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Study on sensitivity of modal parameters for suspension bridges

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Abstract. Safety monitoring systems of structures generally resort to detecting possible changes of dynamic system parameters. Sensitivity analysis of these dynamic system parameters may implement these techniques. Conventional structural eigenvalue problems are discussed in the scope of those systems with deterministic parameters. Large and flexible structures, such as suspension bridges, actually possess stochastic material properties and these random properties unavoidably affect the dynamic system parameters. The sensitivity matrix of structural modal parameters to basic design variables has been established in this paper. Moreover, second order statistics of natural frequencies due to the randomness of material properties have been discussed. It is concluded from numerical analysis of a modern suspension bridge that although the second order statistics of frequencies are small relatively to the change of basic design variables, such as density of mass and modulus of elasticity, the sensitivities of modal parameters to these variables at different locations change in magnitude.

Key words: stochastic finite element method; suspension bridge; sensitivity; free vibration.

1. Introduction

When structures have been in service for many years, the deterioration of structural members under the action of wind load excitations and live loads becomes severe and corresponding inspection of structural safety is necessary. Due to drawbacks of the traditional visual inspection approach, it has become of great interest for civil engineers to perform nondestructive detection on possible damage in large scale structures, such as dams and cable-suspended bridges, by

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system identification technique. In these methods, the structural inherent dynamic properties are well-known parameters to be employed as a measure of monitoring structural changes (Alaylioglu and Alaylioglu 1997, Satake and Yokota 1996). Recent research shows that the measured dynamic properties can be achieved with satisfactory accuracy compared to the theoretical results (Gentile and Cabrera 1997). These dynamic properties can be regarded as a set of "fingerprints" and the original ones can be measured and recorded when the structures are newly built. One approach of detecting damage is to measure the existing structural natural frequencies and modes, i.e., a set of existing "fingerprints", and perform comparison between these results and original "fingerprints". Based on the results of comparisons, possible structural damages will be further located and extent of damage will further be estimated.

This detection approach critically requires the sensitivity analysis of structural dynamic properties, such as natural frequencies and modes, to various design variables (Alampalli *et al.* 1997). Discussions of structural eigenvalue problem are generally performed in the field of structures with deterministic system parameters (Clough and Penzien 1975). These structural parameters are actually indeterministic or stochastic because there are unavoidable errors when fabricating and installing structural members. Therefore, it is necessary to address randomly excited vibration and random eigenvalue problems of structures with stochastic parameters. A detailed discussion of random eigenvalue problems has been made by Scheidt and Purkert (1983) and Kleiber and Hier (1992). As to the random eigenvalue problems of complex structures like suspension bridges, the well known Monte Carlo technique is highly time-consuming. An alternative method is the stochastic finite element method (SFEM), which has been recently developed and applied in the field of structural reliability analysis (Benaroya and Rehak 1988, Der Kiureghian and Liu 1986, Liu and Qin 1996, Shinozuka and Deodatis 1988). This method has been employed as a way to calibrate current probability-based structural design codes, such as Load Resistance Factor Design (LRFD) (Mahadevan and Haldar 1991). Another application of this method is that it can be utilized to perform mechanic analysis of imperfect structures (Bulleit and Yates 1991, Wedel-Heinen 1991). One important and challenging basis of SFEM is random field theory (Vanmarcke 1983). According to this theory, the randomness of structural material properties and external loads can be described by dividing random field meshes. Each mesh represents a random field element or variable. The correlation matrix of these elements or variables can be obtained by means of this kind of numerical algorithm (Ghanem and Spanos 1991).

The objective of this study is to analyze the sensitivities of inherent dynamic properties to systemic parameters on the basis of SFEM for suspension bridges. The statistics of structural natural frequencies are also computed with the assumption of several known random parameters, such as modulus of elasticity and density of mass. A modern suspension bridge is demonstrated as an example and the sensitivities of its structural dynamic properties to variables at different locations have been compared and commented.

2. Discretization of random field

There are several approaches to represent the discretization of random fields, such as local averaging method, midpoint method, interpolation method, and series expansion method. Of these approaches, the local averaging of random field is insensitive to the various correlated types of initial random field. On the basis of this correlation function, SFEM algorithm has fast

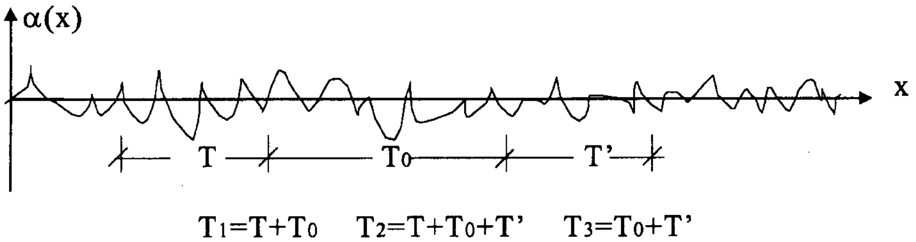


Fig. 1 One-dimensional elements of local averaging random field

convergence and high accuracy.

Fig. 1 shows one-dimensional local averaging random field $\alpha(x)$ with the mean zero and variance σ^2. The local averaging of $\alpha(x)$ in the length T and T' are, respectively, defined as:

$$\alpha_T = \frac{1}{T}\int_{t-T/2}^{t+T/2} \alpha(x)dx \tag{1a}$$

$$\alpha_{T'} = \frac{1}{T'}\int_{t'-T'/2}^{t'+T'/2} \alpha(x)dx \tag{1b}$$

From Eqs. (1), the covariance of α_T and $\alpha_{T'}$ can be derived in the following form:

$$Cov(\alpha_T, \alpha_{T'}) = \frac{\sigma^2}{2TT'}\sum_{k=0}^{3} T_k^2 \Omega(T_k) = \frac{\sigma^2}{2TT'}\sum_{k=0}^{3} T_k^2 \left[\frac{2}{T}\int_0^T \left(1-\frac{\xi}{T}\right)\rho(\xi)d\xi\right] \tag{2}$$

where $\Omega(T)$=the variance function of $\alpha(t)$, and it is a dimensionless function connected with the standard correlation function of $\alpha(t)$; $\rho(\xi)$=correlation function, depending on the distance ξ of two correlated points.

Consider the case of one-dimensional spatial random field $\alpha(S)$ and assume that it is distributed along the curve $z=z(x, y)$, and S is the length of curve, the local averaging of $\alpha(S)$ at the length ΔS is defined as

$$\alpha_S = \frac{1}{\Delta S}\int_{S_1}^{S_2} \alpha(S)\, dS \tag{3}$$

where S_1, S_2=the two end points of curve element ΔS.

The covariance between arbitrary two elements ΔS and $\Delta S'$ can be determined by using

$$Cov(\alpha_S, \alpha_{S'}) = \frac{\sigma^2}{\Delta S\,\Delta S'}\int_{S_1}^{S_2}\int_{S_1'}^{S_2'} \rho(S-S')\, dS'\, dS \tag{4}$$

in which $\rho(\xi)$=correlation function of α_S and $\alpha_{S'}$; ξ=distance of two correlated points S and S'. In general, the correlation function $\rho(\xi)$ is approximately expressed in the form of triangular or exponential type.

Eq. (4) can be solved by using Gaussian numerical integral algorithm. However, it should be mentioned that the local averaging method to represent random field is strictly valid only in the case of stationary random field. In the non-stationary case, alternative discretization method can be chosen to analyze, such as the midpoint method.

3. Perturbation SFEM method for random eigenvalue analysis

Generally, the fundamental equation that describes eigenvalue problems for structures can be expressed as following:

$$\boldsymbol{Ku} = \lambda \boldsymbol{Mu} \tag{5}$$

where $\boldsymbol{K}$=structural stiffness matrix; $\boldsymbol{M}$=structural mass matrix; λ=eigenvalue, $\lambda=\omega^2$ (ω=circular frequency); $\boldsymbol{u}$=modal vector.

As to a given engineering system, the randomness of structural stiffness $\boldsymbol{K}$ and structural mass $\boldsymbol{M}$ comes from uncertain structural parameters α_j (j=1, ..., m, m is the total number of random variables). For the convenience of computation, the randomness of $\boldsymbol{K}$ and $\boldsymbol{M}$ can be expressed as a function of α_j. The first-order expansion of random design variables α_j can be written as following:

$$\alpha_j = \alpha_{dj} + \alpha_{rj} \tag{6}$$

where subscript d denotes deterministic part of α_j and r denotes random part of α_j; mean value of α_{rj} is zero; ε is a very small parameter.

Correspondingly, structural stiffness matrix, mass matrix, eigenvalue, and modal vector can also be expanded in the first order form as follows:

$$\boldsymbol{K} = \boldsymbol{K}_d + \varepsilon \boldsymbol{K}_r \tag{7a}$$

$$\boldsymbol{M} = \boldsymbol{M}_d + \varepsilon \boldsymbol{M}_r \tag{7b}$$

$$\lambda^{(i)} = \lambda_d^{(i)} + \varepsilon \lambda_r^{(i)} \tag{7c}$$

$$\boldsymbol{u}^{(i)} = \boldsymbol{u}_d^{(i)} + \varepsilon \boldsymbol{u}_r^{(i)} \tag{7d}$$

where superscript i of λ and $\boldsymbol{u}$ denotes the corresponding ith mode, i=1, ..., n (n is the total number of natural modes).

Substituting Eqs. (7a)~(7d) into Eq. (5), one obtains:

$$\boldsymbol{K}_d \boldsymbol{u}_d^{(i)} = \lambda_d^{(i)} \boldsymbol{M}_d \boldsymbol{u}_d^{(i)} \tag{8a}$$

$$\boldsymbol{K}_d \boldsymbol{u}_r^{(i)} - \lambda_d^{(i)} \boldsymbol{M}_d \boldsymbol{u}_r^{(i)} = -\boldsymbol{K}_r \boldsymbol{u}_d^{(i)} + \lambda_d^{(i)} \boldsymbol{M}_r \boldsymbol{u}_d^{(i)} + \lambda_r^{(i)} \boldsymbol{M}_d \boldsymbol{u}_d^{(i)} \tag{8b}$$

From Eq. (8a), one can solve for the mean value of eigenvalue $\lambda_d^{(i)}$ and eigenvector $\boldsymbol{u}_d^{(i)}$ and from Eq. (8b), one can obtain the random parts $\lambda_r^{(i)}$ and $\boldsymbol{u}_r^{(i)}$ of $\lambda^{(i)}$ and $\boldsymbol{u}^{(i)}$. When $\lambda_d^{(i)}$ is not repeated, $\lambda_r^{(i)}$ and $\boldsymbol{u}_r^{(i)}$ can be written in the following form:

$$\lambda_r^{(i)} = \boldsymbol{u}_d^{(i)T} (\boldsymbol{K}_r - \lambda_d^{(i)} \boldsymbol{M}_r) \boldsymbol{u}_d^{(i)} \tag{9a}$$

$$\boldsymbol{u}_r^{(i)} = \sum_{j=1}^{m} c_r(i, j)\, \boldsymbol{u}_d^{(i)} \tag{9b}$$

in which $c_r(i, j)$ is a random variable in the following form:

$$c_r(i, j) = \frac{1}{\lambda_d^{(i)} - \lambda_d^{(j)}} \boldsymbol{u}_d^{(j)T} (\boldsymbol{K}_r - \lambda_d^{(i)} \boldsymbol{M}_r) \boldsymbol{u}_d^{(i)} \quad i \neq j \tag{10a}$$

$$c_r(i, j) = -\frac{1}{2} \boldsymbol{u}_d^{(i)T} \boldsymbol{M}_r \boldsymbol{u}_d^{(i)} \tag{10b}$$

For the convenience of solution, expand the random parts $\boldsymbol{K}_r$ and $\boldsymbol{M}_r$ of structural stiffness matrix $\boldsymbol{K}$ and structural mass matrix $\boldsymbol{M}$ at the point of design variables α_{dj} (j=1, ..., m) as follows:

$$\boldsymbol{K}_r = \sum_{j=1}^{m} \boldsymbol{K}_j \alpha_{rj} \tag{11a}$$

$$\boldsymbol{M}_r = \sum_{j=1}^{m} \boldsymbol{M}_j \alpha_{rj} \tag{11b}$$

Substituting Eqs. (11a) and (11b) into Eqs. (9a) and (9b), the random parts of $\lambda_r^{(i)}$ and $\boldsymbol{u}_r^{(i)}$ for the ith mode are obtained as follows:

$$\lambda_r^{(i)} = \sum_{j=1}^{m} \lambda_j^{(i)} \alpha_{rj} \tag{12a}$$

$$\boldsymbol{u}_r^{(i)} = \sum_{j=1}^{m} \boldsymbol{u}_j^{(i)} \alpha_{rj} \tag{12b}$$

in which $\lambda_j^{(i)}$ and $\boldsymbol{u}_j^{(i)}$ are the sensitivities of eigenvalue $\lambda^{(i)}$ and eigenvector $\boldsymbol{u}^{(i)}$ for the ith mode to design variables α_j, respectively.

$$\lambda_j^{(i)} = \boldsymbol{u}_d^{(i)T} (\boldsymbol{K}_j - \lambda_d^{(i)} \boldsymbol{M}_j)\, \boldsymbol{u}_d^{(i)} \tag{13a}$$

$$\boldsymbol{u}_j^{(i)} = -\frac{1}{2} (\boldsymbol{u}_d^{(i)T} \boldsymbol{M}_j \boldsymbol{u}_d^{(i)})\, \boldsymbol{u}_d^{(i)} + \sum_{\substack{s=1 \\ s \neq i}}^{n} \frac{1}{\lambda_d^{(i)} - \lambda_d^{(s)}} [\boldsymbol{u}_d^{(s)T} (\boldsymbol{K}_j - \lambda_d^{(i)} \boldsymbol{M}_j \boldsymbol{u}_d^{(i)})]\, \boldsymbol{u}_d^{(s)} \tag{13b}$$

The covariance matrix of $\lambda^{(i)}$ (i=1, ..., n) based on the first order expansions can be written as follows:

$$Var(\lambda^{(i)}) = \sum_{j=1}^{m} \sum_{k=1}^{m} \frac{\partial \lambda^{(i)}}{\partial \alpha_j} \cdot \frac{\partial \lambda^{(i)}}{\partial \alpha_k} \cdot Cov(\alpha_j, \alpha_k) \tag{14a}$$

$$Cov(\lambda^{(i)}, \lambda^{(s)}) = \sum_{j=1}^{m} \sum_{k=1}^{m} \frac{\partial \lambda^{(i)}}{\partial \alpha_j} \cdot \frac{\partial \lambda^{(s)}}{\partial \alpha_k} \cdot Cov(\alpha_j, \alpha_k) \tag{14b}$$

where the sensitivity $\lambda_j^{(i)} = \partial \lambda^{(i)} / \partial \alpha_j$ can be obtained by Eq. (13a) and the covariance matrix $Cov(\alpha_j, \alpha_k)\,(j, k = 1, ..., m)$ can be computed by the local averaging of random field described previously from Eq. (1) to Eq. (4). In the case of solving covariance of $\omega^{(i)}$, partial derivatives $\partial \lambda^{(i)} / \partial \alpha_j$ in Eqs. (14) can be replaced by $\partial \omega^{(i)} / \partial \alpha_j$, and $\partial \omega^{(i)} / \partial \alpha_j = 1/2\omega^{(i)} \cdot \partial \lambda^{(i)} / \partial \alpha_j$. A Fortran program has been written to accomplish the numerical computation of aforementioned sensitivities and statistics of modal parameters described in Eqs. (13) and (14).

4. Numerical examples

4.1. A simply-supported beam

It is assumed that the mass density ρ and the diameter of cross section d of a simply-supported beam are independent Gaussian variables with standard deviations σ_ρ=156kg/m^3 and σ_d=0.004m, i.

Table 1 Deterministic structural parameters

Mass Density ρ (kg/m^3)	Span Length L (m)	Modulus of Elasticity E (N/m^2)	Diameter d (m)
7.8×10^3	10.0	2.0×10^{11}	0.2

e., coefficient of variation (COV), which is defined as standard deviation divided by mean value, is taken as 0.02. The other structural parameters are deterministic. These deterministic structural parameters are shown in Table 1.

Based on Eq. (14a), the theoretical solution for standard deviation of $\omega^{(i)}$ can be obtained in the following:

$$\begin{aligned}(\sigma_\omega^{(i)})^2 &= Cov(\omega^{(i)}, \omega^{(i)})\\ &= \left(\frac{\partial\omega^{(i)}}{\partial\rho}\right)\left(\frac{\partial\omega^{(i)}}{\partial\rho}\right)Cov(\rho,\rho)+\left(\frac{\partial\omega^{(i)}}{\partial d}\right)\left(\frac{\partial\omega^{(i)}}{\partial d}\right)Cov(d,d)\\ &= \left(\frac{\partial\omega^{(i)}}{\partial\rho}\right)^2\sigma_\rho^2+\left(\frac{\partial\omega^{(i)}}{\partial d}\right)\sigma_d^2 \quad i=1,2,\cdots\end{aligned} \tag{15}$$

Then the standard deviations for the first four natural frequencies $\omega^{(i)}$ (i=1, 2, 3, 4) take the following forms:

$$\sigma_\omega^{(1)}=\sqrt{\left(\frac{\pi^4 d^2 E\,\sigma_\rho}{32\omega^{(1)}\rho^2 L^4}\right)^2+\left(\frac{\pi^4 dE\,\sigma_d}{8\omega^{(1)}\rho L^4}\right)^2} \tag{16a}$$

$$\sigma_\omega^{(2)}=\sqrt{\left(\frac{\pi^4 d^2 E\,\sigma_\rho}{2\omega^{(2)}\rho^2 L^4}\right)^2+\left(\frac{2\pi^4 dE\,\sigma_d}{\omega^{(2)}\rho L^4}\right)^2} \tag{16b}$$

$$\sigma_\omega^{(3)}=\sqrt{\left(\frac{81\pi^4 d^2 E\,\sigma_\rho}{32\omega^{(3)}\rho^2 L^4}\right)^2+\left(\frac{81\pi^4 dE\,\sigma_d}{8\omega^{(3)}\rho L^4}\right)^2} \tag{16c}$$

$$\sigma_\omega^{(4)}=\sqrt{\left(\frac{8\pi^4 d^2 E\,\sigma_\rho}{\omega^{(4)}\rho^2 L^4}\right)^2+\left(\frac{32\pi^4 dE\,\sigma_d}{\omega^{(4)}\rho L^4}\right)^2} \tag{16d}$$

The comparison between the calculated standard deviation and COV and the theoretical standard deviation and COV is shown in Table 2. In Table 2, it can be seen that the maximum error of standard deviation and COV among the first four natural modes is less than 5%.

In the theoretical solution shown in Eq. (16), only the variance of ρ and d is considered but neglecting the correlation among discretized variable meshes in the random fields ρ and d. It is another important factor that may influence the calculated results. To measure the correlation between arbitrary two variables needs a lot of available data and regression analysis as discussed

Table 2 Comparison of theoretical results and numerical results

Mode (i)	Mean Value of Frequency $\omega^{(i)}$ (rad/s)	Theoretical Standard Deviation $\sigma_{\omega}^{(i)}$ (rad/s)	Numerical Standard Deviation $\sigma_{\omega}^{(i)}$ (rad/s)	Errors
1st	24.995	1.03 (0.041)[a]	1.00 (0.040)	-2.91%
2nd	99.978	4.15 (0.042)	3.99 (0.040)	-3.86%
3rd	224.951	9.27 (0.041)	9.01 (0.040)	-2.80%
4th	399.913	16.48 (0.041)	16.90 (0.042)	2.55%

Note: a. Numbers in the parentheses denote coefficient of variation.

by Ang and Tang (1975). Generally, there are several types of correlation expressions widely accepted in the study of random field, for example, the well known exponential and triangle. Although these expressions have different forms, their variance functions in Eq. (2) turn out to be very close. Therefore, the selection of correlation expressions between variables has little influence on the characteristics of local averaging random field. Spatial correlation function between variables within one random field is hereby adopted in the following triangle form:

$$R(\xi)=\begin{cases}1-|\xi|/\theta, & |\xi|\le\theta\\ 0, & |\xi|>\theta\end{cases} \tag{17}$$

in which ξ=distance of two related points; θ=correlation length, which should be in the same unit of ξ. Eq. (17) indicates that a larger correlation length leads to a better correlation coefficient. Fig. 2 demonstrates the variation of calculated standard deviation σ_{ω} with correlation length θ. From Fig. 2, it can be seen that a better correlation among random variables causes a larger standard deviation σ_{ω}. In the analysis, random field meshes are taken the same as those of finite element.

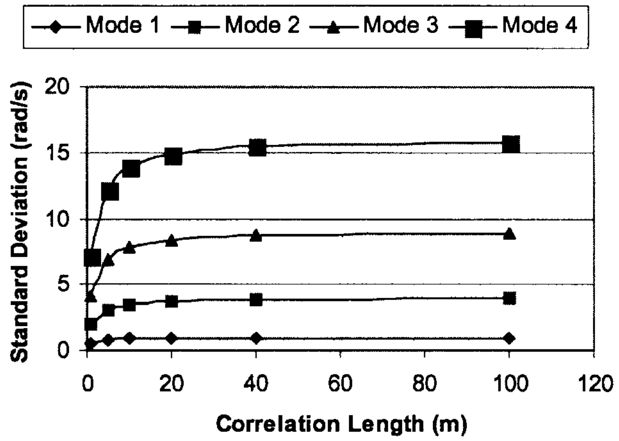


Fig. 2 Standard deviation of $\omega^{(i)}$ vs correlation length

Table 3 Deterministic material properties of Tsing Ma Bridge

Items	E (kN/m^2)	A (m^2)	I_x (m^4)	I_y (m^4)	I_z (m^4)	ρ (kg/m^3)
Cable (each)	1.9×10^8	0.726	0	0	0	8250.00
Hanger(each)	1.3×10^8	0.01	0	0	0	0[a]
Main beam	2.05×10^8	1.41	9.7	145.0	13.2	1.83×10^4
Tower column[b]	2.74×10^7	59.16	582.71	934.79	194.76	2500.0
Lateral beam of tower	3.5×10^7	10.28	42.57	27.33	32.79	2500.00

Note: a. The mass of hangers is merged into that of the main beam and two cables;
b. In fact, section of tower varies with its height, and mean values of section properties are adopted herein.

4.2. Tsing Ma bridge

Tsing Ma bridge, which has been built in Hong Kong, is a modern suspension bridge with a main span of 1377m and it meets the need of both railway and roadway transportation. Deterministic structural parameters of this bridge are shown in Table 3. Its detailed finite element model exactly including every actual hanger is shown in Fig. 3.

The following structural material properties are assumed to be independent Gaussian variables with the other structural parameters remaining deterministic:

1) elasticity modulus E_c, E_b, and E_t of cable, beam, and tower, in which footnotes c, b, and t denote cable, beam, and tower respectively; and
2) mass density ρ_c, ρ_b, and ρ_t of cable, beam, and tower.

The coefficients of variation of these variables are shown in Table 4. To save computation time, number of finite elements is reduced so that about every three hangers in Fig. 3 are merged as one in the present analysis. Random field mesh is taken the same size as that of finite elements.

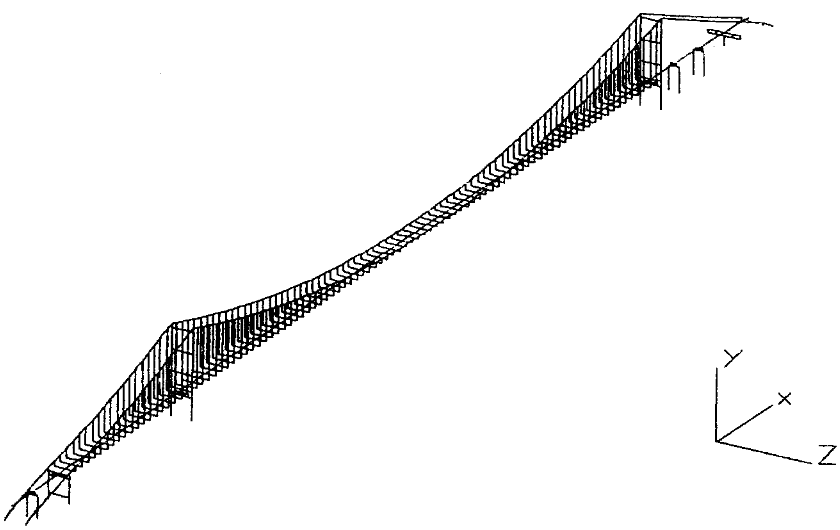


Fig. 3 Finite element model of Tsing Ma Bridge

Table 4 Coefficient of variation

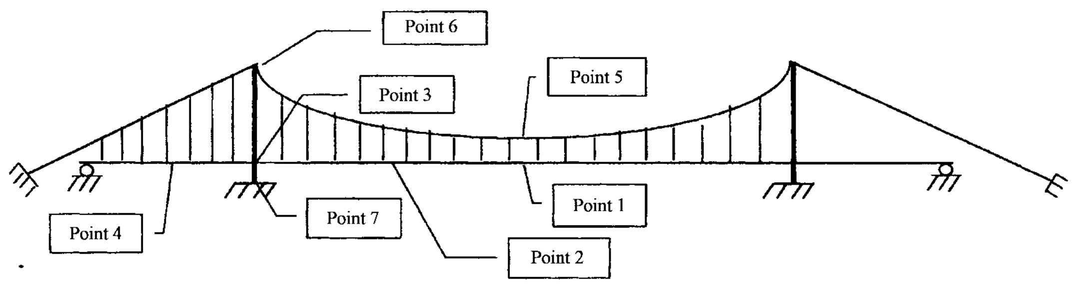
Items	Cable	Beam	Tower
Coefficient of variation	0.12[a] 0.07[b]	0.12 0.07	0.10 0.07

Note: a. Upper number denotes modulus of elasticity.
b. Lower number denotes mass density.

Table 5 Frequency and standard deviation

Vibration Mode	Circular Frequency (rad/s)	Standard Deviation[a] (rad/s)	Standard Deviation[b] (rad/s)	Standard Deviation[c] (rad/s)	Standard Deviation[d] (rad/s)
Mode 1 (Lateral Bending 1)	0.42386	0.00590 (0.01394)[e]	0.00464 (0.01095)	0.00675 (0.01593)	0.01324 (0.03123)
Mode 2 (Vertical Bending 1)	0.74889	0.00553 (0.00738)	0.00335 (0.00447)	0.00441 (0.00589)	0.00843 (0.01126)
Mode 3 (Vertical Bending 2)	0.84540	0.00858 (0.01015)	0.00555 (0.00656)	0.00827 (0.00978)	0.01787 (0.02114)
Mode 4 (Lateral Bending 2)	0.98143	0.01994 (0.02032)	0.015235 (0.015523)	0.02322 (0.02366)	0.04797 (0.04888)
Mode 5 (Vertical Bending 3)	1.10600	0.00188 (0.00170)	0.00145 (0.00131)	0.00231 (0.00210)	0.00198 (0.00179)
Mode 6 (Local Vibration of Cables)	1.15340	0.00920 (0.00798)	0.00575 (0.00499)	0.00903 (0.00783)	0.02076 (0.01800)

Note: a. The correlation length of all random fields is 200m.
b. The correlation lengths of random fields and E_c, and ρ_c, E_b, and E_t and ρ_t are 100m, 100m, and 50m respectively.
c. The correlation lengths of random fields E_c and ρ_c, E_b and ρ_b, E_t, and ρ_t are 300m, 300m, and 20m respectively.
d. The correlation length of all random fields is 3000m.
e. Numbers in the parentheses denote coefficient of variation.



Note: 1. **Points 1, 2, 3, and 4 are located on the main beam.**
2. **Points 5 and 6 are located on one cable.**
3. **Point 7 is located at tower foot of one tower column.**

Fig. 4 Indication of different design variables

Table 6 Sensitivity of frequencies to different design variables

Vibration Mode	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7
Mode 1	0.1850×10^{-13}	0.3965×10^{-14}	0.1668×10^{-13}	0.1602×10^{-14}	0.5061×10^{-18}	0.2506×10^{-18}	0.2183×10^{-14}
(Lateral Bending 1)	0.4383×10^{-7}	0.3963×10^{-7}	-0.725×10^{-10}	0.1905×10^{-8}	0.1600×10^{-6}	-0.5942×10^{-9}	0.8226×10^{-11}
Mode 2	0.7329×10^{-15}	0.8085×10^{-14}	0.2738×10^{-13}	0.1363×10^{-14}	0.2242×10^{-15}	0.2511×10^{-15}	0.4734×10^{-14}
(Vertical Bending 1)	0.2007×10^{-7}	-0.3347×10^{-7}	0.1694×10^{-10}	-0.7846×10^{-8}	0.2027×10^{-7}	-0.246×10^{-10}	-0.1750×10^{-9}
Mode 3	0.8449×10^{-14}	0.3457×10^{-14}	0.4242×10^{-13}	0.5215×10^{-14}	0.1118×10^{-13}	0.3154×10^{-14}	0.2070×10^{-13}
(Vertical Bending 2)	0.9719×10^{-7}	-0.1063×10^{-7}	0.1327×10^{-10}	-0.1433×10^{-7}	0.4960×10^{-7}	0.9405×10^{-10}	-0.3982×10^{-9}
Mode 4	0.5834×10^{-14}	0.9121×10^{-13}	0.5930×10^{-13}	0.4428×10^{-14}	0.8146×10^{-18}	0.2640×10^{-18}	0.5841×10^{-14}
(Lateral Bending 2)	-0.9257×10^{-8}	0.1529×10^{-6}	-0.5172×10^{-9}	0.1048×10^{-7}	0.6429×10^{-8}	-0.3281×10^{-8}	0.4415×10^{-10}
Mode 5	0.3356×10^{-14}	0.1217×10^{-14}	0.1084×10^{-14}	0.2147×10^{-15}	0.1428×10^{-15}	0.4749×10^{-16}	0.7259×10^{-15}
(Vertical Bending 3)	0.2569×10^{-7}	0.1886×10^{-6}	0.2190×10^{-10}	0.5130×10^{-8}	0.1328×10^{-7}	-0.2164×10^{-9}	0.12045×10^{-9}
Mode 6	0.1961×10^{-13}	0.1418×10^{-13}	0.2281×10^{-13}	0.1502×10^{-13}	0.5996×10^{-14}	0.1977×10^{-14}	0.4693×10^{-13}
(Local Vibration of Cables)	-0.4603×10^{-7}	-0.8788×10^{-8}	0.2188×10^{-9}	-0.4244×10^{-7}	-0.2083×10^{-7}	0.1096×10^{-8}	-0.1028×10^{-8}

Note: Upper data is sensitivity to elasticity modulus (unit: rad·m·s/kg) and below data is sensitivity to mass density (unit: rad·m^3/s·kg).

Table 7 Sensitivity of components of vibration mode to different design variables

Vibration Mode	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7
1st	0.228×10^{-20}	-0.178×10^{-21}	0.459×10^{-20}	0.145×10^{-20}	-0.126×10^{-22}	-0.623×10^{-23}	0.250×10^{-20}
	0.2773×10^{-9}	0.1444×10^{-9}	0.3084×10^{-13}	0.2836×10^{-12}	0.1323×10^{-9}	0.9588×10^{-13}	0.1147×10^{-14}
3rd	0.581×10^{-19}	0.218×10^{-18}	-0.131×10^{-17}	-0.266×10^{-19}	-0.599×10^{-20}	-0.122×10^{-19}	-0.119×10^{-18}
	-0.2942×10^{-9}	-0.1145×10^{-9}	0.244×10^{-13}	-0.519×10^{-10}	-0.987×10^{-12}	-0.1497×10^{-9}	-0.142×10^{-11}

Note: Upper data is sensitivity to elasticity modulus and below data is sensitivity to mass density.

The calculated first six natural frequencies and their statistics are shown in Table 5. Herein the variation of axial force in cable due to the COV of mass density (less than 0.3) in beam is neglected since this kind of variation in axial force is small and thus causes little change in the whole structural stiffness. From Table 5, the following conclusions can be drawn:

1. With fixed coefficient of variation of basic variables, the larger the correlation length θ, the more significantly the natural frequencies vary. The tendency is in accordance with the case of simply-supported beam as shown in Fig. 2.

2. The coefficient of variation of the first six natural frequencies is smaller than 5%. This result shows that the first several natural frequencies are insensitive to the selected basic variables compared with the given coefficient of variation listed in Table 4.

Table 6 shows the sensitivity of natural frequencies to various structural damage locations. These locations are shown in Fig. 4 and are denoted by points 1, 2, ..., and 7, which are simulated by finite elements connected to these points. It can be seen from Table 6 that sensitivities of natural frequencies to each variables are all positive. These results indicate that the first six natural frequencies vary proportionally with E_c, E_b, and E_t. In the cases of ρ_c, ρ_b, and ρ_t, some sensitivities are positive but some other ones are negative. Sensitivities of natural frequencies to variables at different locations vary in magnitude, for the first mode (lateral bending motion), in the case of modulus of elasticity, the natural frequency is more sensitive to the change of variable at points 1 and 3; and in the case of mass density, the frequency turns out to be more sensitive to the changes of variable at points 1, 2 and 5.

Table 7 only shows the sensitivities of several components of structural normal modes to various structural damage locations due to the huge amount of available elements in matrix $\boldsymbol{u}_j^{(i)}$ (i= 1, ..., n, j=1, ..., m). It should be noted that every component in the ith mode $\boldsymbol{u}_d^{(i)}$ of Table 7 has been calculated in SI units and divided by $\boldsymbol{u}_d^{(i)T}\boldsymbol{M}\boldsymbol{u}_d^{(i)}$. In the observation of sensitivities of mode components to variables at different locations, it is also found that these sensitivities to variables at different locations vary in magnitude.

5. Conclusions

This paper utilizes a stochastic finite-element-based sensitivity analysis to study the influence of randomness of structural parameters to its natural frequencies and modes based on the variables' locations in the structural configuration. The statistics of natural frequencies of a suspension bridge illustrate that the variation of first several frequencies is very small in comparison with the change of structural parameters. Therefore, it is reasonable nowadays in structural dynamic reliability analysis, e.g., structural aseismic reliability, to consider only variations of external acting loads without including the variations of structural parameters. Since the sensitivities of systematic dynamic parameters (natural frequencies and modes) to variables at different damage locations may vary in magnitude, these results are beneficial for the structural nondestructive damage identification. The present approach is also suitable for other types of bridge and building structures, such as cable-stayed bridges.

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