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Seismic response modification factors for stiffness degrading soil-structure systems

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Abstract. This paper aims to develop response modification factors for stiffness degrading structures by incorporating soil-structure interaction effects. A comprehensive parametric study is conducted to investigate the effects of key SSI parameters, natural period of vibration, ductility demand and hysteretic behavior on the response modification factor of soil-structure systems. The nonlinear dynamic response of 6300 soil-structure systems are studied under two ensembles of accelograms including 20 recorded and 7 synthetic ground motions. It is concluded that neglecting the stiffness degradation of structures can results in up to 22% underestimation of inelastic strength demands in soil-structure systems, leading to an unexpected high level of ductility demand in the structures located on soft soil. Nonlinear regression analyses are then performed to derive a simplified expression for estimating ductility-dependent response modification factors for stiffness degrading soil-structure systems. The adequacy of the proposed expression is investigated through sensitivity analyses on nonlinear soil-structure systems under seven synthetic spectrum compatible earthquake ground motions. A good agreement is observed between the results of the predicted and the target ductility demands, demonstrating the adequacy of the expression proposed in this study to estimate the inelastic demands of SSI systems with stiffness degrading structures. It is observed that the maximum differences between the target and average target ductility demands was 15%, which is considered acceptable for practical design purposes.

Keywords: response modification factor; degrading structures; soil-structure interaction; practical equation; sensitivity analysis

1. Introduction

The observations from severe earthquake events that have occurred recently showed that the earthquake risk in urban areas is increasing rather than decreasing. In this regard, a rational and effective seismic risk mitigation program is necessary to reverse this situation. The development of more reliable seismic-resistant design provisions is one of the most effective steps towards this end. Most of the current seismic design provisions for new civil engineering facilities are based on force based procedure. These provisions allow structures to undergo inelastic deformation during moderate and severe earthquake ground motions. While non-linear time history analysis methods can simulate the actual seismic behavior of structures effectively, due to their high computational costs and complexity, current seismic design codes usually propose alternative simplified analysis methods. Based on this concept, the earthquake input energy dissipation through the hysteretic energy mechanism is achieved by the use of response modification factor (R). response modification factor is generally utilized to decrease the design base shear from those that are required by the structure to remain elastic in the event of severe earthquakes, and is generally defined by the following equation

$$R = R_{\mu} R_{\omega} \tag{1}$$

where R_μ is the conventional ductility-dependent response modification factor (denoted as response modification factor in this study), reflecting nonlinear hysteresis behavior in a structures, and R_w is used to incorporate other reduction factors such as reduction due to element redundancy, overstrength and strain hardening.

Several research studies have investigated the response modification factor (R_μ), especially for fixed-base structures. The pioneer and well known studies on response modification factor were conducted by Veletsos and Newmark (1960) and Newmark and Hall (1969). They proposed simplified formulas for strength reduction as functions of structural period and target ductility ratio. The effect of soil conditions on response modification factor were studied by Elghadamsi and Mohraz (1987). They concluded that response modification factors for systems located on alluvium soils are not significantly different from those for systems located on rock. Based on the results obtained from 124 strong ground motions, Miranda (1993) suggested that the magnitude and epicentral distance have a

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negligible influence on the response modification factors, whereas the effect of soil condition (especially soft soil) can be significant. Subsequently, many studies have been performed on response modification factor of fixed-base SDOF as well as MDOF systems (Fischinger *et al.* 1994, Miranda and Bertero 1994, Ordaz and Pérez-Rocha 1998, Santa-Ana and Miranda 2000, Karmakar and Gupta 2007), which led to different proposed equations.

Previous studies showed that the response modification factors of fixed-base stiffness-degrading structures are generally smaller than those of the corresponding non-degrading systems for period of vibrations less than about 0.4 sec (Nassar and Krawinkler 1991, Rahnama and Krawinkler 1993). In the current design process, in most cases, the SSI effects are neglected. However, a review of the observation from past earthquakes, such as Mexico City earthquake, showed that the soil beneath the structure can considerably affect the seismic demands especially for buildings on soft soil profiles. Generally, soil-structure systems have a longer period of vibration compared to the corresponding fixed-base structures. Moreover, the soil-structure interaction (SSI) effect can increase the effective damping ratio, because of radiation and material damping of soil. Hence, to include the SSI effects in the seismic design process, modern seismic provisions such as FEMA-440 (2005) and ASCE-7-16 (2010) suggest an equivalent fixed-base system with modified fundamental period and damping ratio.

Ghannad and Jahankhah (2007) evaluated the effects of soil-structure interaction on response modification factor using 54 strong motions. They concluded that SSI reduces inelastic R_μ values when compared to their fixed-base counterparts, and hence using the fixed-base response modification factors for soil-structure systems can lead to the underestimation of seismic design forces. The results of their study show that by using fixed-base response modification factor to estimate the inelastic strength demand of a soil-structure system, the systems may exhibit ductility ratios considerably higher than the target value. In another relevant study, Eser *et al.* (2012) investigated the effects of SSI on strength reduction ratio through the use of equal fixed-base SDOF systems. They proposed a new equation for R_μ of soil-structure systems as a function of structural period, target ductility and period lengthening ratio. Several other research efforts have focused on the evaluation of SSI effects on the response modification factor of SDOF and MDOF systems (Avilés J.a Pérez-Rocha 2005, Lu *et al.* 2016). However, the results of these studies were mainly restricted to the structural systems without considering strength and stiffness degradations.

As discussed above, several studies in the past have demonstrated the significant effect of SSI on the seismic demands of structures. However, the effect of SSI on the response modification factor of systems with stiffness degrading hysteretic model has not been well investigated. In this study a comprehensive parametric study is performed to evaluate the effects of SSI on R_μ values of stiffness degraded systems, as a more realistic model for reinforced concrete structures subjected to strong ground motions. By considering a wide range of key design parameters (including three different hysteric models), simplified equations are proposed to estimate the response modification factor of stiffness degraded and non-degraded soil-structure systems.

2. Structural and geotechnical description of the soil-shallow-foundation models

The response of a structure to earthquake strong ground motion is affected by the interaction between three linked subsystems: soil, foundation and structure (Gioncu and Mazzolani 2010). In this study, due to its simplicity and sufficient accuracy, the conventional lumped-mass Cone model which was developed by Wolf (1994) is based on the one dimensional wave propagation theory is utilized to simulate the complex soil-structure interaction phenomenon. The soil is considered as a homogenous half-space medium modeled by an equivalent linear discrete model based on the concept of truncated cone model. The supporting soil is substituted with a three degree of freedoms (3-DOFs) spring and dashpot system. Two DOFs are defined to model the translational (sway) and rotational (rocking) movements of the foundation, respectively, ignoring the insignificant influence of the vertical and torsional movements of the foundation. The third DOF is added to the soil model to incorporate the frequency-dependent rotational dashpot and spring coefficients (Wolf 1994). This extra internal rotational DOF, φ_1, is augmented to a polar mass moment of inertia, M_φ, situated in series with the rotational dashpot. The main parameters of the soil-foundation model are as follows (Wolf 1994)

$$k_h = \frac{8\rho V_s^2 r}{2-\nu} \qquad c_h = \pi\rho V_s r^2 \tag{2}$$

$$k_\varphi = \frac{8\rho V_s^2 r^3}{3(1-\upsilon)} \qquad c_\varphi = \frac{\pi\rho V_p r^4}{4} \tag{3}$$

$$M_{\varphi 1} = \frac{9\rho\pi^2 r^5(1-\nu)}{32}\left(\frac{V_p}{V_s}\right)^2 \tag{4}$$

In Eqs. (2)-(4), k_h= sway stiffness, c_h= sway viscous damping, k_φ = rocking stiffness and c_φ = rocking damping. In addition, ρ= specific mass, υ= Poisson's ratio, V_p and V_s = dilatational and shear wave velocity of soil, respectively, and r = the equivalent circular foundation radius. Moreover, an additional mass moment of inertia ΔM_φ equal to $0.3\pi(\upsilon-1/3)\rho r^5$ is added to I_f for υ greater than 0.3 in order to modify the effect of soil incompressibility (Wolf 1994). The parameters m and h are used to describe the effective mass and effective height of the structure, respectively. The discrete soil-shallow-foundation system utilized in this study is depicted in Fig. 1.

The superstructure is modeled as a nonlinear SDOF system with the same damping ratio and period of vibration as those of the fixed-base structure. In the present study, as shown in Fig. 2, three types of hysteresis behaviors are

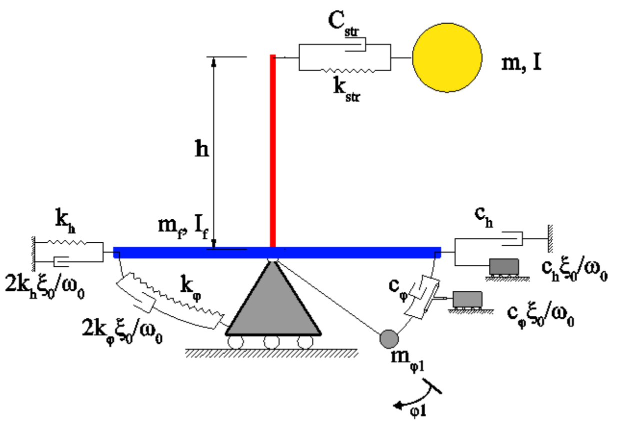

Fig. 1 The discrete shallow-foundation soil-structure system used in this study

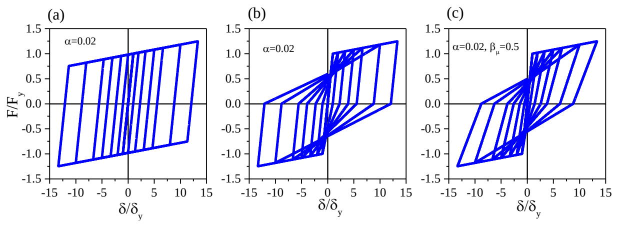

Fig. 2 Hysteretic models used in this study: (a) non-degrading bilinear model; (b) modified Clough Stiffness degrading model; (c) peak-oriented stiffness degrading model

considered, including (a) non-degraded bilinear elsto-plastic with strain hardening (BL), (b) modified Clough hysteretic model (CL) in which the unloading stiffness is kept equal to the initial elastic, and (c) peak-oriented stiffness degrading model (SD) with degradation at both unloading and reloading branches. Strain hardening ratio (α) in all hysteresis behaviors is equal to 0.02.

In this study, to investigate the effect of stiffness degradation on response modification factor of SSI systems two types of aforementioned hysteretic behaviors with stiffness degradation including CL and SD models are considered. In CL model, the stiffness degradation is only dependent on the displacement amplitude. In fact, the model has a bilinear envelop; however after the initial yielding, further loading branches are directed towards the furthest unloading point in the direction of loading (Miranda and Ruiz-Garcia 2002). The unloading stiffness in this model is kept equal to the initial elastic stiffness. On the other hand, in SD model the degradation occurs at both unloading and reloading branches in which unloading stiffness reduces based on the following equation

$$K_{un} = K_0 \left(\frac{u_y}{u_m} \right)^{\beta_\mu} \tag{5}$$

where K_0 is initial stiffness, u_y and u_m are yield displacement and maximum inelastic displacement of the structure, respectively. In addition, K_{un} is unloading stiffness and β_μ is the parameter showing the intensity of stiffness degradation which is taken as 0.5 indicating a moderate degradation (Erberik 2011).

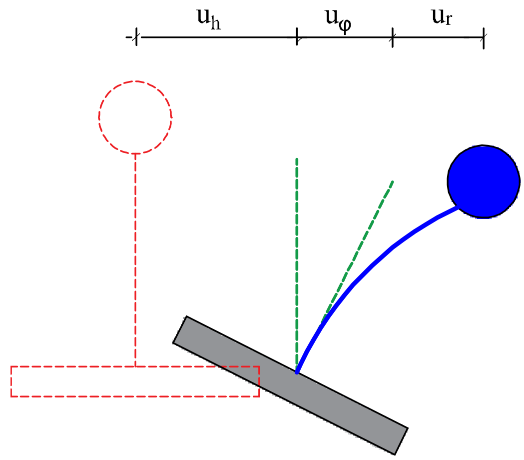

Fig. 3 Deformed shape of the SSI system subjected to earthquake ground motion

The results of Hassani *et al.* (2018) and Bararnia et al. (2018) studies indicate that there are two key design paramtertes that can influence the inelastic displacement ratios of SSI systems. The first parameter is the non-dimensional frequency (a_0), which is defined as structure-to-soil stiffness ratio

$$a_0 = \frac{\omega_{fix} h}{V_s} \tag{6}$$

where ω_{fix} = the natural frequency of vibration for fixed-base systems, V_s = shear wave velocity and h= structure effective height. Note that for ordinary building structures the practical range of a_0 can vary from zero (rigid soil with negligible SSI effect) to three (soft soil with predominant SSI effect). The other SSI influential parameter is the slenderness (aspect) ratio (h/r), which is defined as the structure height (h) normalized to the radius of foundation (r). In this study, other less important parameters such as foundation mass ratio, soil Poisson's ratio and soil material damping ratio are set to the typical values suggested by (Bararnia *et al.* 2018, Hassani *et al.* 2018).

3. Equations of motion for dynamic soil-structure interaction of SDOF oscillator system

The dynamic equations of motion of the 4-DOF SSI system illustrated in Fig. 1 can be written as (Nakhaei 2004)

$$m(\ddot{u}_g + \ddot{u}_l + \ddot{u}_\varphi) + k_{str}(u_l - u_f) + c_{str}(\dot{u}_l - \dot{u}_f) = 0 \tag{7}$$

$$m_f(\ddot{u}_g + \ddot{u}_f) + k_h - k_{str}(u_l - u_f) + c_h \dot{u}_f - c_{str}(\dot{u}_l - \dot{u}_f) = 0 \tag{8}$$

$$mh(\ddot{u}_g + \ddot{u}_l + \ddot{u}_\varphi) + (I_f + I)\frac{\ddot{u}_\varphi}{h} + k_\varphi \frac{u_\varphi}{h} + c_\varphi\left(\frac{\dot{u}_\varphi}{h} - \dot{\varphi}_1\right) = 0 \tag{9}$$

$$m_{\varphi_1} \ddot{\varphi}_1 + c_\varphi\left(\dot{\varphi}_1 - \frac{\dot{u}_\varphi}{h}\right) = 0 \tag{8}$$

As shown in Fig. 3, u_h and u_φ represent the components of horizontal displacement resulted from the sway and the rocking motions at the top story. u_r is the superstructure deformation, and $u_l = u_h + u_r$.

Using Eqs. (7)-(10), the general dynamic equation of the 4-DOF soil-struture system can be expressed as

$$[M]\{\ddot{U}\}+[C]\{\dot{U}\}+[K]\{U\}=-\{L\}\ddot{u}_g(t) \tag{11}$$

Where $\ddot{u}_g$ represents the earthquake input acceleration time histories. U stands for the displacement vector equal to $[u_l, u_f, \varphi, h\varphi_1]^T$. The notation T represents the transpose form of the matrix. The influence coefficient vector, L, is defined as $L = [m, m_f, m, 0]^T$. $[M]$, $[C]$, and $[K]$ respectively denote mass, damping, and stiffness matrices of the SSI system, and are defined as follows

$$[M]=\begin{bmatrix} m & 0 & m & 0 \\ 0 & m_f & 0 & 0 \\ m & 0 & m+\dfrac{I+I_f}{h^2} & 0 \\ 0 & 0 & 0 & \dfrac{m_{\varphi_1}}{h^2} \end{bmatrix},\quad [K]=\begin{bmatrix} k_{str} & -k_{str} & 0 & 0 \\ -k_{str} & k_{str}+k_h & 0 & 0 \\ 0 & 0 & \dfrac{k_\varphi}{h^2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$
$$[C]=\begin{bmatrix} c_{str} & -c_{str} & 0 & 0 \\ -c_{str} & c_{str}+c_h & 0 & 0 \\ 0 & 0 & \dfrac{c_\varphi}{h^2} & -\dfrac{c_\varphi}{h^2} \\ 0 & 0 & -\dfrac{c_\varphi}{h^2} & \dfrac{c_\varphi}{h^2} \end{bmatrix} \tag{12}$$

To take into account for the soil material damping, ζ_0, an extra connected dashpot is augmented to each spring and similarly an extra connected mass is added to each dashpot in the element of soil-foundation system based on the Voigt viscoelasticity model (Wolf 1994). Therefore, the mass and damping matrices of the SSI system are changed to the following equations normalized to the effective structural mass

$$[M]=\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & \dfrac{m_f}{m}+\dfrac{\xi_0}{\omega_0}\dfrac{c_h}{m} & 0 & 0 \\ 1 & 0 & 1+\dfrac{I+I_f}{mh^2}+\dfrac{\xi_0}{\omega_0}\dfrac{c_\varphi}{mh^2} & -\dfrac{\xi_0}{\omega_0}\dfrac{c_\varphi}{mh^2} \\ 0 & 0 & -\dfrac{\xi_0}{\omega_0}\dfrac{c_\varphi}{mh^2} & \dfrac{m_{\varphi_1}}{mh^2}+\dfrac{\xi_0}{\omega_0}\dfrac{c_\varphi}{mh^2} \end{bmatrix} \tag{13}$$

$$[K]=\frac{1}{m}\begin{bmatrix} k_{str} & -k_{str} & 0 & 0 \\ -k_{str} & k_{str}+k_h & 0 & 0 \\ 0 & 0 & \dfrac{k_\varphi}{h^2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\quad C=\frac{c_{str}}{m}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1+\dfrac{c_h}{c_{str}}+\dfrac{2\xi_0}{\omega_0}\dfrac{k_h}{c_{str}} & 0 & 0 \\ 0 & 0 & \dfrac{c_\varphi}{c_{str}h^2}+\dfrac{2\xi_0}{\omega_0}\dfrac{k_\varphi}{c_{str}h^2} & -\dfrac{c_\varphi}{c_{str}h^2} \\ 0 & 0 & -\dfrac{c_\varphi}{c_{str}h^2} & \dfrac{c_\varphi}{c_{str}h^2} \end{bmatrix} \tag{14}$$

Note that the influence coefficient vector, L, is also modified to $L = [1, m_f/m, 1, 0]^T$. As mentioned above, the non-dimensional parameters of a_0 and h/r are commonly treated as the governing parameters in assessing the SSI effects. All components of the mass, stiffness and damping matrices indicated in Eqs. (13) and (14) can be expressed based on these non-dimensional parameters as follows

$$k_h=\frac{8}{2-\nu}k\left(\frac{m}{\rho r^2 h}\right)^{-1}\left(\frac{h}{r}\right)a_0^{-2},\qquad \frac{c_h}{c_{str}}=\pi\frac{1}{2\zeta_{str}}\left(\frac{m}{\rho r^2 h}\right)^{-1}a_0^{-1} \tag{15}$$

$$\frac{k_\varphi}{h^2}=\frac{8}{3(1-\upsilon)}k_{str}\left(\frac{m}{\rho r^2 h}\right)^{-1}\left(\frac{h}{r}\right)^{-1}a_0^{-1},\qquad \frac{c_\varphi}{c_{str}h^2}=\frac{\pi}{4}\frac{1}{2\zeta_{str}}\left(\frac{\upsilon}{\upsilon_s}\right)\left(\frac{m}{\rho r^2 h}\right)^{-1}\left(\frac{h}{r}\right)^{-2}a_0^{-1} \tag{16}$$

Moreover, the additional mass moment of inertia ΔM_φ (for considering soil incompressibility) and the mass moment of inertia $m_{\varphi 1}$ (for the additional internal rotational degree of freedom) can be written as

$$\frac{I+I_f}{mh^2}+\begin{cases} \dfrac{\Delta M_\varphi}{mh^2} & for\ \upsilon>\dfrac{1}{3} \\ 0 & for\ \upsilon\le\dfrac{1}{3} \end{cases}=0.25\left(1+\frac{m_f}{m}\right)\left(\frac{h}{r}\right)^{-2}+\begin{cases} 0.3\pi\left(\upsilon-\dfrac{1}{3}\right)\left(\dfrac{m}{\rho r^2 h}\right)^{-1}\left(\dfrac{h}{r}\right)^{-3} & for\ \upsilon>\dfrac{1}{3} \\ 0 & for\ \upsilon\le\dfrac{1}{3} \end{cases} \tag{17}$$

$$\frac{m_{\varphi_1}}{mh^2}=\frac{9\pi^2}{128}(1-\upsilon)\left(\frac{\upsilon}{\upsilon_s}\right)^2\left(\frac{m}{\rho r^2 h}\right)^{-1}\left(\frac{h}{r}\right)^{-3} \tag{18}$$

The adopted two-dimensional soil-shallow-foundation-structure models introduced in Eqs. (11)-(18) are developed in MATLAB (2014) to perform nonlinear dynamic analysis. The analyses are performed in the time domain, using Newmark's beta method with default parameters $\gamma = 1/2$ and $\beta = 1/4$ as the time stepping method with an event-to-event solution approach. This method has been also implemented in general-purpose finite elements computer programs such as DRAIN-2DX (Prakash and Powell 1993) and PERFORM-3D (Computers and Structures 2006). The dynamic loads are incrementally exerted to the model of the soil-structure systems using a step-by-step solution strategy, while variable load increments are used to control equilibrium errors at each step of the analysis. An event is defined as any state change causing an alteration in the structural stiffness. Using the energy balance and equilibrium force calculations capability implemented in the developed computer program, equilibrium iterations are repeated until convergence. This is achieved through reducing the energy errors (difference between external work (or input energy) and sum of static elastic-plastic work, kinetic energy, and viscous damping work) and the difference between internal force and externally exerted force) are reduced below a target value. Note that in the step-by-step dynamic analysis, energy balance and/or equilibrium may not be fully satisfied at the end of each time step. In such cases, energy balance and equilibrium conditions can be satisfied by modifying the velocities and acceleration, respectively. These corrections will usually improve the accuracy (Prakash and Powell 1993).

4. Selected earthquake ground motions

To perform nonlinear dynamic analysis, an ensemble of 20 earthquake ground motions is selected from the database provided by PEER (http://ngawest2.berkeley.edu/). The selected ground motions have the following characteristics: (i) They correspond to sites of soil profile similar to class D based on FEMA-P-1050 (2015); (ii) They were recorded during 9 different strong earthquake events with moment

Table 1 Earthquake ground motions used in this study based on NEHRP site class D

EQ Index	Record ID	Event	M_w	Station Name	NEHRP site class	Distance (Km)	Strong Component		
							A(g)	V(cm/sec)	D(cm)
1	RSN68	San Fernando	6.61	LA - Hollywood Stor FF	D	22.8	0.2	21.7	15.9
2	RSN162	Imperial Valley-06	6.53	Calexico Fire Station	D	10.5	0.3	22.5	9.9
3	RSN169	Imperial Valley-06	6.53	Delta	D	22	0.3	33	20.2
4	RSN174	Imperial Valley-06	6.53	El Centro Array #11	D	12.6	0.4	44.6	21.3
5	RSN721	Superstition Hills-02	6.54	El Centro Imp. Co. Cent	D	18.2	0.4	48.1	19.3
6	RSN728	Superstition Hills-02	6.54	Westmorland Fire Sta	D	13	0.2	32.3	22.3
7	RSN752	Loma Prieta	6.93	Capitola	D	15.2	0.5	38	7.1
8	RSN776	Loma Prieta	6.93	Hollister - South & Pine	D	27.9	0.4	63	32.3
9	RSN777	Loma Prieta	6.93	Hollister City Hall	D	27.6	0.2	45.5	28.5
10	RSN778	Loma Prieta	6.93	Hollister Differential Array	D	24.8	0.3	44.2	19.7
11	RSN783	Loma Prieta	6.93	Oakland -Outer Harborr	D	74.26	0.3	41.9	9.6
12	RSN953	Northridge-01	6.69	Beverly Hills - 14145 Mulhol	D	17.2	0.5	66.7	12.2
13	RSN960	Northridge-01	6.69	Canyon Country - W Lost Cany	D	12.4	0.4	44.4	11.3
14	RSN1003	Northridge-01	6.69	LA - Saturn St	D	27	0.4	41.6	5
15	RSN1077	Northridge-01	6.69	Santa Monica City Hall	D	26.5	0.9	41.6	15.2
16	RSN1107	Kobe	6.9	Kakogawa	D	22.5	0.3	26.9	8.8
17	RSN1116	Kobe	6.9	Shin-Osaka	D	19.2	0.2	31.3	8.4
18	RSN1158	Kocaeli	7.51	Duzce	D	15.4	0.3	58.9	44.1
19	RSN1203	Chi-Chi	7.62	CHY036	D	16	0.2	44.8	34
20	RSN3749	Cape Mendocino	7.01	Fortuna Fire Station	D	20.4	0.3	38.1	16.7

magnitude larger than 6.5; (iii) Their closest distance to the fault rupture is larger than 10 Kilometers; (iv) At least one of the two horizontal components has a PGA and a PGV larger than 0.2 g and 15 cm/sec, respectively; and (iv) They are not classified as ground motions having pulse like characteristics. In this study, for each event, the horizontal component with larger PGV was used for the analyses (named strong component). The main characteristics of the considered ground motions are provided in Table 1, while their elastic response spectra with their mean values are presented in Fig. 4.

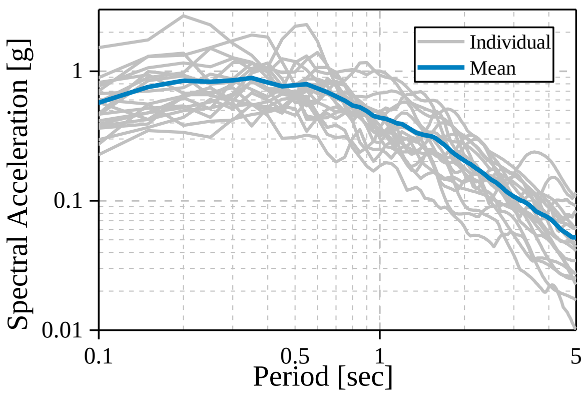

Fig. 4 Elastic response spectra of selected earthquake ground motions

5. Response variable and methodology

Proper structural design is generally achieved when the local ductility demand values of all structural elements are lower than their capacities. Thus, it is necessary to estimate the required lateral strength to ensure that the global displacement ductility demand is limited to a certain target value. To calculate the displacement ductility demands under a given strong ground motion, the lateral yield strength, f_y, is defined as

$$f_y = \frac{f_e}{R} \tag{19}$$

where f_e is the lateral strength required to ensure the structure remains in the elastic range and R is the response modification factor. Displacement-based ductility factor for a given response modification factor R, is the defined as the maximum displacement (u_m) normalized to its yield displacement (u_y)

$$\mu = \frac{u_m}{u_y} \tag{20}$$

For a given strong ground motion and structure, the problem is to calculate the minimum lateral strength ($f_{y\mu}$) in order to assure that the ductility ratio in the structure reaches the predefined target value. In the present study, two iteration algorithms proposed by Song and Gavin (2011), and Ganjavi and Hao (2012) are utilized to achieve the response modification factor (R_μ) corresponding to the

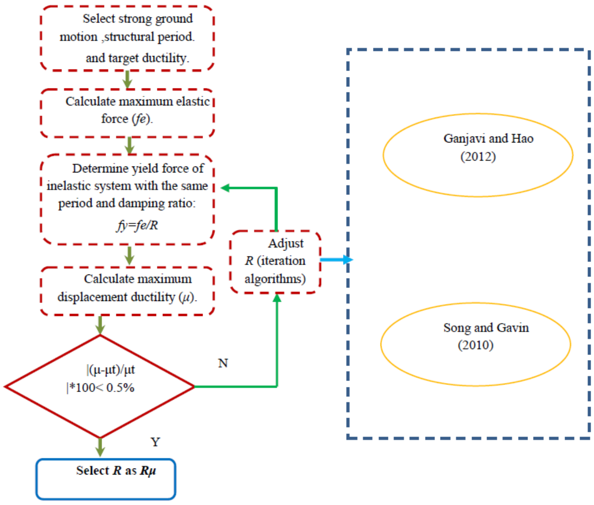

Fig. 5 The flowchart showing the general iteration process to obtain R_μ spectrum based on two algorithms

predefined target ductility. Fig. 5 shows the general iteration process adopted to calculate the minimum lateral strength ($f_{y\mu}$) in the soil-structure systems based on the two aforementioned iteration algorithms.

The relative elastic and inelastic displacements of the structure are used to calculate response modification factor of the soil-structure system. It means that displacements resulting from rigid body motions of the foundation have been removed. In fact, to determine lateral displacements in a soil-structure system, inelastic displacement of fixed-base structure resulting from nonlinear behavior is added to the rigid body displacements resulting from the rocking motion of the foundation. Albeit this approach leads to increasing the absolute structural displacements of SSI system, the role of SSI in nonlinear deflections of superstructure is not considered. Therefore, in computing the inelastic displacement factors of SSI systems, the rigid body displacements of the stories due to sway and rocking motions of the foundation are removed from the total elastic and inelastic deformations of the structure.

An extensive parametric study was performed to evaluate the effects of SSI key design parameters on the response modification factor (R_μ) of structures with different stiffness degrading and non-degrading hysteric behaviors. The results were obtained under 20 ground motions, 35 different periods of vibration ranging from 0.1 to 3 sec, 3 hysteresis models, 3 aspect ratios (h/r=1,3,5), 4 non-dimensional frequencies (a_0 = 0, 1,2,3) and 5 levels of displacement ductility (μ=2,3,4,5,6). The results of the parametric study are explained in details in the upcoming section.

6. Ductility-dependent response modification factor of soil-structure systems with different hysteretic behaviors

The soil-structure systems introduced in the previous section (6300 models in total) were subjected to the 20 earthquake ground motions listed in Table 1. The average

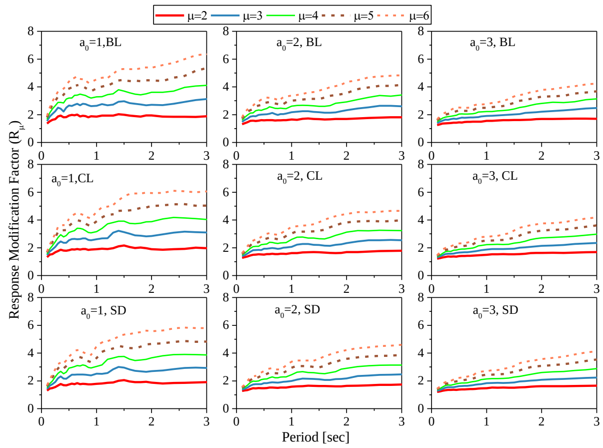

Fig. 6 Effect of ductility demand on response modification factor spectra for SSI systems with different hysteretic behaviors and a_0; (h/r= 1; average of 20 earthquakes)

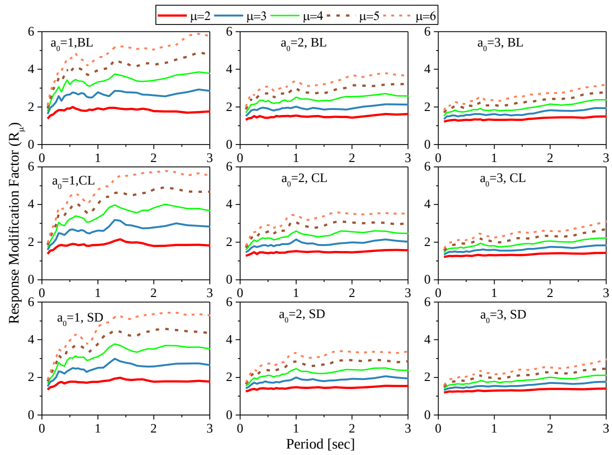

Fig. 7 Effect of ductility demand on mean response modification factor spectra for SSI systems with different hysteretic behaviors and a_0; (h/r= 3)

results for different non-dimensional frequencies, ductility demands and hysteretic behaviors are depicted in Figs. 6-8 for squat (h/r= 1), average (h/r= 3) and slender (h/r= 5) structures, respectively. In all figures provided in this paper, the horizontal axis is the natural period of vibration for fixed-base system, T_{fix}, and the vertical axis is ductility-dependent response modification factor. As seen, a similar trend can be observed for the variations of average response modification factors with respect to the period of vibration for all the hysteretic models considered in this study. For the case of short-period systems with low SSI effect (a_0= 1), the R_μ values are less than the target ductility demands. This behavior is more pronounced as the ductility demand increases. Nevertheless, by increasing the vibration period, response modification factors increase and tend towards the target ductility demands for long-period soil-structure systems. In addition, it is shown that increasing the soil flexibility (i.e., increasing a_0 value) generally results in decreasing the response modification factors. These results are consistent with those reported by Ghannad and Jahankhah (2007) and for respectively SDOF and MDOF soil-structure systems with non-degrading hysteretic

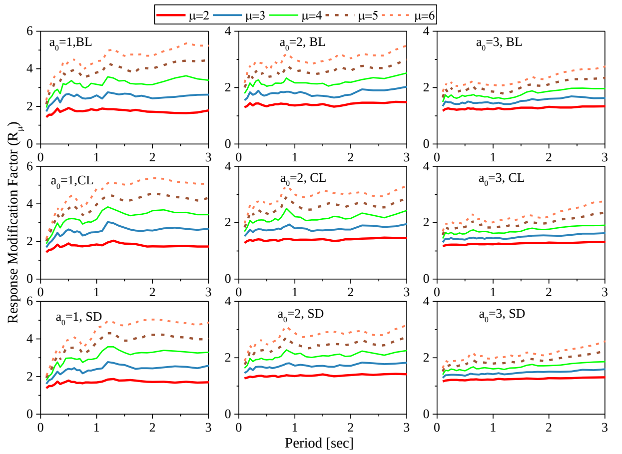

Fig. 8 Effect of ductility demand on mean response modification factor spectra for SSI systems with different hysteretic behaviors and a_0; (h/r= 5)

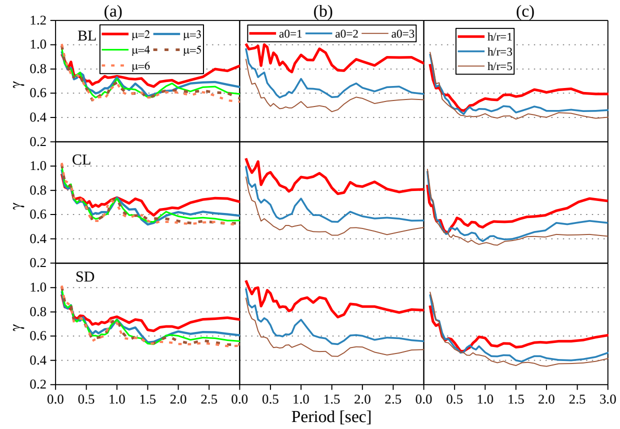

Fig. 9 Effect of SSI interacting parameters on response modification factor spectra for soil-structure systems with different hysteretic behaviors (average of 20 earthquakes)

models. However, the effects of stiffness degradation of the superstructures on the response modification factors of SSI systems were ignored in the previous studies. This phenomenon is especially important for reinforced concrete structures under strong ground motions and will be investigated and discussed in the next subsections.

6.1 Effect of soil flexibility and aspect ratio

As stated in the previous section, the R_μ of a soil-structure system is different from that of its fixed-based system counterpart. For a more detailed study of the SSI effects, a new parameter γ is introduced as the ratio of R_μ for the SSI system to the corresponding fixed-base system, when subjected to the same ground acceleration as follows

$$\gamma = \frac{R_{\mu-SSI}}{R_{\mu-fixed}} \tag{19}$$

The parameter γ, is determined for all the SSI systems having different periods of vibration and ductility demands (μ) under the selected ground motions listed in Table 1. This parameter can be considered as the numeric representation to assess the influence of soil flexibility on the response modification factors of structures, which is usually ignored in the current design approaches.

The value of γ less than 1.0 indicates that using response modification factors determined based on fixed-based assumption will result in an underestimation of the strength demands and hence unconservative (unsafe) design solutions. higher expected ductility demands. Conversely, the γ values larger than 1.0 imply that SSI has beneficial impact on the seismic performance of the system and, hence, using response modification factors calculated based on fixed-base assumption for seismic design of a soil-structure system leads to overestimated strength demands and therefore conservative (safe) design solutions. The results provided for various hysteretic models in Fig. 9 can be used to assess the effects of the following key design parameters: (a) Level of inelastic behavior μ for constant values of a_0 = 2 and h/r = 3, (b) Non-dimensional frequency a_0 for constant values of h/r = 3 and μ = 4, and (c) Aspect ratio h/r for constant values of a_0 = 3 and μ = 6.

As can be seen from Fig. 9(a), ductility demand has negligible effect on the γ value of the short-period structures, whereas for the systems with longer periods increasing the ductility demands leads to decreasing the γ value, implying larger differences between the R_μ of the soil-structure system and the corresponding fixed-base one. Fig. 9(b) shows that regardless of the structural hysteretic model, increasing the soil flexibility (and therefore SSI effects) results in a significant reduction of R_μ for the entire range of periods. In fact, the more the SSI effect the greater is the difference between the response modification factor of the flexible-base and the fixed-base systems. This confirms that using R_μ of fixed-base systems leads to significant underestimation of inelastic strength demands of soil-structure systems especially for the case of predominant SSI effects (i.e., a_0 = 2, 3). Therefore, very large ductility demands can be expected when structures located on soft soil are designed based on the fixed-base response modification factor. Aspect ratio or so-called slenderness ratio is another SSI interacting parameter that can affect the response modification factor of the system when compared to its fixed-base counterpart. As shown in Fig. 9(c), this parameter has imperceptible influence on short-period soil-structure systems with different superstructure hysteretic models. In contrast, for SSI systems with longer-periods (i.e., T_{fix}>0.5 s), R_μ considerably decreases as aspect ratio increases. Figs. 6-9 show a similar trend for all hysteretic models used in this study. However, the results indicate that the influence of each design parameter is affected by the selected hysteretic models as will be discussed in the next section.

6.2 Effect of hysteretic behavior of the superstructure

As stated before, previous studies on the effects of SSI on the inelastic response of structures were mainly restricted to the structural systems without considering strength and stiffness degradations. However, RC structures may exhibit considerable strength and stiffness degradations

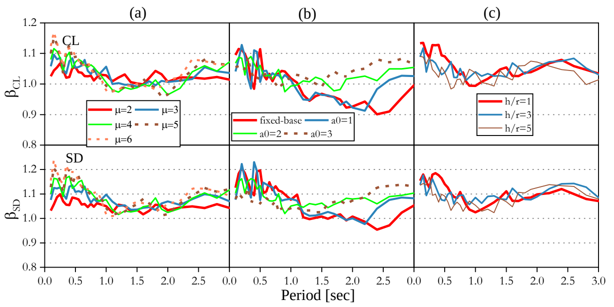

Fig. 10 Effect of hysteretic behavior on response modification factor spectra for soil-structure systems (average of 20 earthquakes)

Table 2 Constant coefficients of Eq. (23) for BL hysteretic model (non-degrading systems)

h/r	a	b	c
1	$-1.2+0.8375\mu+4.095\exp(-a_0)$	$-0.85+0.0254\mu+a_0^{0.703}$	$1.13a_0^{-0.466}$
3	$0.78322+0.323\mu+1.1177\exp(-a_0)$	$-0.766+0.02155\mu+a_0^{1}$	$0.973a_0^{-1.058}$
5	$1.175+0.2064\mu+0.34988\exp(-a_0)$	$-0.6397+0.022768\mu+a_0^{1.154}$	$0.767a_0^{-1.316}$

under strong ground motions. To examine the effect of superstructure stiffness degradation on the response modification factor of the soil-structure system, two new parameters β_{CL} and β_{SD} are introduced representing the ratio of the response modification factor for systems with non-degrading structures ($R_{\mu\text{-}BL}$) to those for stiffness degrading structures using Modified-Clough ($R_{\mu\text{-}CL}$) and Peak-oriented ($R_{\mu\text{-}SD}$) models, respectively.

$$\beta_{CL}=\frac{R_{\mu-BL}}{R_{\mu-CL}} \qquad \beta_{SD}=\frac{R_{\mu-BL}}{R_{\mu-SD}} \tag{20}$$

The β_{CL} and β_{SD} parameters can be efficiently used to assess the influence of the structural stiffness degrading behavior on the seismic response and inelastic strength demands of the SSI system (through Eq. (17)).

The average values of β_{CL} and β_{SD} are calculated for all soil-structure systems subjected to the 20 selected earthquake ground motions. The results are presented in Fig. 10(a) for various μ and constant values of a_0= 2 and *h/r*= 1, in Fig. 10(b) for various a_0 and constant values of *h/r*=3, μ=4, and in Fig. 10(c) for various *h/r* and constant values of a_0= 3, μ= 6. From the Fig. 10(a) it is observed that for short-period systems, increasing the ductility demand leads to an increase in both β_{CL} and β_{SD} values. It is shown that for high inelastic systems, the β_{CL} and β_{SD} can reach up to 1.12 and 1.22 for CL and SD models, respectively. This implies that neglecting the stiffness degradation of superstructures can result in underestimation of inelastic strength demands in soil-structure systems, leading to an unexpected high level of ductility demand in the structures located on soft soil. Fig. 10(b) shows that for short-period systems, non-dimensional frequency (a_0) has a negligible effect on β_{CL} and β_{SD}, whereas for long-period systems increasing a_0 generally results in an increase in the β values. The results also indicate that for fixed-base systems, using R_μ corresponding to non-degrading systems

Table 3 Constant coefficients of Eq. (23) for CL hysteretic model (Modified-Clough stiffness degrading systems)

h/r	a	b	c
1	$-1.86+1.072\mu+9.07\exp(-a_0)$	$-0.95+0.0347\mu+a_0^{0.775}$	$1.39a_0^{-0.56}$
3	$-0.157+0.337\mu+7.83\exp(-a_0)$	$-0.9+0.0311\mu+a_0^{1.1}$	$1.22a_0^{-1.23}$
5	$-0.42-0.032\mu+8.9\exp(-a_0)$	$-0.82+0.044\mu+a_0^{1.44}$	$0.96a_0^{-1.84}$

Table 4 Constant coefficients of Eq. (23) for SD hysteretic model (Peak-oriented stiffness degrading systems)

h/r	a	b	c
1	$-2.044+1.124\mu+8.35\exp(-a_0)$	$-0.85+0.036\mu+a_0^{0.807}$	$1.23a_0^{-0.44}$
3	$-0.6+0.37\mu+7.83\exp(-a_0)$	$-0.77+0.0293\mu+a_0^{1.16}$	$1.073a_0^{-1.09}$
5	$-0.484-0.0152\mu+7.8\exp(-a_0)$	$-0.65+0.04\mu+a_0^{1.51}$	$0.862a_0^{-1.73}$

will provide conservative estimates of strength demands. Nevertheless, for systems with sever SSI effects (i.e., a_0= 2 and 3) using R_μ calculated based on non-degrading systems will lead to underestimation of the strength demands especially in long-period systems. Finally, Fig. 10(c) indicates that the aspect ratio *h/r* has generally an insignificant influence on the β values.

7. Practical equation for estimating R_μ of SSI systems with stiffness degradation

As discussed before, estimating the required strength demand of the design structure to achieve a predefined target ductility is an important step in the current seismic design practices. However, the results of the present study and those reported by other researchers (Avilés J.a Pérez-Rocha 2005, Lu *et al.* 2016) demonstrated that using response modification factors of fixed-base systems for flexible-base structures could lead to underestimation of the actual seismic strength demands and therefore unconservative design solutions. Moreover, the present study showed that ignoring the superstructure stiffness degradation effects can also result in underestimated results. To address these issue, in this section a simplified equation is developed for estimating ductility-dependent response modification factors for soil-structure SDOF systems with stiffness degrading structures through nonlinear regression analyses using Levenberg-Marquardt approach in MATLAB (MATLAB 2014). The proposed equation is a function of fundamental period of fixed-base systems, T_{fix}, ductility level, μ, hysteretic behavior of superstructure, and also SSI key parameters including a_0, and *h/r*

$$R_\mu = 1+\left(\frac{\mu-1}{1-\exp(-aT_{fix})+\dfrac{b}{T^c}}\right) \tag{23}$$

where *a, b, c* are coefficients which depend on a_0, *h/r,* μ and can be calculated from the equations provided in Tables 2-4 for different hysteretic models. Figs. 11-13 show a comparison of mean inelastic response modification factors

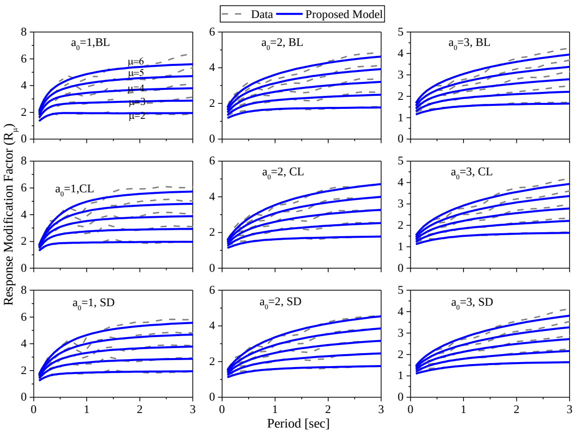

Fig. 11 Comparison of the mean response modification factors obtained from nonlinear dynamic analyses with those calculated from Eq. (23): (h/r=1, Average of 20 earthquakes)

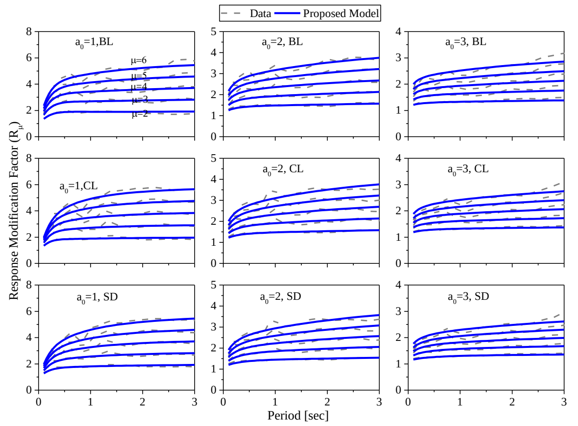

Fig. 12 Comparison of the mean response modification factors obtained from nonlinear dynamic analyses with those calculated from Eq. (23): (h/r=3, Average of 20 earthquakes)

obtained from nonlinear dynamic analyses with those calculated using Eq. (23) for different key SSI parameters, inelastic behaviors and hysteretic models. As observed, despite its simplicity, the proposed expression shows very good agreement with the numerical data, implying that it has the capability to capture efficiently both SSI and structural stiffness degradation effects on response modification factors of soil-shallow-foundation systems.

To assess the efficiency and reliability of the proposed equation to predict the actual values of R_μ, three different statistical parameters were used, including (i) root-mean-square error (RMSE), (ii) *R*-square, and (iii) index of agreement (I_a).

RMSE is a criterion of correctness usually used to compare predicting errors of various methods for specific data which does not exist among the datasets due to its scale dependency, and is calculated for *n* various anticipations as the mean square root of the deviations squares as follows

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}{n}} \tag{24}$$

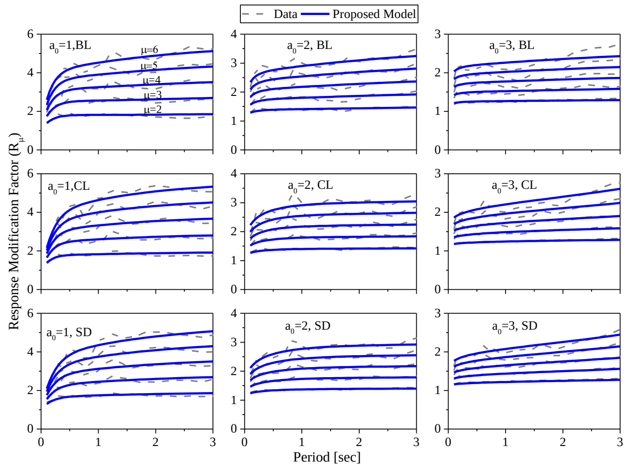

Fig. 13 Comparison of the mean response modification factors obtained from nonlinear dynamic analyses with those calculated from Eq. (23): (h/r=5, Average of 20 earthquakes)

where y_i and $\hat{y}_i$ represent the calculate and the anticipated values, respectively, and *n* is the number of measurements. Lower RMSE values (closer to zero) represent better predictions.

R-square is defined as the square of the interrelationship among the response and the anticipated response values, and is described as the sum of squares of the regression (*SSR*) normalized to the total sum of squares (*SST*). *SSR* and *SST* are expressed as

$$SSR = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 \tag{25}$$

$$SST = \sum_{i=1}^{n}(y_i - \bar{y})^2 \tag{26}$$

where the parameter $\bar{y}$ is the associated mean values of the calculated parameters. According to the above equations, *R-square* can be defined as

$$R\text{-}square = \frac{SSR}{SST} \tag{27}$$

R-square can take on any value between 0 and 1, with a value closer to 1 indicating that the model could efficiently represent the variability of the data around its mean (MATLAB 2014).

The parameter I_a is defined as a measure of the intensity of the model anticipation errors and can be calculated by the following expression

$$Ia = 1 - \frac{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}{\sum_{i=1}^{n}\left(|y_i - \bar{y}| + |\hat{y}_i - \bar{y}|\right)^2} \tag{28}$$

The I_a values are between zero and one, where one shows an excellent match and zero signifies there is no correlation at all (Willmott *et al.* 2012).

The numerical measures of *RMSE*, *R-square* and I_a for the data presented in Figs. 11-13 are summarized in Table 5. The results clearly demonstrate the reliability and accuracy

Table 5 Calculated statistical indices

	h/r	RMSE	R-Squared	Ia
BL	1	0.153	0.976	0.990
	3	0.125	0.980	0.990
	5	0.119	0.982	0.990
CL	1	0.154	0.970	0.994
	3	0.125	0.983	0.996
	5	0.120	0.983	0.995
SD	1	0.126	0.984	0.996
	3	0.118	0.983	0.996
	5	0.119	0.981	0.995

of Eq. (23) in predicting the R_μ values, where RMSE index was always less than 0.153 and R-Squared and I_a indices were above 0.970 and 0.990, respectively.

8. Sensitivity analysis on the accuracy of the proposed equation subjected to an ensemble of synthetic earthquake ground motions

In this section a sensitivity analysis is performed to investigate the adequacy of the proposed practical expression when the stiffness degrading SDOF soil-structure systems are subjected to a set of synthetic spectrum compatible earthquake ground motions. To this end, an ensemble of SSI systems whose inelastic strength demands were already computed by Eq. (23) are analyzed under 7 synthetic earthquake accelerograms compatible with the mean spectrum of the 20 selected ground motions. For each synthetic earthquake, the ductility demands of SSI systems are calculated and then compared with the target ductility ratio. For a selected earthquake $\ddot{u}_g(t)$, the modulating function parameters, ψ, is calculated by matching the expected cumulative energy of the stochastic process with those of preselected target accelerogram over the entire ground motion duration. For this purpose, the cumulative energy of preselected target ground motion accelerogram, $E_a(t)$, and the expected cumulative energy of the synthetic earthquake, $E_x(t)$ are defined by (Rezaeian and Kiureghian 2010)

$$E_a(t) = \int \ddot{u}_g(\tau)d\tau \tag{29}$$

$$E_x(t) = \int_0^t q^2(\tau,\psi)d\tau \tag{30}$$

where $q^2(\tau,\psi)$ is the modulating function that is obtained by matching the two cumulative energy (CE) curves through minimizing the following error function ε_q (Rezaeian and Kiureghian 2010).

$$\varepsilon_q = \frac{\int_0^{t_n} |E_x(t) - E_a(t)|dt}{\int_0^{t_n} E_a(t)dt} \tag{31}$$

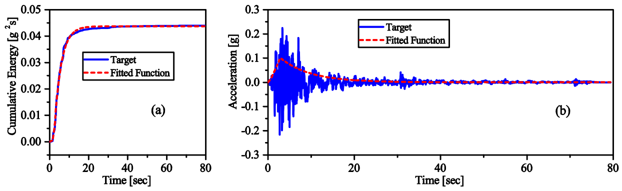

Fig. 14(a) cumulative energy curve of the target accelerogram and the fitted modulating function, (b) associated modulating function superimposed on the target accelerogram

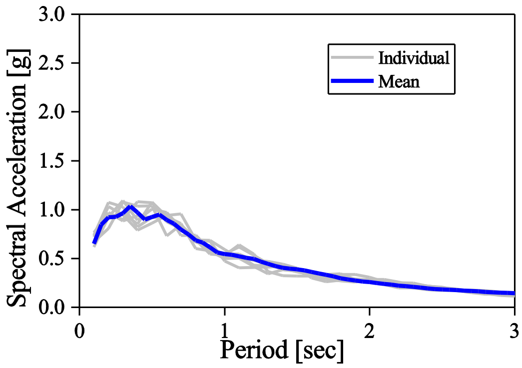

Fig. 15 Individual and mean response spectra of the synthetic earthquake ground motions

The ε_q error function represents the difference between the area underneath the target accelerogram energy curve and the expected synthetic accelerogram. The numerator in Eq. (31) denotes the absolute area between the two aforementioned CE curves, while the denominator states the area underneath the target accelerogram energy curve. Fig. 14 shows the $E_a(t)$ and $E_x(t)$ functions after matching process. The modulation function used in this study is the modified function proposed by Housner and Jennings (1964). To verify the accuracy of the results, after determining the modulation function, the mean spectrum of compatible synthetic earthquake ground motions was calculated by using the computer program, SeismoArtif (2016). Fig. 15 compares the acceleration response spectrum of the individual synthetic earthquakes with the mean spectrum.

To investigate the adequacy of the proposed Eq. (23) for estimating the inelastic strength demands of SSI systems with stiffness degradation, soil-structure systems with various target ductility demands (μ= 2, 3, 4 ,5, 6) and different hysteretic behaviors were analyzed under each of the 7 synthetic ground motions. Fig. 16 compares the resulted ductility demands (grey points) and the mean ductility ratios (blue points) for different target ductility demands. In this figure, the data points below and above the red line indicate that the obtained ductility demands are smaller and larger than the target values, respectively. As observed, there is a good agreement between the results of mean and target ductility demands, demonstrating the adequacy of the proposed expression (Eq. (23)) to estimate inelastic strength demands of SSI systems with stiffness degradation of structures. Based on the results, the maximum difference between the target and average of

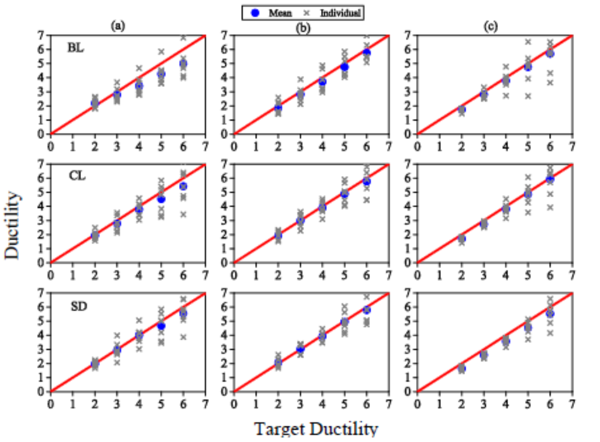

Fig. 16 Comparison of the mean response modification factors obtained from numerical analyses subjected to the selected synthetic earthquakes with those calculated from Eq. (21): (a) T_{fix}= 0.5, a_0= 1 and h/r= 1 ; (b) T_{fix}= 1, a_0= 2 and h/r= 3; (c) T_{fix}= 2, a_0= 3 and h/r= 5

actual values was 15%, which can be considered acceptable for the preliminary design of buildings in common practice.

9. Conclusions

A comprehensive parametric study was performed to investigate the influence of key SSI interacting parameters (non-dimensional frequency and slenderness ratio), natural period of vibration, ductility demand and hysteretic behavior on the response modification factors (R_μ) of soil-structure systems. To achieve this, the nonlinear dynamic response of 6300 soil-structure systems were studied under an ensemble of 20 strong ground motions recorded from pervious earthquake events as well as 7 synthetic spectrum compatible ground motions. The results were then used to propose a practical equation to predict the response modification factors for stiffness degrading soil-shallow-foundation systems. Based on the results of this study, the following conclusions can be drawn:

• Regardless of the structural hysteretic model, increasing the soil flexibility (i.e., increasing a_0 value) generally results in a considerable reduction of R_μ for the entire range of periods. Therefore, using response modification factors of fixed-base systems may lead to underestimation of inelastic strength demands of structures located on soft soil. While the slenderness ratio has negligible effects on the response modification factor of short-period soil-structure systems, R_μ decreases in long-periods systems (i.e., T_{fix}>0.5 s) by increasing the slenderness ratio.

• The stiffness degradation of superstructure can considerably affect the response modification factor of the soil-structure system, especially for high inelastic structures (i.e. μ= 6), where β_{CL} and β_{SD} factors reached up to 1.12 and 1.22 for CL and SD hysteretic models, respectively. This implies that neglecting the stiffness degradation of superstructures can result in underestimation of inelastic strength demands in soil-structure systems, leading to an unexpected high level of ductility demand in the structures located on soft soil.

• Based on nonlinear regression analyses, a simplified equation was proposed for estimating ductility-dependent response modification factors for stiffness degrading SDOF soil-structure systems. The proposed equation is a function of fundamental period of fixed-base systems (T_{fix}), ductility level (μ), hysteretic behavior of superstructure, as well as the SSI key parameters including dimensionless frequency a_0, and slenderness (aspect) ratio of structure h/r. The efficiency and reliability of the proposed equation was demonstrated by using RMSE, R-square and I_a indices.

• A sensitivity analysis was performed to investigate the adequacy of the proposed equation to estimate the inelastic strength demands of stiffness degrading SDOF soil-structure systems subjected to a set of synthetic spectrum compatible earthquakes. A very good agreement was observed between the target and average of actual ductility demands (less than 15% error), demonstrating the adequacy of the proposed expression for practical applications in earthquake engineering design.

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