

## Optimal damping ratio of TLCDs

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**Abstract.** The study of the optimal damping ratio of a tuned liquid-column damper (or TLCD) attached to a single-degree-of-freedom system is presented. The tuned liquid-column damper is composed of two vertical columns connected by a horizontal section in the bottom and partially filled with water. The ratio of the length of the horizontal section to the effective wetted length of a TLCD considered as another important parameter is also presented for investigation. A simple pendulum-like model test is conducted to simulate a long-period motion in order to prove the effectiveness of TLCD for vibrational control. Comparisons of the experimental and analytic results of the TLCD, TLD (tuned-liquid damper), and TMD (tuned-mass damper) are included for discussion.

**Key words:** TLCD; optimal damping ratio; design parameter; experiment and comparison.

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### 1. Introduction

There exists an effective way to control the vibration of a structure by utilizing the water oscillating and sloshing in a tank (Fujino *et al.* 1988, 1992, 1993, Sun *et al.* 1989, Chaiseri *et al.* 1989, Wakahara 1993, Koh *et al.* 1994) or a U-shape vessel (Saoka *et al.* 1998, Sakai *et al.* 1989 and Sun 1994, Won *et al.* 1996). It is different from the conventional method to reduce the vibration level of a structure or building, such as the base isolation (Kelly 1988, Ahmadi *et al.* 1989 and Buckle 1990), the viscoelastic damper (Zhang *et al.* 1969), the added damping and stiffness device (Whittaker *et al.* 1989 and Su *et al.* 1990), the structural bracing and tendon system (Soong 1990), and the tuned mass damper (Hartog *et al.* 1962, Chang *et al.* 1980, Kaynia *et al.* 1981 and Yamaguchi *et al.* 1993). This technique has been successfully employed to control the rolling motion of a ship in waves and the wobbling of a satellite in space (Lewis 1989, Webster *et al.* 1966, Bhuta *et al.* 1966, Alfriend 1974). In this paper the basic characteristics and application of a TLCD is emphasized. It might have many advantages and some of which are as follows: (a) it is easy to build and maintain, (b) it is space saving, (c) there is no need to change the original structure system of a building, (d) it is economical, etc. Therefore it would have a great potential to develop this technique for the high-rise building or other flexible structures in the future. Additionally it would not be much influenced from severe environmental conditions, so it would be

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quite suitable for ocean structures, such as the ship and the offshore platform.

The optimal damping ratio of a simple TLCD system, which is a two-degree-of-freedom system, is derived and presented in this paper. Another important parameter: the ratio of the length of the horizontal section to the effective wetted length of a TLCD is also studied in order to improve the effectiveness of the vibration control. A simple pendulum-like model test is carried out for demonstration including the free and harmonic vibrations. Comparisons of the experimental results of the model tests with the analytic results of the optimal TLCD, TLD, and TMD systems are made to see the effectiveness and potential of the vibrational control of a TLCD in engineering application.

## 2. Fundamentals of TLCDs

A tuned liquid column damper is shown in Fig. 1. It is an U-shape vessel consisting of two vertical columns and a horizontal section. The water can move freely inside the vessel and could be assumed of being an incompressible flow. The cross-sectional dimension of the vertical column in the plane of TLCD is much smaller than the length of the horizontal section; therefore the water surface would be assumed to be flat during moving vertically. In practice, these two vertical columns are usually identical.

If there is a disturbance of the TLCD along  $x$  direction, say  $x(t)$ , the water level would move up and down harmonically. The potential energy of the water with surface movement  $y(t)$  is given by

$$V = \rho g A_y y^2(t) \quad (1)$$

where  $\rho$ ,  $g$ , and  $A_y$  represent the water density, the gravitational acceleration, and the cross-sectional area of the vertical column, respectively.

The continuity equation of the incompressible flow is given by

$$A_y \dot{y} = A(s) \dot{s} \quad (2)$$

where  $A(s)$  and  $\dot{s}$  represent the cross-sectional area and the relative water velocity at any section  $s$ , respectively.

The kinetic energy of water is given by

$$T = \frac{1}{2} \int_0^l A(s) \dot{s}_a^2 ds \quad (3)$$

where  $l$  represents the total wetted length of the vessel i.e.  $l = 2H + B$ , and  $\dot{s}_a$  represents the absolute water velocity at any section  $s$ .

The kinetic energy of water  $T$  can be expressed by

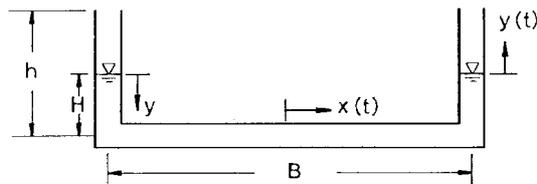


Fig. 1 A TLCD: An U-shape vessel

$$T = \int_0^H \rho A(s)(\dot{s}^2 + \dot{x}^2) dy + \frac{1}{2} \int_0^B \rho A(s)(\dot{s}^2 + \dot{x}^2) dx \quad (4)$$

Substituting Eq. (2) into Eq. (4), Eq. (4) becomes as

$$T = \rho \int_0^H \left[ \frac{A_y^2}{A(s)} \dot{y}^2 + A(s) \dot{x}^2 \right] dy + \frac{\rho}{2} \int_0^B A(s) \left[ \frac{A_y}{A(s)} \dot{y} + \dot{x} \right]^2 dx \quad (5)$$

Lagrange's equation for this case is given by

$$\frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{y}} \right) - \frac{\partial T}{\partial y} + \frac{\partial V}{\partial y} = - \frac{\partial W_d}{\partial y} \quad (6)$$

where  $W_d$  represents the work done by the non-conservative force resulted from the water-head loss due to the friction between the water and vessel surface, and the resistance to water flow by any mechanism installed inside the vessel. The term in the right-hand side of Eq. (6)  $\partial W_d / \partial y$  represents this non-conservative force which would be expressed by a nonlinear function of the velocity of the free-water surface (Saoka *et al.* 1998, Sakai *et al.* 1989, Sun *et al.* 1994). Therefore the equation of motion of the free-surface movement  $y(t)$  due to a  $x$ -directional disturbance  $x(t)$  of the TLCD can be achieved by substituting Eqs. (1) and (5) into Eq. (6) and given as

$$\rho A_y l' \ddot{y} + \frac{1}{2} \rho A_y C_d |\dot{y}| \dot{y} + 2 \rho g A_y y = - \rho A_y B \ddot{x} \quad (7)$$

where  $C_d$  represents the head-loss coefficient, and  $l'$  represents the effective wetted length defined by

$$l' = \int_0^{l'} \frac{A_y}{A(s)} ds \quad (8)$$

If the vessel is uniform along its length, i.e.  $l' = l = 2H + B$ ; if a symmetrical TLCD consisting of two identical uniform columns of the cross-sectional area  $A_y$  and an uniform horizontal section of the cross-sectional area  $A_x$ , i.e.  $l' = 2H + B/R$ , where  $R = A_x/A_y$ .

If the nonlinear damping force would be replaced by an equivalent linear one (Caughey 1963), Eq. (7), would be rewritten as

$$m' \ddot{y} + m' c \dot{y} + \omega_u^2 m' y = - m' \alpha \ddot{x} \quad (9)$$

where  $c$  represents the equivalent linear damping coefficient, and  $m'$ ,  $\alpha$  and  $\omega_u$  are defined as

$$\begin{aligned} m' &= \rho A_y l' \\ \alpha &= \frac{B}{l'} \\ \omega_u &= \sqrt{\frac{2g}{l'}} \end{aligned} \quad (10)$$

in which  $m'$  represents the effective total mass of water,  $\alpha$  the ratio of the length of the horizontal section to the effective wetted length, and  $\omega_u$  the natural frequency of a TLCD, respectively.

Eq. (9) can be expressed in a simple form as

$$\ddot{y} + 2\xi \omega_u \dot{y} + \omega_u^2 y = - \alpha \ddot{x} \quad (11)$$

where  $\xi$  represents the equivalent linear damping ratio of a TLCD and defined as  $\xi = 0.5c/\omega_u$ .

### 3. A simple TLCD system: A two-degree-of-freedom system

A TLCD is added to an one-degree-of-freedom system as shown in Fig. 2. A ground acceleration  $\ddot{u}(t)$  is applied to this simple TLCD system, the equation of motion of this 2 d.o.f. system can be expressed in the following form as

$$\begin{bmatrix} M & m'\alpha \\ m'\alpha & m' \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} C & 0 \\ 0 & m'c \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} K & 0 \\ 0 & m'\omega_u^2 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = - \begin{Bmatrix} M \\ -m'\alpha \end{Bmatrix} \ddot{u} \quad (12)$$

where  $M = m_1 + m'$ , and  $m' = m_2$  the mass of water for an uniform vessel.

The previous equation would be rewritten in a nondimensional form as

$$\begin{bmatrix} 1 & \alpha\mu^2 \\ \alpha\mu^2 & \mu^2 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} 2\eta\omega_p & 0 \\ 0 & 2\xi\omega_u\mu^2 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} \omega_p^2 & 0 \\ 0 & \mu^2\omega_u^2 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = - \begin{Bmatrix} 1 \\ -\mu^2\alpha \end{Bmatrix} \ddot{u} \quad (13)$$

where

$$\begin{aligned} \mu^2 &= \frac{m'}{M} \\ \eta &= \frac{C}{2M\omega_p} \\ \omega_p &= \sqrt{\frac{K}{M}} \end{aligned} \quad (14)$$

If the ground motion is harmonic, i.e.  $u = e^{i\omega t}$ , the response will also be harmonic and can be expressed by

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} X \\ Y \end{Bmatrix} e^{i\omega t} \quad (15)$$

where  $\omega$  represents the frequency,  $X$  and  $Y$  represent the complex amplitudes of  $x(t)$  and  $y(t)$ ,

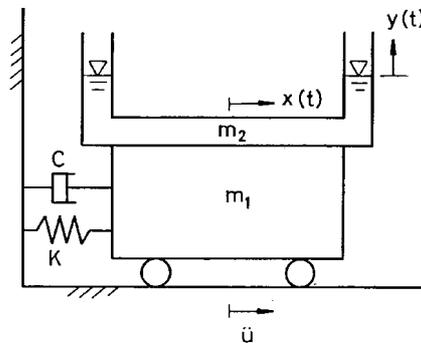


Fig. 2 A simple TLCD system

respectively.

Substituting Eq. (15) into Eq. (13) can yield the following result as

$$\begin{bmatrix} s^2 + 2\eta\omega_p s + \omega_p^2 & \alpha\mu^2 s^2 \\ \alpha\mu^2 s^2 & (s^2 + 2\xi\omega_u s + \omega_u^2)\mu^2 \end{bmatrix} \begin{Bmatrix} X \\ Y \end{Bmatrix} = \begin{Bmatrix} 1 \\ \mu^2 \alpha \end{Bmatrix} \quad (16)$$

where  $s = i\omega$ .

The dynamic magnification factors for responses  $x(t)$  and  $y(t)$  can be obtained and given as

$$D_x = \frac{|X|}{\delta_{st}}, \quad D_y = \frac{|Y|}{\delta_{st}} \quad (17)$$

where  $\delta_{st}$  represents the static displacement of the total mass, i.e.  $\delta_{st} = Mg/K$ .

If  $\eta = 0$ , both  $D_x$  and  $D_y$  can be derived and given as follows:

$$D_x^2 = \frac{b\left(\frac{a}{b} + \xi^2\right)}{d\left(\frac{c}{d} + \xi^2\right)} \quad (18)$$

$$D_y^2 = \frac{p}{q + r\xi^2}$$

where

$$\begin{aligned} a &= (1 - \gamma^2 + \alpha^2 \mu^2 \gamma^2)^2 \\ b &= (2\gamma)^2 \\ c &= [(1 - \beta^2)(1 - \gamma^2) - (\alpha\mu\beta\gamma)^2]^2 \\ d &= [2\gamma(1 - \beta^2)]^2 \\ p &= \alpha^2 \\ q &= (f^2 - \beta^2 - f^2\beta^2 + \beta^4 - \alpha^2\mu^2\beta^4)^2 \\ r &= 4f^2\beta^2(1 - \beta^2)^2 \\ f &= \frac{\omega_u}{\omega_p} = \frac{\beta}{\gamma} \\ \beta &= \frac{\omega}{\omega_p} \\ \gamma &= \frac{\omega}{\omega_u} \end{aligned} \quad (19)$$

#### 4. Optimal damping ratio of TLCD

Based on Hartog's work for the optimal damping ratio of a simple TMD system (Hartog 1962), it is possible to find an optimal damping ratio of a TLCD at proper value of  $f$  in order to achieve a minimum value of  $D_x$  for the case that the damping ratio  $\eta$  of the main system is very small and would be neglected. If  $a/b = c/d$ , then  $D_x^2 = b/d$  as shown in Eq. (18). It means that  $D_x$  is completely nothing to do with  $\xi$ . The equation for this condition can be obtained from Eq. (19) and given as

$$a_1\beta^4 - \alpha_2\beta^2 + f^2 = 0 \quad (20)$$

where

$$\begin{aligned} a_1 &= 1 - \alpha^2 \mu^2 \\ a_2 &= 1 + f^2 - \frac{\alpha^2 \mu^2}{2} \end{aligned} \quad (21)$$

The solutions of  $\beta^2$  are given as

$$\beta^2 = \frac{1}{2a_1} \left[ a_2 \pm (a_2^2 - 4a_1 f^2)^{\frac{1}{2}} \right] \quad (22)$$

Therefore  $D_x^2$  can be determined by  $D_x^2 = b/d = 1/(1-\beta^2)^2$ , or

$$D_x = \frac{1}{1 - \beta^2}, \quad \text{if } \beta^2 < 1$$

or

$$D_x = \frac{1}{\beta^2 - 1}, \quad \text{if } \beta^2 > 1 \quad (23)$$

The curves for the relationship between  $D_x$  and  $\beta$  as given by Eq. (18) always pass through two fixed points assigned as  $(\beta_1, D_{x1})$  and  $(\beta_2, D_{x2})$ , no matter the value of the damping ratio  $\xi$  of the TLCD. If  $D_{x1} = D_{x2}$ , the value of  $f$  can be found from Eqs. (22) and (23) and given by

$$f = (1 - 1.5 \alpha^2 \mu^2)^{\frac{1}{2}} \quad (24)$$

Finally these two fixed points of same magnitude of  $D_x$  can be obtained by substituting Eq. (24) into Eqs. (22) and (23) and given as

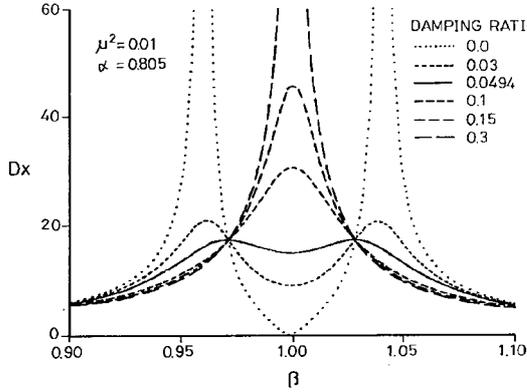
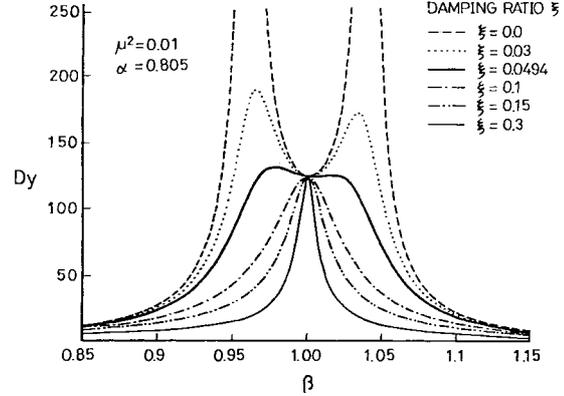
$$\beta_1 \quad \text{or} \quad \beta_2 = 1 \pm \left[ \frac{\alpha^2 \mu^2}{2(1 - \alpha^2 \mu^2)} \right]^{\frac{1}{2}}$$

and

$$D_{x1} = D_{x2} = \left[ 2 \left( \frac{1}{\alpha^2 \mu^2} - 1 \right) \right]^{\frac{1}{2}} \quad (25)$$

$D_x$ - $\beta$  curves for the case of  $\mu^2 = 0.01$  and  $\alpha = B/l' = 0.805$  as an example are shown in Fig. 3. Also shown in this figure are the two fixed points at (0.9710, 17.50) and (1.0282, 17.50) corresponding to  $f = 0.9975$ .

The dynamic magnification factor of the response  $y(t)$  is given by Eq. (18). If  $r = 0$ , then  $D_y^2 = p/q$ , it means that the value of  $D_y$  is nothing to do with the damping ratio  $\xi$  of the TLCD. Therefore the curves for the relationship between  $D_y$  and  $\beta$  always pass through a fixed point, no matter the value of the damping ratio  $\xi$ . Since  $r = 0$ , that is  $\beta = 1$ , and  $D_y = 1/\alpha\mu^2$ .  $D_y$ - $\beta$  curves for the case of  $\mu^2 = 0.01$  and  $\alpha = B/l' = 0.805$  as an example are shown in Fig. 4. Also shown in this figure is the fixed point at (1.0, 124.2). Both the curves of  $D_x$  and  $D_y$  as shown in Figs. 3 and 4 might have one


 Fig. 3 Dynamic response  $D_x$  of a TLCD system

 Fig. 4 Dynamic response  $D_y$  of the free-water surface

or two maximum points dependent on the damping value  $\xi$  of TLCD.

If two maximum points of  $D_x$ - $\beta$  curve are just located at or very close to these two fixed points of same value of  $D_x$ , this TLCD system might be at the optimal condition to achieve the minimum vibrational level of the mass  $m_1$ . The damping ratio  $\xi$  of TLCD at this condition is called the optimal damping ratio and can be derived.

Eq. (18) can give an expression of  $\xi^2$  as

$$\xi^2 = \frac{cD_x^2 - a}{b - dD_x^2} \quad (26)$$

These two fixed points of  $D_x$ - $\beta$  curve are not a function of the damping ratio  $\xi$ ; therefore  $\xi^2$  becomes indeterminate, i.e.,  $\xi^2 = 0/0$ , at these two fixed points as defined by Eq. (26). A neighborhood point is chosen in such a way that  $\beta^2 = \beta_1^2 + \varepsilon$  or  $\beta^2 = \beta_2^2 + \varepsilon$ , where  $\varepsilon$  is a very small quantity. Substituting these two  $\beta$ 's into Eq. (26) and omitting the high order terms of  $\varepsilon$ , and then applying L'Hospital rule, the result of  $\xi^2$  similar to Eq. (26) would be obtained but the parameters  $a$ ,  $b$ ,  $c$  and  $d$  are changed as follows:

$$\begin{aligned} a &= 2f^2(\alpha^2\mu^2 - 1) + 2(1 - 2\alpha^2\mu^2 + \alpha^4\mu^4)\beta^2 \\ b &= 4f^2 \\ c &= -2f^2(1 + f^2) + 2(1 + 4f^2 + f^4 - 2\alpha^2\mu^2f^2)\beta^2 - 6(1 + f^2 - \alpha^2\mu^2 - \alpha^2\mu^2f^2)\beta^4 + 4(1 - 2\alpha^2\mu^2 + \alpha^4\mu^4)\beta^6 \\ d &= 4f^2(1 - 4\beta^2 + 3\beta^4) \end{aligned} \quad (27)$$

Substituting Eqs. (24), (25), and (27) into Eq. (26) would obtain the value of  $\xi$  that the curve of  $D_x$  has two maxima at or very close to the two fixed points. The average of these two values of  $\xi$ 's is assumed to be the optimal value of the damping ratio of the TLCD and assigned as  $\xi_0$ . Assuming  $\mu^2 = 0.01$  and  $\alpha = B/l' = 0.805$  as an example, the values of  $f$ ,  $\beta_1$ ,  $\beta_2$ ,  $D_{x1}$ , and  $D_{x2}$  have already been calculated. Therefore the optimal damping ratio  $\xi_0$  of the TLCD can be determined by Eq. (26) and given as  $\xi_0 = 0.0494$ . The curve of  $D_x$  for  $\xi_0$  is also shown in Fig. 3.

The dynamic magnification factor  $D_x$  given by Eq. (18) would be influenced by the value of  $\alpha$  which is the ratio of the length of the horizontal section to the effective wetted length  $l'$ , and  $l' = 2H + B/R$  for a symmetrical TLCD; therefore the area ratio  $R$  can also affect the value of  $D_x$ . Fig. 5 shows that the maximum value of  $D_x$  would be decreased as  $\alpha$  increases for the same case of  $\mu^2 = 0.01$ .

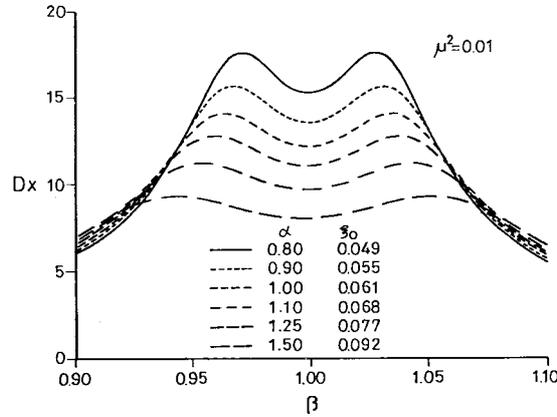


Fig. 5 Dynamic response  $D_x$  at optimal condition

**5. Design consideration**

The following equation can be obtained from the definition of  $\alpha$  given by Eq. (10)

$$H = \frac{1}{2} \left( \frac{1}{\alpha} - \frac{1}{R} \right) B \tag{28}$$

Therefore

$$R > \alpha \tag{29}$$

The relationship between the water mass  $m_2$  and the effective mass  $m'$  of the TLCD is given by

$$m_2 = m' \left[ 1 + \alpha \left( R - \frac{1}{R} \right) \right] \tag{30}$$

If  $\alpha > 1.0$ , then  $m_2 > m'$ . If  $R = 1$  for a uniform TLCD, then  $m_2 = m'$ .

The length of the horizontal section  $B$  can be obtained from Eq. (10)

$$B = \frac{2g\alpha}{\omega_u^2} \tag{31}$$

Substituting Eq. (24) into Eq. (30) yields the result

$$B = \frac{2g\alpha}{(1 - 1.5\alpha^2\mu^2)\omega_p^2} \tag{32}$$

$\mu^2$ ,  $\alpha$ , and  $R$  should be chosen at first in the design of a TLCD system, then  $m_2$ ,  $B$ , and  $H$  can be calculated by Eqs. (30), (32), and (28), respectively. The response  $D_x$  given by Eq. (25) will decrease as  $\alpha$  or  $\mu^2$  increases. But increase of  $\alpha$  or  $\mu^2$  will also increase the water mass  $m_2$ . In practice the water mass should not be much greater than 10% or 20% of the total mass. The maximum water level  $y_{max}$  should also be checked in order to make sure that  $y_{max} \leq H$ .

The optimal damping ratio of a TLCD can be calculated by Eq. (26). The value of the damping ratio strongly depends on the shape and size of the orifice installed inside the horizontal section of the TLCD. The result of the damping ratio from the model test can not be applied to the prototype

directly. Nevertheless, the free vibration test of the free water surface in the columns of a prototype TLCD is also easy to be carried out to determine the damping ratio. In general the damping ratio of a prototype TLCD is smaller than the optimal damping ratio; therefore a proper-design orifice to the TLCD is often necessary. The optimal damping ratio of a TLCD can be achieved easily by adjusting the opening of the orifice. Before the construction of a TLCD, the damping ratio of the TLCD with one or two orifices can be predicted by the finite-element analysis of the water-flow problem. But more research works should be conducted to achieve this goal.

## 6. Experiment and comparison

A simple pendulum-like model test is set up as shown in Fig. 6 for the simulation of a long-period motion. It consists of four 207.8 cm long vertical rods with a heavy rigid platform and a TLCD. The total weight of the platform and TLCD without water is 198 kg. The dimensions of the TLCD made of a uniform plastic pipe are as shown in Fig. 1 and given as follows:  $B=128$  cm,  $h=40$  cm,  $H=0.0\sim 16.5$  cm, and the inner and outer diameters are 4 cm and 5 cm, respectively. The natural frequency and period of this testing model without water is measured as 3.506 rad/sec and 1.7921 sec. The water head and weight of TLCD at this resonant frequency are 15.73 cm and 2 kg, respectively. A rotating motor with an eccentric mass is attached to the platform in order to generate a harmonic force.

### 6.1. Free-vibrational test

The free-vibrational test of this testing model with TLCD is carried out by setting an initial horizontal displacement 4 cm to the platform. The free-vibrational decay curves of the motions of the platform  $x(t)$  and the free-water surface  $y(t)$  for the water heads  $H=0$  (no water inside TLCD) and  $H=16$  cm are shown in Fig. 7. The influence of the decay time of the amplitude of  $x(t)$  decaying to one tenth of the initial displacement from the water head ( $H$ ) of TLCD is shown in Fig. 8. It shows that the decay time of the amplitude of  $x(t)$  decreases tremendously if  $H\neq 0$ . It can also be seen that the effectiveness of TLCD is not very sensitive to the water head in the range of  $10\text{ cm} \leq H \leq 18\text{ cm}$ . The free-vibrational tests are also conducted for different angle ( $\theta$ ) between the plane of TLCD and  $x$  direction. The initial displacement of the platform for these cases are also set 4 cm. The decay time of the amplitude of  $x(t)$  decaying to one tenth of the initial displacement is also shown in Fig. 8. It can be seen in Fig. 8 that TLCD is still effective even for  $\theta = 45^\circ$ . It might

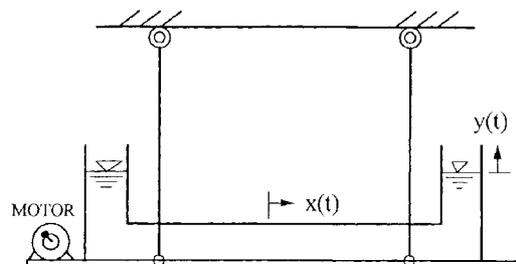


Fig. 6 A pendulum-like testing model

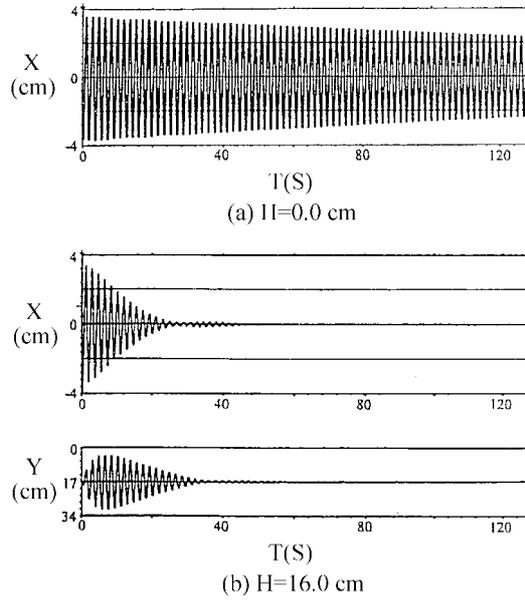


Fig. 7 Free-vibrational decay curves

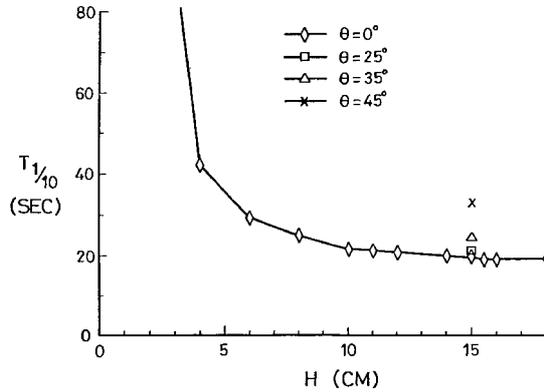


Fig. 8 Decay time of  $x(t)$

be another advantage of TLCD in practice. The damping ratio  $\xi$  of the testing TLCD can be obtained by the free-vibration test of the free-water surface  $y(t)$  of the TLCD alone. In general the damping ratio  $\xi$  of the TLCD depends on the length, size, and water head of the TLCD. The fluid damping is nonlinear in nature; therefore  $\xi$  is also a function of the amplitude of the decay curve of  $y(t)$ . The value of  $\xi$  would be evaluated by the average of  $\xi$  measured from several (generally four to six) important consecutive amplitudes of  $y(t)$ . Of course it is better to repeat the same experiment of the free vibration of  $y(t)$  several times. Finally the average value of  $\xi$  from these experiments gives the the damping ratio of the testing TLCD. The damping ratio of the testing TLCD is then evaluated as 0.0528 at resonance ( $H=15.73$  cm), and it is very close to the optimal damping ratio  $\xi_0=0.0494$ . It is found that the damping ratio is not very sensitive to the water head around  $H=15.73$  cm.

6.2. Harmonic-vibrational test

The harmonic-vibrational tests of the testing model with and without the water inside TLCD are also carried out. The damping ratio of this testing model without water inside TLCD is assumed as  $\eta = 0.002$  according to the result from the harmonic-vibrational test fitted by the analytic result as shown in Fig. 9. The steady-state harmonic responses of both the platform  $x(t)$  and the free-water surface  $y(t)$  are measured for  $H=0.0$  and  $H=16.5$  cm and shown in Figs. 9 and 10, respectively.

If the TLCD on the platform is replaced by a TLD as described by Chen *et al.* (1995). The experimental results of the model test with TLCD or TLD are shown in Fig. 11 for comparison. Also shown in this figure are the analytic results of an optimal TMD system (Chen *et al.* 1995) and an optimal TLCD system for  $\alpha = 1.3$ , respectively. It can be seen obviously that TLCD would compete with TLD or TMD. The effectiveness of the TLCD for the case of  $\alpha > 1.0$  would be further improved properly. Therefore it would be said that the TLCD might have great potential and flexibility in application. Of course more research work should be encouraged.

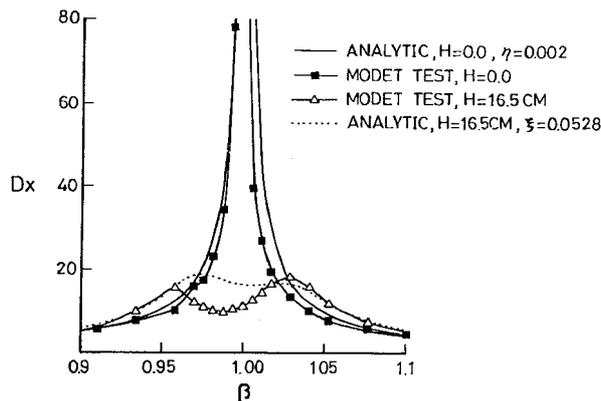


Fig. 9 Harmonic response  $D_x$  of the model test with TLCD for  $H=0.0$  and 16.5 cm

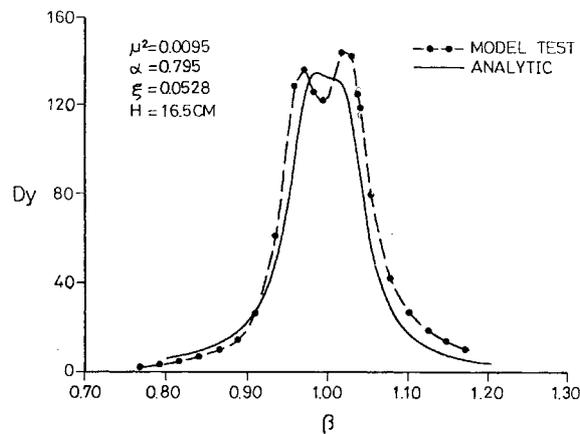


Fig. 10 Harmonic response  $D_y$  of the model test

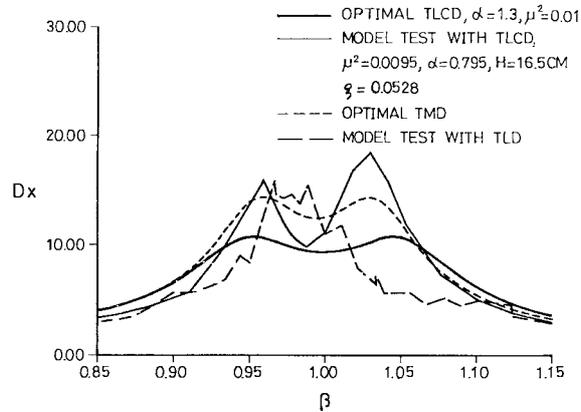


Fig. 11 Comparison of the dynamic responses

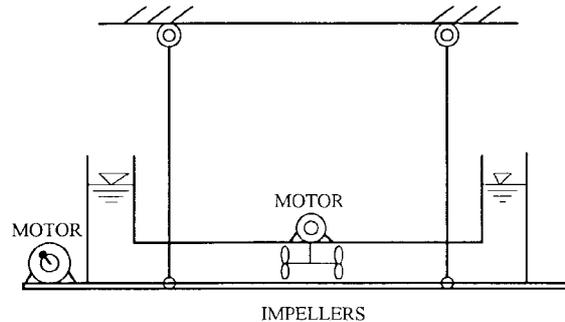


Fig. 12 Active TLCD

## 7. Conclusions

Some important conclusions could be drawn from this study and given as follows:

The natural frequency of a symmetrical TLCD can be adjusted by the water head  $H$ , the length of the horizontal section  $B$ , and the area ratio  $R$  instead of the wetted water length  $l$  (i.e.  $l=2H+B$ ) only for an uniform TLCD.

$\mu^2$ ,  $\alpha$ ,  $R$ , should be chosen at first, then  $m_2$ ,  $B$ , and  $H$  can be calculated by Eqs. (30), (32), and (28), respectively. In application, the water mass  $m_2$  should not be much greater than 10% or 20% of the total mass  $M$  (i.e.  $M = m_1 + m_2$ ).

The optimal damping ratio of a TLCD can be determined easily by Eq. (26), which is a function of  $\mu^2$ ,  $\alpha$ ,  $f$ ,  $\beta$ , and  $D_x$ . Since  $f$ ,  $\beta$ , and  $D_x$  are functions of  $\mu^2$  and  $\alpha$  as given by Eqs. (24) and (25); therefore the optimal damping ratio of a TLCD is eventually a function of these two basic parameters  $\mu^2$  and  $\alpha$ .

In an actual design, the optimal damping ratio would be achieved easily by adjusting the openings of the orifices while performing the free-vibration test of the free water surface in the columns of the TLCD. This should be an important advantage of a TLCD in application.

The responses  $D_x$  and  $D_y$  can be calculated by Eqs. (25) and (18). The vibration of the main mass  $m_1$  can be controlled by  $\mu^2$  and  $\alpha$ . The maximum movement of the water level  $y_{\max}$  has to be checked for the condition that  $y_{\max} \leq H$ .

The effectiveness of a TLCD in vibrational control is not very sensitive to both the water head and the angle between the plane of the TLCD and the ground motion (refer to Fig. 8). This is also an important advantage of a TLCD in application.

In view of the results of the simple pendulum-like model test with TLCD presented in this paper or with TLD (Chen *et al.* 1995), and the analytical result of TMD (Chen *et al.* 1995) (refer to Fig. 11), the effectiveness of a TLCD in vibrational control would be significant and excellent. Of course, more research efforts including the theoretical analysis, the large-scale model test on the shaking table, and even the prototype structural experiment should be encouraged.

Almost all the tall buildings in Taiwan have one or several water-storage tanks on the roof in order to obtain sufficient height of water head for water supply. The total water capacity in the water-storage tanks on the roof is about 50~200 tons. The water-storage tank on the roof of a high-rise building can be designed properly as a TLCD. Probably no extra weight or space is needed to install the TLCD (or several TLCDs) on the roof of the high-rise building, furthermore the original structure of the high-rise building doesn't need to be changed, thus it can be said that TLCD is economical and practical in application.

The water flow inside a TLCD, which is actually a long-and-narrow vessel or a tube-like vessel, can be easily controlled such as by two impellers installed at the center of the horizontal section. Recently the experiment of the same pendulum-like model test of a TLCD with two impellers functioned as the active control as shown in Fig. 12 (Chen 1997) was also carried out, and the result showed excellency in vibrational control. Therefore it may be said that the active TLCD would be expected more efficient in application. Of course more research works should still be done, especially the large scale model test on the shaking table, before the actual design of an active TLCD.

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