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Study on bi-stable behaviors of un-stressed thin cylindrical shells based on the extremal principle

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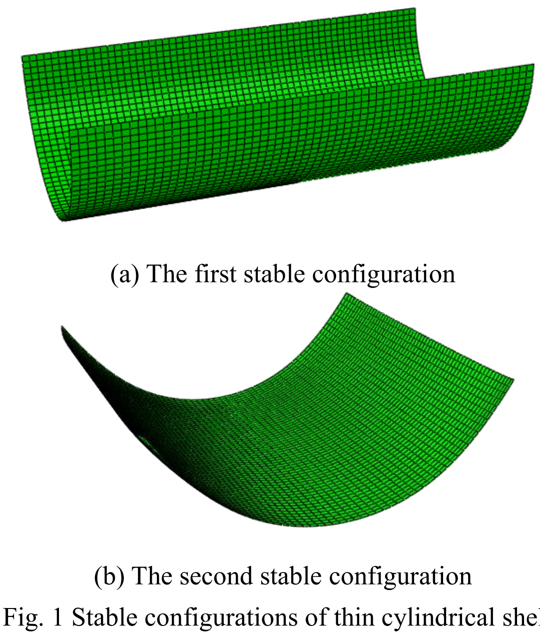
Abstract. Bi-stable structure can be stable in both its extended and coiled forms. For the un-stressed thin cylindrical shell, the strain energy expressions are deduced by using a theoretical model in terms of only two parameters. Based on the principle of minimum potential energy, the bi-stable behaviors of the cylindrical shells are investigated. The results indicate that the isotropic cylindrical shell does not have the second stable configuration and laminated cylindrical shells with symmetric or antisymmetric layup of fibers have the second stable state under some confined conditions. In the case of antisymmetric laminated cylindrical shell, the analytical expressions of the stability are derived based on the extremal principle, and the shell can achieve a compact coiled configuration without twist deformation in its second stable state. In the case of symmetric laminated cylindrical shell, the explicit solutions for the stability conditions cannot be deduced. Numerical results show that stable configuration of symmetric shell is difficult to achieve and symmetric shell has twist deformation in its second stable form. In addition, the roll-up radii of the antisymmetric laminated cylindrical shells are calculated using the finite element package ABAQUS. The results show that the value of the roll-up radii is larger from FE simulation than from theoretical analysis. By and large, the predicted roll-up radii of the cylindrical shells using ABAQUS agree well with the theoretical results.

Keywords: cylindrical shell; bi-stable; two-parameter model; strain energy

1. Introduction

Bi-stable shell structures can be stable in both its extended and coiled forms, as shown in Fig. 1. Bi-stable shells are similar to tape-springs; however, unlike tape-springs, they can be made to be stable in both states and therefore do not require a spindle or casing to hold them.

Bi-stable composite shells are usually manufactured with symmetric (Daynes *et al*. 2008, Li *et al*. 2014) or antisymmetric (Zhang *et al*. 2013, Zhang *et al*. 2014, Zhang *et al*. 2015) layup of fibers. Previous works on bi-stable composite shells have generated a number of analytical and computational models that capture various aspects of their behavior. Galletly and Guest (2004a) presented a beam model to describe the behavior of the bi-stable composite slit tube. The model assumed that the cross-section of the structure remained as an arc of a circle, with a radius that was allowed to vary. Furthermore, the shell model (Galletly and Guest 2004b) was presented, which did not make any assumption about the cross-sectional shape. Instead a differential equation was set up to model the transverse shape, and this was used to find the second equilibrium configuration. Kebadze and Guest (2004) investigated isotropic cylindrical shell which had two stable configurations, due to a particular distribution of residual stresses induced by plastic bending. In addition, an inextensional two-parameter analytical model (Guest and Pellegrino 2006, He 2011) was proposed. For thin cylindrical shell which mid-surface did not stretch, the Gaussian curvature must remain zero. This implied that all possible configurations must be developable, and every possible configuration of the shell could be defined by two variables. Moreover, some researchers (Kumar 2010, Aghajari *et al*. 2011, Guo *et al*. 2015, Javed *et al*. 2016) investigated other mechanical properties of thin cylindrical shells.



(a) The first stable configuration

(b) The second stable configuration

Fig. 1 Stable configurations of thin cylindrical shells

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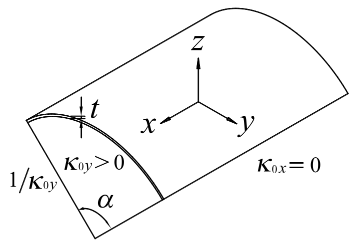


Fig. 2 Initial configuration of the shell

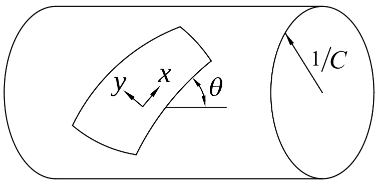


Fig. 3 Deformation of cylindrical shell

In this paper, the bi-stable behaviors of un-stressed thin cylindrical shells are discussed. Using the two-parameter model, the strain energy expressions of isotropic, antisymmetric and symmetric cylindrical shells are derived. Based on the principle of minimum potential energy, the stability of the cylindrical shells is investigated. In addition, a finite element analysis of the process of rolling up a bi-stable cylindrical shell is performed using ABAQUS codes. The results of this simulation provide considerable insights into the structural mechanics of bi-stable shells, as well as predicting the roll-up radii.

2. Theoretical predictions

2.1 Strain energy

All of the work in this paper has assumed that the shells are stress-free in their initial configurations. And studying the bi-stable behaviors of thin cylindrical shells, the following assumptions are useful: (1) Kirchhoff straight normal assumption; (2) linear elastic deformation; (3) no stretching strain of the mid-surface of the shell.

A curvilinear coordinate system will be used in the following analysis. The x and y-directions are defined to be parallel to the cylindrical and circumferential axes, respectively. And z-coordinate is perpendicular to the cylindrical surface. The x and y-directions will be principal directions of curvature throughout, and it will be also assumed that the curvature of the shell is uniform. Coordinate system and the definition for the curvatures are shown in Fig. 2. Note that, because of the uniform curvature assumption, it is unnecessary to choose a specific origin for the coordinate system. And it is assumed that the sign convention for moments follows from the sign convention for curvature, i.e., positive moments apply positive curvatures. Based on the principal, initial curvatures of the cylindrical shell are κ_{0x}=0 and κ_{0y}>0.

For in-extensional deformation, the Gaussian curvature must remain zero. This implies that for every possible configuration of the shell there must be an underlying cylinder about which the shell is wrapped. Thus, the configuration of the shell can be defined by two variables, one defining the curvature of this underlying cylinder, and the other defining the orientation of the shell relative to the cylinder. The angle of the shell relative to the cylinder is defined as θ. The principal curvature of the cylinder is defined as C and its radius is $1/C$ (initial transverse radius, R), as shown in Fig. 3. The two parameters (C and θ) describe the main feature of the cylindrical shell at the deformed configuration.

In the initial configuration of the cylindrical shell (θ=0), its curvatures are κ_{0x}=0, κ_{0y}>0. And in the deformed configuration, the curvatures can be expressed as $\kappa_x=C(1-\cos 2\theta)/2$, $\kappa_y=C(1+\cos 2\theta)/2$, and $\kappa_{xy}=C\sin 2\theta$. Thus, the changes in curvatures are given by

$$\nabla\begin{bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{bmatrix}=\frac{C}{2}\begin{bmatrix}1-\cos 2\theta\\ 1+\cos 2\theta\\ 2\sin 2\theta\end{bmatrix}-\begin{bmatrix}0\\ \kappa_{0y}\\ 0\end{bmatrix} \tag{1}$$

With the integration of the stresses and moments on the entire multilayer, the resultant moments and forces will be achieved as follows (Dogan and Arslan 2012, Ali and Mohammad 2016)

$$\begin{bmatrix}N_x\\ N_y\\ N_{xy}\\ M_x\\ M_y\\ M_{xy}\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16}\\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26}\\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66}\\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16}\\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26}\\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\\ \kappa_x\\ \kappa_y\\ \kappa_{xy}\end{bmatrix} \tag{2}$$

Generally, the shell has both bending and stretching strain energies. But for in-extensional deformation, stretching strain energy is not considered. Therefore, the bending strain energy per unit area can be written as

$$u=\frac{1}{2}\boldsymbol{\kappa}^T\boldsymbol{D}\boldsymbol{\kappa} \tag{3}$$

where

$$\boldsymbol{D}=\begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{12} & D_{22} & D_{26}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}$$

The isotropic cylindrical shell is firstly discussed. From Eq. (3), we can have

$$u_1=\frac{ET^3}{24(1-v^2)}\left[C^2-C\kappa_{0y}(v-v\cos 2\theta+1+\cos 2\theta)+\kappa_{0y}^2\right] \tag{4}$$

where E is the Young's modulus, v is the Poisson's ratio, T is the total thickness of the isotropic cylindrical shell.

For the antisymmetric laminated cylindrical shell, the bending strain energy per unit area can be expressed by

$$\begin{aligned}u_2=&\left(\kappa_{0y}-C\cos^2\theta\right)\left[D_{22}\left(\kappa_{0y}-C\cos^2\theta\right)+D_{12}C\left(\cos^2\theta-1\right)\right]-4D_{66}C^2\cos^2\theta\left(\cos^2\theta-1\right)\\ &+C\left(\cos^2\theta-1\right)\left[D_{12}\left(\kappa_{0y}-C\cos^2\theta\right)+D_{11}C\left(\cos^2\theta-1\right)\right]\end{aligned} \tag{5}$$

Similarly, for the symmetric laminated cylindrical shell, the bending strain energy per unit area can be expressed by

$$\begin{aligned} u_3 &= \left(\kappa_{0y} - C\cos^2\theta\right)\left[D_{22}\left(\kappa_{0y} - C\cos^2\theta\right) - D_{26}C\sin 2\theta + D_{12}C\left(\cos^2\theta - 1\right)\right] \\ &+ C\left(\cos^2\theta - 1\right)\left[D_{12}\left(\kappa_{0y} - C\cos^2\theta\right) - D_{16}C\sin 2\theta + D_{11}C\left(\cos^2\theta - 1\right)\right] \\ &- C\sin 2\theta\left[D_{26}\left(\kappa_{0y} - C\cos^2\theta\right) - D_{66}C\sin 2\theta + D_{16}C(\cos^2\theta - 1)\right] \end{aligned} \tag{6}$$

2.2 Equilibria and stability

In the equilibrium position of the cylindrical shell, the partial derivatives of strain energy for the angle (θ) and the curvature (C) must be zero. So, we can have

$$\begin{cases} \dfrac{\partial u}{\partial C} = 0 \\ \dfrac{\partial u}{\partial \theta} = 0 \end{cases} \tag{7}$$

To check whether these are stable equilibria, we need to check whether the strain energy corresponds to local minima. If they satisfy the conditions ($\dfrac{\partial^2 u}{\partial \theta^2} > 0$, $\dfrac{\partial^2 u}{\partial C^2} > 0$, and $\left(\dfrac{\partial^2 u}{\partial C^2}\right)\left(\dfrac{\partial^2 u}{\partial \theta^2}\right) > \left(\dfrac{\partial^2 u}{\partial C \partial \theta}\right)^2$), the equilibria will be stable. Otherwise, it is unstable equilibrium state.

From Eq. (4), the isotropic cylindrical shell has two equilibrium points: θ_1=0 and θ_2=π/2. In the first equilibrium point (θ_1=0) corresponding to the initial configuration of cylindrical shell, the strain energy is zero. So, the initial configuration of the cylindrical shell is the first stable state. In the second equilibrium point (θ_2=π/2), we can have $\dfrac{\partial^2 u_1}{\partial C^2} > 0$, $\dfrac{\partial^2 u_1}{\partial \theta^2} < 0$, and $\dfrac{\partial^2 u_1}{\partial C \partial \theta} = 0$. Clearly, the local minimum strain energy does not exist. It means that the second equilibrium point is not stable. Therefore, un-stressed isotropic cylindrical shell is not a bi-stable structure.

Similar to isotropic cylindrical shell, the antisymmetric laminated cylindrical shell also has two typical equilibrium points: θ_1=0 and θ_2=π/2. Actually, the antisymmetric laminated cylindrical shell might have four equilibrium points. But numerical investigations show only two equilibrium points can be stable equilibria. In the first equilibrium point (θ_1=0), the strain energy is zero, which corresponds to the first stable configuration. In the second equilibrium point (θ_2=π/2), we can have

$$\begin{cases} \dfrac{\partial^2 u_2}{\partial \theta^2} = \dfrac{4\kappa_{0y}^2 D_{12}\left(D_{12}^2 + 2D_{12}D_{66} - D_{11}D_{22}\right)}{D_{11}^2} \\ \dfrac{\partial^2 u_2}{\partial C^2} > 0 \\ \dfrac{\partial^2 u_2}{\partial \theta \partial C} = 0 \end{cases} \tag{8}$$

If the strain energy satisfy the condition ($\dfrac{\partial^2 u_2}{\partial \theta^2} > 0$), this equilibrium state is also stable state. From Eq. (8), we can have

$$S = D_{12} + 2D_{66} - \frac{D_{11}D_{22}}{D_{12}} > 0 \tag{9}$$

Thus, it can be resulted that the antisymmetric laminated cylindrical shell exists the second stable configuration for S>0.

For the symmetric laminated cylindrical shell, submitting Eq. (6) into Eq. (7), we cannot obtain the explicit solutions to the angle (θ) and the curvature (C). Solving these equations numerically shows that the symmetric laminated cylindrical shell may also have the second stable configuration, as shown in the following chapter.

3. Results and discussion

Based on the works above, the un-stressed isotropic cylindrical shell is not a bi-stable structure. Therefore, only the stabilities of the laminated cylindrical shells with antisymmetric or symmetric layups of fibers are discussed. In the following studying, the influences of the ply angle (β), the layer number (n), the single layer shell thickness (t), and the initial transverse radius (R) will be successively discussed.

Because of the symmetry, the angle range (0º≤θ≤180º) is considered, as shown in Fig. 3. The material properties of the single layer shell are as follows: elastic modulus E_1=27.6 GPa, elastic modulus E_2=2.6 GPa, shear modulus G=0.964 GPa, Poisson's ratio v_{12}=0.305.

3.1 Antisymmetric case

The cylindrical shell is made of five layers ([+β/-β/0/+β/-β]). The thickness of each layer is 0.21mm. The initial transverse radius of the cylindrical shell is 25mm. The influence of the ply angle (β) is shown in Fig. 4, where θ is the angle and u is the strain energy per unit area. We can see that the local minimum value of the strain energy is formed at θ=90º for the ply angles (β=35°, 45°, 55°), which coincides with the above analysis. Solving Eq. (5) numerically shows that the antisymmetric laminated cylindrical shells exist the second stable state for 26.5°<β<63.1°. In addition, Fig. 4 shows that greater ply angle corresponds to greater strain energy in the second equilibrium point (θ=90º).

Given the ply angle (β=45º), the influence of the layer number (n) is considered. The layup models are respectively [+45º/-45º/+45º/-45º], [+45º/-45º/0/+45º/-45º], [+45º/-45º/+45º/-45º/+45º/-45º], [+45º/-45º/+45º/0/-45º/+45º/-45º], and [+45º/-45º/+45º/-45º/+45º/-45º/+45º/-45º]. The thickness of single layer shell is 0.21mm. The initial transverse radius of cylindrical shell is 25mm. Fig. 5 shows how the strain energy per unit area (u) varies with the angle (θ). It can be seen that the antisymmetric laminated cylindrical shell has local minimum value of the strain energy for all the numbers (n=4, 5, 6, 7, 8) in the equilibrium point (θ=90º). So, the layer number has little effect on the stability of the antisymmetric laminated

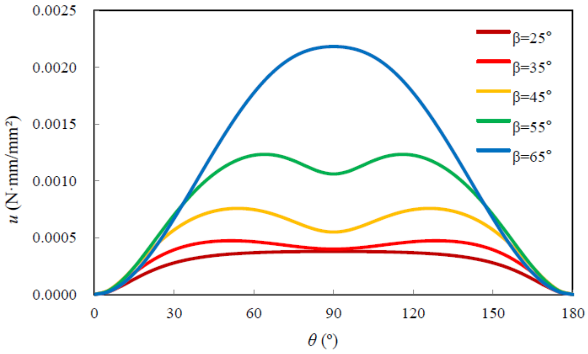


Fig. 4 Influence of the ply angle in the case of antisymmetry

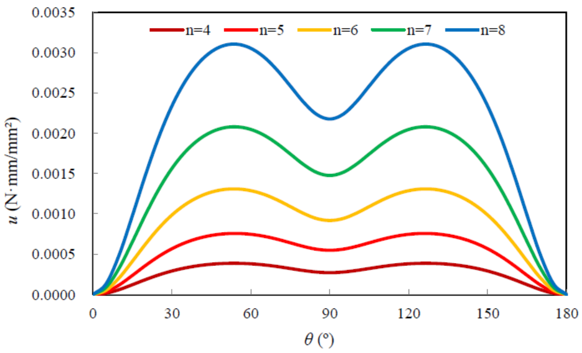


Fig. 5 Influence of the layer number in the case of antisymmetry

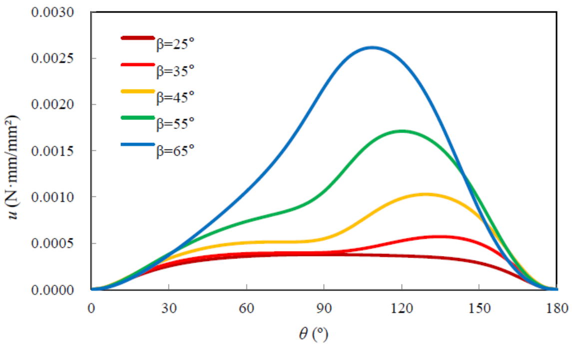


Fig. 6 Influence of the ply angle in the case of symmetry

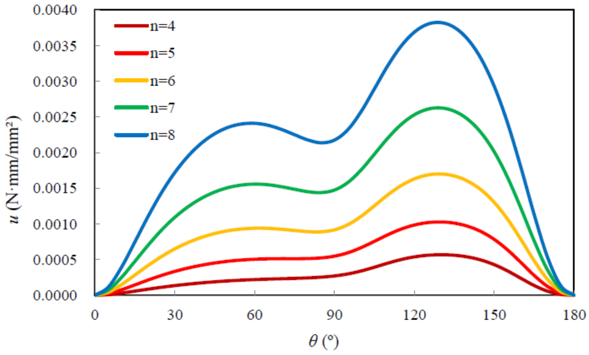


Fig. 7 Influence of the layer number in the case of symmetry

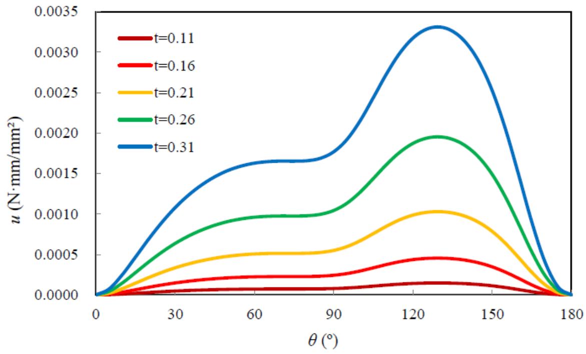


Fig. 8 Influence of the single layer shell thickness in the case of symmetry

cylindrical shell.

The initial transverse radius of the cylindrical shell is 25 mm. For the typical cylindrical shell with the ordering of the laminate ([+45°/-45°/0/-45°/+45°]), the influence of the single layer shell thickness (t=0.11 mm, 0.16 mm, 0.21 mm, 0.26 mm, 0.31 mm) is discussed. The results show that the thickness (t) has little effect on the stability of the antisymmetric laminated cylindrical shell.

In addition, the thickness of the single layer shell is 0.21 mm. For the typical cylindrical shell with the ordering of the laminate ([+45°/-45°/0/-45°/+45°]), the influence of the initial transverse radius (R=15 mm, 20 mm, 25 mm, 30 mm, 35 mm) is considered. The same results can be obtained: the initial transverse radius (R) also has little effect on the stability of the cylindrical shell.

The above results show that: the ply angle has significant influence on the stability of the antisymmetric laminated cylindrical shell; the layer number, the single layer shell thickness, and the initial transverse radius have effect on the strain energy of the cylindrical shell, but little effect on the stability. Furthermore, the antisymmetric laminated cylindrical shell has no twist deformation in the second stable configuration (θ=90º), which makes it possible to compactly roll up a very perfect cylindrical shell.

3.2 Symmetric case

In this chapter, only antisymmetric layup changes into symmetric layup and other parameters are just the same with the previous chapter.

Firstly, the influence of the ply angle (β) is considered, as shown in Fig. 6. Solving Eq. (6) numerically shows that the symmetric laminated cylindrical shells exist local minimum value of the strain energy for 32.7°<β<46.9°. In the case of β=35º, there are two local minimum values of the strain energies corresponding to the angels (θ=79.1º and θ=83.2º). In the case of β=45º, there are three energy minima corresponding to the angels (θ=71.9º, θ=74.8º, and θ=76.9º). But all the strain energies change very little, resulting in the stable configuration being difficult to obtain in actual engineering.

Secondly, the influence of the layer number (n) is considered. Fig. 7 shows that the symmetric laminated cylindrical shells exist local minimum value of the strain energy for the layer number (n=5, n=6, n=7, and n=8). In the case of n=5, there are three energy minima, which are the same as that of the above results (seen Fig. 6). In the

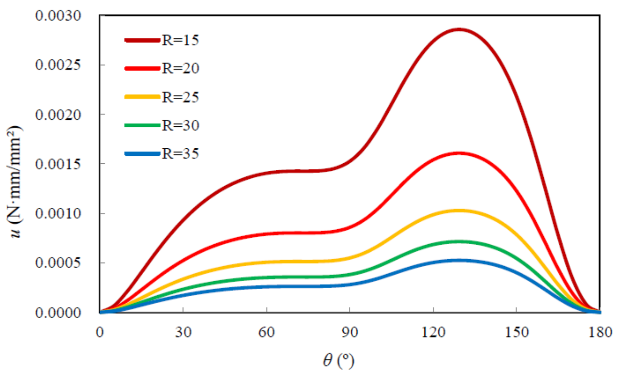


Fig. 9 Influence of the initial transverse radius in the case of symmetry

case of n=6, there are three energy minima corresponding to the angels (θ=83.2º, θ=83.9º, and θ=85.4º). For another two cases (n=7 and n=8), the similar conclusions can be drown. Based on the angel (θ) as shown in Fig. 3, the shells have twist deformation in the other stable configurations ($\theta \neq 90$º) except for the initial configuration (θ=0).

Thirdly, the influence of the single layer shell thickness (t) is considered. Fig. 8 shows that the cylindrical shells have three energy minima. For all the cases (t=0.11 mm, t=0.16 mm, t=0.21 mm, t=0.26 mm, and t=0.31 mm), there are three local minimum values of the strain energies corresponding to the same angels (θ=71.9º, θ=74.8º, and θ=76.9º). It can be concluded that the single layer shell thickness (t) has no effect on the stability of the symmetric laminated cylindrical shell.

Fourthly, the influence of the initial transverse radius (R) is considered. Fig. 9 shows that the cylindrical shells have the local minimum value of the strain energy. In the case of R=15 mm, there are two energy minima corresponding to the angels (θ=71.9º and θ=76.6º). In the case of R=20 mm, there are three energy minima corresponding to the angels (θ=71.9º, θ=75.3º, and θ=76.2º). Another three cases (R=25 mm, R=30 mm, and R=35 mm) are similar to the case of R=20 mm, and the strain energies have three minima. The results show that the initial transverse radius (R) has little effect on the stability of the symmetric laminated cylindrical shell.

4. Numerical simulations

4.1 Deformation process

The cylindrical shell has a great geometric nonlinear deformation during its flattening and curling. ABAQUS software is very good at dealing with nonlinear problems so that it will be used in the numerical simulations.

The symmetric laminated cylindrical shell has poor stability in its second stable state and it is difficult to achieve the numerical simulation results. Therefore, only the computational study of the antisymmetric laminated cylindrical shell is carried out using ABAQUS here. Taking the typical cylindrical shell with the ordering of the laminate ([+45°/-45°/0/-45°/+45°]) as example, the layup

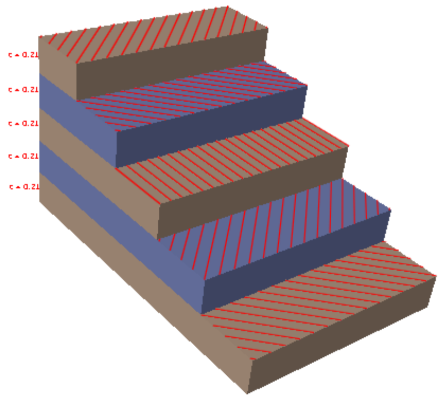


Fig. 10 Layup method

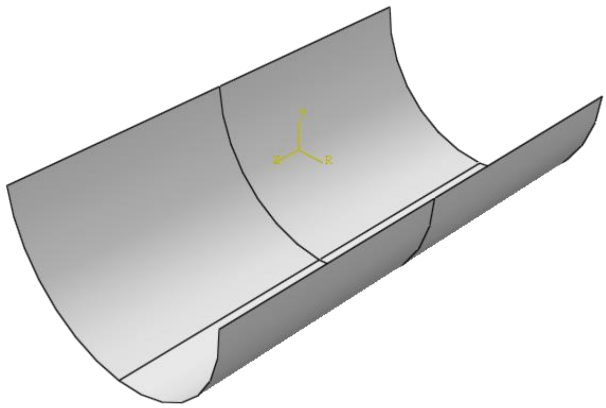
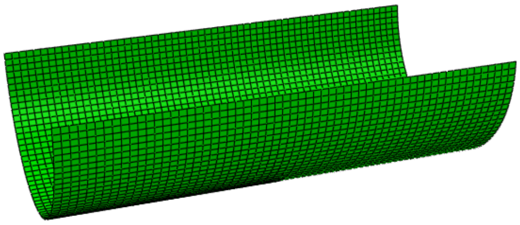


Fig. 11 Numerical model

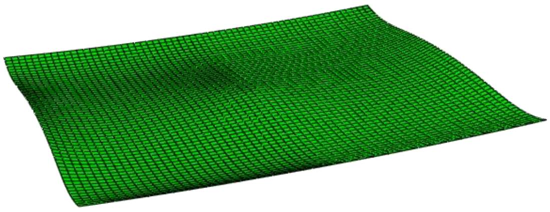
method is shown in Fig. 10. The properties of the cylindrical shell are as follows: the initial transverse radius R=25 mm, the thickness of the single layer shell t=0.21 mm. This layup model is similar to the existing method (Dogan *et al*. 2010). In order to compare with the theoretical results, the material constants are the same as the previous values: elastic modulus E_1=27.6 GPa, elastic modulus E_2=2.6 GPa, shear modulus G=0.964 GPa, Poisson's ratio ν_{12}=0.305.

The whole shell is analyzed, but only a single node is restrained in all six degrees of freedom. No stress concentrations arise at this node, because the loads applied to the shell are self-equilibrated. The particular node that is constrained is the node right at the center of the shell but, of course, any other node can be chosen. The shell is modeled using S4R shell element of ABAQUS (Patel *et al*. 2011), which is found to be the robust for this type of analysis. Meanwhile, the automatic stabilization is successfully used. This method carries out a pseudo-dynamic simulation as soon as a negative pivot is detected during the inversion of the stiffness matrix. Thus, the algorithm is able to cope with localized snaps that occur in the mesh. Fictitious nodal masses and a small amount of numerical damping are introduced by ABAQUS to stabilize the snap. The numerical model of the cylindrical shell is shown in Fig. 11.

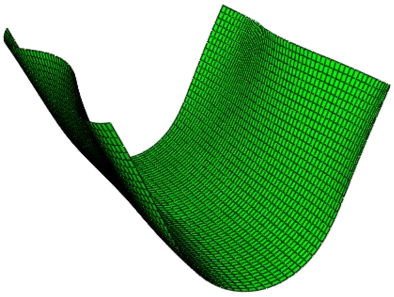
On the first issue, attempts are firstly made to drive the change of configuration by imposing edge rotations along the initially straight edges of the shell. This seems the obvious way of proceeding, based on the standard practice



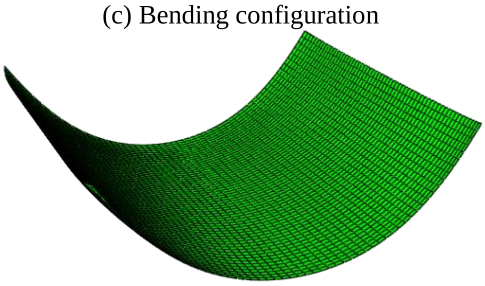
(a) Initial configuration



(b) Flattening configuration



(c) Bending configuration



(d) Final configuration

Fig. 12 Bi-stable deformation process of the shell

Table 1 Roll-up radii considering the influence of the ply angle

β (°)	r (mm)	
	theoretical analysis	FE simulation
35	61.7	78.7
40	46.3	55.5
45	35.9	42.2
50	28.8	34.7
55	24.3	-

of using nodal displacements or rotations as control parameters in the analysis of structures that exhibit snapping. The problem with imposing edge rotations in the present case is that they are about fixed axes, which is incompatible with the large rotations that occur during rolling-up of the shell. It is found that applying edge moments using the follower option works much better.

Table 2 Roll-up radii considering the influence of the layer number

n	r (mm)	
	theoretical analysis	FE simulation
4	35.0	42.7
5	35.9	42.2
6	35.0	43.2
7	35.3	43.7
8	35.0	44.0

Table 3 Roll-up radii considering the influence of the single layer shell thickness

t (mm)	r (mm)	
	theoretical analysis	FE simulation
0.11	35.9	40.9
0.16	35.9	41.6
0.21	35.9	42.2
0.26	35.9	42.8
0.31	35.9	43.4

The maximum value of the edge moment that is assigned is greater than the moment required to flatten the shell. Fig. 12 shows a complete bi-stable deformation process of the shell. Fig. 12(a) is the initial configuration of the cylindrical shell. In the configuration of Fig. 12(b), the shell is flattened. Then the shell bends reversely, as shown in Fig. 12(c). In the end, all loads are removed, and the final configuration is shown in Fig. 12(d). The radius of the final configuration is the roll-up radius (r) of the cylindrical shell.

4.2 Comparison

Based on the above results from theoretical analysis and FE simulation, the roll-up radii of the cylindrical shells will be compared.

Firstly, the influence of the ply angle (β) is considered. The cylindrical shells are made of five layers ([+β/-β/0/+β/-β]). The thickness of each layer is 0.21 mm. The initial transverse radius of cylindrical shell is 25 mm. The roll-up radii of the cylindrical shells are listed in Table 1. The results show that the values of the roll-up radii from FE simulation are larger than that from theoretical analysis. It is because that the theoretical assumptions limit the possible displacements of the cylindrical shell. And the two-parameter analytical model may also affect the values of the roll-up radii. In addition, Table 1 shows that greater ply angle (β) corresponds to smaller roll-up radius (r). In the case of β=55°, the cylindrical shell has poor stability in its second stable state, and even the roll-up radius cannot be simulated using the finite element software.

Secondly, the influence of the layer number (n) is considered. The roll-up radii of the cylindrical shells are listed in Table 2. Because of the same reasons as above, the values of the roll-up radii from FE simulation are larger than that from theoretical analysis. Overall, the layer number has little effect on the roll-up radii of the cylindrical shells.

Table 4 Roll-up radii considering the influence of the initial transverse radius

R (mm)	r (mm)	
	theoretical analysis	FE simulation
20	28.7	34.2
25	35.9	42.2
30	43.0	50.1
35	50.2	57.9
40	57.4	65.4

Thirdly, the influence of the single layer shell thickness (t) is considered. The roll-up radii of the cylindrical shells are listed in Table 3. The theoretical results show that the single layer shell thickness has no effect on the roll-up radii of the cylindrical shells. But the simulation results show that the roll-up radii slightly rise with the increasing single layer shell thickness.

Fourthly, the influence of the initial transverse radius (R) is considered. The roll-up radii of the cylindrical shells are listed in Table 4. The results show that the roll-up radii of the cylindrical shells almost increases linearly with the increasing initial transverse radius.

By and large, the predicted roll-up radii of the cylindrical shells using ABAQUS agree well with the theoretical results.

5. Conclusions

The strain energy expressions of the un-stressed thin cylindrical shells are deduced by using a two-parameter analytical model. Based on the principle of minimum potential energy, the following conclusions can be drawn:

• The isotropic cylindrical shell has two equilibrium states. The first equilibrium position is corresponding to the initial configuration of the cylindrical shell, which is the first stable configuration. But in the second equilibrium position, the cylindrical shell has no local minimum strain energy. It means that the second equilibrium position is not stable. So, the isotropic cylindrical shell is not a bi-stable structure.

• The antisymmetric laminated cylindrical shell has four equilibrium points. But the numerical investigations show that only two equilibrium points can be stable equilibria. The initial state, whose strain energy is zero, is the first stable configuration of cylindrical shell. In another equilibrium position of the shell, the analytical expressions of the stability are derived based on the extremal principle. The results show that: the ply angle has significant influence on the stability of the cylindrical shell; the layer number, the single layer shell thickness, and the initial transverse radius have effect on the strain energy of the cylindrical shell, but little effect on the stability. In addition, the antisymmetric laminated cylindrical shell has no twist deformation in its second stable state, which can achieve a compact coiled configuration.

• The explicit solutions for the stability conditions of the symmetric laminated cylindrical shell cannot be derived because of the complex strain energy equations. Solving these equations numerically shows that the symmetric laminated cylindrical shell may also be bi-stable, but its stability is difficult to achieve. And the symmetric laminated cylindrical shell has twist deformation in its second stable configuration, which tends to coil into a helix shell rather than a compact coiled one.

• The values of the roll-up radii from FE simulation are larger than that from theoretical analysis. By and large, the predicted roll-up radii of the cylindrical shells using ABAQUS agree well with the theoretical results. In addition, the roll-up radius of the cylindrical shell decreases with the increasing ply angle or with the decreasing initial transverse radius; the layer number and the single layer shell thickness have little effect on the roll-up radius.

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