Structural Engineering and Mechanics, Vol. 9, No. 4 (2000) 375-382 375
DOI: http://dx.doi.org/10.12989/sem.2000.9.4.375

Cyclic compressive loading-unloading curves of brick masonry

Milad M. AlShebani† and S.N. Sinha‡

Department of Civil Engineering, I.I.T. Delhi, New Delhi-110 016, India

Abstract. Experimental investigation into the cyclic behaviour of sand plast brick masonry was performed on forty two square panels. The panels were subjected to cyclic uniaxial compression for two cases of loading: normal to bed joint and parallel to bed joint. Experimental data were used to plot the unloading-reloading curves for the entire range of the stress-strain curve. Mathematical expressions to predict the reloading and unloading stress-strain curves at various values of residual strain are proposed. A simple parabola and an exponential type formula are found adequate to model the unloading and reloading curves respectively. The models account for the potential effects of residual strain on these curves. Comparison of test results with the proposed mathematical expression shows good correspondence.

Key words: cyclic loading; unloading; reloading; plastic strain; envelope curve.

1. Introduction

Extensive studies have been reported over a long period of time on the behaviour of brick masonry under monotonic loading. Work on cyclic behaviour of clay brick masonry or concrete block masonry has been done in connection with seismic design of buildings with no particular emphasis on cyclic deformation characteristics of masonry assemblage (e.g., Chen *et al.* 1978, Macchi 1985 and Tomazevic *et al.* 1996). Design of brick masonry structures having a live to dead load ratio needs to account for the cyclic compressive response of such structures. Repeated loading-unloading cycles can cause accumulation of strains which eventually produce failure as strain level increases with increasing number of cycles. Tests on masonry under cyclic loading are vital for information related to material ductility, stiffness degradation and energy dissipation characteristics. Analytical models to predict the behaviour of concrete under uniaxial cyclic compressive loading have been reported by a number of researchers (Sinha *et al.* 1964, Karsan and Jirsa 1969, Yenkelevsky and Reinhardt 1987, and Bahn and Hsu 1998). Laboratory tests on solid clay brick masonry subjected to uniaxial compressive loading have been reported only in the last decade (Abrams *et al.* 1985, Naraine and Sinha 1989). Studies on the cyclic behaviour and deformation characteristics of high strength sand plast brick masonry is currently lacking.

This paper presents the results of laboratory tests on half-scale sand plast (a form of calcium silicate) brick panels subjected to uniaxial cyclic compression with particular emphasis on loading-reloading behaviour. Analytical models are proposed to predict the loading and reloading curves for both directions of loading; i.e., normal to bed joint and parallel to bed joint.

† Research Scholar
‡ Professor

2. Cyclic models

Cyclic models to simulate stress-strain behaviour of concrete have been previously reported as mentioned before. More recently, Bahn and Hsu (1998) have proposed a general cyclic model to describe the behaviour of random cycles on concrete. They have expressed the unloading curve as a parabola and reloading curve as a straight line.

For brick masonry, two mathematical models have been previously reported to obtain reloading and unloading stress-strain curves of brick masonry under uniaxial compressive loading. These models are for fired clay bricks with low compressive strength (13.6 N/mm^2). In the first model, exponential functions are proposed for the reloading and unloading curves (Naraine and Sinha 1989). Test data showed that the reloading curve can be represented by a family of parabolas and unloading curves can be represented by a family of straight lines. The equations of a parent parabola and a parent straight line are then used to generate family of parabolas and family of straight lines respectively which in turn are used to obtain the reloading and unloading curves. In the second model, polynomial functions are proposed to obtain the reloading and unloading curves (Subramaniam and Sinha 1995). Their proposed analytical model transfers the experimental reloading and unloading curves to a normalized stress-strain coordinate system on which both the unloading and reloading curves plot within their narrow range bounded by a lower and upper limits. A parent polynominal expression is then developed to fit the curves. The individual reloading and unloading curves can then be produced by transferring the parent equation to the stress-strain coordinate system.

The behaviour of brick masonry in general depends on its load history. The path of unloading for any cycle depends primarily on the plastic strain accumulated in that cycle, and reloading path depends on the previous unloading path. This implies that for a model to predict unloading and reloading curves, plastic strain needs to be incorporated into the mathematical formulations.

3. Test program

Sand plast brick masonry panels of dimension 360 mm $\times$ 360 mm $\times$ 115 mm have been constructed from sand plast half brick units each measuring 110 mm $\times$ 55 mm $\times$ 35 mm. The average compressive strength of the brick unit was 23.4 N/mm^2 and the average compressive cube strength of the mortar used for the joints at 28 days was 10.2 N/mm^2. X-Y plotters have been used to monitor the displacements and the applied load through LVDTs and a load cell respectively. The loading-unloading was controlled by a Universal Testing Machine. The laboratory experiments consist of three types of test. The first test is a monotonic one in which loads are steadily increased until failure. This test establishes the monotonic stress-strain curve. The second test is a cyclic test in which loading originates at zero stress level and terminates at the envelope stress-strain curve. Unloading, then, commences from the envelope curve and terminates at zero stress level for each cycle. The stress-strain hysteresis so obtained possesses a locus of common points. A common point is defined as the intersection point of the reloading curve of any cycle with the unloading curve of the previous cycle. The reloading curve is terminated when its peak approximately coincides with envelope curve. This is done by monitoring the incremental increase of axial strain in the ascending branch of the envelope curve. In the descending branch of the envelope curve, the load was released when the reloading curve tends to descend. The third test is also a cyclic test in which for each cycle reloading and unloading are repeated when reloading curve intersects the original unloading curve of that cycle. The process forms locii of common points in descending order until it stabilizes at lower lucus. The locus of the lower bound points termed as the stability point curve.

The envelope stress-strain curve is established by superimposing the stress-strain peaks of the second and third cyclic tests on the monotonic stress-strain curve. The envelope curve was found to follow an exponential formula (AlShebani and Sinha 1999). The parameters of this formula depend on the direction of loading being normal or parallel to the bed joint.

4. Stress-strain hysteresis

Sand plast brick panels have been tested cyclically in uniaxial compression normal to the bed joints and parallel to the bed joints. Cyclic loading has been performed with the peak of each reloading coinciding with the monotonic stress-strain curve. Therefore, the envelope stress-strain curve was obtained by superimposing the points of peak stress-strain curve under cylic loading on monotonic stress-strain curve. The stress-strain hysteresis comprising unloading and reloading curves are plotted by expressing stress and strain in non-dimensional form. The stress coordinate is normalized with respect to peak (failure) stress and the strain coordinate is normalized with respect to the strain corresponding to peak (failure) stress for each specimen.

5. Unloading curves

Unloading of any cycle originates from the envelope stress-strain curve and terminates at zero stress level at a particular value of residual (plastic) strain. It exhibits a single curvature at lower value of residual strain at which unloading terminates. The softening phenomena of unloading curves are more pronounced at larger value of residual strain. The shape of the unloading path is greatly influenced by the residual strain, ε_r. An analytical model for unloading curve is developed first by approximating it by straight line from unloading strain on envelope curve to plastic strain at zero stress level, as shown in Fig. 1, and then by expressing it in non-linear form.

A linear expression for the line joining unloading strain on envelope curve to plastic strain at zero stress level, (Fig. 1), can be written as

$$\frac{\sigma}{\sigma_{eu}}=\frac{\varepsilon-\varepsilon_r}{\varepsilon_{eu}-\varepsilon_r}=x \tag{1}$$

where x is a linear function and σ and ε represent the normalized stress and strain respectively at any point on the assumed straight line, and σ_{eu} and ε_{eu} are the normalized stress and the strain respectively at the start of unloading on the envelope curve, and ε_r is the normalized plastic strain at the end of unloading. The envelope stress-strain curve for sand plast brick masonry for both cases of loading has been proposed earlier (AlShebani and Sinha 1999) as:

For normal to bed joint, $\sigma_e=\varepsilon_e^{\beta}\mathrm{Exp}(1-\varepsilon_e/\alpha)$ (2a)

For parallel to bed joint, $\sigma_e=\varepsilon_e^{\beta}\mathrm{Exp}[(1-\varepsilon_e).\varepsilon_e/(\alpha+\beta)]$ (2b)

Where α and β both equal unity in Eq. (2a) while in Eq. (2b) α equals 0.73 and β equals 0.70.

The actual shape of unloading curve exhibits nonlinear behaviour for both cases of loading. The

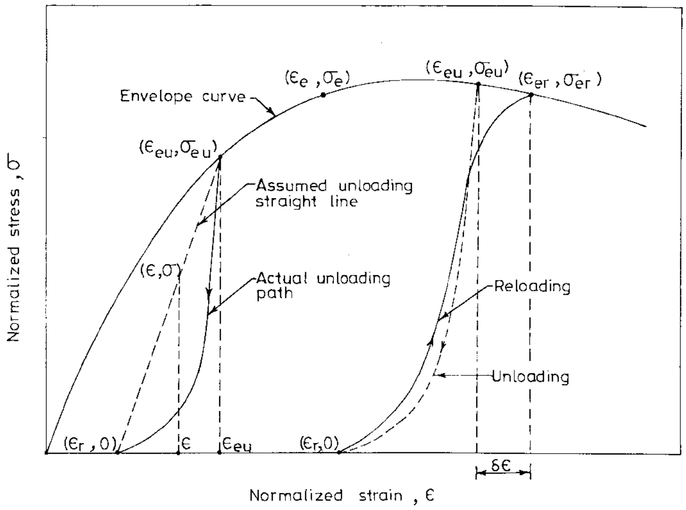

Fig. 1 Unload-reload curves

linear function x in Eq. (1) is therefore required to be modified to reflect the nonlinearity of the unloading curve.

A mathematical expression to obtain the unloading curve is therefore proposed as follows:

$$\sigma = C_u \sigma_{eu} \left(\frac{\varepsilon - \varepsilon_r}{\varepsilon_{eu} - \varepsilon_r} \right)^{n_u} \tag{3}$$

where C_u is a parameter and n_u is dependent on the normalized plastic strain (plastic strain ratio).

Based on test data, the values of C_u and n_u for loading normal to bed joint case have been determined as 0.98 and $2.0+1.5\sqrt{\varepsilon_r}$ respectively, and for loading parallel to bed joint case as 0.97 and $3.0+2.0\sqrt{\varepsilon_r}$ respectively.

The relation between the envelope unloading strain, ε_{eu} and the plastic strain can be expressed as,

$$\varepsilon_r = C_r (\varepsilon_{eu})^{n_r} \tag{4}$$

where C_r and n_r are constants which are derived from the test data. The values of C_r and n_r for normal to bed joint case are 0.47 and 1.2 respectively. The corresponding values for parallel to bed joint case are 0.35 and 2.0 respectively.

The development of the expressions reported in this investigation are based on the best correspondence between the experimental data and the proposed mathematical models. The correspondence between expressions and data is measured by the correlation index.

The unloading curves for the two cases of loading can now be obtained using Eq. (3) and Eq. (4) and the envelope stress-strain curves given in Eq. (2). A comparison of the analytical model with experimental curves are shown in Fig. 2(a) and Fig. 2(b) in a normalized stress-strain coordinate system, in which the model and experimental curves show good correspondence.

6. Reloading curves

The full reloading curve originates at a certain plastic strain level and terminates as it coincides

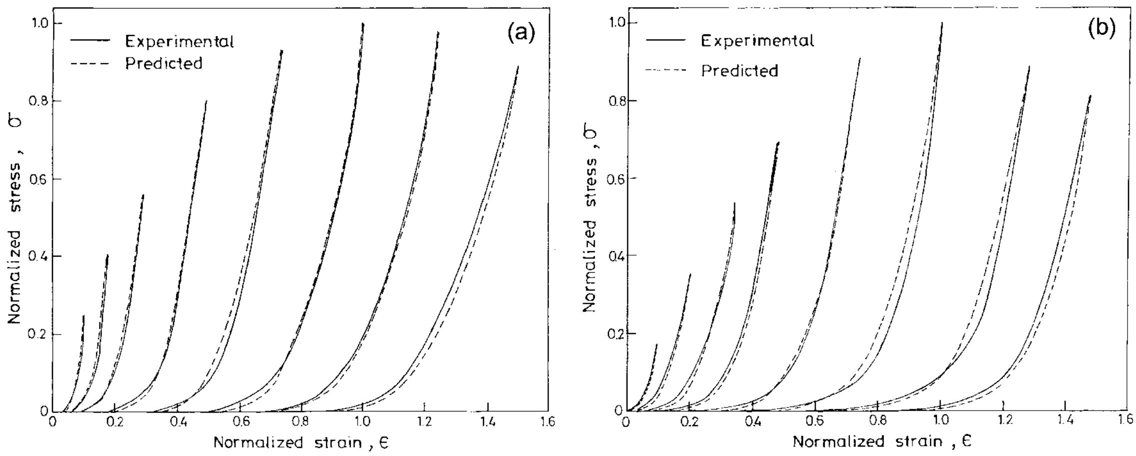

Fig. 2 Unloading curves

with envelope stress-strain curve. It exhibits double curvatures at low stress level and when it crosses the common point. It has been observed that the envelope reloading strain of any cycle is always greater than the envelope unloading strain of the previous cycle. Therefore, a straight line representation of reloading curve (Bahu and Hsu 1998) neither reflects the actual nonlinear behaviour nor does it give a proper representation of the influence of plastic strain level on the behaviour of reloading curves.

To model the reloading curves, all reloading curves were first plotted from a common origin by transferring these curves from normalized coordinate, ε versus σ to other normalized coordinate, $(\varepsilon-\varepsilon_r)_n$ versus σ_n, in which $\varepsilon-\varepsilon_r$ is normalized with respect to $(\varepsilon_e-\varepsilon_r)$, and σ is normalized with respect to σ_e for each curve. The coordinates $(\varepsilon_e-\varepsilon_r)$ and σ_e correspond to the maximum value of $(\varepsilon-\varepsilon_r)$ and σ respectively for each curve, where it coincides with the envelope curve. On the new normalized coordinate system, each curve originates from (0,0) and terminates at (1,1), as shown in Fig. 3(a) and Fig. 3(b). It is observed that all reloading curves plot within a narrow band which can be reasonably represented by the following expression:

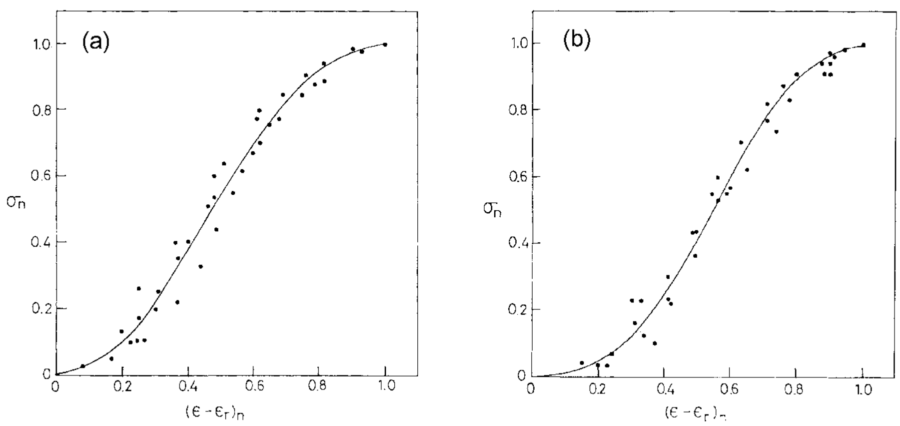

Fig. 3 Reloading curves normalized to a common point

Table 1 Values for C_0, C_1 and n_1

Direction of Loading	Level of plastic strain	C_0	C_1	n_1
Normal to bed joint	$\varepsilon_r \leq 0.1$	0.09	2.2	0.8
	$\varepsilon_r > 0.1$	0.2	0.1	0.8
Parallel to bed joint	$\varepsilon_r \leq 0.1$	0.1	1.0	0.5
	$\varepsilon_r > 0.1$	0.2	0.1	0.5

$$\sigma_n = [(\varepsilon - \varepsilon_r)_n]^{n_r}\ \mathrm{Exp}[1-(\varepsilon - \varepsilon_r)_n^{\ n_r}] \tag{5}$$

where n_r is a function of plastic strain ratio and $(\varepsilon - \varepsilon_r)_n$ is the value of $\varepsilon - \varepsilon_r$ normalized with respect to $(\varepsilon_e - \varepsilon_r)$. The power term n_r is derived from the test data as $1.5 + \sqrt{\varepsilon_r}$ for loading normal to bed joint and $2.0 + \sqrt{\varepsilon_r}$ for loading parallel to bed joint.

The reloading envelope strain ε_{er}, of any cycle is greater than the unloading envelope strain, ε_{eu} of the previous cycle by an increment of $\delta\varepsilon$ as shown in Fig. 1. This implies that:

$$\varepsilon_{er} = \varepsilon_{eu} + \delta\varepsilon \tag{6}$$

The difference between reloading envelope strain and unloading envelope strain, $\delta\varepsilon$ depends on the plastic strain level and unloading envelope strain. Therefore, $\delta\varepsilon$ can be approximated as $C_0 + C_1\varepsilon_{eu}(\varepsilon_r)^{n_1}$ for $\varepsilon_r \leq 0.1$ and $C_0 + C_1(\varepsilon_r/\varepsilon_{eu})^{n_1}$ for $\varepsilon_r > 0.1$.

The constants C_0 and C_1 and the power term n_1 are given in Table 1 for the two cases of loading. Once ε_{er} is determined from Eq. (6), σ_e can be found from envelope stress-strain relationship using Eq. (2).

The reloading curve can now be obtained as follows:

1. For a given ε_r, ε_{eu} is evaluated using Eq. (4) and then ε_{er} can be evaluated using Eq. (6).

2. For the computed value of ε_{er}, which is equal to ε_e, the value of σ_e can be evaluated using Eq. (2a) or Eq. (2b).

3. The value of σ_n can be evaluated using Eq. (4) for different values of ε on the reloading curve originating from the given ε_r and terminating at ε_e.

4. The value of σ_n corresponding to a value of ε can be obtained by multiplying σ_n by σ_e. The computed reloading curves are shown with experimental curves in Fig. 4(a) and Fig. 4(b) and the

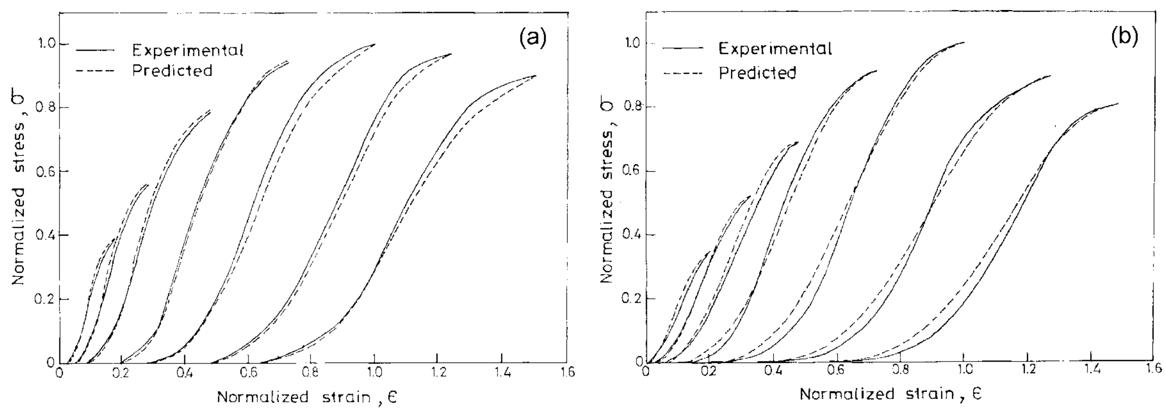

Fig. 4 Reloading curves

comparison showed good agreement.

Although the procedures outlined above to produce the reloading and unloading curves were based on experimental investigation conducted on sand plast brick masonry, it was found to provide an approximation to reloading and unloading curves of clay brick masonry reported by Subramaniam and Sinha (1995). Application of the model to clay masonry was found to fit better for specimens loaded normal to bed joint. However, some deviation of the model from the experimental data was noticed for clay masonry loaded parallel to bed joint especially over the post peak zone. This may suggest that material physical properties can have some effect on the generalization of the model. Nevertheless, it is stipulated that the mathematical models reported in this study can be generally used for brick masonry based on normalized coordinate system.

7. Conclusions

An analytical model for unloading and reloading curves of sand plast brick masonry under cyclic uniaxial compression is presented. The model for the unloading curve is a simple parabola while reloading curve is expressed by exponential formula. The model for the reloading curve involves plotting the reloading curves from a common origin by transferring them to a new normalized coordinate. Unloading curves are more influenced by plastic strain at which they terminate than strain on envelope curve from which they originate. Reloading curves are dependent on the history of unloading paths that preceded each of them and on the strain level at which reloading is terminated on envelope curve. Good correspondence between the analytical curves and the experimental data was obtained. Testing the proposed models on clay brick masonry produced better approximation of reloading and unloading curves for specimens loaded normal to bed joint than specimens loaded parallel to bed joint.

References

Abrams, D., Noland, J. and Atkinson, R. (1985), "Response of clay unit masonry to repeated compressive forces", *Proc. 7th Int. Brick Masonry Conf.*, Melbourne, 565-576.

AlShebani, M.M. and Sinha, S.N. (1998), "Influence of panel aspect ratio on the cylic residual strains of brick masonry", *Proc. 8, 5th Int. Masonry Conf.*, Stoke-on-Trent, 316-319.

AlShebani, M.M. and Sinha, S.N. (1999), "Stress-strain characteristics of brick masonry under uniaxial cyclic loading", *J. of Struct. Engrg., ASCE,* **125**(6), 600-604.

Bahn, B.Y. and Hsu, C.T. (1998), "Stress-strain behaviour of concrete under cyclic loading", *ACI Material J.,* **95**(2), 178-193.

Chen, S.W.J., Hidalgo, P.A., Mayes, R.L., Clough, R.W. and McNiven, H.D. (1978), "Cyclic loading tests of masonry single piers, Volume 2-height to width ratio of 1", EERC Report No. 78/28, Univ. of Clif., Berkely.

Karsan, J.K. and Jersa, J.O. (1969), "Behaviour of concrete under compressive loadings", *J. of Struct. Div., ASCE,* **95**(ST 12), **95**, 2543-2563.

Macchi, G. (1985), "Behaviour of masonry under cyclic actions and seismic design", *Proc. of 6th Int. Brick Masonry Conf.*, Rome, Italy, LI-LXXIV.

Naraine, K. and Sinha, S.N. (1989), "Behaviour of brick masonry under cyclic compressive loading", *J. of Struct. Engg., ASCE,* **115**(6), 1432-1445.

Naraine, K. and Sinha, S.N. (1989), "Loading and unloading stress-strain curves for brick masonry", *J. of Struct. Engg., ASCE,* **115**(10), 7631-7644.

Sinha, B.P., Gerstle, K.H. and Tulin, L.G. (1964), "Stress-strain relations for concrete under cyclic loading", *ACI Journal*, **61**, 195-210.
Subramaniam, K.V. and Sinha, S.N. (1995), "Analytical model for cyclic compressive behaviour of brick masonry", *ACI Struct. J.*, **92**(3), 288-294.
Tomazevic, M., Lutman, M. and Petkovic, L. (1996), "Seismic behaviour of masonry walls: experimental simulation", *J. of Struct. Engg., ASCE*, **122**(9), 1040-1047.
Yankelevsky, D.Z. and Reinhardt, H.W. (1987), "Model for cyclic compressive behaviour of concrete", *J. of Struct. Engg., ASCE*, **113**, 225-240.

Notation

(σ_e, ε)	: Normalized stress and corresponding normalized strain on the envelope curve at any point
(σ, ε)	: Normalized stress and corresponding normalized strain at any point on the unloading curve
$(\sigma_{eu}, \varepsilon_{eu})$	: Normalized stress and corresponding normalized strain on envelope curve at the start of unloading
$(\sigma_{er}, \varepsilon_{er})$	: Normalized stress and corresponding normalized strain on envelope curve at the end of reloading
ε_r	: Normalized residual (plastic) strain at the end of unloading
σ_n	: Normalized stress in terms of σ/σ_e
$(\varepsilon-\varepsilon_r)_n$	: Normalized strain difference in terms of $(\varepsilon-\varepsilon_r)/(\varepsilon_e-\varepsilon_r)$