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Free vibration analysis of elliptic and circular plates having rectangular orthotropy

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Abstract. The natural frequencies and modes of free vibration of specially orthotropic elliptic and circular plates are analysed using the Rayleigh-Ritz method. The assumed functions used are two-dimensional boundary characteristic orthogonal polynomials which are generated using the Gram-Schmidt orthogonalization procedure. The first five natural frequencies are reported here for various values of aspect ratio of the ellipse. Results are given for various boundary conditions at the edges i.e., the boundary may be any of clamped, simply-supported or free. Numerical results are presented here for several orthotropic material properties. For rectilinear orthotropic circular plates, a few results are available in the existing literature, which are compared with the present results and are found to be in good agreement.

Key words: elliptic; orthotropic; vibration; plate; orthogonal polynomials; Rayleigh-Ritz.

1. Introduction

In recent years lightweight structures have been widely used in many engineering fields and so vibration analyses of different shaped plates have been studied extensively for its practical applications. The application of composite materials in engineering structures require information about the vibration characteristics of anisotropic materials. The free vibration of orthotropic plates is an important area of such behaviour. Orthotropic materials have extensive application in modern technology such as in modern missiles, space crafts, nuclear reactors, printed circuit boards etc. A vast amount of work has been done for theoretical and experimental results for vibration of orthotropic skew, triangular, circular, annular and polygonal plates as mentioned by Leissa (1969, 1978, 1981, 1987) and Bert (1976, 1979, 1980, 1982, 1985, 1991). But the authors have found very little work on the vibration of elliptic and circular plates with rectangular orthotropy. The investigation presented here gives extensive, accurate and a wide variety of new results to study the free vibration of specially rectilinear orthotropic (i.e., whose symmetrical axes coincide with the principal elastic axes of the plate material) elliptic and circular plates.

For a circular plate with rectangular orthotropy, only a few results are available in the existing literature, namely Rajappa (1963), Leissa (1969), Sakata (1976), Narita (1983), Dong and Lopez

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(1985) and also some of the references mentioned therein. Rajappa (1963) has used Galerkin's method and reported only the fundamental frequency of circular orthotropic plates with clamped and simply supported boundaries. Reduction methods have been used by Sakata (1976), who has given only the fundamental frequency for a clamped orthotropic circular plate. Narita (1983) gives some higher modes for a circular plate with clamped boundary by a series type method. Dong and Lopez (1985) analysed a clamped circular plate with rectilinear orthotropy by a modified application of the interior collocation method.

Compared with circular plates, even less work has been carried out for elliptical plates with rectangular orthotropy. The only two references known to the authors are Sakata (1976) and Narita (1985). Sakata (1976) deals with only the fundamental frequency for a clamped elliptic plate. He obtained his results by using a simple co-ordinate transformation of a clamped orthotropic circular plate to give a reduction formula for the fundamental frequency of an elliptic orthotropic plate with the same boundary. A Ritz method analysis is carried out by taking a complete power series as a trial function by Narita (1985) to obtain the first few natural frequencies for an orthotropic elliptical plate with a free boundary. Numerical results are illustrated there by two figures only, for two types of orthotropic material properties.

Sakata (1979) in a two-part article, describes in Part I three exact reduction methods. Part II describes a generalised reduction method. The reduction method is used to derive an approximate formula for estimating the natural frequencies of orthotropic plates. Vibration of an orthotropic elliptical plate with a similar hole has been analysed by Irie and Yamada (1979). In another paper Irie *et al.* (1983) deals with the free vibration of circular-segment-shaped membranes and clamped plates of rectangular orthotropy. An interesting paper is that of Narita (1986), who has analysed the free vibration of orthotropic elliptical plates with point supports of arbitrary location. Only those papers which deal with rectangular orthotropic circular or elliptical geometries, are mentioned here.

Here two dimensional boundary characteristic orthogonal polynomials have been used in the Rayleigh-Ritz method. Recently orthogonal polynomials have been used extensively to find the vibration characteristics of different types of plate geometries with various boundary conditions at the edges. Some of the references in this connection are Bhat (1985,1987), Dickinson and Blasio (1986), Kim and Dickinson (1987, 1989), Laura *et al.* (1989), Liew and Lam (1990) and Liew *et al.* (1990). Chakraverty (1992) and Singh and Chakraverty (1991, 1992, 1993, 1994) have already applied this method to free vibration of isotropic circular, elliptic, annular, skew and triangular plates and also elliptic plates with variable thickness and have obtained excellent results. They have also reported the orthogonal polynomials generated over all the above mentioned domains in a recent paper (1994). One can directly use those polynomials for solving the related problem without generating them again and again for each specific problems. Chakraverty and Chakrabarti (1993) and Chakraverty (1996) have also applied this method to the determination of the static deflection of circular and elliptic plates. In a more recent paper the authors (1997) use this method for non-homogeneous elliptic plates, viz. when the modulus of elasticity and the density of the material are non-homogeneous. The main aim of the present study is to use this method to analyse elliptic and circular plates with rectangular orthotropy and to report many new results, which are not found in the open literature. Here the same orthogonal polynomials generated by Chakraverty (1992) and Singh and Chakraverty (1991, 1992) have been used to determine the transverse vibration of elliptic and circular plates with rectangular orthotropy having clamped, simply-supported or free boundary at the edges.

To use the method, three steps have to be followed. The first step consists of the generation of the orthogonal polynomials in the domain occupied by the plate. Since there exists no three term recurrence relation, as in the one dimensional case, the well-known Gram-Schmidt orthogonalization procedure has been used. The second step is to use these generated polynomials in the Rayleigh-Ritz method, which produces a standard eigenvalue problem rather than a generalized one. This is the extra advantage of using these polynomials which make the analysis numerically efficient, straight forward and simple. It also gives a faster rate of convergence. The third and the last step is to solve this standard eigenvalue problem to give the vibration characteristics.

2. Basic equations and method of solution

Let the domain occupied by the elliptic plate be

$$S=\{(x,y),\, x^2/a^2+y^2/b^2\leq 1,\quad x,y\in R\}, \tag{1}$$

where a and b are the semi major and minor axes of the ellipse respectively as shown in Fig. 1.

The maximum strain energy V_{max} of the deformed orthotropic plate is given by Timoshenko and Woinowsky (1953) as

$$V_{max}=(1/2)\iint_R [D_x W_{xx}^2+2\nu_x D_y W_{xx} W_{yy}+D_y W_{yy}^2+4D_{xy} W_{xy}^2]\, dydx \tag{2}$$

where $W(x, y)$ is the deflection of the plate, W_{xx} is the second derivative of W with respect to x. The D coefficients are bending rigidities defined by,

$$\left.\begin{array}{l} D_x=E_x h^3/(12(1-\nu_x\nu_y)) \\ D_y=E_y h^3/(12(1-\nu_y\nu_x)) \\ D_y\nu_x=D_x\nu_y \\ \text{and } D_{xy}=G_{xy}h^3/12 \end{array}\right\} \tag{3}$$

where E_x, E_y are Young's moduli and ν_x, ν_y are Poisson's ratios in the x, y directions, G_{xy} is shear modulus and h is the uniform thickness.

The maximum kinetic energy is given by,

$$T_{max}=(1/2)\rho h\omega^2\iint_R W^2 dydx \tag{4}$$

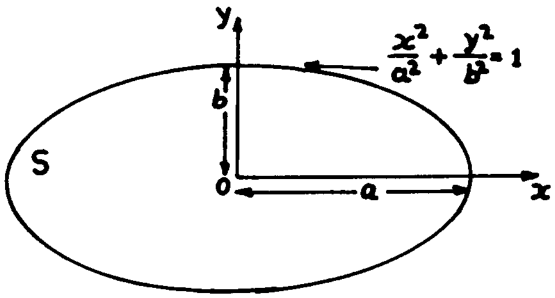

Fig. 1 Geometry of the elliptic plate

where ρ is the mass density per unit volume and ω is the radian natural frequency of the plate. Now equating the maximum strain and kinetic energies we have the Rayleigh quotient as,

$$\omega^2 = \frac{\iint_R [D_x W_{xx}^2 + 2\nu_x D_y W_{xx} W_{yy} + D_y W_{yy}^2 + 4D_{xy} W_{xy}^2]\, dydx}{h\rho \iint_R W^2\, dydx} \tag{5}$$

Substituting the N-term approximation

$$W(x,y) = \sum_{j=1}^{N} c_j \phi_j(x,y) \tag{6}$$

and minimizing ω^2 as a function of the coefficients c_j's we have,

$$\sum_{j=1}^{N} (a_{ij} - \lambda^2 b_{ij}) c_j = 0, \quad i = 1, 2, \ldots, N \tag{7}$$

where,

$$a_{ij} = \iint_{R'} [(D_x/H)\phi_i^{XX}\phi_j^{XX} + (D_y/H)\phi_i^{YY}\phi_j^{YY} + \nu_x(D_y/H)(\phi_i^{XX}\phi_j^{YY} + \phi_i^{YY}\phi_j^{XX}) + 2(1 - \nu_x(D_y/H))\phi_i^{XY}\phi_j^{XY}]\, dYdX \tag{8}$$

$$b_{ij} = \iint_{R'} \phi_i \phi_j\, dYdX \tag{9}$$

$$\lambda^2 = a^4 \rho h \omega^2 / H \tag{10}$$

$$\text{and } H = D_y \nu_x + 2D_{xy} \tag{11}$$

The ϕ_i's are orthogonal polynomials and are described in the next section. ϕ_i^{XX} is the second derivative of ϕ_i with respect to X and the new domain R' is defined by

$$R' = \{(X,Y), X^2 + Y^2/m^2 \leq 1, \quad X, Y \in R\}$$

where, $X=x/a$, $Y=y/a$ and $m=b/a$.
If the ϕ_i's are orthogonal, Eq. (7) reduces to

$$\sum_{j=1}^{N} (a_{ij} - \lambda^2 \delta_{ij}) c_j = 0, \quad i = 1, 2, \ldots, N \tag{12}$$

where, $\delta_{ij}=0$, if $i \neq j$
$=1$, if $i=j$.

The three parameters D_x/H, D_y/H and ν_x define the orthotropic property of the material under consideration. It is interesting to note here that for an isotropic plate these parameters reduce to $D_x/H = D_y/H = 1$ and $\nu_x = \nu_y = \nu$. Eq. (12) is a standard eigenvalue problem and can be solved for the vibration characteristics.

3. Generation of orthogonal polynomials

Although the present polynomials have been generated by exactly the same way as described in earlier studies (Chakraverty 1992, Singh and Chakraverty 1991, 1992), the method is described below for the sake of completeness.

We start with a linearly independent set

$$F_i(x,y)=f(x,y)\{f_i(x,y)\},\quad i=1,2,\cdots,N \tag{13}$$

where $f(x, y)$ satisfies the essential boundary conditions and $f_i(x, y)$ are taken as the combinations of terms of the form $x^{l_i} y^{n_i}$ where l_i and n_i are nonnegative integers. The function f is defined by,

$$f(x,y)=(1-x^2-y^2/m^2)^p \tag{14}$$

If we take right hand side of Eq. (14) as u^p where $u=1-x^2-y^2/m^2$ then it is clear that the boundary of the ellipse ∂S is given by $u=0$ and at the centre $u=1$. The curves u=constant will be concentric ellipses. From Eq. (14), it is to be noted that

i) if $p=0$, $f=1$ on ∂S
ii) if $p=1$, $f=0$ and $\partial f/\partial n=0$ on ∂S
iii) if $p=2$, $f=0$ on ∂S

Hence the functions ff_i also satisfy the same conditions on ∂S. When $p=0$, $f=1$ on ∂S and so ff_i are free since their values on ∂S depend upon l_i and n_i. Therefore it is clear that p takes the value of 0, 1 or 2 according as the boundary of the elliptic (or circular) plate is free, simply-supported or clamped.

From $F_i(x, y)$, we generate an orthogonal set by the well-known Gram-Schmidt process. For this we define the inner product of two functions f and g by,

$$\langle f,g\rangle=\iint_R f(x,y)g(x,y)\,dxdy \tag{15}$$

The norm of f is therefore given by

$$\|f\|=\langle f,f\rangle^{1/2} \tag{16}$$

Proceeding as in Chakraverty (1992) and Singh and Chakraverty (1991, 1992) the Gram-Schmidt orthogonalization process can be written as,

$$\left.\begin{aligned}&\phi_1=F_1,\\ &\phi_i=F_i-\sum_{j=1}^{i-1}\alpha_{ij}\phi_j\\ &\alpha_{ij}=\langle F_i,\phi_j\rangle/\langle\phi_j,\phi_j\rangle,\quad j=1,2,\cdots,(i-1)\end{aligned}\right\},\quad i=2,\cdots,N \tag{17}$$

where ϕ_i's are orthogonal polynomials. The normalized polynomials are generated by

$$\hat{\phi}_i=\phi_i/\|\phi_i\| \tag{18}$$

All the integrals involved in the inner product are evaluated in closed form by the formulas given in earlier works (Chakraverty 1992, Singh and Chakraverty 1991, 1992).

4. Numerical work and discussions

In all there are five parameters viz., D_x/H, D_y/H, ν_x, p and m. It would be a gigantic task to present the results for all possible combinations of these parameters. So seven different types of material have been selected, the properties of which are given in Table 1. The first three (M1, M2 and M3) have been taken from a paper by Lam *et al.* (1990), M4 from Kim and Dickinson (1990)

Table 1 Material properties

Material	D_x/H	D_y/H	ν_x
M1: Graphit-epoxy	13.90	0.79	0.28
M2: Glass-epoxy	3.75	0.80	0.26
M3: Boron-epoxy	13.34	1.21	0.23
M4: Carbon-epoxy	15.64	0.91	0.32
M5: Kevlar	2.60	2.60	0.14
M6	2.0	0.5	0.3
M7	0.5	2.0	0.075

and M5 from Dong and Lopez (1985). Lastly, M6 and M7 are taken from Narita (1985). In order to make comparison with the few known results for circular plates, a few more materials are considered in Table 3.

In the following subsections, the results for elliptic and circular plates with different boundary conditions are discussed.

4.1. Clamped boundary

Figs. 2 to 8 show the first five natural frequencies for various values of m=0.2, 0.4, 0.5, 0.6, 0.8 and 1.0 for the materials M1 to M7 respectively. The clamped boundary results are denoted by 'C'. All the results except for the fundamental mode for an elliptic plate (i.e., when $m \neq 1.0$) with a clamped boundary are new and are not found elsewhere. So, comparison can only be made for an elliptic plate for the fundamental frequencies. A few results are available for a clamped circular plate as discussed in Section 1, which are also compared here.

Sakata (1976) has given a formula for finding the fundamental frequency for a clamped elliptic plate as

$$\omega_s^2 = \frac{41.52}{\rho h a^4} \{D_x + \frac{2}{3}(a/b)^2 H + (a/b)^4 D_y\} \tag{19}$$

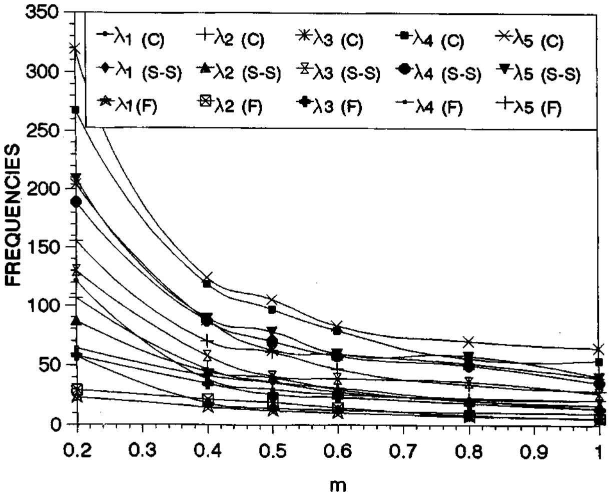

Fig. 2 First five frequencies for M1 for C: clamped, S-S: simply-supported and F: free boundary

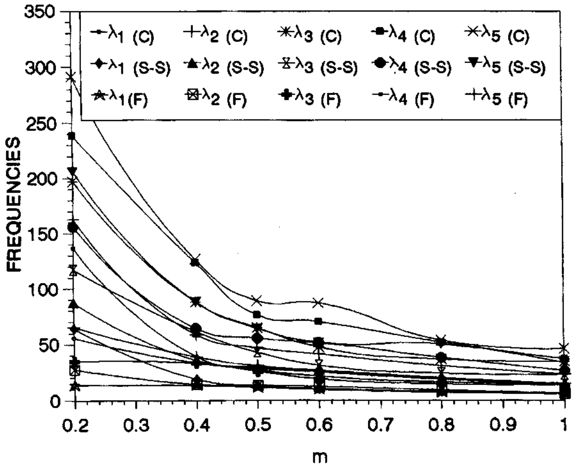

Fig. 3 First five frequencies for M2 for C: clamped, S-S: simply-supported and F: free boundary

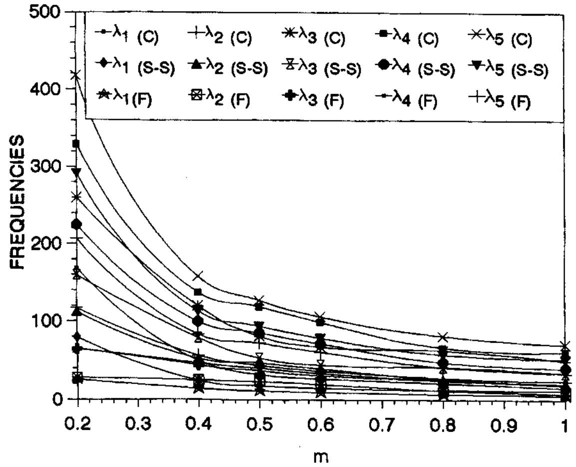

Fig. 4 First five frequencies for M3 for C: clamped, S-S: simply-supported and F: free boundary

Sakata mentions that Rajappa (1963) indicates that an accurate value of the numerical coefficient in Eq. (19) should be 40.0. He also reported that, the coefficient is 39.22 according to McNitt's solution (1962) for an isotropic clamped elliptical plate, which is obtained by the use of Galerkin's method. Finally he mentioned that upon taking the deflection function used in each analysis and the resultant solution into consideration, one should assume that the more accurate coefficient of Eq. (19) is 39.22.

As per the above discussions by taking the numerical coefficient of (19) as 41.52 due to Sakata (1976), 40.0 due to Rajappa (1963) and 39.22 due to McNitt (1962), the fundamental frequencies for M1 to M7 for various values of m have been computed and reported in Table 2. In this table λ_S,

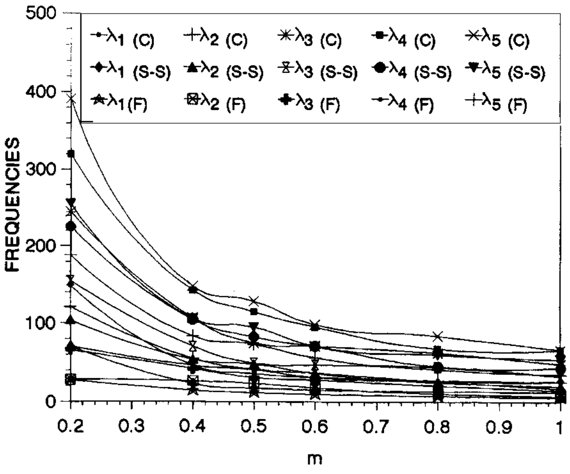

Fig. 5 First five frequencies for M4 for C: clamped, S-S: simply-supported and F: free

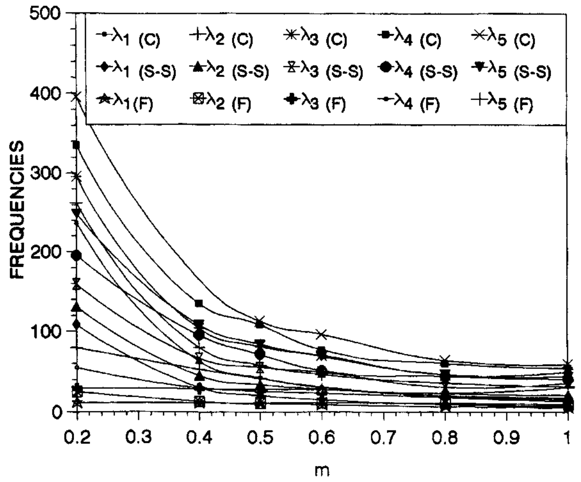

Fig. 6 First five frequencies for M5 for C: clamped, S-S: simply-supported and F: free boundary

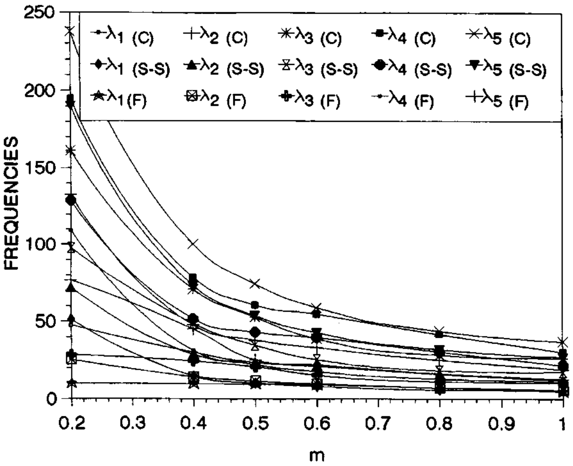

Fig. 7 First five frequencies for M6 for C: clamped, S-S: simply-supported and F: free boundary

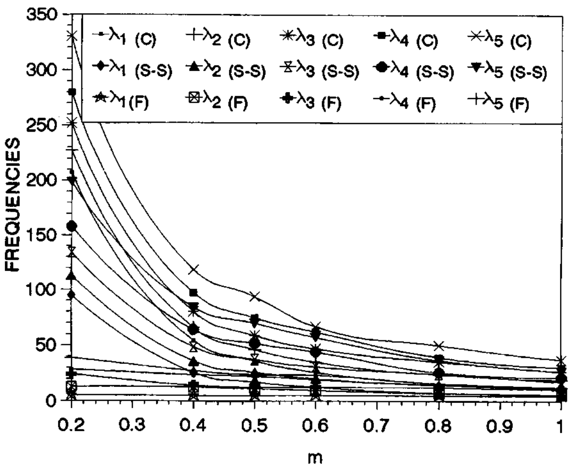

Fig. 8 First five frequencies for M7 for C: clamped, S-S: simply-supported and F: free boundary

λ_R and λ_M denote the frequencies computed by the numerical coefficients given by Sakata, Rajappa and McNitt in Eq. (19) respectively and the last column corresponds the results computed by the present method. It would appear from Table 2 that using the coefficient of McNitt one obtains results which are closer to the present one than the other two.

Before discussing further comparison, it is important to mention here that to find all the first five natural frequencies the results have been obtained first for various combinations of symmetric-symmetric, symmetric-antisymmetric, antisymmetric-symmetric and antisymmetric-antisymmetric modes about the two axes of the ellipse. Then the first five frequencies have been chosen from them to ensure that none will be left out.

Narita (1983) has reported a few higher modes for a clamped circular plate for five types of orthotropic material properties. Comparisons have been made with these in Table 3 which indicate good agreement. In this table for each of the combinations of symmetric and/or antisymmetric motion the first row cited is obtained using the present method.

Table 4 gives the comparison of the present results with those of Dong and Lopez (1985), who have reported the results for a Kevlar (material M5) clamped circular plate. They have given a table for a factor K in the frequency formula

$$\omega = K \frac{1}{a^2} \sqrt{\frac{D_x}{\rho h}} \tag{20}$$

Our present frequency parameter is given by (10). In order to compare with Dong and Lopez the present changed frequency parameter will be used

$$\lambda_P = \lambda / \sqrt{D_x / H} = K$$

Dong and Lopez (1985) gives results for five modes for sym./sym., sym./anti. and anti./anti. We have found that all the results compared show good agreement.

Table 2 Comparison of fundamental frequencies for clamped boundary with Sakata(1976), Rajappa (1963) and McNitt (1962)

MAT.	m	λ_S	λ_R	λ_M	λ
M1	0.2	128.96	126.58	125.34	121.58
	0.4	39.722	38.988	38.606	38.515
	0.5	30.716	30.148	29.853	29.820
	0.6	26.535	26.045	25.790	25.727
	0.8	23.230	22.801	22.578	22.362
	1.0	22.098	21.690	21.477	21.053
M2	0.2	147.24	144.52	143.11	136.51
	0.4	40.383	39.637	39.249	38.849
	0.5	28.280	27.757	27.485	27.376
	0.6	22.131	21.722	21.509	21.476
	0.8	16.742	16.433	16.272	16.242
	1.0	14.720	14.448	14.306	14.225
M3	0.2	180.60	177.27	175.53	168.53
	0.4	51.841	50.883	50.385	50.152
	0.5	38.308	37.601	37.232	37.181
	0.6	31.904	31.315	31.008	30.966
	0.8	26.823	26.327	26.069	25.907
	1.0	25.130	24.666	24.424	24.057
M4	0.2	158.10	155.18	153.66	148.67
	0.4	47.966	47.079	46.618	46.501
	0.5	36.953	36.270	35.915	35.876
	0.6	31.909	31.319	31.012	30.937
	0.8	28.016	27.498	27.229	26.959
	1.0	26.735	26.241	25.984	25.442
M5	0.2	261.08	256.26	253.75	236.43
	0.4	67.016	65.778	65.133	63.077
	0.5	44.080	43.266	42.842	42.043
	0.6	31.880	31.291	30.984	30.666
	0.8	20.351	19.975	19.779	19.726
	1.0	15.596	15.308	15.158	15.142
M6	0.2	117.26	115.09	113.96	109.24
	0.4	32.664	32.061	31.747	31.435
	0.5	22.932	22.509	22.288	22.195
	0.6	17.891	17.561	17.389	17.358
	0.8	13.303	13.057	12.929	12.910
	1.0	11.466	11.254	11.144	11.097
M7	0.2	229.37	225.13	222.93	207.23
	0.4	58.630	57.547	56.983	54.622
	0.5	38.211	37.505	37.138	36.017
	0.6	27.173	26.671	26.409	25.847
	0.8	16.332	16.030	15.873	15.717
	1.0	11.466	11.254	11.144	11.097

Table 3 Comparison with Narita (1983) for clamped circular plate

Mode	$(D_x/H,\ D_y/H)$				
	(1.469, .735)	(0.5, 0.5)	(0.5, 2)	(2, 0.5)	(2, 2)
SS-1	10.591	8.0762	11.097	11.097	13.514
*	10.59	8.090	11.05	11.05	13.51
AS-1	20.066	16.801	26.462	19.062	28.119
*	20.09	16.82	19.09	26.22	28.06
SA-1	23.836	16.801	19.062	26.462	28.119
*	-	16.82	26.22	19.09	28.06
SS-2	33.285	26.157	30.255	30.255	47.756
*	-	26.09	30.20	30.20	47.31
AA-1	35.891	28.917	37.178	37.178	44.459
*	-	28.97	37.21	37.21	44.59
SS-3	43.728	31.359	49.134	49.134	52.542
*	-	31.42	48.75	48.75	52.48

*Taken from Narita (1983)

Table 4 Comparison of results with Dong and Lopez (1985) for Kevlar circular plate

M	K					
	Sym/Sym		Sym/Anti		Anti/Anti	
	*	**	*	**	*	**
1	9.4396	9.3963	19.549	19.548	30.368	30.587
2	33.433	33.503	46.210	46.357	63.922	64.073
3	35.522	36.502	55.465	56.299	73.356	73.932
4	62.556	63.064	88.838	82.978	106.50	104.56
5	79.491	80.038	102.79	99.755	132.84	128.40

*Taken from Dong and Lopez (1985), **Results from present method

4.2. Simply-supported boundary

Again the first five natural frequencies are presented for various values of m for the mentioned material properties and the corresponding results are cited in Figs. 2 to 8 and denoted by 'S-S'. The authors have found no results in the existing literature for this boundary condition except one for the fundamental frequency of a circular plate with material M6 given by Rajappa (1963). A value of 5.3591 is reported which compares with 5.1628 obtained with the present method.

4.3. Free boundary

The results for this boundary condition are denoted by 'F' in Figs. 2 to 8. Only one reference by Narita (1985) is known to the authors for this boundary condition, who has presented the results in two figures for materials M6 and M7. The authors have obtained approximate results from the graphs given by Narita for m=0.5 and 1.0. All other results are entirely new. These values are slightly larger than the ones obtained with the present method.

4.4. Special case (isotropic plate)

As mentioned in earlier sections, the present problem reduces to that of an isotropic plate if $D_x/H=D_y/H=1$ and $\nu_x=\nu_y=\nu$, for which results are already reported by Chakraverty (1992) and Singh and Chakraverty (1991, 1992), where they have already made comparison with all the existing results for the isotropic case. Using the present computer program taking $D_x/H=D_y/H=1$ and $\nu_x=\nu_y=0.3$, we have again computed the results for various values of m which are given in Fig. 9. These are found to be exactly the same as reported in earlier papers (Chakraverty 1992, Singh and Chakraverty 1991, 1992).

4.5. Discussion of the results

It is seen from Figs. 2 to 8 (for materials M1 to M7), that for any boundary conditions i.e., for clamped simply-supported or free, the frequencies decrease as m is increased. For a clamped boundary the frequencies are a maximum and for a free boundary these are a minimum for each m for all the materials considered here.

As mentioned by Narita (1985) for a free boundary, the frequencies for m=1.0 in cases M6 and M7 are identical, since one case is just that of a 90 degree rotation of the other. This is true for all the boundary conditions (clamped, simply-supported and free) for m=1.0. Also Narita rightly mentions that as m is decreased, both cases show different variations. The results for M6 are lower than the corresponding ones for M7, due to the smaller bending rigidity in the x-direction.

To fix the number of approximations N needed, calculations were carried out for different values of N until the first five significant digits had converged. It was found that the results converged in about 8 to 10 approximations for clamped and simply-supported boundary and in 12 to 15 approximations for free boundary. Fig. 10 gives results for the convergence of the first three natural frequencies for clamped, simply-supported and free (i.e., p=2, 1 and 0 respectively) boundaries with N increasing from 2 to 15. These results were obtained for the material M6 and

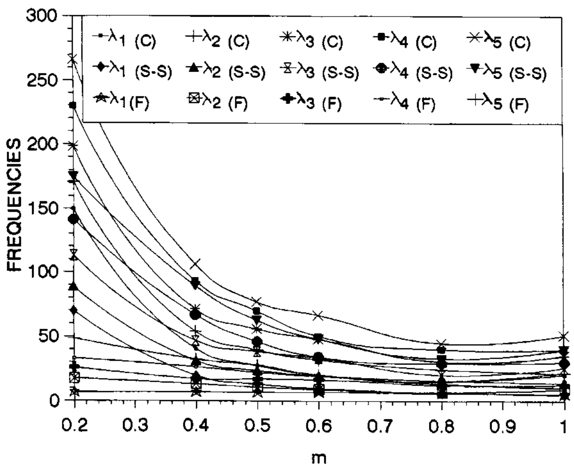

Fig. 9 First five frequencies for isotropic case for C: clamped, S-S: simply-supported and F: free boundary

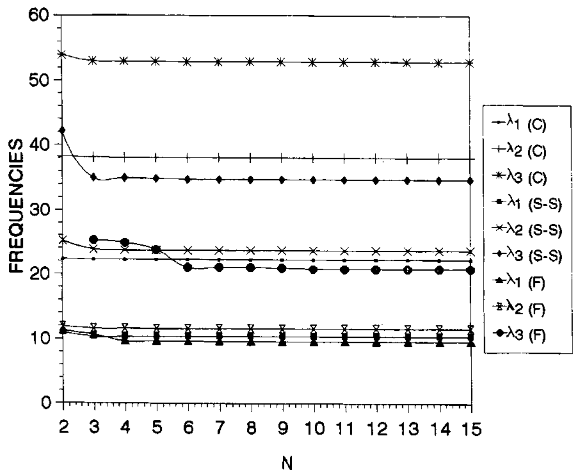

Fig. 10 Convergence pattern of first three frequencies for M6 material for C: clamped, S-S: simply-supported and F: free boundary

taking the aspect ratio of the ellipse to be 0.5.

5. Concluding remarks

Although this approach has been extensively used in isotropic plate vibration problems, it has been applied successfully in this study for the first time to the present problem of an orthotropic plate with a curved boundary (especially for elliptical). The effect of orthotropy has been fully investigated over a wide range of material properties. Most of the results are new and not found elsewhere. The use of two dimensional boundary characteristic orthogonal polynomials in the Rayleigh-Ritz method makes the problem a computationally efficient and simple numerical technique for finding vibration characteristics. It is important to mention here (as reported in earlier papers also) that the generation of orthogonal polynomials is very much sensitive to the numerical errors as the approximations are increased due to the rounding errors, which grow to an extent rendering the results to diverge. Therefore, all the computations have been carried out in double precision arithmetic and then the results converge as shown in Fig. 10. The present study can be generalized to other type of orthotropic plate geometries considering various complicating effects, such as plates with variable thickness, elastic foundation, considering in-plane forces etc.

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Notations

a, b	semi major and minor axes of the elliptic plate (Fig. 1)
x, y	cartesian coordinates
X, Y	non-dimensional coordinates
$R=S$	the interior of the plate
R'	the interior of the transformed plate after the transformation $X=x/a$, $Y=y/a$
∂S	boundary of the transformed plate
m	aspect ratio of the elliptic plate
W	displacement
E_x, E_y	Young's moduli in x and y directions
ν_x, ν_y	Poisson's ratio in x and y directions
D_x, D_y	Bending rigidities in x and y directions
G_{xy}	shear modulus
h	uniform thickness of the plate
ρ	mass density per unit volume
ω	circular frequency
c_j	constants defining mode shapes
a_{ij}, b_{ij}	constants defined by Eqs. (8), (9)
N	order of approximation
λ	non-dimensional frequency parameter
H	defined in equation (11)
f_i	suitably chosen functions
F_i	ff_i
f	$(1-x^2-y^2/m^2)^p$
p	integer which can have values 0, 1 or 2

$\langle f, g \rangle$	inner product of functions f and g
$\lVert f \rVert$	norm of the function f
$i=\phi_i$	orthogonal polynomials
$\hat{\phi}_i$	orthonormal polynomials
α_{ij}	constants defined in Eq. (17)
C	clamped boundary
S *or* S-S	simply-supported boundary
F	free boundary