Structural Engineering and Mechanics, Vol. 7, No. 6 (1999) 533-550
DOI: http://dx.doi.org/10.12989/sem.1999.7.6.533

Optimization approach applied to nonlinear analysis of raft-pile foundations

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Abstract. Optimal design of raft-pile foundations is examined by combining finite element technique and the optimization approach. The piles and soil medium are modelled by three dimensional solid elements while the raft is modelled by shell elements. Drucker-Prager criterion is adopted for the soil medium while the raft and the piles are assumed to be linear elastic. For the optimization process, the approximate semi-analytical method is used for calculating constraint sensitivities and a constraint approximation method which is a combination of the extended Bi-point approximation and Lagrangian polynomial approximation is used for predicting the behaviour of the constraints. The objective function of the problem is the volume of materials of the foundation while the design variables are raft thickness, pile length and pile spacing. The generalized reduced gradient algorithm is chosen for solving the optimization process. It is demonstrated that the method proposed in this study is promising for obtaining optimal design of raft-pile foundations without carrying out a large number of analyses. The results are also compared with those obtained from the previous study in which linear analysis was carried out.

Key words: optimization; raft-pile foundation; finite element; semi-analytical method.

1. Introduction

Raft-pile foundations are usually adopted when it is desired to reduce the maximum displacement and the differential displacement under the structures subjected to heavy loading.

Many researchers have investigated the behaviour of raft-pile foundations using semi-analytical and numerical methods. Approximate procedures combining finite elements and simplified closed form expressions for displacement of raft-pile foundation have been recently developed (Clancy and Randolph 1993, 1996). An approximate numerical analysis of raft-pile foundations in which the raft is modelled as a thin plate, the piles as interacting springs of appropriate stiffness and the soil as an isotropic elastic continuum consisting of horizontal layers has been proposed by Poulos (1994). Yu (1993) has investigated the behaviour of raft- pile foundations using three dimensional finite element analyses in which Drucker-Prager criterion was used for modelling plastic behaviour in the soil medium. In spite of the fact that many investigations have been conducted in this area, only a small number of investigations has been carried out to find an optimum solution of such foundations.

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Chow and Thevendran (1987) presented a simple optimization technique applied to pile groups with rigid caps. The objective function was to apportion the length oı piles in a group subject to the constraints which make the total load evenly distributed over the piles while the stiffness of the group remains within 1% of the stiffness of the original pile group of uniform length. The subgrade reaction method was used to model the soil response at the individual piles while the interaction of pile-soil was based on Mindlin solutions. Hoback and Truman (1993) investigated the least weight design of steel piles using basic pile group behaviour and optimality criteria. However, the two studies are limited to assumptions of fully rigid or flexible raft, free standing piles and linear elastic problems. Tandjiria *et al.* (1996) proposed a combined finite element method and optimization technique to obtain a minimum design of 2×2 raft-pile foundation. The sensitivity analyses in this study were performed using the finite difference method. In spite of the fact that a linear elastic condition was applied in the above study, the constraints selected indirectly accounted for nonlinearities in the problem.

This paper describes a combination of the finite element method and the optimization technique applied to a raft-pile foundation system in order to obtain an optimum design of such foundation without performing a large number of analyses. In this case, the soil behaviour is assumed to be nonlinear. In addition, due to implicit relationship between the behaviour of the foundation such as displacement, differential displacement and moments in the raft and the design variables, an approximate semi-analytical sensitivity method for nonlinear analysis and a new approximation constraint procedure are also presented.

2. Finite element analysis

In this study, the soil medium and the piles have been modelled using three dimensional solid elements whereas three dimensional shell elements formed from isoparametric plane stress elements and 'thick' Reissner-Mindlin plate elements have been used to model the raft. The stiffness matrix formulation of such elements has been discussed elsewhere and hence will not be repeated here.

Special consideration is given to different degrees of freedom involved between the solid element and the shell element. A special algorithm is used to satisfy the compatibility condition. The general stiffness equation is expressed as:

$$[K]\,[u] = [P] \tag{1}$$

where $[K]$ is the global stiffness matrix, $[u]$ is the nodal displacement vector and $[P]$ is the nodal force vector. Eq. (1) can be partitioned as follows:

$$\begin{bmatrix} K_{rr} & K_{rd} & 0 \\ K_{dr} & K_{dd}+K_{bb} & K_{bs} \\ 0 & K_{sb} & K_{ss} \end{bmatrix} \begin{bmatrix} U_r \\ U_b \\ U_s \end{bmatrix} = \begin{bmatrix} P_r \\ P_b \\ P_s \end{bmatrix} \tag{2}$$

where:

K_{rr} = stiffness matrix related to rotation in the shell elements
K_{rd}, K_{dr} = coupling between rotation and displacement of shell elements
K_{dd} = stiffness matrix related to displacement of shell element except the interface

K_{bb} = stiffness matrix of solid elements related to interface
K_{bs}, K_{sb} = coupling of stiffness matrix between interface and outer medium
K_{ss} = stiffness matrix of solid elements
u_r, u_b, u_s = rotation and displacement vectors corresponding to their stiffness matrices
P_r, P_b, P_s = force vectors corresponding to rotation and displacement vectors

To take into account the nonlinear behaviour of the soil medium, Drucker-Prager criterion was used while the raft and piles were assumed to be linear elastic. The initial stress method was adopted as an iterative technique for the elasto-plastic analysis.

3. Semi-analytical sensitivity method

The semi-analytical sensitivity method has been widely applied in many structural optimizations (Haftka *et al.* 1990). However, most of the studies were related to linear elastic cases. A brief description of the method is as follows: The method begins with the differentiation of Eq. (1) with respect to the design variable, x.

$$[K]\left[\frac{du}{dx}\right]+\left[\frac{dK}{dx}\right][u]=\left[\frac{dP}{dx}\right] \tag{3}$$

By adopting finite difference approximation of the first derivatives of the stiffness matrix and the force vectors of Eq. (3), it can be written as:

$$[K]\left[\frac{du}{dx}\right]+\left[\frac{\Delta K}{\Delta x}\right][u]=\left[\frac{\Delta P}{\Delta x}\right] \tag{4}$$

or the sensitivity formula for the displacement is

$$\left[\frac{du}{dx}\right]=[K]^{-1}\left\{\left[\frac{\Delta P}{\Delta x}\right]-\left[\frac{\Delta K}{\Delta x}\right][u]\right\} \tag{5}$$

where:

$[K]$ = Initial stiffness matrix for the original design variables
$\left[\frac{du}{dx}\right]$ = Displacement sensitivity vectors

The forward finite difference equation applied to the original design variables and its perturbed variables is used for calculating the first derivative of the load vector and the stiffness matrix.

Besides the displacement sensitivity mentioned previously, the stress sensitivity can be obtained using a finite difference approximation of the first derivative of the stress vector. The stresses within an element can be written in terms of displacements, as:

$$[\sigma]=[D][B][u] \tag{6}$$

where:

$[\sigma]$ = stress vector
$[D]$ = matrix representing the constitutive relationship for the material
$[B]$ = strain-displacement matrix

Then, using the chain rule of differentiation with respect to the design variable, the stress sensitivity is:

$$\left[\frac{d\sigma}{dx}\right] = \left[\frac{\Delta(DB)}{\Delta x}\right][u] + [DB]\left[\frac{du}{\Delta x}\right] \tag{7}$$

where:

$\left[\frac{d\sigma}{dx}\right]$ = stress sensitivity vector

In nonlinear analyses, external loading is generally applied in increments. For every increment of loading, a number of iterations is performed in the initial stress method and a convergent criterion is adopted for satisfying the degree of accuracy required.

For nonlinear analysis, the equilibrium equation can be stated as:

$$[K_i]\,[\Delta u_i] = [\Psi_i . P] \tag{8}$$

where:
$[K_i]$ = chord stiffness matrix of the original design variables in the ith increment
Ψ_i = ith incremental factor

Similar to the sensitivity analysis for linear cases as given in Eq. (4), for nonlinear cases,

$$[K_i]\left[\frac{d(\Delta u_i)}{dx}\right] + \left[\frac{\Delta K_i}{\Delta x}\right]_x [\Delta u_i] = \left[\frac{\Delta(\Psi_i . P)}{\Delta x}\right] \tag{9}$$

Thus,

$$\left[\frac{d(\Delta u_i)}{dx}\right] = [K_i]^{-1}\left\{\left[\frac{\Delta(\Psi_i . P)}{\Delta x}\right] - \left[\frac{\Delta K_i}{\Delta x}\right][\Delta u_i]\right\} \tag{10}$$

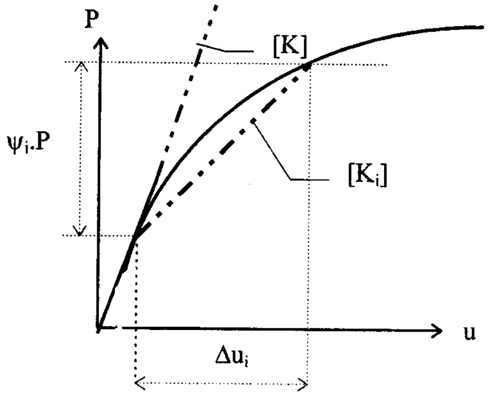

Fig. 1 Initial and chord stiffness matrix

Based on the previous study (Valliappan, Tandjiria and Khalili 1997), for the case where the external loads are independent to the design variables, the chord stiffness matrix of the original design variables can be approximately expressed in terms of the initial stiffness matrix of the original design variables for the purpose of an optimization process. Eventhough this approximation is not an exact expression, the results obtained are accurate enough to predict the responses of a structure under a given loading condition. Therefore, Eq. (10) can be written as follows:

$$\left[\frac{d(\Delta u_i)}{dx}\right] = [K]^{-1}\left\{-\left[\frac{\Delta K}{\Delta x}\right][\Delta u_i]\right\} \tag{11}$$

The total displacement sensitivity vector for M number of increments is

$$\left[\frac{du}{dx}\right] = \sum_{i=1}^{M}\left[\frac{d\,\Delta u_i}{dx}\right] \tag{12}$$

Furthermore, the stress sensitivity in a certain element for the ith incremental loading can also be derived as:

$$\left[\frac{d\,\Delta\sigma_i}{dx}\right] = \left[\frac{\Delta(DB)}{\Delta x}\right][\Delta u_i] + [DB]\left[\frac{d\,\Delta u_i}{dx}\right] \tag{13}$$

Thus,

$$\left[\frac{d\sigma}{dx}\right] = \sum_{i=1}^{M}\left[\frac{d\,\Delta\sigma_i}{dx}\right] \tag{14}$$

It should be noted that the sensitivity expression given by Eq. (14) is applicable only for the elements which are still linear elastic. For the elements which have reached the plastic stage, it is not necessary to determine the sensitivity since it has already been taken care of by Drucker-Prager yield criterion.

4. Constraint approximation

In an optimization problem, constraints should be stated in terms of design variables. However, for complex problems, such as raft-pile foundations, the responses of a structure such as displacements and stresses are implicitly stated in terms of the design variables. To overcome this problem, some constraint approximations for the responses of the structure are required.

Several constraint approximations, such as linear approximation and reciprocal approximation are often used for structural optimization under linear elastic condition. In this paper, the combination of two constraint approximation methods, extended Bi-point and Lagrangian polynomial approximations, is presented.

4.1. Bi-point approximation

Let x_1 and x_2 be two initial points of a design variable with values of $g(x_1)$ and $g(x_2)$ and their first derivatives be $g'(x_1)$ and $g'(x_2)$. Ranges and conditions which are valid for the Bi-point approximation are shown in Fig. 2.

- for the range between x_1 and x_2

$$g_{BP}(X) = \alpha_1 x^3 + \alpha_2 x^2 + \alpha_3 x + \alpha_4 \tag{15}$$

where:

$$\alpha_1 = -(x_1 - x_2)^{-4}\,\{2[x_1 - x_2]\,[g(x_1) - g(x_2)] + [2x_1x_2 - x_1^2 - x_2^2][g'(x_1) - g'(x_2)]\}$$

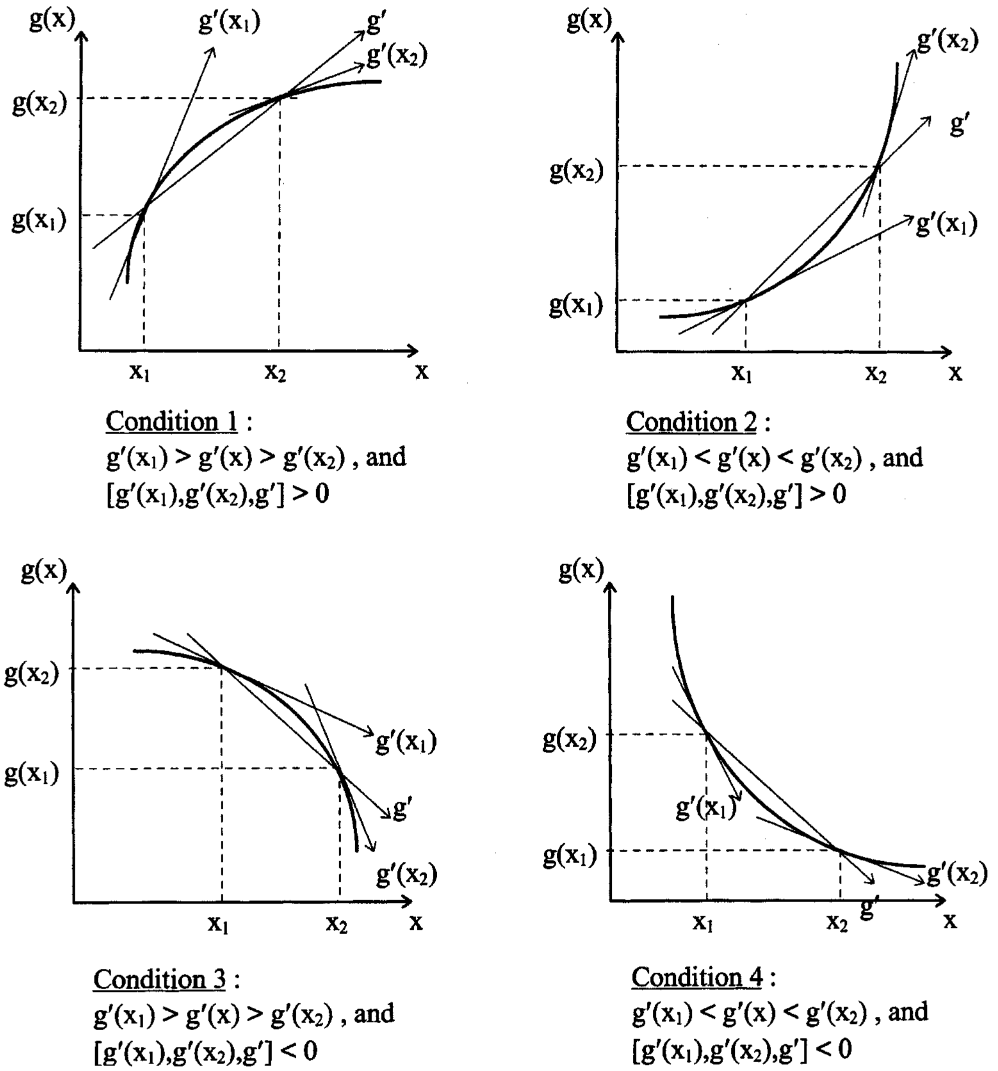

Fig. 2 Ranges and conditions of Bi-point constraint approximation

$$\alpha_2 = -(x_1-x_2)^{-4}\ \{3[x_1^2-x_2^2]\ [g(x_2)-g(x_1)]+[x_1^3+2x_2^3-3x_1x_2^2]\ g'(x_1)+[x_2^3+2x_1^3-3x_1^2x_2]\ g'(x_2)\}$$

$$\alpha_3 = -(x_1-x_2)^{-4}\ \{6[x_1^2x_2-x_1x_2^2]\ [g(x_1)-g(x_2)]+[3x_1^2x_2^2-2x_1^3x_2-x_2^4]\ g'(x_1)+[3x_1^2x_2^2-2x_1x_2^3-x_1^4]\ g'(x_2)\}$$

$$\alpha_4 = -(x_1-x_2)^{-4}\ \{[4x_1x_2^3-3x_1^2x_2^2-x_2^4]\ g(x_1)+[4x_1^3x_2-3x_1^2x_2^2-x_1^4]\ g(x_2)+[x_1x_2^4+x_1^3x_2^2-2x_1^2x_2^3]\ g'(x_1) + [x_1^4x_2+x_1^2x_2^3-2x_1^3x_2^2]\ g'(x_2)] \tag{16}$$

- beyond x_1 and x_2

$$g_{BP}(X) = g(x_1) + (x - x_1)\, g'(x_1)\, \theta_1 \qquad \text{for } x_L \le x \le x_1 \tag{17a}$$

$$g_{BP}(X) = g(x_2) + (x - x_2)\, g'(x_2)\, \theta_1 \qquad \text{for } x_2 \le x \le x_U \tag{17b}$$

where:

$g_{BP}(X)$ = Bi-point approximation of the constraint at x_i
x_L = Lower bound of the design variable
x_U = Upper bound of the design variable

$$\theta_1 = \left(\frac{g'(x_1)}{g'}\right)^{\beta_1} \tag{18a}$$

$$\theta_2 = \left(\frac{g'(x_2)}{g'}\right)^{\beta_2} \tag{18b}$$

$$g' = \left(\frac{g(x_2) - g(x_1)}{x_2 - x_1}\right) \tag{19}$$

$$\beta_1 = -k_1\, ln(x/x_1) \tag{20a}$$

$$\beta_2 = -k_2\, ln(x_2/x) \tag{20b}$$

$$k_1 = \left|\frac{g'(x_2)}{g'(x_1) - g'}\right| \tag{21a}$$

$$k_2 = \left|\frac{g'(x_1)}{g'(x_2) - g'}\right| \tag{21b}$$

Eqs. (17) to (21) are valid for the condition where $[g'(x_1).\ g'(x_2)] > 0$

4.2. Extended Bi-point approximation

When three initial design points, x_1, x_2 and x_3 including their values and their first derivatives are available, the extended Bi-point approximation can be obtained by providing two ranges of design space, i.e., $x \le x_2$ and $x > x_2$. For each range, the Bi-point approximation can be calculated separately. By combining the results from the two ranges, global results of the extended Bi-point approximation are found.

4.3. Lagrangian polynomial approximation

The variable, g, which has certain known values at n design points can be used to approximate any other design point by fitting a polynomial of order (n-1). Lagrangian polynomial constraint approximation can be expressed as:

$$g_{LP}(X) = g(X_P) + \sum_{i=1}^{N} \left[\sum_{j=1}^{n_i} N_{ij} \, (g_{ij} - g_P) \right] \tag{22}$$

where:

N = number of design variables
$g_{LP}(X)$ = Lagrangian polynomial approximation of the constraint at x_i
n = number of design points of each design variable
N_{ij} = Lagrangian shape function which is

$$N_{ij} = \frac{\prod_{k=1,n\ k\neq j} (x - x_k)}{\prod_{k=1,n\ k\neq j} (x_j - x_k)}$$

4.4. Combination of extended Bi-point and Lagrangian polynomial approximations

Although the extended Bi-point approximation needs only three initial design points including their values and their first derivatives for each design variable, two equations are required for each range. These discontinuous equations are usually not preferable in optimization problems. On the other hand, more data points improve the accurary of the the Lagrangian polynomial approximation results. However, in large problems and nonlinear analysis, to adopt a large number of design points is not economical.

A special procedure of combining the Lagrangian polynomial approximation with either the Bi-point or the extended Bi-point approximation may improve the accuracy of the results. Fig. 3 shows the algorithm for the combined method of the Bi-point approximation and the Lagrangian approximation.

5. Optimization procedure

The following steps are involved in the optimization process of the raft-pile foundation:

1. Provide an optimization form which consists of the objective function and the constraints.
2. Select a set of initial design variables and its perturbed design variables, e.i., raft thickness, pile length and pile spacing.
3. Based upon these variables, carry out a nonlinear finite element analysis of the raft-pile foundation. Sensitivity analyses for this initial design variables are automatically included in this step.
4. Perform two other specified basic design points for each design variable and carry out finite element analysis as well as the sensitivity analyses corresponding to these points.
5. Based on the sensitivity results of the constraints, stated in step 1 which are obtained from steps 3 and 4, derive the extended Bi-point constraint approximations.

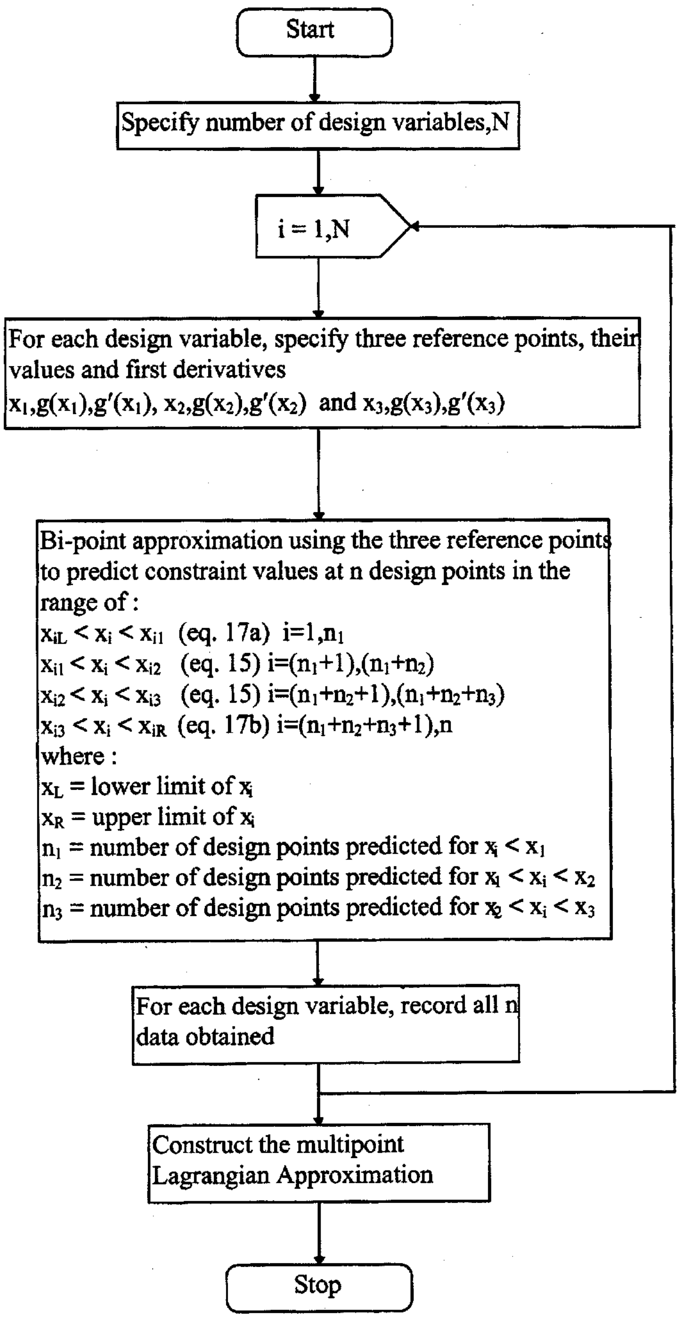

Fig. 3 Algorithm for the combined method of extended Bi-point approximation and Lagrangian polynomial approximation

6. Perform Lagrangian polynomial constraint approximation by selecting the values of design variables at a number of points from the results obtained in step 5.
7. Solve the optimization problem.
8. Use the set of design variables obtained from step 7 as a new set of design variables to be

input for the finite element analysis.

9. Check to see whether the convergence criteria have been satisfied. If so, stop. Otherwise go to step 3.

The two criteria which should be satisfied during the optimization procedure are that the constraints involved in the model should not be violated and the error between the optimization solution and finite element solution is within a specified limit. The error is calculated using

$$e = \frac{\sum_{i=1}^{m} \left| \frac{FEM_i - OPT_i}{g_i} \right|}{m} \tag{23}$$

where e is error, m is number of constraints, FEM_i is constraint value obtained from the finite element analysis, OPT_i is constraint value from the optimization solver and g_i is the allowable limit for the constraints.

6. Numerical results

A 2 by 2 raft-pile foundation with a raft width of 10 m and pile diameter of 0.5 m under an uniform loading, $q = 20$ kPa, was investigated. The piles and raft have a compressive strength, $f_c' = 25$ MPa, modulus of elasticity, $E_c = 24000$ MPa and Poisson's ratio, $\mu_c = 0.2$. The soil medium has a modulus of elasticity, $E_s = 15$ Mpa, Poisson's ratio, $\mu_s = 0.3$ and angle of friction, $\phi' = 26°$ kPa.

Fig. 4 shows the raft-pile foundation system. As the initial design variables, the raft thickness, length of piles and pile spacing were taken as 0.5 m, 10 m and 5.0 m, respectively. For the finite element mesh, the extent of the soil medium is 6B × 6B with a depth of 3L from the surface.

The volume of the material required for piles and raft is selected as the objective function while raft thickness, pile length and pile spacing are design variables. The general form of the optimization of raft-pile foundation is as follows:

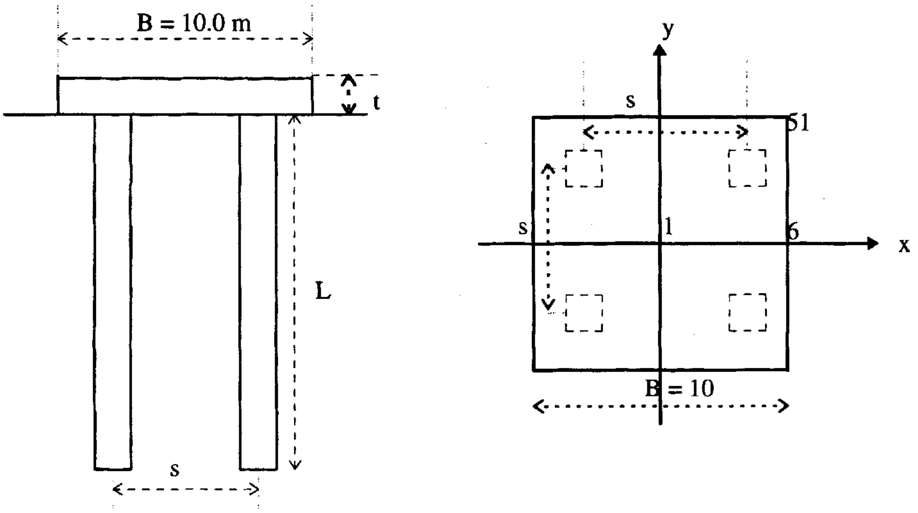

Fig. 4 Raft-pile foundation system

Table 1 Comparison of the linear and nonlinear semi-analytical sensitivity method

	Thickness		Length		Spacing	
	linear analysis	nonlinear analysis	linear analysis	nonlinear analysis	linear analysis	nonlinear analysis
δ	0.00219	0.00234	0.00021	0.00023	-0.00017	-.000249
Δ	0.00383	0.00387	9.81E-05	0.0001	-.000358	-.000399
θ	-0.00077	-0.00078	-1.9E-05	-1.9E-5	1.57E-05	7.94E-5
$M_{(+)}$	-47.2084	-53.2337	2.33456	2.3968	3.805712	5.1118
$M_{(-)}$	-64.7552	-61.7454	0.85543	0.8770	8.58858	6.5903

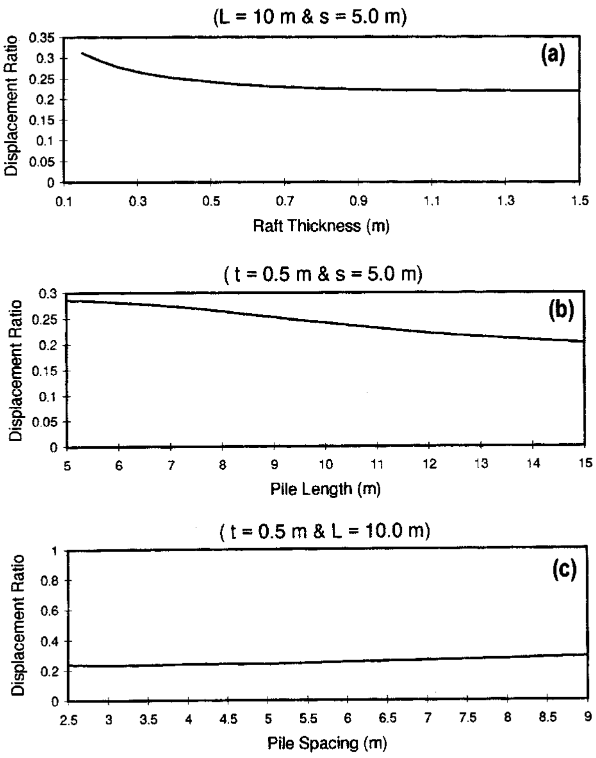

Fig. 5 (a) Displacement ratio versus raft thickness obtained from extended bi-point approximation, (b) Displacement ratio versus pile length obtained from extended bi-point approximation, (c) Displacement ratio versus pile spacing obtained from extended bi-point approximation

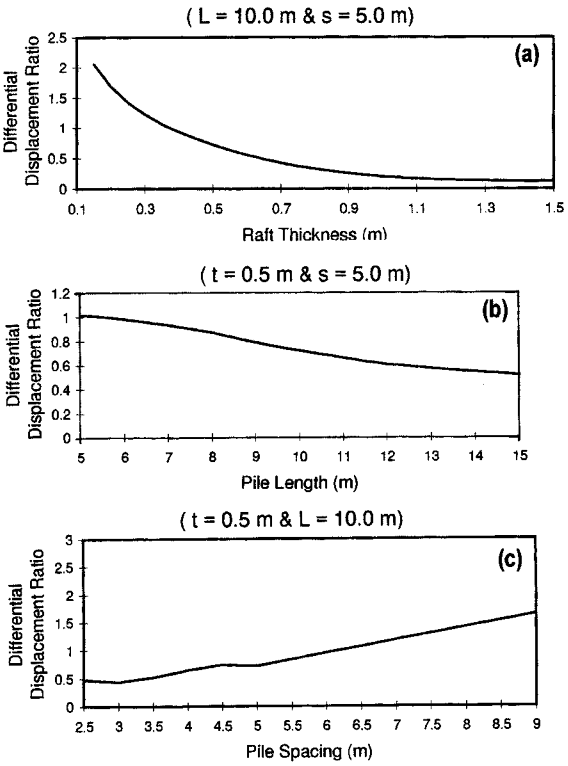

Fig. 6 (a) Differential displacement ratio versus raft thickness obtained from extended bi-point approximation, (b) Differential displacement ratio versus pile length obtained from extended bi-point approximation, (c) Differential displacement ratio versus pile spacing obtained from extended bi-point approximation

Minimise : Volume of material
Subject to :

Displacement	:	$\delta_{max} / \delta_{all}$	$\leq$	1.0
Differential displacement	:	$\Delta_{max} / \Delta_{all}$	$\leq$	1.0
Rotation	:	$\theta_{max} / \theta_{all}$	$\leq$	1.0
Positive moment in the raft	:	$M_{(+)\,max} / M_{(+)\,all}$	$\leq$	1.0
Negative moment in the raft	:	$M_{(-)\,max} / M_{(-)\,all}$	$\leq$	1.0
and				
Side constraints	:	$D_L \leq D \leq D_U$		

D_L and D_U are lower and upper limit of the design variables. They are 0.2 m and 1.5 m for raft thickness, 8 m and 12 m for pile length and 2.5 and 9 m for pile spacing. Based on the previous study (Tandjiria, Valliappan and Khalili 1996) which showed that the stresses in the piles are not significant, they are not considered here, and hence they are not included as constraints.

Generalized Reduced Gradient (GRG) algorithm developed by Lasdon *et al.* (1978) has been chosen to solve the nonlinearly constrained optimization problem in this study.

The comparison between the sensitivity results such as maximum displacement, differential

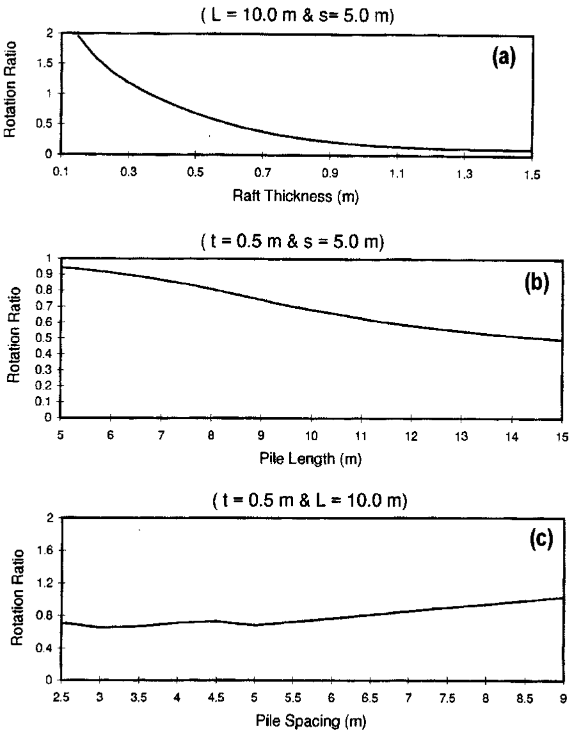

Fig. 7. (a) Rotation ratio versus raft thickness obtained from extended bi-point approximation, (b) Rotation ratio versus pile length obtained from extended bi-point approximation, (c) Rotation ratio versus pile spacing obtained from extended bi-point approximation

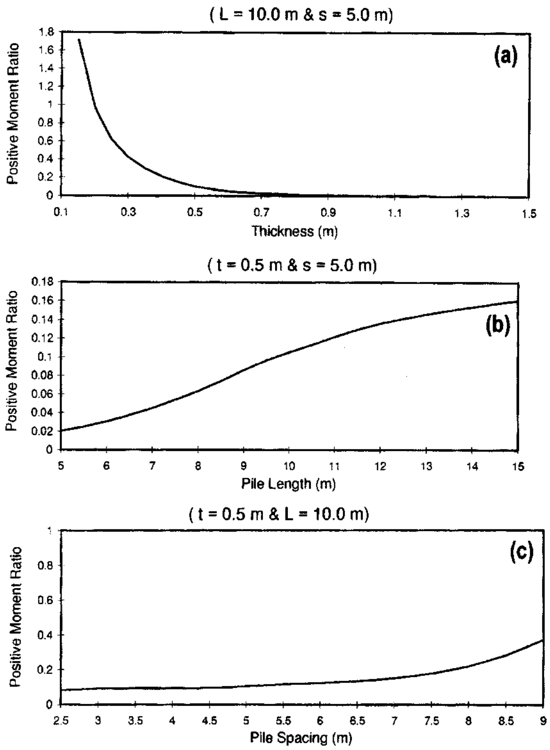

Fig. 8 Positive moment ratio versus raft thickness obtained from extended bi-point approximation, (b) Positive moment ratio versus pile length obtained from extended bi-point approximation, (c) Positive moment ratio versus pile spacing obtained from extended bi-point approximation

displacement etc with respect to raft thickness, pile length and pile spacing obtained from both linear and nonlinear semi-analytical sensitivity method is tabulated in Table 1. It can be seen that nonlinear analyses give higher sensitivity values for most of the constraints.

6.1. Results of extended bi-point approximation

Besides the initial values of the design variables mentioned previously, for each design variable two other values are required for deriving the extended Bi-point approximation. Thus, for thickness, the three values are 0.3 m, 0.5 m and 1.0 m, for pile lengths, 8.0 m, 10.0 m and 12.0 m and for pile spacing, 2.5 m, 5.0 m and 8.0. Based on the constraint values and their sensitivity results at those design points, the constraint approximation for each design variable can be found as shown in Figs. 5 to 9. The constraints are presented in terms of a ratio of maximum value of each constraint to the corresponding "allowable" value.

Displacement ratios at the centre of the raft versus raft thickness, pile length and pile spacing are shown in Figs. 5a to 5c, respectively. The displacement ratios smoothly decrease with increasing raft thickness. At thickness of 0.15 m, the displacement ratio is about 0.3 and then

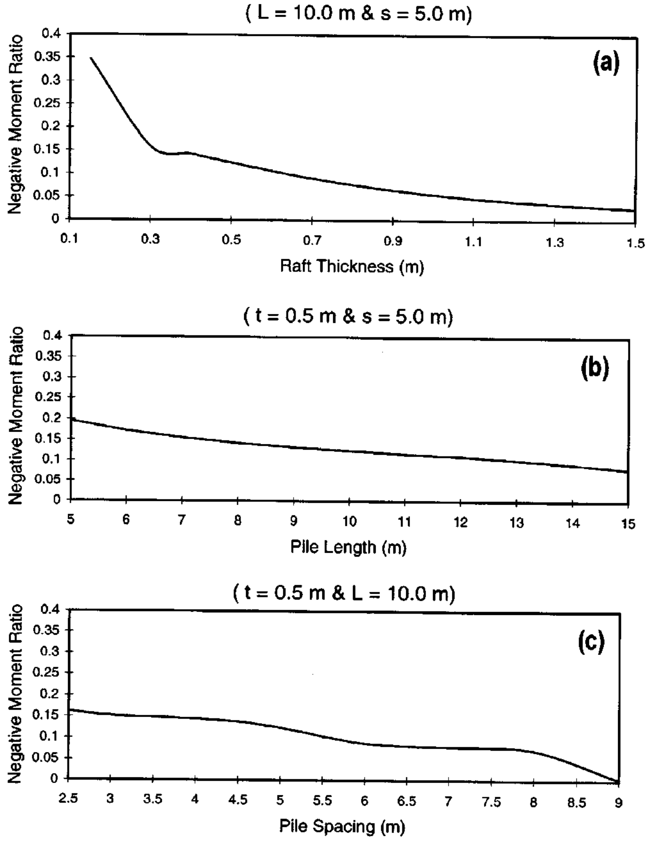

Fig. 9 (a) Negative moment ratio versus raft thickness obtained from extended bi-point approximation, (b) Negative moment ratio versus pile length obtained from extended bi-point approximation, (c) Negative moment ratio versus pile spacing obtained from extended bi-point approximation from thickness of 1.0 m, the displacement ratio becomes steady with little change. Figs. 5b and 5c respectively show that longer piles produce less displacements and larger pile spacings increase the displacements. This is as expected.

Differential displacement ratios between the central nodes and corner nodes of the raft versus raft thickness, pile length and pile spacing are shown in Figs. 6a to 6c, respectively. Raft thickness predominantly influences the differential displacement ratio, especially for thinner raft. Differential displacement ratio of 2 is reached for raft thickness of 0.15 m. For thicker raft, smaller differential displacements are obtained. As in the displacement ratios, the longer piles reduce the differential displacement ratio and the larger pile spacings increase the differential displacement ratio. For pile spacing, the prediction obtained from the extended Bi-point approximation gives slightly unusual pattern between pile spacing of 3.0 m and 5.0 m. This is because a saddle point occurs in that range during the approximation process.

Maximum rotation occuring at the corner nodes of the raft versus raft thickness, pile length and pile spacing can be seen in Figs. 7a to 7c, respectively. The rotation behaviour in terms of the design variables is basically identical to those of displacement and differential displacement, i.e., thicker raft, longer piles and smaller spacing reduce the rotation ratio. Again for pile spacing, the extended Bi-point approximation gives a saddle point at the spacing ranges of 3.0 m and 5.0 m.

Maximum positive moment ratios versus raft thickness, pile length and pile spacing can be seen in Figs. 8a to 8c, respectively. For values smaller than 0.5 m, raft thickness influences the moment values while for values larger than 0.5 m, the raft thickness is less sensitive to the positive moment ratio. Longer piles and larger spacing increase the positive moment ratios.

Figs. 9a to 9c shows the maximum negative moments occuring in the raft versus raft thickness, pile length and pile spacing. The ratios of negative moment is significantly influenced by raft thickness, especially for the raft thickness less than 0.3 m. Opposite to the case of the positive moment ratios, longer piles and larger pile spacings decrease the negative moment ratio.

6.2. Results of the combined extended Bi-point and Lagrangian polynomial constraint approximation

From the results obtained using the extended Bi-point approximation, for each design variable

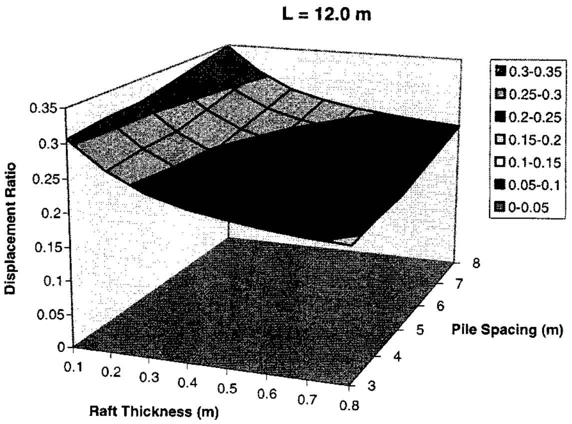

Fig. 10 Displacement ratio in terms of raft thickness and pile spacing

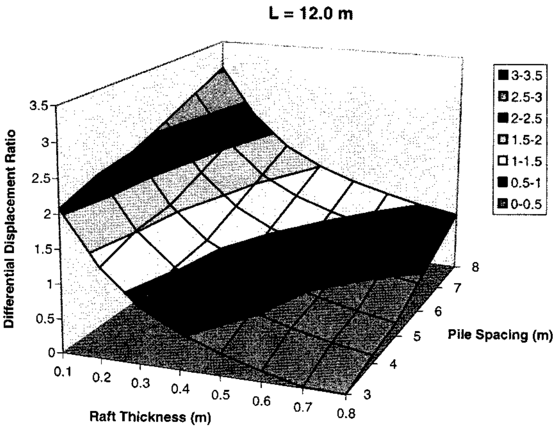

Fig. 11 Differential displacement ratio in terms of raft thickness and pile spacing

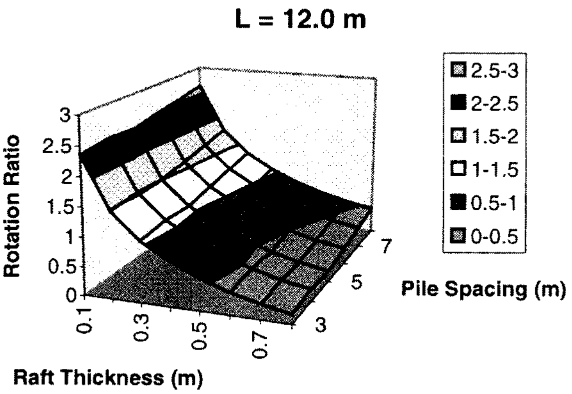

Fig. 12 Rotation ratio in terms of raft thickness and pile spacing

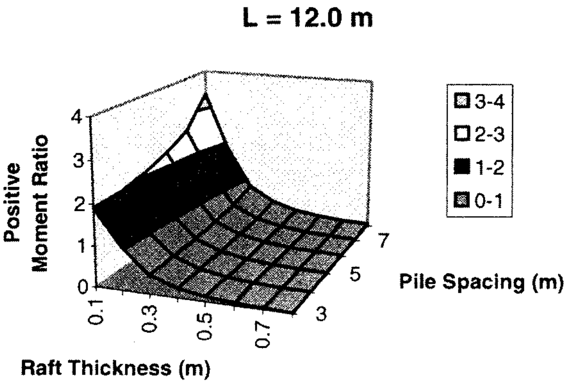

Fig. 13 Positive moment ratio in terms of raft thickness and pile spacing

one basic point and eight other supplementary data points including their constraint values and their first derivatives at the basic point are used. For raft thickness, a basic thickness of 0.5 m and supplementary thicknesses of 0.15 m, 0.3 m, 0.4 m, 0.7 m, 0.9 m, 1.1 m, 1.3 m and 1.5 m are chosen. For pile length, a basic pile length of 10.0 m and supplementary lengths of 8.0 m, 8.5 m, 9.0 m, 9.5 m, 10.0 m, 10.5 m, 11.0 m, 11.5 m and 12.0 m are selected. For pile spacing, a basic pile spacing of 5.0 m and supplementary spacings of 2.5 m, 3.0 m, 4.0 m, 4.5 m, 6.0 m, 7.0 m, 8.0 m and 9.0 m are used. In order to eliminate the influence of the saddle points occuring in the predicted constraints in terms of the pile spacing, the values at the locations of the supplementary points should be made judiciously.

The results of the combined extended Bi-point and Lagrangian polynomial constraint approximation for pile length of 12 m are shown in Figs. 10 to 14. Fig. 10 shows that the displacement ratios at the centre of the raft are less than 1 for all ranges of both raft thickness and pile spacing. The maximum displacement ratio is about 0.36 at thickness of 0.1 m and spacing of 8 m. The differential displacement ratio is very sensitive to both raft thickness and pile spacing. The values increase with decreasing the raft thickness and increasing pile spacing as shown in Fig. 11. Fig. 12 shows the rotation ratio versus raft thickness and pile spacing. A similar pattern to the

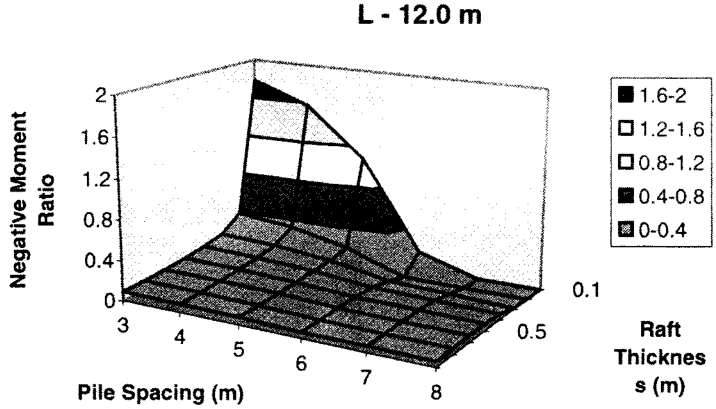

Fig. 14 Negative moment ratio in terms of raft thickness and pile spacing

Table 2 Comparison of constraint values from optimization solver and finite element

	Optimization	Finite Element
Displacement Ratio	0.241	0.241
Differential Displacement Ratio	1.000	0.987
Rotation Ratio	0.974	0.935
Positive moment Ratio	0.415	0.389
Negative moment ratio	0.259	0.290

differential displacement ratio is also found. As shown in Fig. 13, for thick raft the pile spacing is insensitive to the positive moment while for thin raft, the greater the pile spacing, the higher the positive moment. In general, the pattern of this positive moment ratio is the same as the other patterns mentioned previously. Fig. 14 shows the negative moment ratio in terms of the raft thickness and pile spacing. It is interesting note that for large values of pile spacing, the maximum moment is almost zero. It should be pointed out that within all ranges of the design variables, only the displacement constraint never reaches its limit and hence it is not critical.

6.3. Optimization results

Based on the constraint formulations built from the combination of the extended Bi-point approximation and the Lagrangian polynomial approximation in terms of the design variables, an optimization problem is defined. It is known that the Lagrangian polynomial approximation has multi minima and maxima points. Therefore, initial values that should be input in the optimization solver influence the result. In other words, the optimum results for certain initial design variables are only local. However, the original design variables that have been applied for creating the extended Bi-point approximation may be used for this case as adopted in this study. In addition, the authors have tried several initial design variables for the optimization process and so far the original design variables are the best option. The optimum value for the volume of materials required for the piles and the raft is 45.170 m^3 and the corresponding design variables are 0.332 m for the raft thickness, 12 m for the pile length and 5.08 m for the pile spacing. Table 2 shows the comparison of the constraints calculated during the optimization process and the finite element method. It can be noted from the table that the critical constraint is the differential displacement.

It is noted that the optimization of this raft-pile foundation requires only 1 iteration with only about 2% of error and provides a more economical solution. In the previous investigation (Tandjiria *et al.* 1996) where linear analysis of the raft-pile foundation was carried out by considering a constraint based on the yield function due to the Drucker-Prager criterion, 5 iterations were necessary. The previous investigation resulted in the optimum volume for the materials to be 50.62 m^3. The optimum design corresponding to this was 0.3861 m for the raft thickness, 12 m for the pile length and 2.5 m for the pile spacing. The reason for the different pile spacings obtained from the two studies is that in the nonlinear analysis, several points were allowed to yield and the excessive stresses at these points were distributed to other points while in the linear analysis of the previous study, only a constraint based on the yield function was included.

7. Conclusions

A combined nonlinear analysis using the finite element method and the optimization technique has been proposed for the design of raft-pile foundations. Raft thickness, pile length and pile spacing are chosen as design variables. The volume of the material for the piles and the raft is minimised by considering certain constraints, such as displacement, differential displacement, rotation, moments in the raft and side constraints of the design variables. In order to construct the constraints in terms of the design variables, the combination of extended Bi-point and Lagrangian polynomial constraint approximations is used. In addition, the modified semi-analytical sensitivity method is adopted.

Compared to the constraints obtained from the elastic linear case, most of the sensitivity results of the constraints obtained from the nonlinear ones are larger.

The extended Bi-point constraint approximation shows good results in predicting the responses of the raft-pile foundation. Increasing the raft thickness reduces the displacement, the differential displacement, the rotation in the raft and the moments in the raft. Increasing the pile length reduces the displacement, the differential displacement, the rotation in the raft and the negative moment but increases the positive moment. By increasing the pile spacing, the displacement, the differential displacement, the rotation and the moments are increased.

The combination of extended Bi-point and Lagrangian polynomial constraint approximation provides a better constraint approximation method. The combined method together with the Generalized Reduced Gradient (GRG) method for nonlinear optimization problems provides an optimum result of the raft-pile foundation efficiently. Only one iteration is required for solving the problem with a small error. This indicates that the proposed constraint approximation is efficient. It was found that the differential displacement is the most critical constraint.

It is also shown that using only three reference points for each design variable, the proposed constraint approximation is in a good agreement with the results from the finite element method.

The combined approach of the finite element method and the optimization technique provides an economic design of the raft-pile foundation without performing a number of separate finite element analyses.

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