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Optimisation of bridge deck positioning by the evolutionary procedure

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Abstract. This paper presents some simple thinking on an age-old question that given a bridge of a certain span and loading, from the point of view of the structural efficiency, where should the bridge deck be positioned? Generally, this decision is made for other reasons than structural efficiency such as aesthetics and the analyst is often presented with a *fait accompli.* Using the recently invented Evolutional Structural Optimisation (ESO) method, it is possible to demonstrate that having the deck at different vertical locations can lead to a very different mass and shape for each structural form resembling cable-stayed and cable-truss bridges. By monitoring a performance index which is the function of stresses and volume of discretised finite elements, the best optimised structure can be easily determined and the bridge deck positioning problem can be efficiently solved without resorting to any complex analysis procedures.

Key words: bridge deck positioning; evolutionary optimal design; performance index; shape and topology optimisation.

1. Introduction

This paper deals with an age-old problem that has puzzled engineers over many centuries. If there is a bridge to be built, whereabouts in the height of the bridge should the deck be placed so as to make the best utilisation of the materials used. In general, if the deck is built at the top such as in an arch bridge, most of the material could be in compression which is good for concrete. On the other hand, if the deck is at the bottom of the available space such as in a cable-stayed bridge,

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the support structure is mostly in tension. The latter is good for high strength cable without the occurrence of buckling.

Rather than fix on a certain style of bridge, the question will be solved in a general manner. If some general conclusions can be made, further detailed optimisation can then be conducted. For a specific geography of bridge location and materials available, it would generally be necessary to undertake this study again.

The state-of-the-art techniques for structural optimisation include the following six categories: (1) the classical design parameter search method developed by Vanderplaats (1984); (2) the optimality criteria derived by applying the dual theory of convex programming to a separable approximation of the design problem (Rozvany 1995); (3) the optimal material distribution algorithm with the homogenisation method (Bensøe and Kikuchi 1988), where the design domain is filled with a porous medium of infinitely many microscale voids and the composite material is employed as a basis for defining shape in terms of material density; (4) the biological growth method introduced by Mattheck and Burkhardt (1990) where the structures attempt to adapt to their external loadings by changing their geometry and reducing stress peaks; (5) the genetic algorithm on the basis of the theory of natural selection where the structural topology optimisation can be achieved by evolving a population of chromosomes and each of which represents a possibly-optimal topology (Goldberg 1989); and, (6) the material removal algorithm associated with the evolutionary structural optimisation method (ESO) (Xie and Steven 1993, 1994, 1997), where the most inefficiently used materials are removed step by step until the optimal design is achieved.

The ESO method has been demonstrated to be capable of solving the whole range of structural optimisation problems including statics, dynamics and buckling (Xie and Steven 1997). It is a very versatile design tool which is capable of performing shape, topology as well as size optimisation. Due to its simplicity and effectiveness compared to other methods for structural optimisation, the ESO method was adopted in this study. Its capabilities in shape and topology optimisation were demonstrated by discovering the shape and layout of a given object (i.e., the bridge model) with the optimum structural performance on all its environments.

2. Evolutionary structural optimisation procedure

In the ESO method, a piece of material which is large enough to cover the whole area of the final design is subdivided into a fine mesh of finite elements. Being one of the most frequently used criteria for isotropic materials, the von Mises stress (σ_{VM}) can therefore be selected as the optimisation criterion. Subsequently, the initial rejection ratio RR_0 needs to be defined. After carrying out the linear static finite element analysis of the current structure, the stress level of each element can be determined. All the elements that satisfy the condition

$$\sigma_{VM,e} \leq RR_{ss} \times \sigma_{VM,max} \tag{1}$$

are removed from the model. In Eq. (1), $\sigma_{VM,e}$ and $\sigma_{VM,max}$ are respectively the element von Mises stress and the maximum von Mises stress of the structure and,

$$RR_{ss} = a_0 + a_1 \times SS + a_2 \times SS^2 + \cdots + a_4 \times SS^4 \tag{2}$$

in which a_i $(i = 0, 4)$ are the coefficients determined from experience with the ESO method and SS is the steady state number. The cycle of finite element analysis and element removal is repeated using the same value of RR_{ss} until a steady state is reached, which means that there are no more elements being deleted at the current iteration. At this stage a local optimum is also attained. To continue the optimisation process, the number SS is incremented by one and the cycle of finite element analysis and element removal is repeated using the incremented RR_{ss+1}. Such an evolutionary process continues until a desired optimum is obtained. Finally by examining the structural volume and the stress field, or the performance index, of all steady state designs, the most suitable and logical topology can be selected.

A non-dimensionalised performance index number (*PI*) has been successfully used to determine the best optimised 2D and 3D Michell type structures using plate or brick elements (Querin *et al.* 1996a). This was accomplished by monitoring the *PI* number during the optimisation cycle and selecting the structure which has the lowest value of *PI*.

The performance index is a function of the individual stresses ($\sigma_{VM,e}$) carried by each of the discretised finite elements and their corresponding volume (V_e) (Querin *et al.* 1996a). Or,

$$PI = \frac{1}{FL} \sum_{e=1}^{e=N} \sigma_{VM,e} V_e \tag{3}$$

where F is the applied load and L is a nominal length of the structure; N is the number of total remaining elements during the evolutionary process. For the bridge deck positioning problem presently under consideration, only dead load is taken into account, therefore,

$$F = \sum_{e=1}^{e=N} V_e \rho_e g \tag{4}$$

where ρ_e is the material density of individual element and g is the gravity acceleration.

Substituting Eq. (4) into Eq. (3) yields

$$PI = C\left(\sum_{e=1}^{e=N} \sigma_{VM,e} V_e\right) \Big/ \left(\sum_{e=1}^{e=N} \rho_e V_e\right) \tag{5}$$

where C is a constant and has the value of g/L.

The ESO method was implemented into the *EVOLVE* program (Querin *et al.* 1996b) and with which the present study was conducted.

3. Finite element analysis modelling of the bridge deck

When dealing with the bridge deck positioning problem, the ESO method was used starting with the space allowed for the whole bridge. The ESO process eliminated the lowly stressed elements step-by-step until an optimum was reached. A portion of the design region was designated as the bridge deck and was therefore non-removable, or what is called 'non-design'. The non-design part of the total design space-the bridge deck, corresponds to a horizontal layer of elements. It can be seen from the subsequent results that this deck was preserved for all the models.

As a generic situation, the span of the bridge was taken as 4 units and the available height was 1.2 units, as illustrated in Fig. 1. In the present study the only loading was 'dead' load due to gravity and the non-design space was assumed to have the same density as the bridge material.

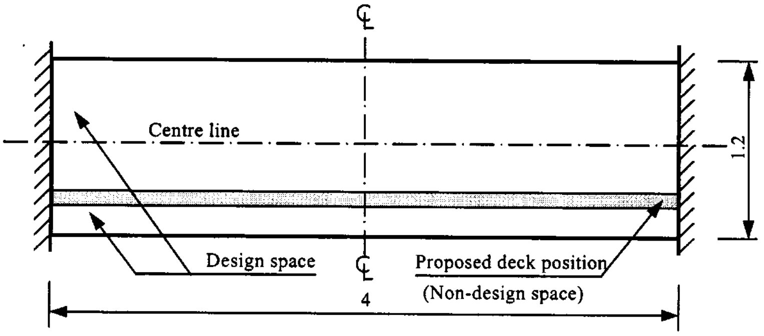


Fig. 1 Bridge design domain

The width of the bridge deck could be of any specified value, e.g., 1 unit.

Owing to the symmetry of the design domain and the target structure, only the left hand symmetric half of the structure was considered and the selected domain was discretised into 100× 60 4-node square finite elements. The left hand side of the modelling region had the rigid abutment. However, the elements adjoining this clamped end could also be removed and those that remained had the fixed freedom conditions on their left hand side.

In total, six positions were available for the bridge deck, designated as Positions 1 to 6 which locate subsequently from the bottom margin to the horizontal centre line of the modelling region (see Fig. 2). The six positions were all below the horizontal centre line, the other possible six positions above the centre line would produce the inverse result, i.e., as if the structure was turned upside down. Thus all stresses switch sign when the deck was at the same distance but opposite side of the centre line. Since the von Mises stress rejection criterion was used, the optimised design will be the same for the deck at the same distance either side of the centre line.

The analysis was carried out by using 2D plane stress elements simply to save on time for this initial study. If the technique was being seriously applied, then as the final optimised design emerged, a 3D optimisation could be undertaken as well as a dynamic and a stability optimisation. The model was optimised with a linear static solution.

4. Presentation and discussion of the results

Figs. 3 to 8 plot the performance index (*PI*) (given in Eq. (5)) versus steady state number (*SS*) for six bridge deck positions. They all have similar features as evident in the figures. However, due to the nature of the evolutionary process, the volume ratios V/V_0 (current structural volume/ initial structural volume) are all slightly different. This can be seen from Figs. 9 to 14 which show

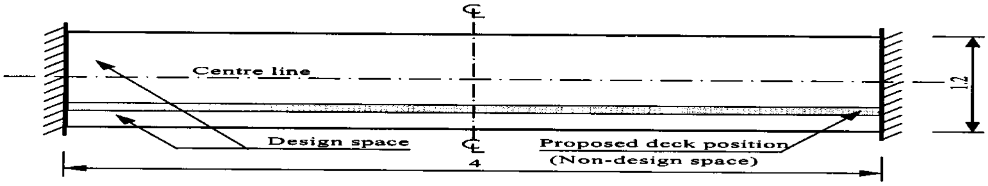


Fig. 2 Assumed bridge deck positions

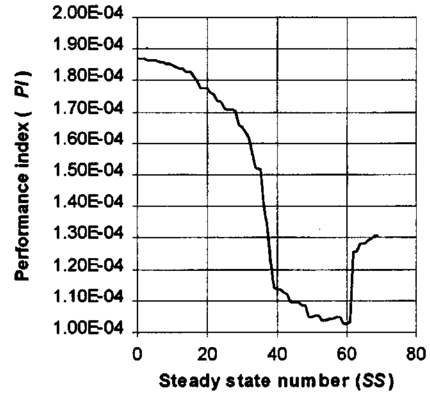


Fig. 3 *PI* for deck position 1

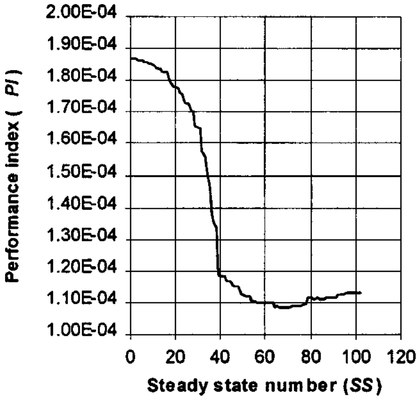


Fig. 4 *PI* for deck position 2

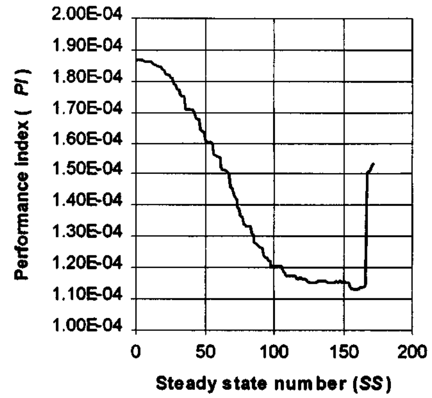


Fig. 5 *PI* for deck position 3

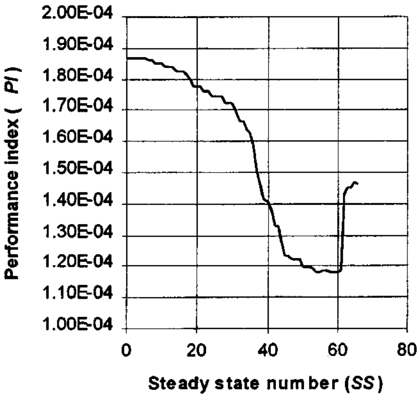


Fig. 6 *P*I for deck position 4

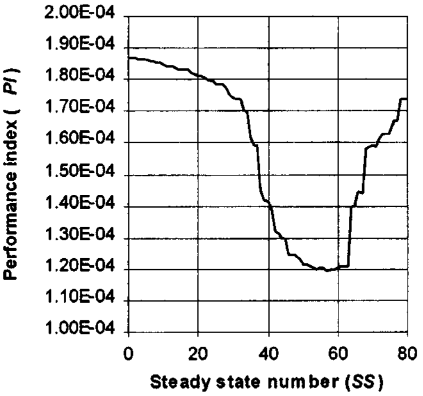


Fig. 7 *PI* for deck position 5

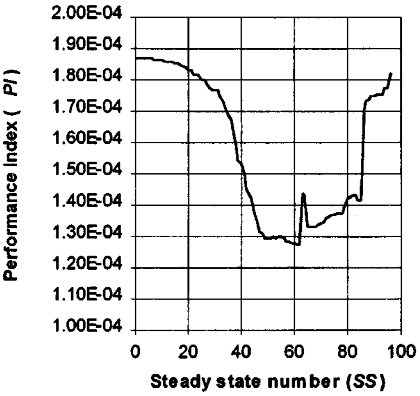


Fig. 8 *P*I for deck position 6

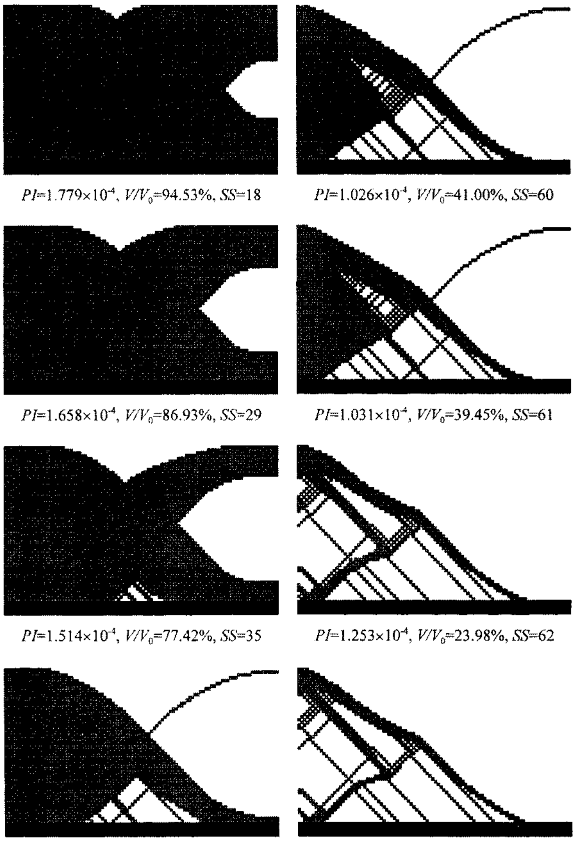
the evolutionary path for the six deck positions. To fully demonstrate the difference the bridge deck position makes to the structural volume, Table 1 summarises the lowest performance index and its corresponding volume ratio as well as steady state number for each of the six deck

Table 1 Comparison of lowest performance index for six deck positions at optimal stage

Deck Position	1	2	3	4	5	6
PI ($\times 10^{-4}$)	1.026	1.084	1.131	1.179	1.196	1.275
V/V_0 (%)	41.00	41.85	43.90	46.67	46.93	43.77
SS	60	69	157	58	57	62

positions at optimal stage. Note that the lowest *PI* indicates the absolute optimum structure without significant voids or cavities formed inside. In order for the optimisation process to continue, cavities have to be formed, thus changing the continuum to a discretised structure. This in turn would lead to a sudden evolution jump in the gradually descending *PI* curves as shown in Figs. 3 to 8. Note also that in Fig. 4, the sudden jump after the last point (steady state 102) was omitted. This was because there was a very large increase in *PI* number at steady state 103 whereby the structure became scattered and practically meaningless.

Amongst the lowest performance indices presented in Table 1, Deck Position 1 produces the minimum value which implies that the bridge deck should be placed at the bottom of the design region so the least amount of material would be used. A 59% volume reduction was obtained for



$PI=1.779\times10^{-4}$, $V/V_0=94.53\%$, $SS=18$

$PI=1.658\times10^{-4}$, $V/V_0=86.93\%$, $SS=29$

$PI=1.514\times10^{-4}$, $V/V_0=77.42\%$, $SS=35$

$PI=1.139\times10^{-4}$, $V/V_0=55.77\%$, $SS=39$

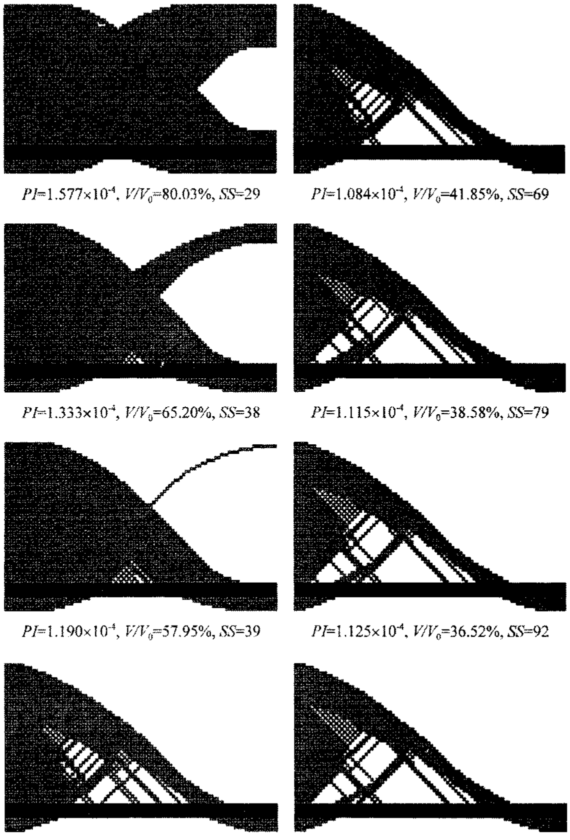
$PI=1.026\times10^{-4}$, $V/V_0=41.00\%$, $SS=60$

$PI=1.031\times10^{-4}$, $V/V_0=39.45\%$, $SS=61$

$PI=1.253\times10^{-4}$, $V/V_0=23.98\%$, $SS=62$

$PI=1.308\times10^{-4}$, $V/V_0=21.32\%$, $SS=69$

Fig. 9 Evolutionary path for bridge deck position 1



$PI=1.577\times10^{-4}$, $V/V_0=80.03\%$, $SS=29$

$PI=1.333\times10^{-4}$, $V/V_0=65.20\%$, $SS=38$

$PI=1.190\times10^{-4}$, $V/V_0=57.95\%$, $SS=39$

$PI=1.086\times10^{-4}$, $V/V_0=43.05\%$, $SS=64$

$PI=1.084\times10^{-4}$, $V/V_0=41.85\%$, $SS=69$

$PI=1.115\times10^{-4}$, $V/V_0=38.58\%$, $SS=79$

$PI=1.125\times10^{-4}$, $V/V_0=36.52\%$, $SS=92$

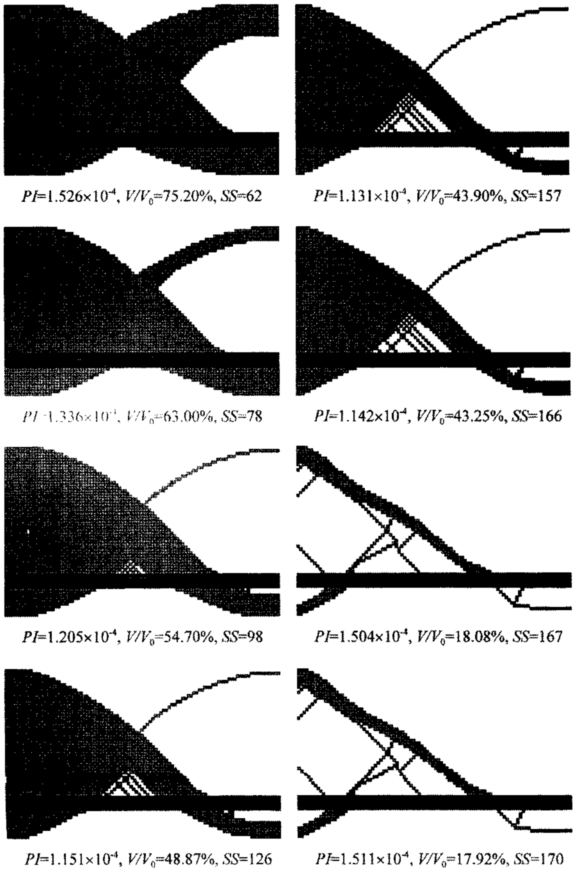
$PI=1.132\times10^{-4}$, $V/V_0=35.80\%$, $SS=102$

Fig. 10 Evolutionary path for bridge deck position 2

Deck Position 1 that was the highest among all six cases.

For all six proposed bridge deck positions, Figs. 9 to 14 clearly show the evolutionary path towards the optimum configuration. The grey area denotes the design space and the darker area represents the non-design domain, i.e., the bridge deck. It can be seen that after the lowest performance index has been reached, the more segmented structure could be optimised further. In Fig. 9, the best discretised structure for Deck Position 1 was the one corresponding to $PI=1.253\times10^{-4}$, because this is the smallest PI value of the non-continuous structure and it gives the best indication of the area distribution of all discretised members. The final topology which indicates the location of each member was that having the smallest volume ratio (of 21.32%). At this stage, the structure has evolved to a shape similar to a harp-shaped cable-stayed bridge.

In Figs. 10 to 14, the best discretised structures were those corresponding to $PI=1.115\times10^{-4}$, 1.504×10^{-4}, 1.430×10^{-4}, 1.398×10^{-4} and 1.330×10^{-4}, respectively, for Deck Positions 2 to 6. As can be seen, a truss-like structure gradually emerged with the deck moving from the bottom margin towards the horizontal centre line of the design domain. It is also interesting to note that the final topology of Deck Position 6 (having volume ratio of 15.58% in Fig. 14) is akin to a cable truss bridge, as shown in Fig. 15.



$PI=1.526\times10^{-4}$, $V/V_0=75.20\%$, $SS=62$

$PI=1.131\times10^{-4}$, $V/V_0=43.90\%$, $SS=157$

$PI=1.336\times10^{-4}$, $V/V_0=63.00\%$, $SS=78$

$PI=1.142\times10^{-4}$, $V/V_0=43.25\%$, $SS=166$

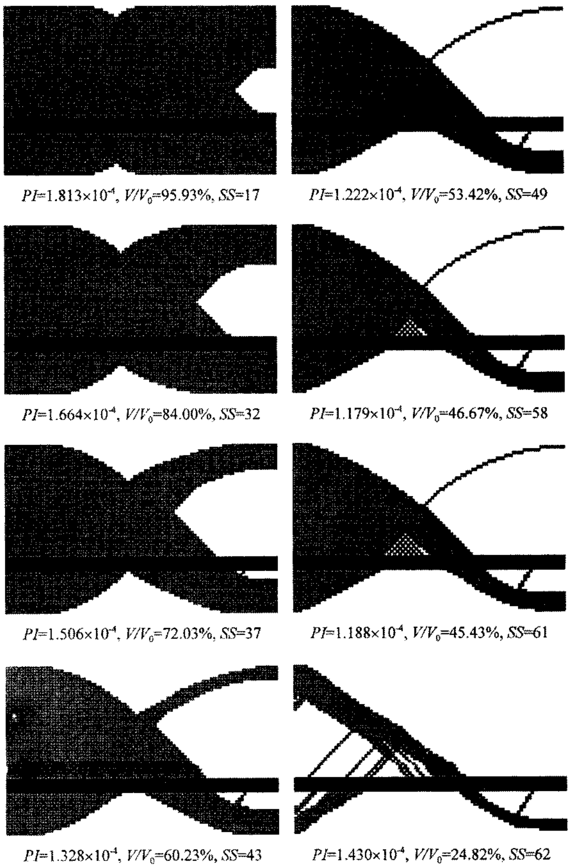
$PI=1.205\times10^{-4}$, $V/V_0=54.70\%$, $SS=98$

$PI=1.504\times10^{-4}$, $V/V_0=18.08\%$, $SS=167$

$PI=1.151\times10^{-4}$, $V/V_0=48.87\%$, $SS=126$

$PI=1.511\times10^{-4}$, $V/V_0=17.92\%$, $SS=170$

Fig. 11 Evolutionary path for bridge deck position 3



$PI=1.813\times10^{-4}$, $V/V_0=95.93\%$, $SS=17$

$PI=1.222\times10^{-4}$, $V/V_0=53.42\%$, $SS=49$

$PI=1.664\times10^{-4}$, $V/V_0=84.00\%$, $SS=32$

$PI=1.179\times10^{-4}$, $V/V_0=46.67\%$, $SS=58$

$PI=1.506\times10^{-4}$, $V/V_0=72.03\%$, $SS=37$

$PI=1.188\times10^{-4}$, $V/V_0=45.43\%$, $SS=61$

$PI=1.328\times10^{-4}$, $V/V_0=60.23\%$, $SS=43$

$PI=1.430\times10^{-4}$, $V/V_0=24.82\%$, $SS=62$

Fig. 12 Evolutionary path for bridge deck position 4

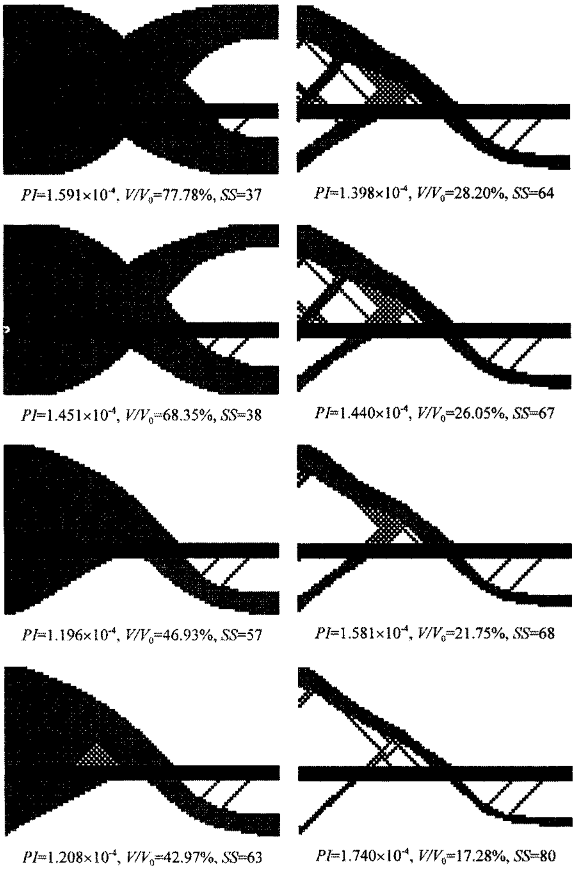


Fig. 13 Evolutionary path for bridge deck position 5

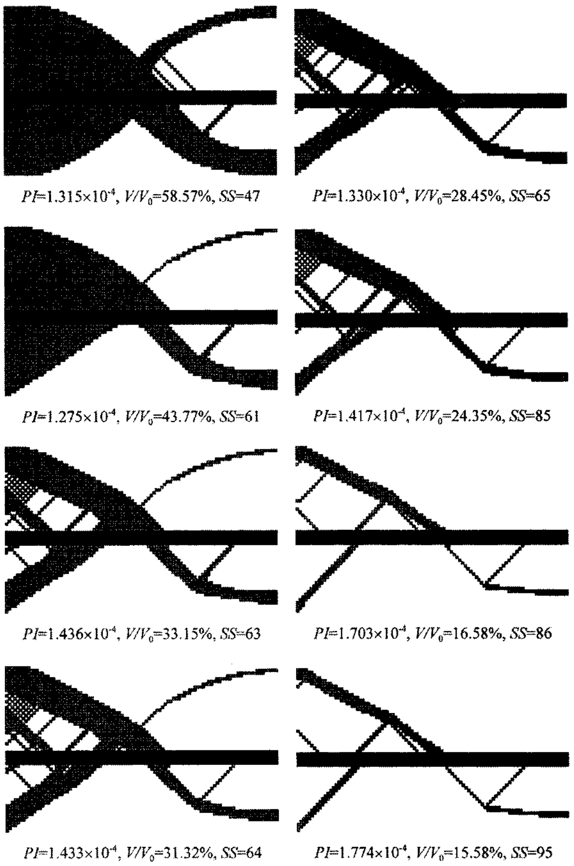


Fig. 14 Evolutionary path for bridge deck position 6

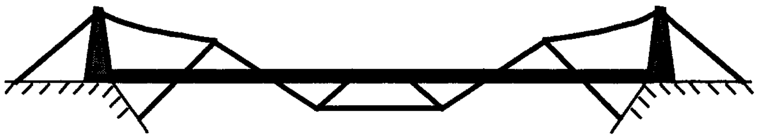


Fig. 15 Impressionistic sketch of final topology of deck position 6

5. Conclusions

The problem of bridge deck positioning has been investigated using the ESO method which is capable of performing shape and topology optimisation simultaneously. It is evident that the procedure is able to offer the possibility of solving the apparently difficult structural optimisation problems quite easily using the standard finite element analysis.

Due to the feature of this initial study, we have not taken into account other factors that may influence the position of the bridge deck, such as the variation of the thickness of the deck and the overall span and height of the bridge, as well as the support and the moving load (traffic) conditions. Note, however, that this initial study has provided a path of the shape and layout of the bridge from every intermediate evolutionary stage towards the desired optimum. On the basis of this study, further detailed optimisation can be readily conducted.

In the present study, the six deck positions were assumed below the horizontal centre line. Due to the adoption of von Mises stress as the rejection criterion, the other possible six positions above the centre line would produce the inverse design topogolies. However, when deck is built at the top of the design space with appropriate kinematic boundary conditions, an arch-type bridge will be expected but not a reversed cable-stayed bridge. Therefore, further investigation is desirable to differentiate tension and compression regions in a specified design domain and to achieve optimum design that makes the best utilisation of the available materials.

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