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Simultaneously evolutionary optimization of several natural frequencies of a two dimensional structure

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Abstract. This paper presents a solution method, which can be regarded as the further extension of the generalized evolutionary method (Zhao *et al.* 1998a), for the simultaneous optimization of several different natural frequencies of a structure in general and a two dimensional structure in particular. The main function of the present method is to optimize the topology of a structure so as to simultaneously make several different natural frequencies of interest to be of the corresponding different desired values for the target structure. In order to develop the present method, the new contribution factor of an element is proposed to consider the contribution of an element to the gaps between the currently calculated values for the different natural frequencies of interest and their corresponding desired values in a weighted manner. Using this new contribution factor of an element, the most inefficiently used material can be detected and removed gradually from the design domain of a structure. Through applying the present method to optimize two and three different natural frequencies of a two dimensional structure, it has been demonstrated that it is possible and applicable to use the generalized evolutionary method for tackling the simultaneous optimization of several different natural frequencies of a structure in the structural design.

Key words: simultaneous optimization; several natural frequencies; generalized evolutionary method.

1. Introduction

Simultaneous optimization of several natural frequencies of a structure is often encountered in

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the structural design. For instance, in civil engineering, a structure may be subjected to several dynamic loads of different predominant frequencies. An earthquake load and a wind load are typical loads of different predominant frequencies. In mechanical engineering and vehicle industry, a machine or a vehicle may work at several speeds. This means that the machine or the vehicle has several different working frequencies. From the structural design point of view, it is desirable that the natural frequencies of a structure be of certain desired values, which are reasonably away from the predominant frequencies in the case of a civil engineering building or the working frequencies in the case of a machine or a vehicle, so that the material used for the structure can be minimized because the dynamic response of the structure is kept minimum under the given design condition. Thus, optimizing several natural frequencies of a structure to different desired values is not only of the academic significance in the field of structural optimization, but also of very strong engineering background. For this reason, simultaneous evolutionary optimization of several natural frequencies of a structure becomes the main purpose of this study.

Although the simultaneous optimization of several natural frequencies of a structure can be achieved by structural shape or sizing optimization approaches, it will be carried out in this research by the structural topological optimization approach since optimizing the topology of a structure usually produced a much better structural configuration. So far several methods, such as the homogenization method (e.g., Bendsøe and Kikuchi 1988, Bendsøe 1989, Olhoff *et al.* 1991, Suzuki and Kikuchi 1991, Tenek and Hagiwara 1993, Diaz and Kikuchi 1992, Ma *et al.* 1995), the discretized continuum type optimality criteria (DCOC) method (Zhou and Rozvany 1992, 1993), the simple evolutionary procedure (Xie and Steven 1993) and the generalized evolutionary method (Zhao *et al.* 1996a, 1996b, 1997a, 1997b, 1997c, 1998a, 1998b) have been available for solving structural topological optimization problems in the literature. Owing to the simplicity and soundness of the generalized evolutionary method, it will be further extended and used to deal with the simultaneous optimization of several natural frequencies of a two dimensional structure in this paper.

2. Statement of the problem and solution method

In order to tackle the simultaneous optimization of several natural frequencies of a structure using the generalized evolutionary method (Zhao *et al.* 1996a, 1998a), the problem statement and the solution method need to be given as follows.

2.1. Statement of the problem

For the simultaneous optimization of several natural frequencies of a structure, as shown in Fig. 1, what we know is the size of the design domain, the properties of the structural material, the boundary conditions of the structure, and the desired values of several different natural frequencies for the target structure. What we need to know is to find out the optimal topology of the target structure, which not only satisfies the above-mentioned conditions but also leads to the lightest weight of the structure. Therefore, the objective of the structural design in this case is to optimize the topology of a structure using as less material as possible within the prescribed design domain so as to make several concerned natural frequencies of the target structure to be of the desired values at the same time.

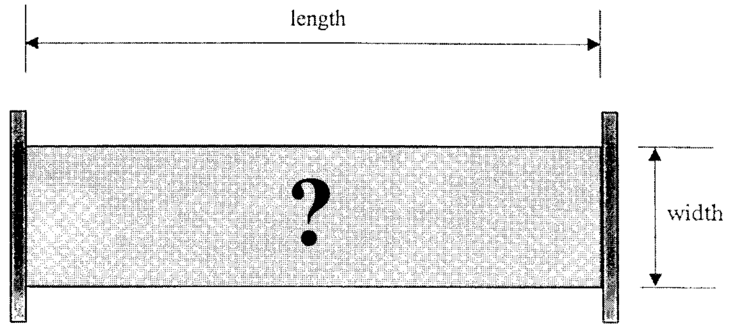


Fig. 1 Design domain for a problem with unknown topology

2.2. Solution method

The generalized evolutionary method (Zhao *et al.* 1996a) is employed to solve this kind of optimization problem. In the generalized evolutionary method, firstly the design domain of a structure is fully filled with the given structural material, and then the most inefficiently used material is gradually detected and removed from the design domain until the prescribed design requirement is met. For this particular kind of optimization problem, the most inefficiently used material in the design domain is the material which makes the concerned natural frequencies of the system furthest away from their corresponding desired values. In other words, the most inefficiently used material in the design domain makes the greatest contribution to distance the concerned natural freqencies of the system from their desired values. Thus, by removing such most inefficiently used material gradually from the design domain, the gaps between the currently calculated values of the concerned natural frequencies and their desired values will be reduced accordingly. Since the most inefficiently used material is detected and removed step by step, it forms an evolutionary process for the structural design. This is the reason why the method used here is called the generalized evolutionary method.

In order to detect which part of material in the design domain is the most inefficiently used material, the whole design domain is fully filled with the structural material and discretized into a fine mesh of finite elements. This makes it possible to evaluate the contribution factor of a finite element to the gaps between the currently calculated values of the concerned natural frequencies and their desired values using the following formula.

$$\Delta\omega_i = \sum_{j=1}^{n} W_j \Delta\omega_{ij} \tag{1}$$

where $\Delta\omega_i$ is the exact contribution factor of element i to the gaps between the currently calculated values of the concerned natural frequencies and their desired values; $\Delta\omega_{ij}$ is the exact contribution factor of element i to the jth natural frequency of the system; n is the maximum order of the concerned natural frequency of the system; W_j is the weight of the jth natural frequency of the system and can be expressed as

$$W_j = \frac{\omega_j^* - \omega_j}{\omega_j} \qquad \text{for} \qquad |\omega_j^* - \omega_j| > \beta\omega_j$$

$$W_j = 0 \qquad \text{for} \qquad |\omega_j^* - \omega_j| < \beta\omega_j \tag{2}$$

where ω_j and ω_j^* are the currently calculated values and the desired value of the jth natural frequency of the system; β is a parameter to reflect the accuracy requirement for the solution.

Using the definition for the contribution factor of an element to the concerned natural frequency of a system (Zhao *et al.* 1996a), $\Delta\omega_{ij}$ can be expressed as

$$\Delta\omega_{ij} = \sqrt{(\omega_j)^2 + \alpha_j} - \omega_j \tag{3}$$

where

$$\alpha_j = \frac{(\omega_j)^2\{d_j^e\}^T[m^e]\{d_j^e\} - \{d_j^e\}^T[k^e]\{d_j^e\} + f_1 - f_2}{M_j - f_3} \tag{4}$$

where

$$\begin{aligned} f_1 &= (\omega_j)^2\{\delta d_j\}^T([M]_o - [M^e])\{\delta d_j\} \\ f_2 &= \{\delta d_j\}^T([K]_o - [K^e])\{\delta d_j\} \\ f_3 &= \{\delta d_j\}^T[M^e]\{\delta d_j\} + f_1/(\omega_j)^2 \end{aligned} \tag{5}$$

where $\{d_j^e\}$ is the jth modal shape vector of the element in the old system; $\{d_j\}$ is the jth nodal shape vector of the old system; $\{\delta d_j\}$ is the jth modal shape vector difference between the old and the new systems; $[m^e]$ and $[k^e]$ are the mass matrix and stiffness matrix of the element; $[M^e]$ and $[K^e]$ are the enlarged mass matrix and stiffness matrix of the element; $[M]_o$ and $[K]_o$ are the global mass matrix and global stiffness matrix of the old system; M_j is the modal mass corresponding to the jth natural frequency of the system. It needs to be pointed out that the old system refers to the system before an element removal in the beginning of an iteration, while the new system refers to the system after an element removal at the end of the iteration.

If the first order variations are only considered, Eq. (1) can be further simplified as

$$\Delta\omega_i^* = \sum_{j=1}^{n} W_j \Delta\omega_{ij}^* \tag{6}$$

where

$$\Delta\omega_{ij}^* = \frac{(\omega_j)^2\{d_j^e\}^T[m^e]\{d_j^e\} - \{d_j^e\}^T[k^e]\{d_j^e\}}{2(\omega_j)^2 M_j} \tag{7}$$

where $\Delta\omega_{ij}^*$ is the approximate contribution factor of element i to the jth natural frequency of the old system; $\Delta\omega_i^*$ is the approximate contribution factor of element i to the gaps between the currently calculated values of the concerned natural frequencies and their desired values.

The reason for using β in Eq. (2) is that in the numerical calculation, it is very difficult to make the calculated value of a natural frequency exactly equal to its desired value due to numerical errors in using computers. To avoid this difficulty, β is used to determine the extend to which the solution accuracy is acceptable for the structural design. In this sense, the smaller β is, the more accurate the solution. However, from the computational point of view, the smaller β is, the more CUP time it takes to solve the same problem. On the other hand, from the engineering design point of view, the solution is only required to satisfy the particularly prescribed accuracy for a particular structure. This is the reason why β is used in Eq. (2).

3. Simultaneous optimization of several natural frequencies of a two dimensional structure

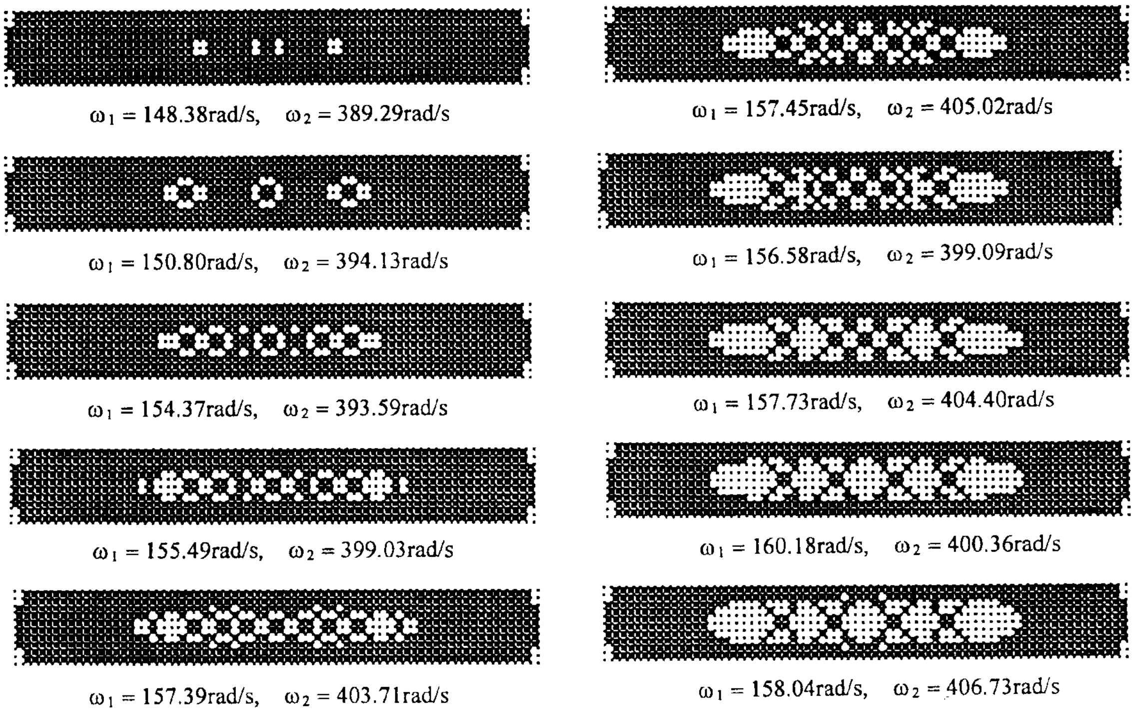
Based on the solution method presented in the last Section, the simultaneous evolutionary optimization of two or three natural frequencies of a two dimensional structure under plane stress conditions is carried out in this Section. The problem stated in Section two is considered in the following calculations. For the purpose of using the generalized evolutionary method, the initial design domain of the problem, which is shown in Fig. 1, is fully filled with the structural material and clamped on its left and right sides. Since the finite element method is used to calculate the natural frequencies of the structure at any stage of structural topology optimization in the generalized evolutionary method, this initial design domain is discretized into a fine mesh of 700 4-node square elements. The following parameters are used in the calculation: the length, width and thickness of the initial design domain are 14m, 2m and 0.1m; the elastic modulus of the design domain materials is 25×10^6 kPa; Poisson's ratio is 0.3; the unit weight is 2.5×10^3 kg/m^3; the weight of the initial design domain is 7000 kg. In order to keep the solution accuracy and speed up the solution convergency at the same time, β is chosen as 0.03 in the calculation. As mentioned in a previous paper (Zhao *et al.* 1996a), the minimum number of elements to be removed in an iteration is equal to four so as to maintain the symmetric nature of the target structure, because there are two symmetric axes in the system.

3.1. Simultaneous optimization of two natural frequencies

In this case, the fundamental and the second natural frequencies of a two dimensional structure under plane stress conditions is optimized to the desired values. The fundamental and the second natural frequencies of the initial design domain prior to the evolutionary optimization are 185.46 rad/s and 453.11 rad/s, while their corresponding desired values for the target structure are 160 rad/s and 410 rad/s, respectively. This implies that in order to find out the optimal topology of the target structure, the most inefficiently used material in the initial design domain needs to be gradually removed so that the fundamental and the second natural frequencies of the initial design domain decrease by about 13.73% and 9.51% at the same time.

Fig. 2 shows some typical structural topologies in the simultaneously evolutionary process of optimizing the first two natural frequencies of the two dimensional structure. The topology shown at the bottom of the right column in this figure is the optimal topology of the target structure, the fundamental and second natural frequencies of which are 158.04 rad/s and 406.73 rad/s. Compared with their desired values, the errors in these two natural frequencies of the target structure are 1.23% and 0.80%, which are smaller than the allowable error of β=3%. In order to check whether the topology shown at the bottom of the right column in Fig. 2 is the optimal topology for the target structure, the evolutionary process is further continued but the fundamental and second natural frequencies of the consequent topologies of the structure do not satisfy the solution accuracy requirement. This fact indicates that the topology shown at the bottom of the right column in Fig. 2 is the optimal topology of the target structure in the finite element sense because it not only results in the fundamental and second natural frequencies with the required accuracy, but also leads to the lightest weight of the structure.

Fig. 3 shows the variation of normalized natural frequencies of the two dimensional structure during evolutionary optimization. In this figure, φ and ψ are defined as follows:



$\omega_1 = 148.38$rad/s, $\omega_2 = 389.29$rad/s

$\omega_1 = 157.45$rad/s, $\omega_2 = 405.02$rad/s

$\omega_1 = 150.80$rad/s, $\omega_2 = 394.13$rad/s

$\omega_1 = 156.58$rad/s, $\omega_2 = 399.09$rad/s

$\omega_1 = 154.37$rad/s, $\omega_2 = 393.59$rad/s

$\omega_1 = 157.73$rad/s, $\omega_2 = 404.40$rad/s

$\omega_1 = 155.49$rad/s, $\omega_2 = 399.03$rad/s

$\omega_1 = 160.18$rad/s, $\omega_2 = 400.36$rad/s

$\omega_1 = 157.39$rad/s, $\omega_2 = 403.71$rad/s

$\omega_1 = 158.04$rad/s, $\omega_2 = 406.73$rad/s

Fig. 2 Evolutionary process of optimizing two natural frequencies of a two dimensional structure

$$\varphi = \frac{W_d - W_s}{W_d}$$

$$\psi = \frac{\omega}{\omega_1^*} \tag{8}$$

where φ is the ratio of the weight of material removed from the initial design domain to that of the initial design domain; W_d and W_s are the weights of the initial design domain and the structure, respectively; ψ is the normalized natural frequency of the structure; ω is the natural frequency of the structure and ω_1^* is the desired value of the fundamental natural frequency of the target structure.

It is noted from Fig. 3 that with the increase in the value of ϕ, the normalized fundamental and second natural frequencies approach 1 and 2.56, which are the desired values for the target structure. In this figure, φ=28% corresponds to the optimal topology of the target structure in the finite element sense.

If the desired values of the fundamental and second natural frequencies of the target structure are prescribed as 200rad/s and 400rad/s, the corresponding optimal topology of the target structure can be found out in the same manner as mentioned above. Fig. 4 shows some typical topologies in the simultaneously evolutionary process of optimizing the first two natural frequencies of the structure. Fig. 5 shows the variation of normalized natural frequencies of the structure during evolutionary optimization. It is clear that when 24% of the material is gradually removed from the initial design domain, the topology of the structure is the optimal one for the target structure in the finite element sense because the errors in the fundamental and second natural frequencies of the structure are 0.07% and 0.47%, which are much smaller than the allowable error of β=3%.

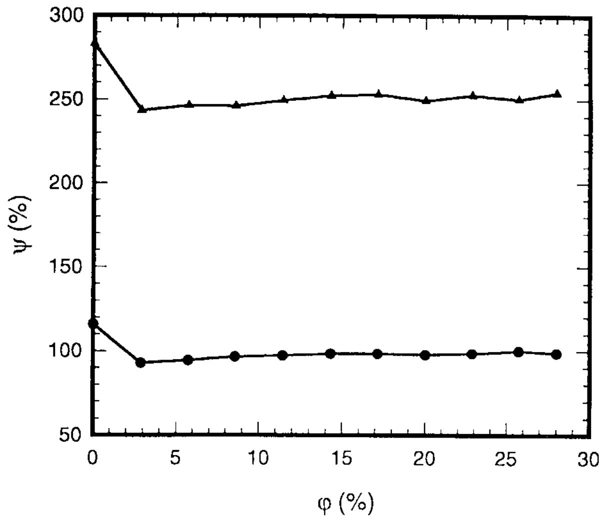


Fig. 3 Variation of normalized natural frequencies of a two dimensional structure

3.2. Simultaneous optimization of three natural frequencies

The proposed solution method in the early Section of this paper is applied to the simultaneous optimization of three natural frequencies of a structure. In this case, the fundamental, the second and the third natural frequencies of the two dimensional structure (see Fig. 1) under plane stress conditions are optimized to the desired values. The fundamental, the second and the third natural frequencies of the initial design domain prior to the evolutionary optimization are 185.46rad/s, 453. 11rad/s and 711.70rad/s, while their corresponding desired values for the target structure are 100 rad/s, 300 rad/s and 500 rad/s, respectively. This means that in order to find out the optimal

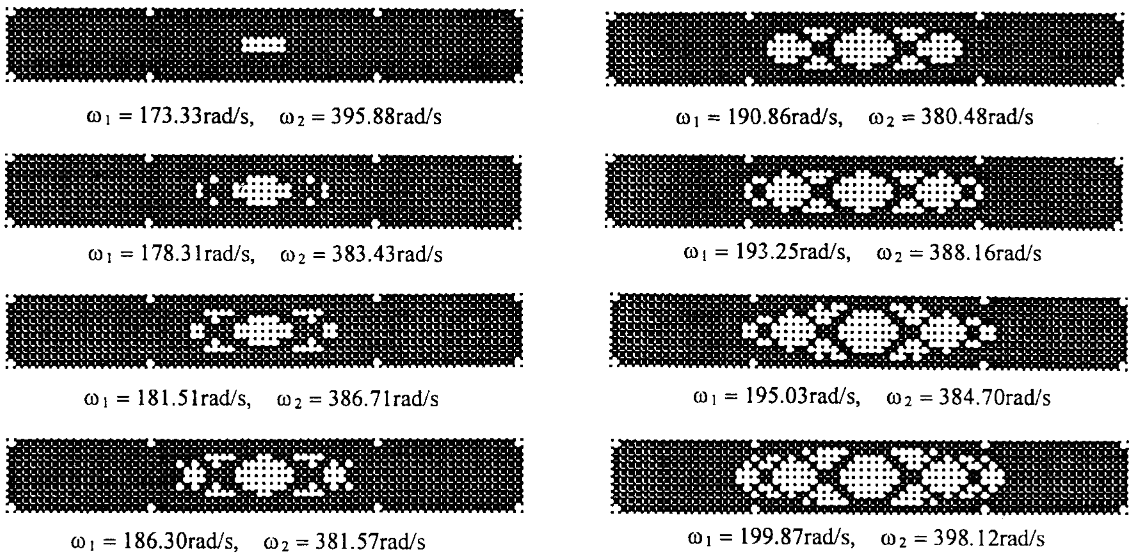


Fig. 4 Evolutionary process of optimizing two natural frequencies of a two dimensional structure

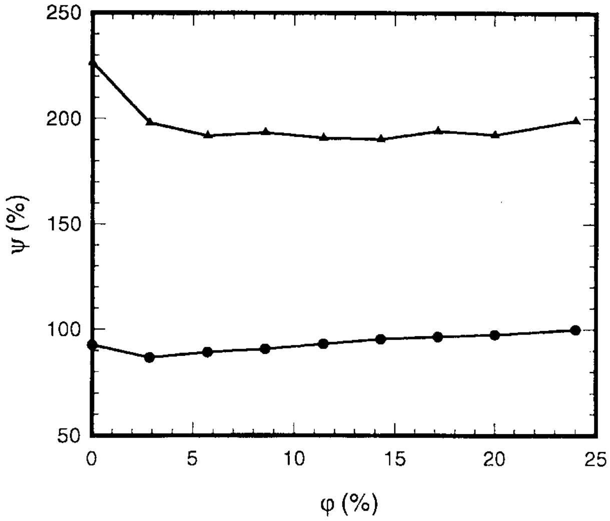


Fig. 5 Variation of normalized natural frequencies of a two dimensional structure

topology of the target structure in the finite element analysis, the most inefficiently used material in the initial design domain needs to be gradually removed so that the fundamental, the second and the third natural frequencies of the initial design domain decrease by about 46.08%, 33.79% and 29.78% at the same time.

Fig. 6 shows some typical structural topologies in the simultaneously evolutionary process of optimizing the first three natural frequencies of the two dimensional structure. Fig. 7 shows the variation of normalized natural frequencies of the target structure during evolutionary optimization. Compared with the solutions obtained from the simultaneously evolutionary optimization of the first two natural frequencies of the structure, the optimal topology of the target structure in the case of simultaneously evolutionary optimization of the first three natural frequencies is much different from that obtained in the preceding cases. It can be seen from the related results that when φ is equal to 25.14%, the fundamental, the second and the third natural frequencies of the structure are 98.75rad/s, 293.65rad/s and 492.05rad/s, the errors in which are 1.25%, 2.12% and 1.59%, respectively. Since these errors are smaller than the allowable error, which is 3% for the solution, the topology in correspondence with φ=25.14% is the optimal topology for the target structure in the finite element sense. This optimal topology is shown at the bottom of the right column in Fig. 6.

4. Conclusions

The simultaneous optimization of several different natural frequencies of a structure is often encountered in the structural design. To solve this kind of optimization problem, a solution method, which can be regarded as the further extension of the generalized evolutionary method, is presented in this paper. The main function of the present method is to optimize the topology of a structure so as to simultaneously make several different natural frequencies of interest in the design of the structure to be of the corresponding different desired values for the target structure. In the process of developing the present method, the new contribution factor of an element is

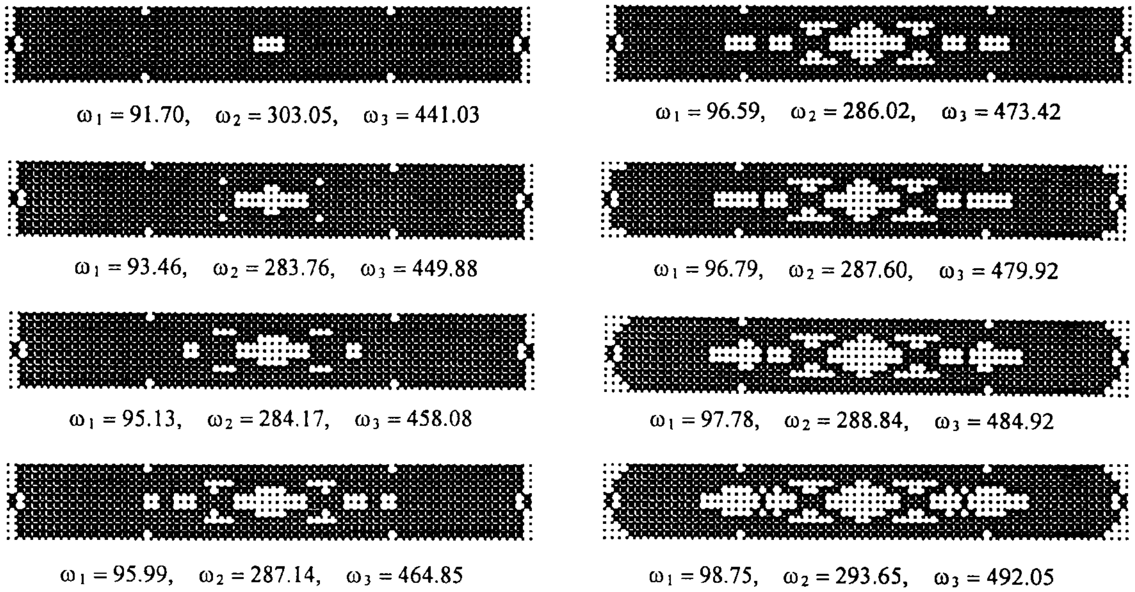


Fig. 6 Evolutionary process of optimizing three natural frequencies of a two dimensional structure

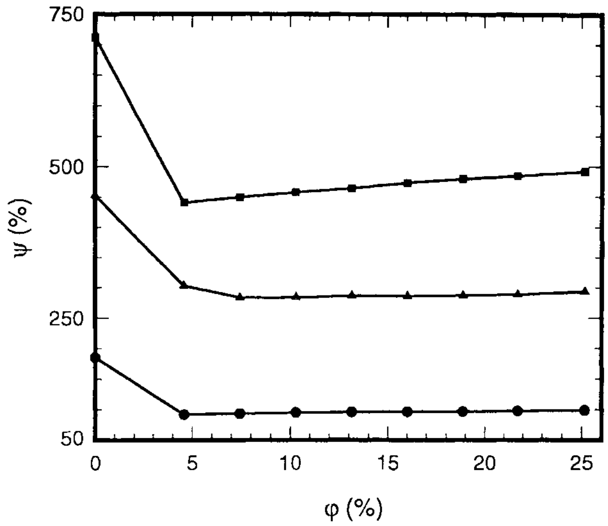


Fig. 7 Variation of normalized natural frequencies of a two dimensional structure

proposed to consider the contribution of an element to the gaps between the currently calculated values for the different natural frequencies of interest and their corresponding desired values in a weighted manner. Using this new contribution factor of an element, the most inefficiently used material in the design domain of a structure can be detected and removed gradually from the design domain of the structure. Through applying the present method to optimize two and three different natural frequencies of a two dimensional structure, it has been demonstrated that it is possible and applicable to use the generalized evolutionary method for tackling the simultaneous optimization of several different natural frequencies of a structure, although the resulting optimal topology of the structure is only meaningful in the finite element sense.

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