Structural Engineering and Mechanics, Vol. 9, No. 2 (2000) 181-193

DOI: http://dx.doi.org/10.12989/sem.2000.9.2.181

Damage assessment of linear structures by a static approach, I: Theory and formulation

Shih-Shong Tseng†

Department of Civil Engineering, National Kaohsiung Institute of Technology, 415 Cheng Kung Road, Kaohsiung 807, Taiwan, R.O.C.

Abstract. The objective of this research is to propose a new global damage detection parameter, termed as the static defect energy (SDE). This candidate parameter possesses the ability to detect, locate and quantify structural damage. To have a full understanding about this parameter and its applications, the scope of work can be divided into several tasks: theory and formulation, numerical simulation studies, experimental verification and feasibility studies. This paper only deals with the first part of the task. Brief introduction will be given to the dynamic defect energy (DDE) after systematically reviewing the previous works. Process of applying the perturbation method to the oscillatory system to obtain a static expression will be followed. Two implementation methods can be used to obtain SDE equations and the diagrams. Both results are equally good for damage detection.

Key words: damage assessment; static defect energy; global method; perturbation method.

1. Introduction

For large-scale public structures, safety has always been the most important issue to be concerned. Great effort has to be put forth by engineers to ensure the integrity of a structure throughout its service lifetime. Therefore, long term security monitoring systems are required for this purpose. Stress concentration at the damaged area has been recognized as one of the main factors contributing to the collapse of structures. Before any repair works can be done to prevent the disasters, locations of damage have to be first detected. Sometimes, budget or other reasons prevent repair work from proceeding immediately even if the damage locations are known. Decisions on the priority of the repairing works then depend on the severity of the damage. To have a full understanding and control over the problems, information about presence of damage in the early stage, precise locations and severity of damage becomes very important.

Several global damage detection parameters have been proposed in the past few years, by using dynamic responses of the system such as, the natural frequencies, frequency response functions (FRFs), mode shapes and shifted energy functions. The word “global” is used to distinguish from the “local” NDE methods, such as, the ultrasonic test, magnetic tests, radio-graphic test, X-ray, and eddy current test etc. When a global method is applied, access to the entire structure is not needed. The potential of using experimental modal analysis for damage detection help global parameters and techniques to grow. In our recent studies (Tseng 1996, 1998), a DDE parameter was proposed by

†Associate Professor

using the shifted dynamic response concept. Results from the numerical studies show its stability and sensibility to reflect damage information. We attempt to simplify the formulation to a static equation in this research.

2. Literature review

Traditional non-destructive evaluation methods can be employed to detect damage of different types for various materials. But generally speaking, they are more suitable for mechanical than for civil structures because of their localized nature of operation. For large-scale civil structures, global damage detection parameters and its application methods are more desired. These parameters can be generally obtained by detecting the shifted quantities under vibration, and can be approximately classified, according to the use of the dynamic characteristics, into four categories:

2.1. Using dynamic responses of the system

Ju *et al.* (1982) evaluated damage by using electrical analogy method. They started to study changes in the dynamic stiffness parameters. Later on (Akgun 1985), frames were analyzed by the same electrical analogy method to get the transmissibility from the dynamic responses. “Relative transmissibility change” was used to detect the presence of damage. Then, another damage index called “relative inertance change” was created (Akgun 1990) by using the transmissibility difference between the intact and the damaged system to detect the presence of damage within one or two cells of a frame. A local method can be used to locate damage position precisely. This method requires the response station to be very close but not at the pseudo node point. Some of the successive research works also went along this line (Ju 1987, Akgun and Ju 1987). The latter discussed the diagnosis of multiple cracks on a beam structure.

Biswas *et al*. (1991) studied several dynamic parameters for damage detection in a full scale modal testing. A probable failure mode of a large fatigue crack was simulated by unfastening a set of high-strength bolts of a splice connection of a highway steel bridge girder. An experimental modal testing was performed for the intact as well as the cracked cases. Changes in frequency spectra are detectable but are difficult to quantify while changes in frequency response function are detectable and quantifiable. In the mean time, Samman *et al.* (1991) applied the Freeman’s code for pattern recognition and image processing to accentuate the differences in the frequency response function between the intact and cracked bridge signal. Since one signal is enough for each girder to detect damage, one response station is all that is needed. Significant slope and curvature differences were found whenever a crack was introduced especially near the natural frequency range. The FRFs are also widely used in other structures, for instance, the offshore platforms by Springer *et al.* (1982), Kenley and Dodds (1980), and Mataraja (1980).

Time domain analysis can also be used for structural damage detection. A state variable method of structural response analysis to random excitations in time domain was developed successfully by Chang and Gu (1986). Another model for damped linear systems was achieved by Fang *et al.* (1986) around the same time. These extracted parameters provide the closest description of the actual structural behaviors and, therefore, the change in the values before and after structural damage can be used for damage detection. In the paper presented by Tsai *et al*. (1985), the cross random decrement method was formulated. The free decay responses contain many structural modes. The modal frequencies, damping, and the complex amplitudes were resolved by curve fitting. These parameters were used for damage detection. The descritization time interval and the number of sampled data points were found to be the important factors affecting the numerical accuracy. This technique was also applied to an offshore model structure by Yang *et al*. (1984) and detected the presence of damage successfully. The advantage is that it requires only measurement of the dynamic response of the structure and not the input forces.

2.2. Using eigenvalues or natural frequencies

Natural frequency is a function of stiffness, mass and damping ratio by its nature. Any damage that occurs either in a single member or in part of the system, will lead to a decrease in the stiffness, mass and increase the damping. Therefore, change in natural frequency is expected. This behavior was reported by Adams *et al.* (1975) in fiber-reinforced plastics and other materials (Adams *et al*. 1978, Cawley and Adams 1979a, 1979b). From their derivation, the ratio of the natural frequency changes in two modes is a function of the damage location. However, because the frequency changes tend to be very small and the model of damage is unsophisticated, another damage parameter is desired. The “error function” was created to be that parameter. Then, by using the error function in a location chart, damage can be approximately located.

The relationship between the fractional changes in the modal eigenvalues and the fractional change in stiffness, mass and damping parameters was studied by Stubbs *et al*. (1990a). In order to get the sensitivity matrix, change in the mass and damping matrices must be either neglected or assumed. Then, predictions for damage locations and magnitudes can be made. The presence of multiple damage locations is successfully predicted in a simply-supported beam by using finite element analysis simulation. Later on, aluminum cantilever beams were tested to provide the support to their conclusions (Stubbs *et al.* 1990b). Regretfully, no experiments were conducted to confirm their multiple damage prediction.

Change in natural frequencies was widely applied for damage detection of offshore structures, for instance by Coppolino and Rubin (1980), Vanhonacker (1980), Stubbs and Osegueda (1987), Loland and Dodds (1976), and Chen and Garba (1988). It can also be applied to composite materials (Sanders 1989). Nevertheless, there were still no mathematical expressions to quantify damage severity even if the relative severity could be plotted from the experimental results.

2.3. Using eigenvectors or mode shapes

For each eigenvalue, there is a corresponding set of eigenvectors. Each mode of vibration or resonance has a mode shape associated with it, which describes spatially the predominant motion of the structure at or near the frequency of the mode. Since mode shapes are not unique in value, but only in “shape”, they can be scaled and plotted together. This is quick and simple and can be applied for damage detection. The Modal Assurance Criterion (MAC) method was first proposed by Allemang and Brown (1982) for comparing mode shapes. This calculation, simply a dot product between two complex unit vectors, results in a single number scalar for comparing shapes. The MAC value equals one, if they are identical shapes, and zero if they are orthogonal. This method was also used by Wolff and Richardson (1989), and Biswas *et al*. (1991). But a further study by Pandey *et al*. (1991) found that this method did not have a good performance for cantilever beam displacement mode shapes. A similar method called the Coordinate Modal Assurance Criterion

(COMAC) was studied by Lieven and Ewins (1988) later on. The COMAC correlates two sets of mode shapes, either from test or Finite Element models, and identifies the coordinates at which the test/model or test/test do not agree.

Yuen proposed displacement and rotation eigenparameters for cantilever beams in 1985. Model displacement and rotation data/output obtained from free vibration analysis were used to formulate eigenparameters, which are the absolute differences of the displacement and rotation mode shapes normalized with respect to their corresponding modal frequencies. Significant differences were found at the prescribed damage location. Qian *et al.* (1990) tried to modify Yuen's method by changing the assumptions and using new stiffness matrix at the damaged elements. The same eigenparameters defined by Yuen were obtained and the identified results agree quite well with the experimental data.

Displacement mode shape was also used by Rizos and Aspragathos (1990). A transverse surface crack was introduced in a rectangular cantilever beam. The beam was forced by a harmonic vibration exciter to vibrate at one of the natural modes of vibration. The amplitude was measured at two arbitrarily chosen positions. From the non-linear equations derived, damage position and depth was obtained. It is a simple method if the experiment is to be carried out in situ, yet it lacks accuracy for small cracks.

In structural analysis, stress and strain fields have been a prime objective to obtain. If strain modes can be measured in a dynamic system, then the stress modes can be calculated. Using strain gauges as the transducer, the strain mode shapes can be obtained. Feng *et al.* (1989) proposed that strain mode is more sensitive to the damage of a structure than natural frequency. From their study, the quantity of the relative change of the damped natural frequency of a damaged beam is proportional to the square of the strain mode value. Yao *et al.* (1992) also applied strain mode shapes to a frame model. A steel frame was excited with a white noise motion on a shaking table. They concluded that this method is more sensitive than using the displacement mode shapes. The absolute changes in curvature mode shapes were investigated by Pandey *et al.* (1991). Since curvature is equal to the summation of the compressive and tensile strain divided by depth of the beam, curvature and strain mode shapes are exactly the same, only to some scale.

2.4. Using energy-related parameters

Gudmundson (1982) derived equations between frequency ratios and strain energy correction ratios of the structure under geometrical changes. A circular hole and a longitudinal straight cut were imposed to a bar in the FEM model. Close results were observed between his method and the FEM simulation result. By using the force-deformation records, Jerry and Yao (1987) created a cumulative plastic deformation damage function and a maximum deformation/cumulative dissipated energy damage function to evaluate the condition of damaged multistory reinforced concrete structures during an earthquake. Correlation were found between damage indices and damage states. Damage information can be revealed based on these correlation. DiPasquale *et al.* (1990) found that damage to engineering materials results in a decrease of the free energy stored in the body with consequent degradation of the material stiffness. They proposed that the parameter-based global damage indices can be related to local damage variables through operations of averaging over the body volume. A parameter study was presented by Leger and Dussault (1992) on the influence of the mathematical modeling of viscous damping on seismic-energy dissipation of multi-degree-of-freedom structures. All the energy terms including the kinetic and strain energy plus the energy dissipated by both hysteretic action of the structural elements and other non-yielding mechanisms were considered in the damage evaluation parameters.

3. Formulation

The SDE formulation is originated from the extended study of the DDE. In this section, we intend to simplify calculation as well as the experimental procedures by continuous research on this parameter. Equations related to DDE will be listed and kept as simple as a reasonable complete treatment of this part allows. Details of the theory can be found in reference by Tseng and Saleeb (1998).

For the spatial continuous solid shown in Fig. 1, assume it to be linear, elastic and its deformation very small under external loading. The governing equations of an oscillatory system for the *n*th eigenmode can be expressed as:

$$\sigma^n_{ij,j}+\sigma\omega_n^2 u_i^n+f_i=0 \quad \text{in} \quad D^n \tag{1a}$$

$$\bar{T}_i^n=\sigma_{ij}n_j \quad \text{on} \quad C_T^n \tag{1b}$$

$$\bar{u}^n=u_i \quad \text{on} \quad C_u^n \tag{1c}$$

where super- and subscripts "*n*" is used to represent parameters of the *n*th eigenmode of the homogeneous material to distinguish from the same notations without "*n*" of the non-homogeneous material. σ_{ij} is the stress tensor, ρ is the density, ω is the angular eigenfrequency, u_i is the displacement vector, f_i is the body force, T_i is the surface traction, and n_i is the unit outward normal vector on the boundary C_T. C_T is part of the boundary where the traction are zero and C_u is part of the boundary where the displacements are zero. D is the body domain. Superscript "bar" is used as an indication of the prescribed boundary condition. A comma in the subscript means the derivative of that quantity with respect to a spatial coordinate. Let x_j be the spatial coordinate, then

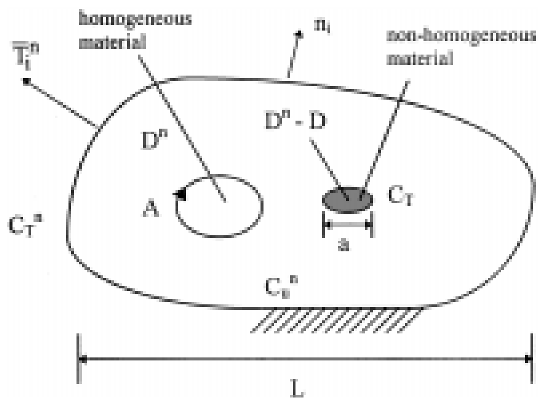

Fig. 1 Elastic solid with a small concentrated cut-out of material

$$\sigma_{ij,j}^{n}=\frac{\partial \sigma_{ij}^{n}}{\partial x_j} \tag{2}$$

It is well known in fracture mechanics that the J integral has been related to potential energy release rate associated with cracks in linear or nonlinear materials. Its integration path must be a closed loop. If there exist any crack within the explicit integration paths, J is not equal to zero. On the other hand, if there is no crack or material non-homogeneity within the paths, J is equal to zero. Assume that the body forces were neglected and Young's modulus is a constant. Similar to the J integral defined in fracture mechanics, the rate of energy release per unit of crack extension vector, F_i, can be defined to satisfy the governing equations:

$$F_i=\int_A \left[(W-T)\delta_{ij}-\sigma_{kj}\frac{\partial u_k}{\partial x_j}\right]n_j dA \tag{3}$$

in which A is a closed integration loop, W and T are, respectively, the total strain energy density and the kinetic energy of an unit volume, and δ_{ij} is the Kronecker delta. The function of vector F_i is similar to the J integral. By changing path of integration, the existence of the non-homogeneous material can be found if F_i value is non-zero. By narrowing down the path of integration, location of the non-homogeneous material can be determined precisely. If Young's modulus of this non-homogeneous area is set to be zero for a special case, it is the situation of cracking. For any member of a 2-D beam/frame structure shown in Fig. 2, the vector F_i can be reduced to a scalar:

$$F=[-(W+V\theta+T)]_{x_1}^{x_2} \tag{4}$$

where V is the shear force and θ is the rotation. The integration area of vector F_i in a 3-D case can now be reduced to the evaluation at two random points on the member, x_1 and x_2 only. The corresponding kinetic energy, and the total strain energy density are

$$T=\frac{1}{2}\omega_n^2\rho A\left(u^2+\frac{I}{A}\theta^2\right) \tag{5}$$

$$W=W(\kappa)+W(\gamma)+W(\varepsilon) \tag{6}$$

where κ, γ and ε are curvature, in plane shear strain, and axial strain of a member, respectively. Suppose P is an energy related scalar, whose contributions are energy terms of F evaluated at a single point x_1 or x_2 only. It is generally non-zero. Then, the quantity of P at location "i" for the "jth" dynamic mode becomes

$$P_{ij}=[W+V\theta+T]_{ij} \tag{7}$$

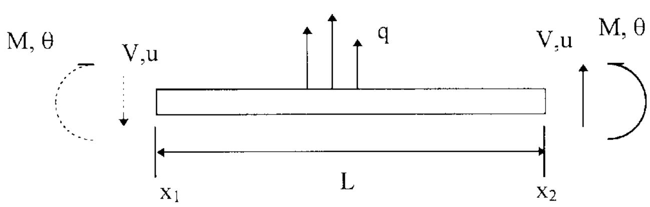

Fig. 2 A 2-D beam/frame member and its sign convention

For a homogeneous material, F_i = 0, that is the same expression as

$$[P]_{x_1}=[P]_{x_2} \tag{8}$$

While for a non-homogeneous material, $F_i \neq 0$, that is

$$[P]_{x_1} \neq [P]_{x_2} \tag{9}$$

P is now an energy scalar quantity. For each individual stress or strain quantity, it may vary in its magnitude and sign, but the summation of the energy components would simply become a constant within the homogeneous area. Plot of *P* along location of a member would appear as two different energy constants separated by a vertical step right at the damaged location as shown in Fig. 3.

Since changes in the geometry are very small, they can be considered as perturbations of an undisturbed boundary. The first application of perturbation theory to changes in geometry in eigenvalue problems was made by Brillouin in (1937). Several papers have been published on this subject since then. Most of the applications of this theory were used for the eigenvalue-change related predictions. For simplification of the DDE equations, the perturbation techniques adopted by Gudmundson (1982) can be applied again. Referring to Fig. 1, for a linear elastic structure with no surface traction, from Eq. (1), the undisturbed eigenvalues and eigenvectors are solutions to the problem:

$$\sigma^n_{ij,j}+\rho\omega^2_n u^n_i=0 \quad \text{in} \quad D^n \tag{10a}$$

$$T^n_i=0 \quad \text{on} \quad C^n_T \tag{10b}$$

$$u^n_i=0 \quad \text{on} \quad C^n_u \tag{10c}$$

and are assumed to be known. Consider changes in geometry which introduce new traction-free boundaries. The disturbed eigenvalue problem takes the following form:

$$\sigma_{ij,j}+\rho\omega^2 u_i=0 \quad \text{in} \quad D \tag{11a}$$

$$T_i=0 \quad \text{on} \quad C_T \tag{11b}$$

$$u_i=0 \quad \text{on} \quad C_u \tag{11c}$$

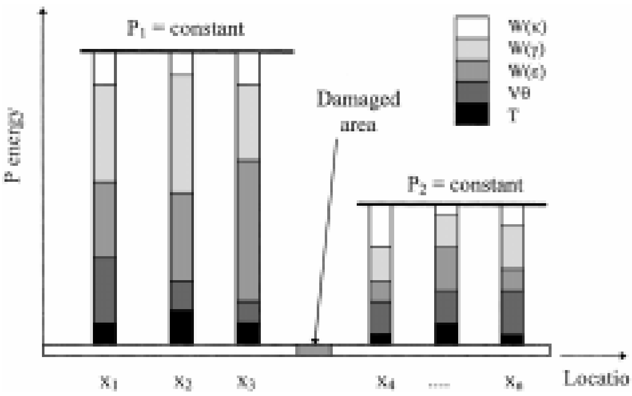

Fig. 3 Components of *P* energy along a damaged member

Suppose that the shift in the nth resonance frequency is of primary interest. Define the shift of the following quantities:

$$\Delta u_i = u_i - u_i^n \tag{12a}$$

$$\Delta \sigma_{ij} = \sigma_{ij} - \sigma_{ij}^n \tag{12b}$$

$$\Delta \varepsilon_{ij} = \varepsilon_{ij} - \varepsilon_{ij}^n \tag{12c}$$

where Δu_i, $\Delta \sigma_{ij}$ and $\Delta \varepsilon_{ij}$ are, respectively, corrections to u_i^n, σ_{ij}^n and ε_{ij}^n. By combining Eqs. (10) to (12), the equations for the correction, Δu_i, are obtained.

$$\rho(\omega^2 - \omega_n^2)u_i^n + \Delta \sigma_{ij,j} + \rho \omega^2 \Delta u_i = 0 \quad \text{in} \quad D \tag{13a}$$

$$\Delta T_i = -T_i^n \quad \text{on} \quad C_T \tag{13b}$$

$$\Delta u_i = 0 \quad \text{on} \quad C_u \tag{13c}$$

All the Δs represent the correction quantities. Assume that the size of the undisturbed body is of order L and the cut-out is of order a, and that

$$\frac{a}{L} << 1 \tag{14a}$$

If the undisturbed eigenmode, u_i^n, is of the order L, the correction, Δu_i, can be assumed to be of order a. A typical distance close to the cut-out is a; hence the correction is assumed to vary over a distance a. The following dimensionless variables can now be introduced:

$$\Delta u_i = a \Delta u_i^* \left(\frac{x_i}{a}\right) \tag{15a}$$

$$\Delta \sigma_{ij} = E \Delta \sigma_{ij}^* \left(\frac{x_i}{a}\right) \tag{15b}$$

$$u_i^n = L u_i^{*n} \left(\frac{x_i}{L}\right) \tag{15c}$$

$$\sigma_{ij}^n = E \sigma_{ij}^{*n} \left(\frac{x_i}{L}\right) \tag{15d}$$

$$\omega_n^2 = \frac{E}{\rho} \frac{1}{L^2} \omega^{*2} \tag{15e}$$

$$\omega^2 = \frac{E}{\rho} \frac{1}{L^2} \omega^{*2}(1 + \delta) \tag{15f}$$

in which x_i is a position vector of a point of interest and δ is the shift quantity. If these dimensionless variables are introduced in Eq. (13), the equations for the unknown Δu_i^* take the following form:

$$\Delta \sigma_{ij,j}^* \left(\frac{x_i}{a}\right) + \left(\frac{a}{L}\right)^2 (1 + \delta) \omega^{*2} \Delta u_i^* + \frac{a}{L} \delta \omega^{*2} u_i^{*2} = 0 \quad \text{in} \quad D \tag{16a}$$

$$\Delta \sigma_{ij}^* n_j = -\Delta \sigma_{ij}^* n_j \quad \text{on} \quad C_T \tag{16b}$$

$$\Delta u_i^* = 0 \quad \text{on} \quad C_u \tag{16c}$$

By neglecting the second and higher order terms in Eq. (16), the first order approximation to Δu_i can be calculated as:

$$\Delta\sigma_{ij,j}=0 \quad \text{in} \quad D \tag{17a}$$

$$\Delta T_i=-T_i^n \quad \text{on} \quad C_T \tag{17b}$$

$$\Delta u_i=0 \quad \text{on} \quad C_u \tag{17c}$$

From Eq. (17), two important conclusions can be drawn: (1) the inertia forces of the disturbed structure has been removed, therefore, it becomes a static problem for the unknown correction Δu_i (2) all the contribution terms of the governing equations have been changed to the shifted quantities, therefore, equations can be treated as the incremental problems.

4. The implementation methods

The idea of using F_i for damage detection is to show the non-zero quantities across the damaged locations. An alternative way of presenting is to plot the individual P energy along the member. The following two implementation methods can be applied by using P energy to reveal damage information.

4.1. The first method

By applying the first conclusion to the DDE equations, remove the inertia forces, P energy will be reduced to a scalar.

$$P=W(\kappa)+W(\gamma)+W(\varepsilon)+V\theta \tag{18}$$

The calculation procedures will be reduced greatly as compared to Eq. (7) since the kinetic energy has been discarded. What is more important is that it has become a static problem for both numerical calculations and practical measurement. Components needed for P energy can be obtained directly from static analysis or static experiments.

For numerical simulation in dynamic problems, calculation errors are inevitable especially when dealing with very small changes in the stiffness matrices. It was found that the accuracy of the calculation results is sensitive to the applied finite element code itself as well as the post-processing procedures. It is always possible to obtain an oscillating result even for a homogeneous material. The same calculation error will also happen to static simulation problems. Actually, the deflected shape of a beam under static load is similar to the first mode under vibration. The static situation can be treated as a special case of the dynamic analysis.

Errors can also be found in both static and dynamic experimental measurements. Good results cannot be obtained without eliminating the errors. Either from numerical simulation or from the field implementation point of view, using only one set of P value for damage evaluation may not obtain very clear information as it is expected from the theory. Therefore, it is suggested here to take the differences between two states: at a reference time "*t*" and after a duration "*dt*". Let $Us1$ be the difference of P energy between the two stages.

$$Us1 = \{P\}_t-\{P\}_{t+dt} \tag{19}$$

It shows that the amount of $Us1$ will be identical for any two random evaluated points within which there is no structural or material deficiencies. The amount will not be the same if deficiencies occurs between these two points within the monitoring duration. By plotting the magnitude of $Us1$ along the structure, a sharp vertical step can be seen right at the damaged location. Since this quantity is used to evaluate change of energy induced by the structural/ material deficiencies by applying static load, it is called the static defect energy to distinguish from the dynamic defect energy proposed previously.

4.2. The second method

From the second conclusion of Eq. (17), F can be changed to incremental quantities.

$$F = [-(\Delta W + V\Delta\theta)]_{x_1}^{x_2} \tag{20}$$

where

$$\Delta W = W(\Delta\kappa) + W(\Delta\gamma) + W(\Delta\varepsilon) \tag{21}$$

Components of P are now changed to incremental quantities which also takes differences between two states: at a reference time “t” and after a duration “dt”. A second static defect energy formula can be defined.

$$Us2 = [\Delta W + V\Delta\theta]_{x = x_i} \tag{22}$$

where x_i is a random point of the member. Results from $Us2$ can also provide damage information. The diagram of $Us2$ along the axial direction shows a similar pattern as the first method. They are equally good for damage detection.

5. Conclusions

If the dynamic experiment is hard to achieve, a static one is preferred. By using the SDE alone, damage information of a structure can be obtained and assessed. To use SDE for damage detection, the following steps should be taken:

(1) measure the first set of data, i.e., curvature k, shear strain γ, axial strain ε, rotation θ, and shear force V, at time reference, t, from a fixed location (element) under static load
(2) record the same data sets for every single element of the structure
(3) measure the second set of data for all the elements under the exact same loading condition after time dt
(4) use the first implementation method
    (i) calculate energy components and then P energy at reference time t
    (ii) calculate energy components and then P energy at time $t+dt$
    (iii) use Eq. (19) to calculate $Us1$ for every single measuring location
    (iv) plot $Us1$ along the structure to form a SDE diagram
(5) use the second implementation method
    (i) calculate the incremental quantities for each set of data
    (ii) calculate the incremental energies for every single measuring location

(iii) use Eq. (22) to calculate *Us*2 for every single measuring location
(iv) plot *Us*2 along the structure to form a SDE diagram

A horizontal line in the SDE diagram indicates no damage occurred within the measuring locations; a vertical step indicate the location of damage; the higher the vertical rise appears, the more sever the damage to the member.

There are some advantages about the SDE parameter:

(1) It is a simple, stable and reliable damage detection parameter. Only a few quantities are required to be measured, none of them is needed from the damaged location.
(2) It provides better sensitivity to a localized damage.
(3) Calculation is very simple, no integration process is involved. It can be done simply by an ordinary calculator.
(4) Only a few measuring stations are needed to obtain damage information.
(5) Load can be applied at any convenient locations with arbitrary magnitude as long as it can produce measurable strain and displacement quantities.

Acknowledgements

This research work is supported by the National Science Council of Taiwan, the Republic of China, under grant No. NSC-87-2211-E-151-003.

References

Adams, R.D., Walton, D., Fletcroft, J.E. and Short, D. (1975), "Vibration testing as a nondestructive test tool for composite materials", *Composite Reliability: ASTM STP 580*, Philadelphia, 159-175.

Adams, R.D., Cawley, P., Pye, C.J. and Stone, B.J. (1978), "Vibration technique for nondestructive assessing the integrity of structures", *J. Mech. Engr. Sci.*, **20**(2), 93-100.

Akgun, M.A., Ju, F.D. and Paez, T.L. (1985), "Transmissibility as a means to diagnose damage in structures", *Proceedings of the 3rd Int. Modal Analysis Conference*, Orlando.

Akgun, M.A. and Ju, F.D. (1990), "Damage diagnosis in frame structures with a dynamic response", *Mech. of Structures and Machines*, **18**(2), 175-196.

Akgun, M.A. and Ju, F.D. (1987), "Diagnosis of multiple cracks on a beam structure", *Int. J. Analytical and Experimental Modal Analysis*, **2**(4), 149-154.

Allemang, R.J. and Brown, D.L. (1982), "A correlation coefficient for modal vector analysis", *Proceedings of the 1st IMAC, Orlando.*

Biswas, M., Pandey, A.K. and Samman, M.M. (1991), "Diagnostic experimental spectral/ modal analysis of a highway bridge", *Int. J. Analy. and Exp. Modal Analysis*, **5**(1), 33-42.

Cawley, P. and Adams, R.D. (1979), "The location of defects in structures from measurements of natural frequencies", *J. of Strain Analysis*, **14**(2), 49-57.

Cawley, P. and Adams, R.D. (1979), "Vibration technique for nondestructive testing of fiber-composite structures", *J. Composite Materials*, **13**, 161-175.

Chang, J. and Gu, Y. (1986), "Response of structures to random excitations in time domain", *Proceedings of the 4^{th}* IMAC, Los Angeles.

Chen, J.C. and Garba, J.A. (1988), "Structural damage assessment using system identification technique", *Proceedings of Structural Safety Evaluation Based On System Identification Approaches*, Germany.

Coppolino, R.N. and Rubin, S. (1980), "Detectable of structural failures in offshore platforms by ambient vibration monitoring", *Proceedings of the 12th Annual Offshore Tech. Conference*, Houston, May.

DiPasquede, E. and Ju, J.W. (1990), "Relation between global damage indices and local stiffness degradation", *J. Struct. Eng., ASCE*, **116**(5), 1440-1456.

Fang, T. and Wang, Z.N. (1986), "Time domain modal analysis of random vibration", *Proceedings of the 4th* IMAC, Los Angeles.

Feng, W.Q., Zhang, K.Y. and Wu, X.Y. (1989), "Research on the change of modal parameters of a beam resulted from a slot", *Proceeding of the 7th Int. Modal Analysis Conference*, Las Vegas.

Gudmudson, P. (1982), "Eigenfrequency changes of structures due to crack, notches or other geometrical changes", *J. of the Mech. and Physics of Solids*, **30**(5), 339-352.

Jerry, S.E. and Yao, T.P. (1987), "Damage assessment using response measurements", *J. Stru. Eng., ASCE*, **113**(4), 787-801.

Ju, F.D., Akgun, M.A., Paez, T.L. and Wang, T.E. (1982), "Model method in diagnosis of fracture damage in simple structures", Vol. editors Merchant, H.C. and Geers, T.L., Productive Applications of Mechanical Vibrations, *ASME AMD* **52**, 113-125.

Ju, F.D. (1987), "Model theory of fracture damage diagnosis in structures", Vol. editors Sih G.C., Nisitani, H. and Ishihara, T., Role of Fracture Mechanics in Modern Technology, *Amsterdam: Elsevier Science Publishers.*

Kenley, R.M. and Dodds, C.J. (1980), "West sole we platform: detection of damage by structural response measurements", *Proceedings of the 12th Annual Offshore Tech. Conference*, Houston, May.

Leger, P. and Dussault, S. (1992), "Seismic-energy dissipation in mdof structures", *J. Stru. Eng., ASCE*, **118**(5),

Lieven, N.A.J. and Ewins, D.J. (1988), "Spatial correlation of mode shapes, the coordinate modal assurance criterion (COMAC)", *Proceedings of the 6th Int. Modal Analysis Conference*.

Loland, O. and Dodds, C.J. (1976), "Experiences in developing and operating integrity monitoring system in north sea", *Proceeding of the 8th Annual Offshore Technology Conference*, Dallas, May.

Mataraja, R. (1980), "Structural integrity monitoring in real seas", *Proceedings of the 12th Annual Offshore Tech. Conference*, Houston, May.

Pandey, A.K., Biswas, M. and Samman, M.M. (1991), "Damage detection from changes in curvature mode shapes", *J. Sound and Vibration*, **145**(2), 321-332.

Qian, G.L., Gu, S.N. and Jiang, J.S. (1990), "The dynamic behavior and crack detection of a beam with crack", *J. Sound and Vibration*, **138**(2), 233-243.

Rizos, P.F. and Aspragathos, N. (1990), "Identification of crack location and magnitude in cantilever beam from the vibration modes", *J. Sound and Vibration*, **138**(3), 381-388.

Samman, M.M., Biswas, M. and Pandey, A.K. (1991), "Employing pattern recognition for detecting cracks in a bridge model", *Int. J. Analytical and Exp. Modal Analysis*, **6**(1).

Sanders, D.R., Stubbs, N. and Kim, Y.I. (1989), "Global nondestructive damage detection in composite structures", *Proc. 7th Int. Modal Annual Conference*, **2**, 1501-1507, Las Vegas, NV.

Springer, W.T., Lawrence, K.L. and Lawley, T.J. (1982), "Damage assessment based on structural frequency response function", *Proceedings of Joint Conference on Exp.l Mechanics*, Hawaii.

Stubbs, N. and Osegueda, R. (1990), "Global non-destructive damage evaluation in solids", *Int. J. Analytical and Exp. Modal Analysis*, **5**(2), 67-80.

Stubbs, N. and Osegueda, R. (1990), "Global damage detection in solids: experimental verification", *Int. J. Analytical and Experimental Modal Analysis*, **5**(2), 81-97.

Stubbs, N. and Osegueda, R. (1987), "Global nondestructive evaluation of offshore platforms using modal analysis", *Proc. 6th Int. Offshore Mechanics And Arctic Engineering Symposium*, **II**, ASME, New York, 517-524.

Tsai, T., Yang, J.C.S. and Chen, R.Z. (1985), "Detection of damage in structures by the cross random decrement method", *Proceedings of the 3rd Int. Modal Analysis Conference*, Orlando, FL., **2**, 691-700.

Tseng, S.S. and Saleeb, A.F. (1998), "Defect energy for global damage detection method", *J. Chinese Institute of Civil and Hydraulic Engineering*, **10**(2), 389-396.

Tseng, S.S., Huang, L.J. and Lin, M.C. (1996), "A numerical study of the dynamic responses of a damaged continuous bridge girder", *Proceedings of the 1st International Conference on Computing and Information Technology for Architecture, Engineering and Construction*, Singapore, 369-376.

Vanhonacker, P. (1980), "Differential and difference sensitivities of natural frequencies and mode shapes of mechanical structures", *AIAA J.*, **18**(12), 1511-1514.

Wolff, T. and Richardson, M. (1989), “Fault detection in structures from changes in their modal parameters”, *Proc. 7th Int. Modal Analysis Conference*, **1**, 87-94, Las Vegas, NV.
Yang, J.C.S., Chen, J. and Dagalakis, N.G. (1984), “Damage detection in offshore structures by the random decrement technique”, *J. Energy Resources Technology*, **106**, 38-42.
Yao, G.C., Chang, K.C. and Lee, G.C. (1992), “Damage diagnosis of steel frames using vibrational signature analysis”, *ASCE J. Engineering Mechanics*, **118**(9).
Yuen, M.M.F. (1985), “A numerical study of the eigenparameter of a damaged cantilever”, *J. Sound And Vibration*, **103**(3), 301-310.

Notations

a	crack length
C_T	part of the boundary where the tractions are zero
C_u	part of the boundary where the displacements are zero
D	body domain
E	Young’s modulus
F_i	the rate of energy release per unit of crack extension
f_i	body force vector
I	second moment of inertia
L	length of elastic solid or member
M	moment; mass
n_j	unit outward normal vector
P_{ij}	defined energy quantity
T	kinetic energy
T_i	surface traction vector
t	a reference time
$Us1$	static defect energy from the first implementation
$Us2$	static defect energy from the second implementation
u_i	displacement vector
V	shear force
W	total strain energy density
x_j	spatial coordinate
γ	shear strain
δ_{ij}	Kronecker delta
ε_{ij}	strain tensor
σ_{ij}	stress tensor
ρ	mass density
ω	natural frequency
θ	rotation
κ	curvature