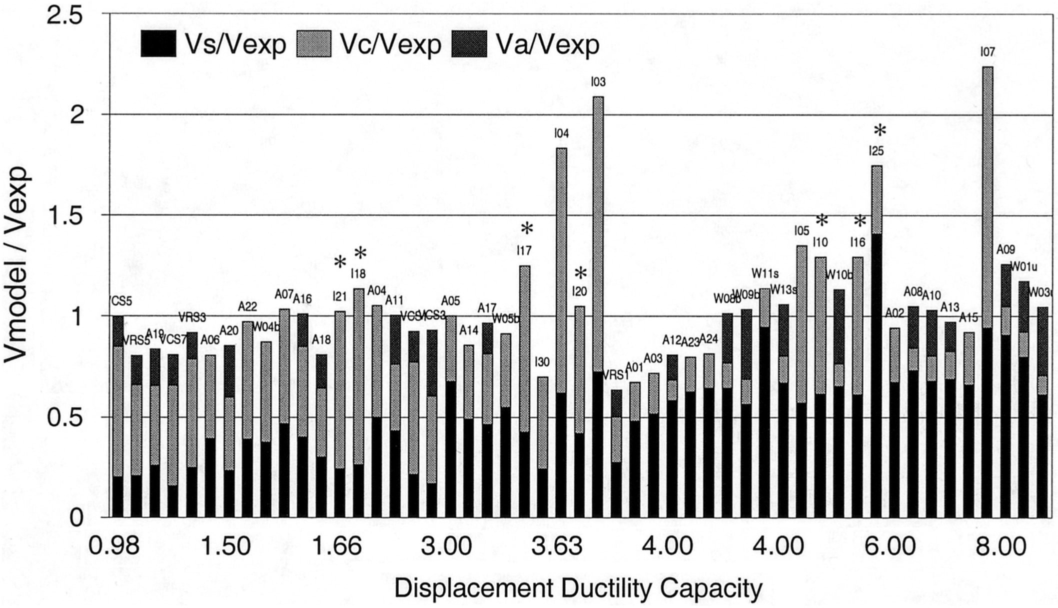


Fig. 2 San Diego model

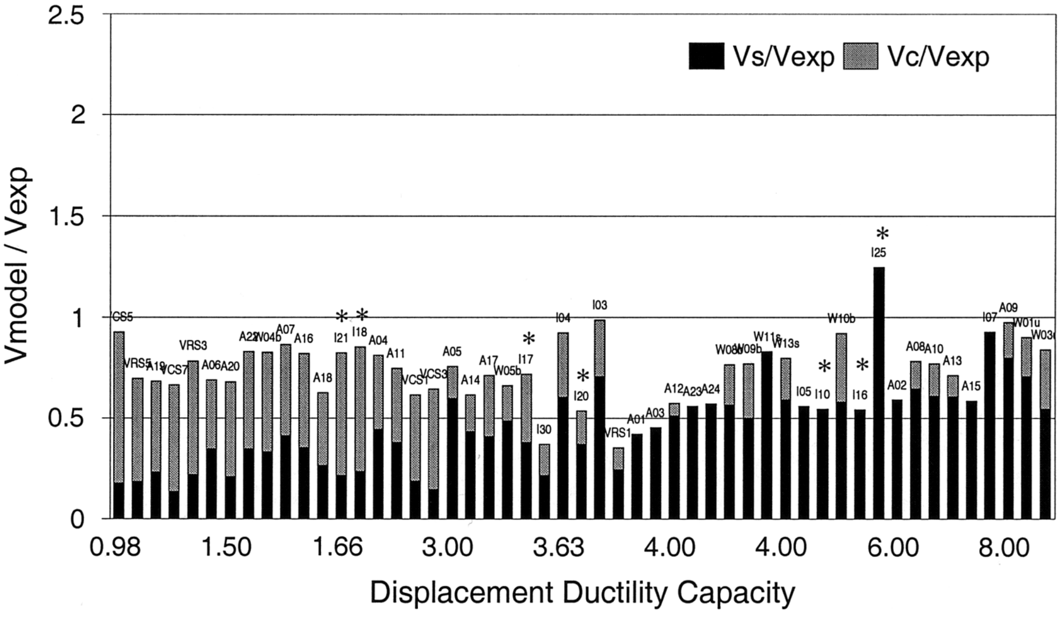


Fig. 3 Berkeley model

presence of lateral confining pressure. Several models have been developed that express the effect of confinement on concrete stress-strain response (for example, Mander 1988); the findings of these researchers are generally consistent with those of Richart (1929). From tests of 10-inch diameter concrete cylinders confined by spiral reinforcement, Richart found the enhanced compressive strength,

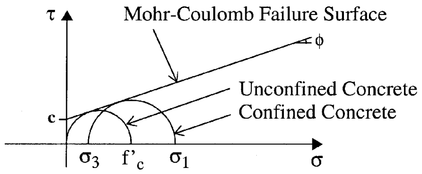


Fig. 4 Mohr-Coulomb failure surface for concrete

$f_{c,\max}$, was approximately equal to

$$f_{c,max}=f_c'+4.1\,\sigma_3 \tag{1}$$

where f_c' = unconfined concrete compressive strength and σ_3= lateral confining pressure.

A Mohr-Coulomb failure criterion may be used to express conditions causing failure in concrete (e.g., Braestrup 1981). Other, more advanced 3-, 4-, or 5- parameter models may provide more accurate predictions of concrete strength (e.g., Willam and Warnke 1974, and Han and Chen 1987), but for the purposes of this paper, the simpler Mohr-Coulomb model is sufficient to identify if these approaches hold promise for modeling shear strength degradation.

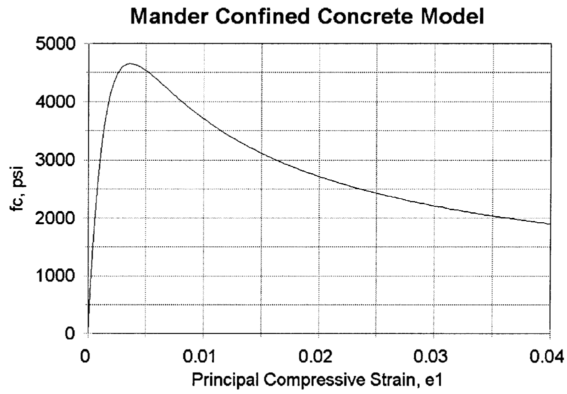
With reference to Fig. 4, principle stresses σ_1 and σ_3 can be related to the cohesion, c, and internal friction angle, ϕ, by means of Eq. (2):

$$\sigma_1=\sigma_3\left(\tan\left(45+\frac{\phi}{2}\right)\right)^2+2c\tan\left(45+\frac{\phi}{2}\right) \tag{2}$$

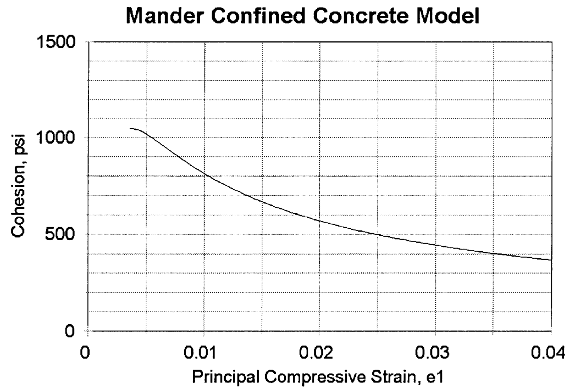
The enhancement of concrete strength resulting from a confining stress σ_3 is shown in Fig. 4 by the increase in strength from f_c' to σ_1. Mohr's circles describe the state of stress at a point, and various combinations of normal and shear stresses can produce a given Mohr's circle. Inspection of Fig. 4 reveals that the failure surface may be reached for any of the following changes: an increase in the major principal (compressive) stress, a reduction in the minor principal (confining) stress, or an increase in the shear acting on non-principal stress planes.

Eqs. (1) and (2) can be equated to present Richart's findings for peak strength in terms of the cohesion and angle of internal friction. Doing so gives $c \cong f_c'/4$ and $\phi \cong 37°$. This result is confirming, because the internal friction angle for typical aggregates is known to be about 37°. In test columns constructed of normal strength concrete and with normal-weight aggregate, the aggregate generally remains intact, with the bulk of damage occurring within the cement matrix. If the shear strength degrades, it seems that ϕ should change little, allowing degradation to be represented simply as a reduction in the cohesion alone. Reductions in c and ϕ seem appropriate where a greater proportion of the aggregate fails. Focusing here on normal strength and normal weight concretes, the critical question is defining how the cohesion degrades.

To address this question, at least initially, we can look further at the behavior of confined concrete. In place of test data, it is more convenient to use an existing model for confined concrete. For example, the Mander (1988) confined concrete model for a 4000 psi (27.6 MPa) concrete having confinement stress σ_3 equal to 100 psi (0.69 MPa) results in the stress-strain curve of Fig. 5a. By definition, the initial failure surface is reached precisely at the peak strength. As the principal compressive strain continues to increase, the stress carried by the concrete decreases. Strain hardening of the transverse reinforcement may cause the confining stress to increase. This situation



(a) Confined Concrete Stress-Strain Relation



(b) Degradation of Cohesion with Increasing Strain

Fig. 5 Degradation of confined concrete

is represented schematically in the diagram of Fig. 6, where the Mohr's circle contracts in the post-peak range. Inspection of Fig. 6 suggests that if the material stays on the failure surface as its strength reduces, then the cohesion must reduce with increasing principal strain ε_1, assuming the internal friction angle remains constant. Eq. (2) can be applied to the data of Fig. 5a to obtain an expression for the degrading cohesion as a function of the major principal strain, and the result is shown in Fig. 5b. Here, it is assumed that σ_3 and ϕ remain constant.

The above description of degradation is derived for confined concrete in which the principal stresses are aligned with the cylinder geometry. Through Mohr's circle transformations, other alignments of principal stress relative to physical geometry can be considered. Before continuing in this direction, it is worth noting that basic research done by Pantazopoulou (1995) indicates that degradation is controlled by the areal strain (sum of the principal strains orthogonal to the direction under consideration) rather than by confinement pressure. Thus, application of the above model in contexts where the presence of shear rotates the principal stress axes relative to the specimen geometry may cause the areal strain to depart from the values implicit in tests of spirally-confined concrete cylinders. Use of data from confined concrete tests may, however, lead to acceptable results despite this departure from analytical rigor.

5. Application to columns

In applying the above concepts to modeling shear strength degradation in reinforced concrete

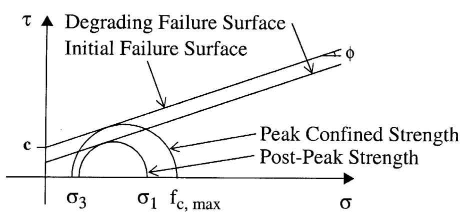


Fig. 6 Contraction of Mohr's circle with degradation

members, the analyst-modeler faces two coupled tasks. The first is to develop a suitable mechanical model for actions in the plastic hinge zone. The second is to identify the degradation relationship to be used with the mechanical model. Given sufficient data from experimental tests, it would be possible to assess strains and confinement stresses in the plastic hinge zone, and together with measured shear strengths and deformation capacities, determine a model which works for as broad a set of experimental data as is available. If this approach were successful, however, it would still be of limited value given that our models for the response of plastic hinge zones are fairly crude and are not necessarily applicable to columns dominated by shear failure. (For example, see Kinugasa 1996 for a discussion of the role of shear deformations in the plastic hinge zone.) Thus, such an "ideal" model would not lend itself to practical application, given the absence of detailed understanding of the response of hinge zones.

As previously stated, the objective of this paper is to show how a phenomenologically-based shear strength degradation model can be developed, and to show that results obtained for a particularly simple model give credence to this approach. It is hoped that improved models may be developed following this approach.

5.1. Development of a tentative mechanical model

It is desired to develop a model that addresses the degradation of shear strength within plastic hinge zones under reversed cyclic inelastic loading. Several assumptions will be made to try to represent the major effects present under these conditions. It is assumed that the concrete cover has spalled, and that the compression zones have been repeatedly cracked in tension by previous load reversals. Whether other potential sources of shear resistance, such as aggregate interlock and dowel action of the longitudinal reinforcement, contribute significantly to the shear strength of the cross section is uncertain; for simplicity these potential contributions are neglected.

Pilakoutas (1995) suggested that the shear resistance of reinforced concrete members can be calculated using a "shear resisting surface" model. The model posits that the concrete contribution to shear resistance is zero in the flexural tension zone, and is equal to the value determined from a Mohr-Coulomb failure criterion in the flexural compression zone of the member. Reference is made to enhancement of the concrete shear capacity in the presence of transverse reinforcement, but this effect was not explicitly considered. Even so, the model provided reasonably good estimates of the strengths of structural walls obtained in the experimental program.

A modified version of this model is used here, incorporating the effects of confinement and degradation on the strength of the concrete in the flexural compression zone. The model is shown in Fig. 7 for the plastic hinge zone of a rectangular cross-section column having two layers of longitudinal reinforcement, one near the tension face and one near the compression face. The model accounts for tension and compression forces that might develop in the longitudinal reinforcement, and provides for a compressive stress block located within the confined region of the concrete core. The cover concrete, if present, is considered to provide a negligible contribution. It is assumed that the concrete contribution to shear resistance occurs entirely in the flexural compression zone, and that this contribution may be enhanced by confining stress generated by the transverse reinforcement. The transverse reinforcement also contributes directly to the shear strength of the cross section, but fewer stirrups are engaged relative to the traditional truss model.

Internal and external forces acting on the free body diagram are shown in Fig. 7. For the sake of simplicity it is assumed that the shear and normal stresses within the confined concrete compression

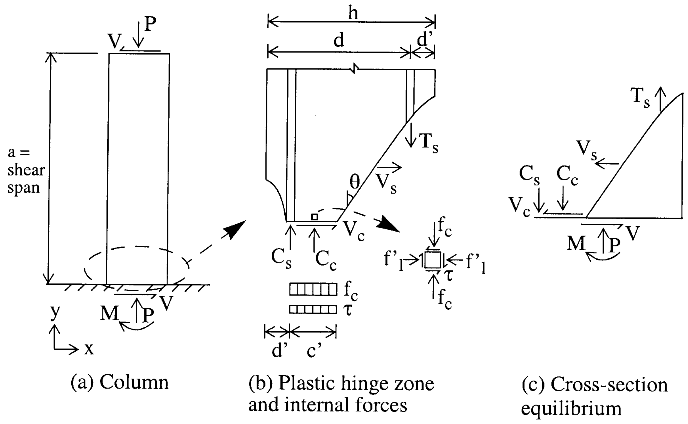


Fig. 7 Tentative model for shear and flexural strength in the column plastic hinge zone

block are uniform. This assumption is clearly a gross simplification which belies the complex, but uncertain, kinematics of deformation within the plastic hinge zone. Using the standard approach and notation, the transverse steel contribution to shear strength, V_s, is given as

$$V_s = \frac{A_v f_y (d - d' - c')}{s \tan\theta} \tag{3}$$

where c'=the depth of the confined compression zone from the centroid of the extreme compression reinforcement.

Equilibrium leads to the following equations:

$\Sigma F_y = 0$

$$C_s + f_c c' b = P + T_s \tag{4}$$

$\Sigma M = 0$

$$C_s\left(\frac{h}{2} - d'\right) + f_c c' b\left(\frac{h}{2} - d' - \frac{c'}{2}\right) + T_s\left(\frac{h}{2} - d'\right) + \frac{V_s(d - d' - c')}{2\tan\theta} = M \tag{5}$$

$\Sigma F_x = 0$

$$\tau c' b + V_s = V \tag{6}$$

Note that f_c and τ are the uniform normal and shear stresses carried by the confined compression block. Unlike standard cross section analyses, V_s contributes to the flexural strength of the cross section in Eq. (5).

In this analysis, the strengths are contrived to match the experimental test data. If the model has some validity, it will apply to a broad range of test parameter values, and material properties required to fit the test data will not depart significantly from expectations based on other studies of concrete degradation.

5.2. Description of experimental data set

The above relationships were applied to the 7 square cross section specimens tested by Iwasaki

(1985) for which the presence of shear cracks in the plastic hinge zone (observed in photographs of the specimens) suggested that shear played a role in the failure of the specimen. The specimens cantilevered up from a monolithic base on a strong-floor, and no axial load was applied. The longitudinal reinforcement ratio was 2% for all the specimens, and the bars were arranged in two layers, each layer located near the extreme tension or compression face of the member. Longitudinal and transverse reinforcement yield strengths were approximately 47 and 37 ksi (320 and 260 MPa), and concrete compressive strengths were between approximately 4500 and 5500 psi (31 and 38 MPa), respectively. The transverse reinforcement ratio, defined here as $\rho''=2A_v/(b_w d)$, was 0.2% for 6 columns and 1.02% for the seventh column, and aspect ratios (M/VD) ranged between approximately 2 and 5, as shown in Table 1.

The specimens were subjected to reversed cyclic loading. The number of cycles in Table 1 was applied at each displacement level before advancing to the next, larger, displacement amplitude.

These specimens are identified by asterisks in the plots of Figs. 1, 2, and 3. It can be observed that the large transverse steel content in Specimen I25 causes the Berkeley and San Diego models to overestimate the shear that can be sustained at the measured displacement ductility capacity.

5.3. Degradation relationship

Degradation in the cohesion, c, was derived for the 7 columns using the tentative model of Fig. 7 and Eqs. (3), (4), (5), and (6). The cohesion required to obtain the measured shear strength was determined as follows.

Table 1 Summary of test data and results

Specimen ID	Aspect Ratio M/VD	Transverse Steel Content ρ''	Number of Cycles	Displacement Ductility Capacity, μ_δ
I10	5.0	0.20%	10	4.8
I16	5.0	0.20%	10	5.1
I17	3.5	0.20%	10	3.3
I18	2.0	0.20%	10	1.9
I20	3.5	0.20%	3	3.6
I21	2.0	0.20%	3	1.7
I25	2.3	1.02%	10	5.8

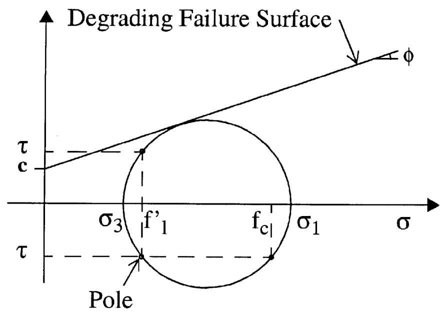


Fig. 8 Mohr's circle for confined compression zone

Using a computer spreadsheet, Eq. (4) was used to determine f_c for an assumed initial value of c'. Values of c' were iteratively adjusted so that both Eq. (4) and Eq. (5) were satisfied. Then τ was determined to satisfy Eq. (6), which also depends on c' through the V_s term. Because the specimen is, by definition, at failure, the construction of Fig. 8 pertains.

From the geometry of Fig. 8, the cohesion can be determined from the quadratic equation:

$$4\cos(\phi)c^2+4\sin(\phi)\cos(\phi)(f_c+f_l')c+2f_cf_l'(1+\sin(\phi)^2)-\cos(\phi)^2(f_c^2+f_l'^2)-4\tau^2=0 \qquad (7)$$

The effective confining pressure, f_l', provided by the transverse reinforcement was estimated as follows. First, the confining pressure exerted by the transverse reinforcement on the core concrete was calculated by standard methods assuming the transverse reinforcement had reached yield. The assumption of yield is reasonable given the damage observed in photographs of the specimens. Second, a factor to account for confinement effectiveness was obtained from Fig. 7 of Sheikh and Uzumeri (1982), and this factor was applied to the calculated confining pressure to obtain f_l'.

Because failure of the specimens was defined as the displacement or displacement ductility capacity attained when the measured strength decreased to about 80% of the peak experimental strength, the values of M and V used in Eqs. (5) and (6) were equal to 80% of the peak values determined in the experiment. The calculation of V_s was based on an assumed θ=30°. Also, ϕ was assumed equal to 37°. The solution is sensitive to values assumed for T_s and C_s. A credible solution was found assuming the tension reinforcement is at yield and the compression reinforcement is stressed to 50% of the yield strength. Other assumptions would be more relevant to columns having externally imposed axial load. Detailed strain and displacement measurements would support the use of more sophisticated techniques in place of these relatively coarse assumptions.

The cohesion was determined using the above assumptions, and is plotted as a function of specimen displacement ductility capacity in Fig. 9. While the specimen displacement ductility capacity is not likely to be a generally robust measure of the degradation of the confined concrete compressive block in the plastic hinge zone, it is used here because it is easily determined and does not involve the arbitrary assumptions that would be required to estimate other indices of local demands in the hinge zone.

The calculated cohesion was positive and less than $f_c'/4$ in all cases, and tended to decrease with

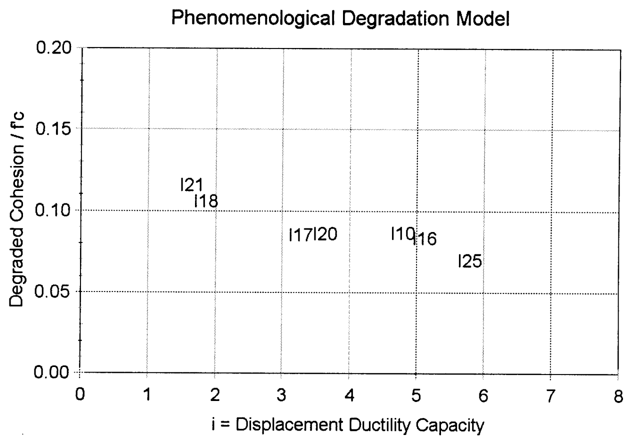


Fig. 9 Cohesion plotted as a function of specimen displacement ductility capacity

increasing displacement ductility capacity. This result is consistent with expectation based on confined concrete, and suggests that improved shear strength degradation models can be developed that incorporate well-established material degradation phenomena. The improved models will have a stronger conceptual basis, and thus should be more broadly applicable and more reliable for general use than the first generation models discussed earlier.

6. Conclusions

Existing models for the shear strength degradation of reinforced concrete members provide varied conceptual approaches to represent empirical data. When applied to test data external to the dataset used in the model's calibration, substantial unconservatism may result. Moreover, datasets used to calibrate these models are not ideal for establishing the merits of one conceptual basis or another. These datasets are lacking in columns with high axial load ratios and do not vary specimen parameters such that the plastic shear demand is held constant as various parameters are varied (such as axial load ratio, aspect ratio, and longitudinal and transverse reinforcement contents).

Degradation of concrete as a material is well known and has been represented in various confined concrete models. Use of a degrading Mohr-Coloumb failure surface shows promise for developing improved models of shear strength degradation in reinforced concrete members. A tentative model applied to 7 specimens revealed clearly that greater specimen ductility capacities can be obtained when the shear imposed on confined concrete compression zone is reduced, as shown in Fig. 9. Consequently, a degrading Mohr-Coulomb failure surface shows promise for developing improved models for shear strength degradation.

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