Structural Engineering and Mechanics, *Vol. 68, No. 2 (2018) 215-226*
DOI: https://doi.org/10.12989/sem.2018.68.2.215

Effect of heat source and gravity on a fractional order fiber reinforced thermoelastic medium

Kavita Jain[a], Kapil Kumar Kalkal* and Sunita Deswal[b]

Department of Mathematics, Guru Jambheshwar University of Science and Technology, Hisar-125001, Haryana, India

(Received November 8, 2017, Revised August 20, 2018, Accepted August 26, 2018)

Abstract. In this article, the theory of fractional order two temperature generalized thermoelasticity is employed to study the wave propagation in a fiber reinforced anisotropic thermoelastic half space in the presence of moving internal heat source. The whole space is assumed to be under the influence of gravity. The surface of the half-space is subjected to an inclined load. Laplace and Fourier transform techniques are employed to solve the problem. Expressions for different field variables in the physical domain are derived by the application of numerical inversion technique. Physical fields are presented graphically to study the effects of gravity and heat source. Effects of time, reinforcement, fractional parameter and inclination of load have also been reported. Results of some earlier workers have been deduced from the present analysis. **2010MSC: 74A15, 80A20.**

Keywords: fiber reinforcement; gravity; heat source; two temperature; fractional order theory

1. Introduction

In the last three decades, the analysis of stress and deformation of fiber reinforced composite materials has been an important research area of solid mechanics. Wave propagation in a reinforced medium plays a very interesting role in civil engineering and geophysics. Fiber reinforced composites are used in a variety of structures due to their low weight and high strength. The components of a reinforced composite act together as a single anisotropic unit as long as they remain in elastic condition and this is the main characteristic property of reinforced composites. Belfield *et al.* (1983) introduced the idea of continuous self reinforcement at every point of an elastic solid. The problem of surface waves in fiber reinforced anisotropic medium was discussed by Sengupta and Nath (2001). Singh and Singh (2004) discussed the problem of reflection of plane waves at the free surface of a fiber reinforced elastic half space. Abbas *et al.* (2011) studied wave propagation in a fiber reinforced anisotropic thermoelastic half space under the effect of magnetic field. Sarkar *et al.* (2016) investigated the influences of fractional parameter, hydrostatic initial stress and magnetic field on the plane waves in a fiber reinforced generalized thermoelastic solid half space.

In classical theory of elasticity, gravity field is generally neglected. The effect of gravity in the problem of wave propagation in solids, particularly on vibrations in an elastic globe, was first studied by Bromwich (1898). Subsequently, an investigation on the effect of gravity was discussed by Love (1911), who showed that the velocity of Rayleigh waves is increased to a significant extent by gravitational field when wavelengths are large. The effects of rotation and gravity in a generalized thermoelastic medium were analyzed by Ailawalia and Narah (2009). Abd-Alla *et al.* (2017) investigated the propagation of waves in a homogeneous, orthotropic thermoelastic medium under the effect of gravity field.

Due to extensive engineering applications, such as pulsed laser cutting and welding, high speed machining and grinding, several research works have been devoted to problems involving a moving heat source. Danilovskaya (1950) was the first who solved a dynamical heat source problem under the purview of coupled thermoelasticity. It is worthwhile to mention the contributions of Eason and Sneddon (1959) and Nowacki (1959) under the coupled theory with instantaneous and moving heat source. Baksi *et al.* (2008) studied a thermoviscoelastic problem in an infinite isotropic medium in the presence of a point heat source by using joint Laplace-Fourier transform technique and eigen value approach. Ailawalia and Singla (2015) employed the dual-phase lag heat transfer model to study the problem of isotropic generalized thermoelastic medium with internal heat source.

Chen and Gurtin (1968) and Chen *et al.* (1968, 1969) presented a new theory including two temperatures of heat conduction in deformable bodies. This theory suggests that heat conduction comprises of two different temperatures, *i.e.*, the conductive temperature ϕ and the thermodynamical temperature θ. The first is due to the thermal process and the second is due to the mechanical process inherent between the particles and layers of elastic materials. For time independent situations, the difference between these two temperatures is proportional to the heat

*Corresponding author, Ph.D.
E-mail: kapilkalkal_gju@rediffmail.com
[a]M.Sc.
E-mail: kavitaj217@gmail.com
[b]Ph.D.
E-mail: spannu_gju@yahoo.com

ISSN: 1225-4568 (Print), 1598-6217 (Online)

supply and in the absence of heat supply, the two temperatures are identical. Ailawalia *et al.* (2009) studied the deformation of a rotating generalized thermoelastic medium with two temperature under the influence of gravity. Said and Othman (2016) applied the two-temperature theory of generalized thermoelasticity to study the wave propagation in a fiber-reinforced magneto-thermoelastic medium in the context of the three-phase-lag model and Green-Naghdi theory without energy dissipation. Recently, Yadav *et al.* (2017) analyzed the thermoelastic interactions in a homogeneous isotropic electro-microstretch semi-space caused by mechanical source acting on the initially stressed surface under the purview of two temperature thermoelasticity theory without energy dissipation.

Nowadays, fractional calculus is playing a crucial role in developing several models and it has been verified that the use of fractional order derivatives/ integrals leads to the formulation of certain physical problems, which are more economical and appropriate than the classical approach. Furthermore, fractional calculus has also been proved to be very useful in the areas of diffusion, heat conduction, continuum mechanics, viscoelasticity and electromagnetic etc. The first application of fractional derivatives was given by Abel, who applied fractional calculus in the solution of tautochrone problem. Povstenko (2005) developed a quasi-static uncoupled thermoelastic model based on the heat conduction equation with fractional order time derivatives. Sherief *et al.* (2010) has constructed a new model of thermoelasticity using fractional calculus with second sound. Taking into consideration the new Taylor series expansion of time fractional order developed by Jumarie (2010), Ezzat (2010) established a new model of fractional order generalized thermoelasticity. El-Karamany and Ezzat (2011) introduced the two temperature fractional thermoelasticity theory for non-homogeneous anisotropic elastic solid, proved uniqueness and reciprocal theorems and established the convolutional variational principle. Deswal and Kalkal (2013) studied a half space problem in the context of fractional order micropolar thermo-viscoelasticity with two temperatures. Youssef (2013) solved a one dimensional problem of fractional order two temperature generalized thermoelastic medium subjected to moving heat source. Recently, Deswal *et al.* (2017) studied the magneto-thermo-viscoelastic interactions in a homogeneous, isotropic medium under generalized thermoelasticity theory without energy dissipation with fractional order strain.

In the present analysis, the propagation of plane waves in a fiber reinforced generalized thermoelastic medium in the presence of moving internal heat source and gravity has been investigated by employing the fractional order two temperature theory. Laplace Fourier double transform technique has been used to solve the non-dimensional field equations. By using numerical inversion technique, expressions for displacement, temperature and stresses are computed in the physical domain. Effects of different parameters on various fields inside the medium are analyzed graphically. Some special cases are also derived.

2. Basic governing equations

Following Belfield *et al.* (1983) and El-Karamany and Ezzat (2011), the constitutive equation and the field equations for the proposed model of generalized thermoelasticity are given as:

(i) The constitutive relation

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha\left(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}\right) + 2(\mu_L - \mu_T) \times (a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j - \beta_{ij}\theta\delta_{ij} \tag{1}$$

(ii) The strain-displacement relation

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{2}$$

(iii) Equation of motion

$$\sigma_{ji,j} + F_i = \rho\ddot{u}_i . \tag{3}$$

(iv) Two-temperature fractional order heat conduction equation

$$K_{ij}\phi_{,ij} = \frac{\partial}{\partial t}\left(1 + \frac{\tau_0^m}{\Gamma(m+1)}\frac{\partial^m}{\partial t^m}\right)\left(\rho c_E\theta + T_0\beta_{ij}u_{i,j}\right) - \left(1 + \frac{\tau_0^m}{\Gamma(m+1)}\frac{\partial^m}{\partial t^m}\right)Q . \tag{4}$$

In the above equation, the fractional order derivative proposed by Caputo is defined as

$$D^m f(t) = \frac{1}{\Gamma(1-m)}\int_0^t (t-\tau)^{-m}\frac{\partial f(\tau)}{\partial\tau}d\tau, \qquad m \in [0,1) .$$

(v) Relation between thermodynamical and conductive temperature

$$\phi - \theta = a_{11}\phi_{,11} + a_{22}\phi_{,22} . \tag{5}$$

Here σ_{ij}'s are the components of stress tensor, e_{ij}'s are the components of strain tensor, e_{kk} is the dilatation, λ, μ_T are the elastic constant, β_{ij} is thermoelastic coupling tensor, $\alpha, \beta, (\mu_L - \mu_T)$ are reinforcement parameters, δ_{ij} is the Kronecker delta and $\vec{a} = (a_1, a_2, a_3)$, $a_1^2 + a_2^2 + a_3^2 = 1$. We choose the fiber reinforcement direction as $\vec{a} = (1, 0, 0)$. ρ is the density of the medium, c_E is specific heat at constant strain and K_{ij} is thermal conductivity tensor. α_{ij}'s are the coefficients of linear thermal expansion, Q is the heat source, τ_0 is the thermal relaxation time, F_i is the gravity force, m is the fractional order parameter and Γ denotes the Gamma function. a_{11} and a_{22} are two temperature parameters, $\theta = T - T_0$ represents the thermodynamical temperature where T is the absolute temperature, T_0 is the reference temperature and $\phi = \phi - T_0$ stands for the conductive temperature. Comma notation denotes partial derivatives with respect to spatial co-ordinates and dot notation represents derivative with respect to time.

3. Problem formulation

We consider a two dimensional problem in a fiber reinforced anisotropic half space. We shall use the rectangular Cartesian co-ordinate system (x, y, z) having the surface of the half-space as the plane $x = 0$ and x axis is assumed to be pointing vertically into the medium $(x \geq 0)$. The whole medium is considered under the influence of gravity. The bounding surface of the half space is assumed to be isothermal and subjected to a mechanical type load F_0 with an inclination δ to x axis.

For two dimensional problem in Cartesian co-ordinates x and y, the displacement vector $\vec{u}$, conductive temperature ϕ, thermodynamical temperature θ and gravity force $\vec{F}$ takes the form

$$\vec{u} = (u, v, 0),\ u = u(x, y, t),\ v = v(x, y, t),$$
$$\phi = \phi(x, y, t),\ \theta = \theta(x, y, t),\ F_1 = \rho g \frac{\partial v}{\partial x},$$
$$F_2 = -\rho g \frac{\partial u}{\partial x},\ F_3 = 0.$$

The governing Eqs. (1) and (3)-(5) in two dimensional case assume the shape

$$\sigma_{xx} = A_{11}\frac{\partial u}{\partial x} + A_{12}\frac{\partial v}{\partial y} - \beta_{11}\theta, \tag{6}$$

$$\sigma_{yy} = A_{12}\frac{\partial u}{\partial x} + A_{13}\frac{\partial v}{\partial y} - \beta_{22}\theta, \tag{7}$$

$$\sigma_{xy} = A_{14}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{8}$$

$$A_{11}\frac{\partial^2 u}{\partial x^2} + A_{15}\frac{\partial^2 v}{\partial x \partial y} + A_{14}\frac{\partial^2 u}{\partial y^2} - \beta_{11}\frac{\partial \theta}{\partial x} + \rho g \frac{\partial v}{\partial x} = \rho\left(\frac{\partial^2 u}{\partial t^2}\right), \tag{9}$$

$$A_{14}\frac{\partial^2 v}{\partial x^2} + A_{15}\frac{\partial^2 u}{\partial x \partial y} + A_{13}\frac{\partial^2 v}{\partial y^2} - \beta_{22}\frac{\partial \theta}{\partial y} - \rho g \frac{\partial u}{\partial x} = \rho\left(\frac{\partial^2 v}{\partial t^2}\right), \tag{10}$$

$$K_{11}\frac{\partial^2 \phi}{\partial x^2} + K_{22}\frac{\partial^2 \phi}{\partial y^2} = \frac{\partial}{\partial t}\left(1 + \frac{\tau_0^m}{\Gamma(m+1)}\frac{\partial^m}{\partial t^m}\right)$$
$$\times\left(\rho c_E \theta + T_0\beta_{11}\frac{\partial u}{\partial x} + T_0\beta_{22}\frac{\partial v}{\partial y}\right) - \left(1 + \frac{\tau_0^m}{\Gamma(m+1)}\frac{\partial^m}{\partial t^m}\right)Q, \tag{11}$$

$$\phi - \theta = a_{11}\frac{\partial^2 \phi}{\partial x^2} + a_{22}\frac{\partial^2 \phi}{\partial y^2}, \tag{12}$$

where

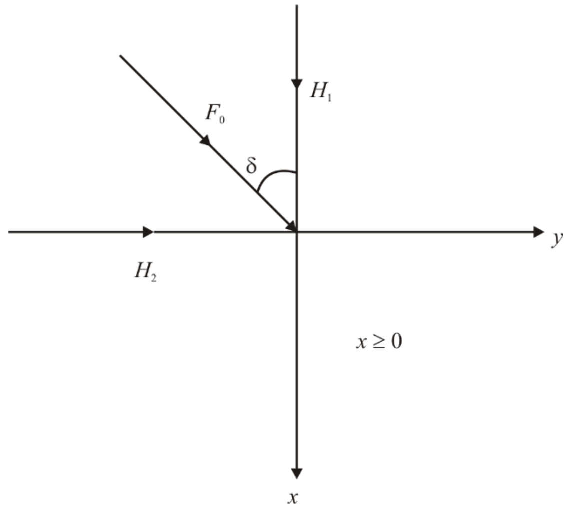

Fig. 1 Inclined load over a fiber reinforced thermoelastic half-space

$$A_{11} = \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta,\ A_{12} = \lambda + \alpha,$$
$$A_{13} = \lambda + 2\mu_T,\ A_{14} = \mu_L,\ A_{15} = A_{12} + A_{14},$$
$$\beta_{11} = (2\lambda + 3\alpha + 4\mu_L - 2\mu_T + \beta)\alpha_{11} + (\lambda + \alpha)\alpha_{22},$$
$$\beta_{22} = (2\lambda + \alpha)\alpha_{11} + (\lambda + 2\mu_T)\alpha_{22}.$$

Now, we transform the above equations into non-dimensional forms by introducing the following dimensionless parameters

$$(x', y', u', v') = c_1\chi(x, y, u, v),\ (t', \tau_0') = c_1^2\chi(t, \tau_0),$$
$$(\phi', \theta') = \frac{\beta_{11}}{\rho c_1^2}(\phi, \theta),$$
$$(\sigma_{xx}', \sigma_{xy}', \sigma_{yy}') = \frac{1}{\rho c_1^2}(\sigma_{xx}, \sigma_{xy}, \sigma_{yy}),$$
$$(H_1', H_2') = \frac{1}{\rho c_1^2}(H_1, H_2),\ (a_{11}', a_{22}') = c_1^2\chi^2(a_{11}, a_{22}),$$
$$Q' = \frac{1}{\rho c_E T_0 c_1^2 \chi}Q,\ g' = \frac{1}{c_1^3\chi}g,$$

where

$$\chi = \frac{\rho c_E}{K_{11}} \text{ and } c_1^2 = \frac{A_{11}}{\rho}.$$

Eqs. (6) and (8)-(12) may now be reduced to the following system of dimensionless equations (after removing the primes for clarity)

$$\sigma_{xx} = \frac{\partial u}{\partial x} + B_1\frac{\partial v}{\partial y} - \theta, \tag{13}$$

$$\sigma_{xy} = B_3\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{14}$$

$$\frac{\partial^2 u}{\partial x^2} + B_2\frac{\partial^2 v}{\partial x \partial y} + B_3\frac{\partial^2 u}{\partial y^2} - \frac{\partial \theta}{\partial x} + B_4\frac{\partial v}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{15}$$

$$B_3 \frac{\partial^2 v}{\partial x^2} + B_2 \frac{\partial^2 u}{\partial x \partial y} + B_5 \frac{\partial^2 v}{\partial y^2} - B_6 \frac{\partial \theta}{\partial y} - B_4 \frac{\partial u}{\partial x} = \frac{\partial^2 v}{\partial t^2}, \quad (16)$$

$$\frac{\partial^2 \phi}{\partial x^2} + \varepsilon_1 \frac{\partial^2 \phi}{\partial y^2} = \frac{\partial}{\partial t}\left(1 + \frac{\tau_0^m}{\Gamma(m+1)} \frac{\partial^m}{\partial t^m}\right)\left(\theta + \varepsilon_2 \frac{\partial u}{\partial x} + \varepsilon_3 \frac{\partial v}{\partial y}\right) - \left(1 + \frac{\tau_0^m}{\Gamma(m+1)} \frac{\partial^m}{\partial t^m}\right)\varepsilon_4 Q \quad (17)$$

$$\phi - \theta = a_{11} \frac{\partial^2 \phi}{\partial x^2} + a_{22} \frac{\partial^2 \phi}{\partial y^2}, \quad (18)$$

where

$$B_1 = \frac{A_{12}}{A_{11}}, \quad B_2 = \frac{A_{15}}{A_{11}}, \quad B_3 = \frac{A_{14}}{A_{11}}, \quad B_4 = g,$$

$$B_5 = \frac{A_{13}}{A_{11}}, \quad B_6 = \frac{\beta_{22}}{\beta_{11}}, \quad \varepsilon_1 = \frac{K_{22}}{K_{11}},$$

$$\varepsilon_2 = \frac{T_0 (\beta_{11})^2}{\rho A_{11} c_E}, \quad \varepsilon_3 = \frac{T_0 \beta_{11} \beta_{22}}{\rho A_{11} c_E}, \quad \varepsilon_4 = \frac{T_0 \beta_{11}}{A_{11}}.$$

4. Solution in the transformed domain

Following the solution methodology through integral transform technique, we now operate Laplace and Fourier transforms on Eqs. (15)-(18). The Laplace and Fourier transforms of a function $f(x, y, t)$ with parameters s and ξ are defined by the relations

$$\bar{f}(x, y, s) = \int_0^\infty f(x, y, t) e^{-st} dt \quad \text{and} \quad (19)$$

$$\hat{f}(x, \xi, s) = \int_{-\infty}^{\infty} \bar{f}(x, y, s) e^{\iota \xi y} dy \quad (20)$$

where over-bar and over-cap denote the Laplace and Fourier transforms respectively. Applying Laplace and Fourier transforms on Eq. (18), we get

$$\hat{\phi} - \hat{\theta} = \left(a_{11} D^2 - a_{22} \xi^2\right) \hat{\phi} \quad (21)$$

Application of Laplace and Fourier transforms on Eqs. (15)-(17) and the usage of $\hat{\theta}$ in terms of $\hat{\phi}$ from Eq. (21) provides

$$\left(D^2 - G_{11}\right)\hat{u} + G_{12} D \hat{v} - \left(G_{13} - G_{14} D^2\right) D \hat{\phi} = 0 \quad (22)$$

$$G_{21} D \hat{u} - \left(G_{22} D^2 - G_{23}\right)\hat{v} - G_{24}\left(G_{13} - G_{14} D^2\right)\hat{\phi} = 0 \quad (23)$$

$$G_{31} D \hat{u} - G_{32} \hat{v} - \left(D^2 - G_{33} - G_{34}\left(G_{13} - G_{14} D^2\right)\right)\hat{\phi} = G_{35} \quad (24)$$

where

$$\hat{\theta} = \left(G_{13} - G_{14} D^2\right)\hat{\phi} \quad (25)$$

and $G_{11} = B_2 \xi^2 + s^2$, $G_{12} = B_4 - \iota \xi B_2$, $G_{13} = 1 + a_{22} \xi^2$, $G_{14} = a_{11}$, $G_{21} = B_4 + \iota \xi B_2$, $G_{22} = B_3$, $G_{23} = B_5 \xi^2 + s^2$, $G_{24} = \iota \xi B_6$, $G_{31} = sk\varepsilon_2$, $G_{32} = \iota \xi sk \varepsilon_3$, $G_{33} = \varepsilon_1 \xi^2$, $G_{34} = sk$, $G_{35} = k \varepsilon_4 \hat{Q}$, $k = 1 + \frac{\tau_0^m}{\Gamma(m+1)} s^m$.

Moving source is located at the origin and time $t = 0^+$ starts moving along the positive y axis with uniform velocity v_0. The source is assumed to be of the form

$$Q(y, t) = Q_0 \delta(y - v_0 t) H(t), \quad (26)$$

where Q_0 is the heat source strength (constant), δ is the Dirac delta function and H is the Heaviside unit step function.

Taking Laplace and Fourier transforms of Eq. (26), we have

$$\hat{Q} = \frac{Q_0}{s - \iota \xi v_0}.$$

Eliminating $\hat{u}$ and $\hat{\phi}$ from Eqs. (22)-(24), we get the following sixth order ordinary differential equation satisfied by $\hat{v}$

$$\left(D^6 + R D^4 + N D^2 + I\right)\hat{v} = P, \quad (27)$$

where

$$R = \frac{Q_1}{P_1}, \quad N = \frac{R_1}{P_1}, \quad I = \frac{S_1}{P_1}, \quad P = \frac{T_1}{P_1},$$

$$P_1 = -\left(H_{13} H_{21} + H_{23} H_{11}\right),$$

$$Q_1 = H_{22} H_{13} - H_{14} H_{21} + H_{24} H_{11} + H_{23} H_{12},$$

$$R_1 = H_{14} H_{22} - H_{25} H_{11} - H_{12} H_{24}, \quad S_1 = H_{12} H_{25},$$

$$T_1 = H_{12} H_{26}, \quad H_{11} = G_{24} - G_{21}, \quad H_{12} = G_{11} G_{24},$$

$$H_{13} = G_{22}, \quad H_{14} = G_{12} G_{24} - G_{23},$$

$$H_{21} = G_{21} G_1 + G_{24} G_{31} G_{14}, \quad H_{22} = G_{21} G_2 + G_{24} G_{31} G_{13},$$

$$H_{23} = G_{22} G_1, \quad H_{24} = G_{22} G_2 + G_{23} G_1 - G_{32} G_{24} G_{14},$$

$$H_{25} = G_{23} G_2 - G_{32} G_{24} G_{13}, \quad H_{26} = G_{24} G_{13} G_{35},$$

$$G_1 = 1 + G_{34} G_{14}, \quad G_2 = G_{33} + G_{34} G_{13}.$$

Adopting the same procedure, we can establish the following equation satisfied by $\hat{u}$ and $\hat{\phi}$

$$\left(D^6 + R D^4 + N D^2 + I\right)\left(\hat{u}, \hat{\phi}\right) = (0, P). \quad (28)$$

Eq. (27) can be factorized as

$$\left(D^2 - \lambda_1^2\right)\left(D^2 - \lambda_2^2\right)\left(D^2 - \lambda_3^2\right)\hat{v} = P, \quad (29)$$

where

$$\lambda_1 = \sqrt{\frac{1}{3}\left[2n \sin(q) - R\right]},$$

$$\lambda_2 = \sqrt{\frac{1}{3}\left[-R - n\left(\sqrt{3}\cos(q) + \sin(q)\right)\right]}$$

$$\lambda_3 = \sqrt{\frac{1}{3}\left[-R + n\left(\sqrt{3}\cos(q) - \sin(q)\right)\right]}$$

are the roots with positive real part of the characteristic equation

$$\lambda^6 + R\lambda^4 + N\lambda^2 + I = 0$$

with $n = \sqrt{R^2 - 3N}$, $q = \dfrac{\sin^{-1}(r)}{3}$,

$$r = -\frac{-2R^3 + 9RN - 27I}{2n^3}$$

satisfying the relation

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = -R,\quad \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_1^2\lambda_3^2 = N,$$
$$\lambda_1^2\lambda_2^2\lambda_3^2 = -I.$$

Since the intent is that the solution vanishes at infinity so as to satisfy the regularity condition at infinity, we now consider the following solutions of Eqs. (27) and (28)

$$\left(\hat{u}, \hat{v}, \hat{\phi}\right) = \sum_{i=1}^{3} (d_i, 1, l_i) A_i e^{-\lambda_i x} + (0, 1, l_4) A_4 \tag{30}$$

where

$$d_i = \frac{H_{13}\lambda_i^3 + H_{14}\lambda_i}{H_{11}\lambda_i^2 - H_{12}},\quad l_i = \frac{G_{22}\lambda_i^2 - G_{23} + G_{21}\lambda_i d_i}{G_{24}\left(G_{14}\lambda_i^2 - G_{13}\right)}\ (i = 1, 2, 3),$$

$l_4 = \dfrac{G_{23}}{G_{24}G_{13}}$, $A_4 = \dfrac{P}{I}$ and $A_i\,(i = 1, 2, 3)$ are arbitrary constants.

5. Application

We have considered a fiber reinforced, anisotropic thermoelastic half space $x \geq 0$ with fractional order heat conduction. The bounding surface $x = 0$ is assumed to be isothermal and is subjected to a mechanical type inclined load F_0 with an inclination δ to x axis as shown in Fig. 1. Hence, the normal line load H_1 and tangential line load H_2 are expressed as $H_1 = F_0 \cos\delta$ and $H_2 = F_0 \sin\delta$ respectively. In order to solve the problem, mathematically, the boundary conditions at the surface $x = 0$ are expressed as

$$\begin{aligned} \sigma_{xx}(0, y, t) &= -H_1\delta(y)H(t), \\ \sigma_{xy}(0, y, t) &= -H_2\delta(y)H(t), \\ \phi(0, y, t) &= 0, \end{aligned} \tag{31}$$

where $\delta(y)$ is the Dirac delta function and $H(t)$ is the Heaviside unit step function.

Making use of (13) and (14) in boundary conditions (31), applying the transformations defined by (19) and (20) and then substituting the expressions from (30) in the resulting equations and Eq. (25), we obtain the following expressions for the different field quantities in non-dimensional form

$$\begin{aligned} \left(\hat{u}, \hat{v}, \hat{\phi}, \hat{\theta}, \hat{\sigma}_{xx}, \hat{\sigma}_{xy}\right) = {} & \frac{1}{\Delta}\sum_{i=1}^{3}\left(d_i, 1, l_i, \left(G_{13} - G_{14}\lambda_i^2\right)l_i, -M_i, -N_i\right)\Delta_i e^{-\lambda_i x} \\ & + \left(0, 1, l_4, G_{13}l_4, -\left(\iota\xi B_1 + G_{13}l_4\right), 0\right)A_4 \end{aligned} \tag{32}$$

where

$$\begin{aligned} \Delta &= l_1\left(M_2N_3 - M_3N_2\right) - l_2\left(M_1N_3 - M_3N_1\right) \\ &\quad + l_3\left(M_1N_2 - M_2N_1\right), \\ \Delta_1 &= -l_4A_4\left(M_2N_3 - M_3N_2\right) - l_2\left(M_4N_3 - M_3N_4\right) \\ &\quad + l_3\left(M_4N_2 - M_2N_4\right), \\ \Delta_2 &= l_1\left(M_4N_3 - M_3N_4\right) + l_4A_4\left(M_1N_3 - M_3N_1\right) \\ &\quad + l_3\left(M_1N_4 - M_4N_1\right), \\ \Delta_3 &= l_1\left(M_2N_4 - M_4N_2\right) - l_2\left(M_1N_4 - M_4N_1\right) \\ &\quad - l_4A_4\left(M_1N_2 - M_2N_1\right) \end{aligned}$$

and

$$M_i = d_i\lambda_i + \iota\xi B_1 + \left(G_{13} - G_{14}\lambda_i^2\right)l_i,\ N_i = B_3\left(\iota\xi d_i + \lambda_i\right)\ (i = 1, 2, 3),$$
$$M_4 = \frac{H_1}{s} - \left(\iota\xi B_1 + G_{13}l_4\right)A_4,\quad N_4 = \frac{H_2}{s}.$$

6. Special cases

6.1 Without internal heat source

Neglecting the influence of internal heat source *i.e.*, Q_0=0, the expressions of displacements, conductive temperature, thermodynamical temperature and stresses will be obtained from relation (32). If we remove gravitational and two temperature effects also, then our results match with those of Sarkar *et al.* (2016) with appropriate changes in boundary conditions and solution technique (avoiding magnetic effect and initial stress).

6.2 Ignoring the effect of gravity

The gravitational effect can be neglected by taking $g = 0$ in the equation of motion. Then we shall be left with the relevant problem in a fractional order fiber reinforced thermoelastic half space with two temperature. Neglecting reinforcement, two temperature and fractional effect also and making suitable changes in boundary conditions and solution technique, our results coincide with those of Ailawalia and Singla (2015).

6.3 Without fiber reinforcement

By setting reinforcement parameters $\alpha = \beta = 0$ and $\mu_L = \mu_T$ in the constitutive relation, we get corresponding expressions of the field variables in the context of fractional order two temperature generalized thermoelasticity theory with internal heat source and gravity.

6.4 Without fractional order

By taking $m = 1$ in the heat conduction equation, the problem reduces to a two dimensional problem in a fiber reinforced thermoelastic medium with internal heat source under gravity in the context of Lord-Shulman theory with two temperature.

7. Inversion of the transforms

The transformed displacements, stresses and temperature distributions are functions of x and the parameters s and ξ of Laplace and Fourier transforms respectively and hence are of the form $\hat{f}(x, \xi, s)$. To get the function in the physical domain, first we invert the Fourier transform using

$$F^{-1}\left[\hat{f}(x, \xi, s)\right] = \bar{f}(x, y, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{f}(x, \xi, s)e^{-\iota\xi y}d\xi$$
$$= \frac{1}{\pi}\int_{0}^{\infty}\left\{\cos(\xi y)f_e - \iota\sin(\xi y)f_o\right\}d\xi \tag{33}$$

where f_e and f_o denote the even and odd parts respectively of the function $\hat{f}(x, \xi, s)$.

We shall now outline the numerical inversion method used to find the solution in the physical domain. The inversion formula of the Laplace transform is defined as

$$L^{-1}\left[\bar{f}(x, y, s)\right] = f(x, y, t) = \frac{1}{2\pi\iota}\int_{c-\iota\infty}^{c+\iota\infty}e^{st}\bar{f}(x, y, s)ds, \tag{34}$$

where $\bar{f}(x, y, s)$ be the Laplace transform of function $f(x, y, t)$, c is an arbitrary real number larger than the real parts of all the singularities of $\bar{f}(x, y, s)$.

Taking $s = c + \iota w$, the preceding integral takes the form

$$f(x, y, t) = \frac{e^{ct}}{2\pi}\int_{-\infty}^{\infty}\bar{f}(x, y, c+\iota w)e^{iwt}dw. \tag{35}$$

Expanding the function $h(x, y, t) = e^{-ct}f(x, y, t)$ in a Fourier series in the interval $[0, 2t_1]$, we obtain the approximate formula (Honig and Hirdes 1984)

$$f(x, y, t) = f_\infty(x, y, t) + E_D, \tag{36}$$

where

$$f_\infty(x, y, t) = \frac{1}{2}c_0 + \sum_{k=1}^{\infty}c_k, \quad 0 \le t \le 2t_1 \tag{37}$$

and

$$c_k = \frac{e^{ct}}{t_1}\left[e^{\frac{\iota k\pi t}{t_1}}\bar{f}\left(x, y, c + \frac{\iota k\pi t}{t_1}\right)\right]. \tag{38}$$

The discretization error E_D can be made arbitrarily small by choosing c large enough (Honig and Hirdes 1984). Since the infinite series in Eq. (37) can be summed upto a finite number N of terms, the approximate value $f(x, y, t)$ becomes

$$f_N(x, y, t) = \frac{1}{2}c_0 + \sum_{k=1}^{N}c_k, \quad 0 \le t \le 2t_1. \tag{39}$$

Using the preceding formula to evaluate $f(x, y, t)$, we introduce a truncation error E_{t_1} that must be added to the discretization error to produce the total approximation error. Two methods are used to reduce the total error. First, the 'Korrektur' method is applied to reduce the discretization error. Next, the ε-algorithm is used to accelerate convergence (Honig and Hirdes 1984).

The Korrektur method uses the following formula to evaluate the function $f(x, y, t)$

$$f(x, y, t) = f_\infty(x, y, t) - e^{-2ct_1}f_\infty(x, 2t_1 + t) + E'_D, \tag{40}$$

where the discretization error $|E'_D| \le |E_D|$. Thus, the approximate value of $f(x, y, t)$ becomes

$$f_{NK}(x, y, t) = f_N(x, y, t) - e^{-2ct_1}f_{N'}(x, y, 2t_1 + t), \tag{41}$$

where N' is an integer such that $N' < N$.

We shall now describe the ε-algorithm that is used to accelerate the convergence of the series in Eq. (39). Let $N = 2q + 1$, where q is a natural number and $s_m = \sum_{k=1}^{m}c_k$ is the sequence of the partial sum of the series in Eq. (39).

We define the ε-sequence by

$$\varepsilon_{0,m} = 0, \ \varepsilon_{1,m} = s_m, \tag{42}$$

and

$$\varepsilon_{p+1,m} = \varepsilon_{p-1,m+1} + \frac{1}{\varepsilon_{p,m+1} - \varepsilon_{p,m}}, \quad p = 1, 2, 3, \ldots \tag{43}$$

It can be shown that (Honig and Hirdes 1984) the sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \varepsilon_{5,1}, \ldots, \varepsilon_{N,1}$, converges to $f(x, y, t) + E_D - \frac{c_0}{2}$ faster than the sequence of partial sums $s_m, m = 1, 2, 3, \ldots$.

The actual procedure used to invert the Laplace transform consists of using Eq. (41) together with the $\grave{o}$-algorithm. The values of c and t_1 are chosen according to the criteria outlined by Honig and Hirdes (1984).

Following Rakshit and Mukhopadhyay (2007), simultaneous computations of the inversion of the Fourier transform are performed by evaluating the infinite integral (33) numerically by seven-point Gaussian quadrature formula for several prescribed values of the variables x and y.

8. Numerical results and discussions

To illustrate and compare the theoretical results, obtained in Section 5, we now present some numerical results which depict the variations of displacement, temperature and stress fields. Following Abbas *et al.* (2011), we take the following values of the physical constants

$$\lambda = 5.65\times10^{10}\, Nm^{-2},\ \ \mu_T = 2.46\times10^{10}\, Nm^{-2},$$
$$\mu_L = 5.66\times10^{10}\, Nm^{-2},\ \ \alpha = -1.28\times10^{10}\, Nm^{-2},$$
$$\beta = 220.90\times10^{10}\ Nm^{-2},\ \ c_E = 0.787\times10^3\ Jkg^{-1}K^{-1},$$
$$K_{11} = 0.0921\times10^3\ Jm^{-1}K^{-1}s^{-1},$$
$$K_{22} = 0.0963\times10^3\ Jm^{-1}K^{-1}s^{-1},\ \ \alpha_{11} = 0.017\times10^{-4}\, K^{-1},$$
$$\alpha_{22} = 0.015\times10^{-4}\, K^{-1},\ \ \tau_0 = 0.02s,\ \ g = 0.2\ ms^{-2},$$
$$\rho = 2660\ kgm^{-3},\ T_0 = 293\, K,$$
$$a_{11} = 0.02\, m^2,\ \ a_{22} = 0.04\ m^2.$$

Other parameters of the problem are taken as

$$Q_0 = 10,\ v_0 = 1,\ \ m = 0.5,\ \ F_0 = 1,\ \ \delta = 45°.$$

With these numerical values of the parameters, expressions of the non-dimensional field variables have been evaluated and results are presented in the form of graphs at different positions of x at $t = 0.1$ and $y = 1.0$. From clarity point of view, we have divided the graphical representation into four groups. In first group (Figs. 2-6), we have explored the effects of heat source and gravity on the spatial variations of physical fields in fractional order two temperature generalized thermoelastic medium. The variation of different dimensionless variables are represented for three different cases:

1. Fiber reinforced thermoelastic solid with internal heat source and gravity (FRTQG),
2. Fiber reinforced thermoelastic solid with gravity (FRTG),
3. Fiber reinforced thermoelastic solid with internal heat source (FRTQ).

Second group (Figs. 7-11) exhibits the behaviour of field variables for different values of fractional order parameter which are taken as 0.1, 0.5, 1.0. Without fractional order case corresponds to $m = 1.0$. Third group (Figs. 12-16) is meant for analysing the effects of reinforcement parameter (under the cases, with reinforcement (WRE) and no reinforcement (NRE)) and time ($t = 0.1, 0.2$) on various fields. Fourth group (Figs. 17-21) shows the effect of different inclinations $(\delta = 0°, 45°, 90°)$ of the inclined load on various field quantities.

Group I

In Fig. 2, we have depicted the normal displacement distribution with distance x to investigate the effects of heat source and gravity, by taking all other parameters as constant. Displacement field exhibits significant sensitivity towards heat source and gravity. Absence of heat source (FRTG) causes decrement in the magnitude of displacement. In the absence of gravity (FRTQ), displacement is negative initially and numerical values are small as compared to FRTQG case. Ultimately, in all the cases, displacement field tends to zero.

Figs. 3 and 4 describe that the conductive temperature starts with zero value for all the three cases, which is in accordance with the boundary condition, while thermodynamical temperature starts with non zero value. For both the temperatures, their profiles are quite similar for FRTQG and FRTQ cases and tend to non zero value.

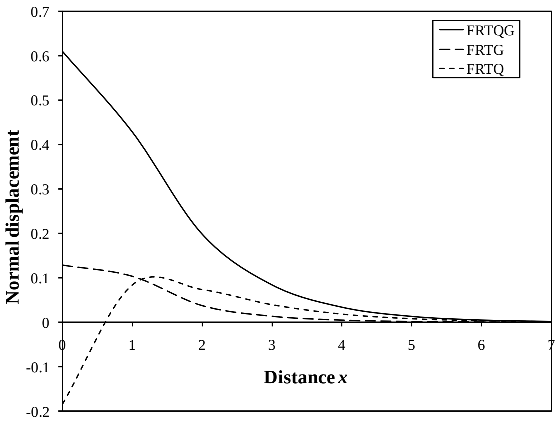

Fig. 2 Effect of heat source and gravity on normal displacement

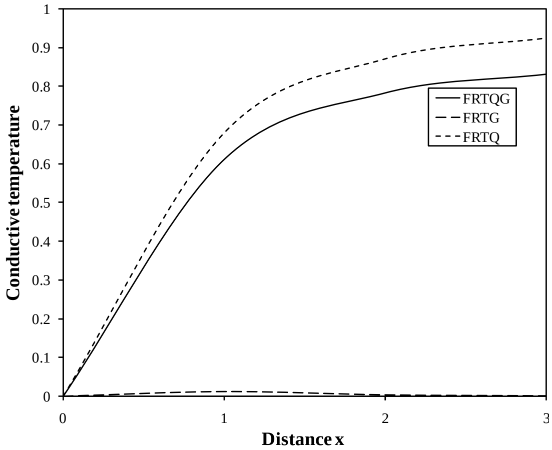

Fig. 3 Effect of heat source and gravity on conductive temperature

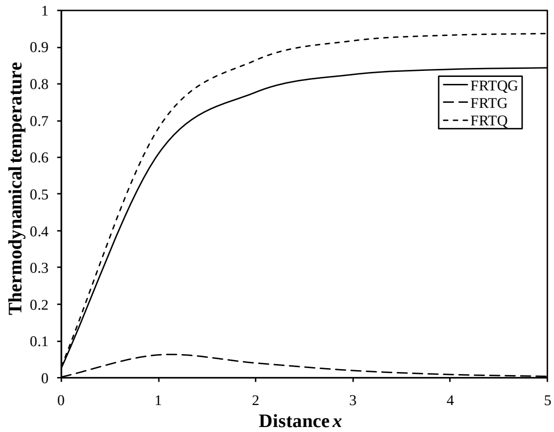

Fig. 4 Effect of heat source and gravity on thermodynamical temperature

But the magnitude of both the temperatures is very small in the case FRTG and tends to zero value. Also, presence of heat source increases and presence of gravity decreases both the temperatures numerically.

Figs. 5 and 6 demonstrate the distributions of normal and tangential stress respectively. According to the boundary conditions and numerical data, both the stresses

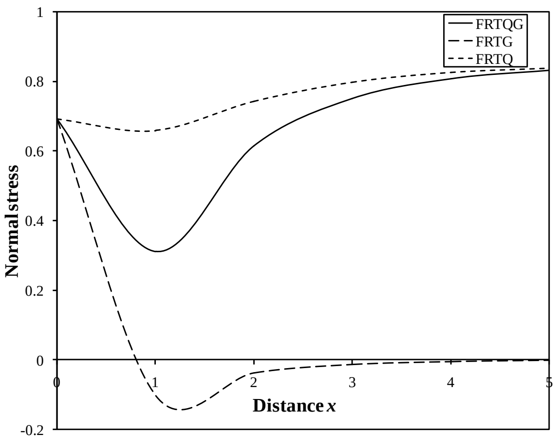

Fig. 5 Effect of heat source and gravity on normal stress

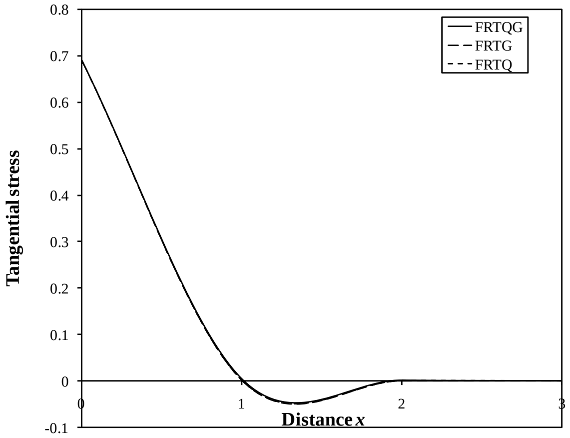

Fig. 6 Effect of heat source and gravity on tangential stress

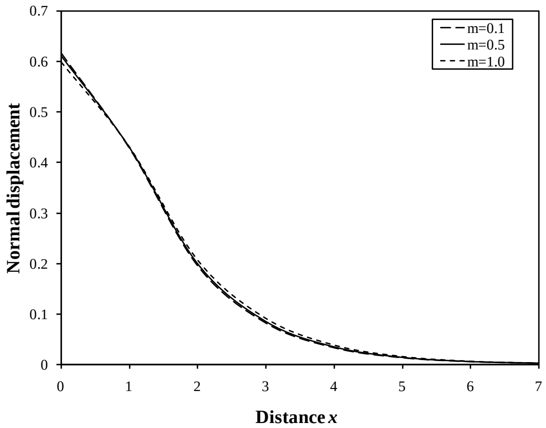

Fig. 7 Effect of fractional parameter on normal displacement

start with the same value. Presence of heat source causes increasing effect while presence of gravity causes decreasing effect on normal stress. It is clear from Fig. 6 that heat source and gravity exhibit very small effect on tangential stress. Normal stress tends to non zero value in the presence of heat source and tends to zero in the absence of heat source while tangential stress tends to zero in the absence and presence of heat source. This can be verified from the expression of both the stresses in relation (32).

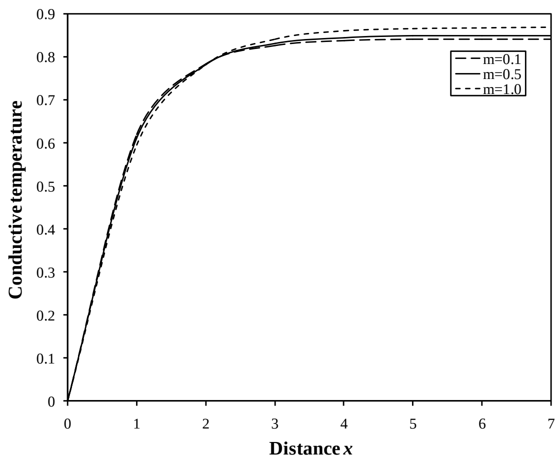

Fig. 8 Effect of fractional parameter on conductive temperature

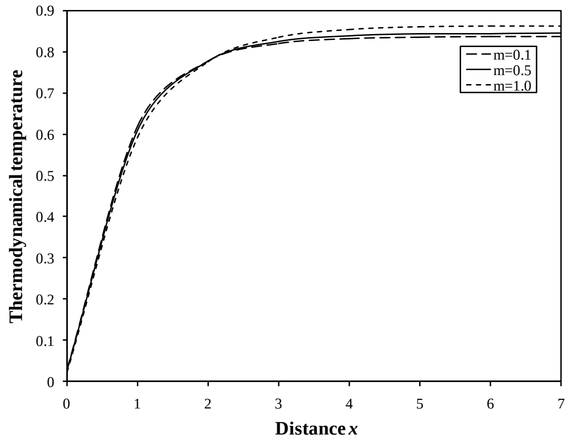

Fig. 9 Effect of fractional parameter on thermodynamical temperature

Group II

In Fig. 7, we have plotted the normal displacement against distance x at three different mentioned values of fractional parameter m. Fractional parameter exhibits decreasing effect in the range $0 \leq x \leq 1$ and increasing effect for $1 < x \leq 7$. Removal of fractional parameter causes decrement in the value of normal displacement before $x = 1$ and increment after $x = 1$. Finally, displacement tends to zero for $x > 7$.

Figs. 8 and 9 are drawn to demonstrate the profiles of conductive and thermodynamical temperatures respectively for the considered values of fractional parameter. For all the three cases, initially, conductive temperature begins with zero value which is in accordance with the boundary condition and thermodynamical temperature starts with non-zero value. For both the temperatures, fractional parameter shows decreasing effect before $x = 2$ and increasing effect after $x = 2$. Absence of fractional parameter numerically decreases both the temperatures for $0 \leq x \leq 2$ and increases for $x > 2$.

Figs. 10 and 11 depict the space variation of normal stress and tangential stress respectively. As we increase the value of fractional parameter, normal stress is decreased numerically in the range $0 \leq x \leq 2$ and is increased for

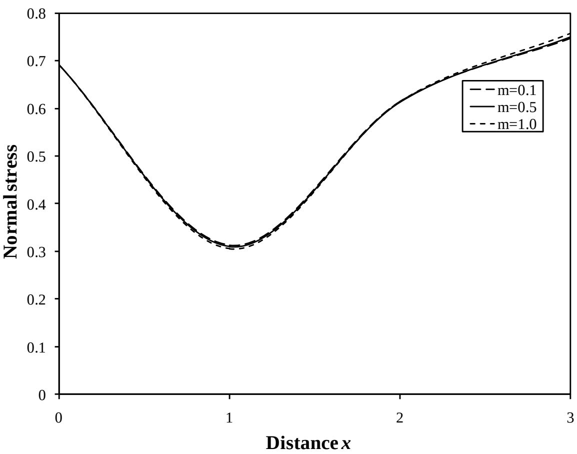

Fig. 10 Effect of fractional parameter on normal stress

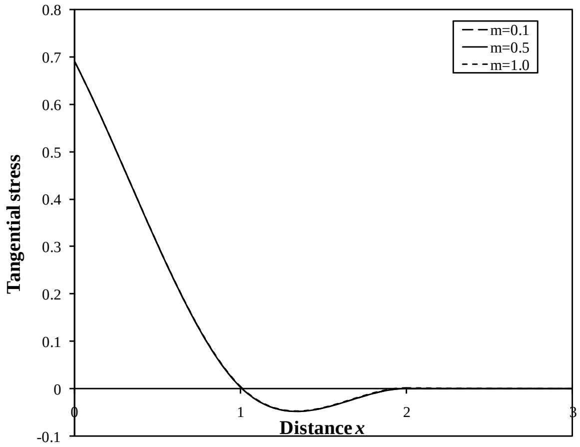

Fig. 11 Effect of fractional parameter on tangential stress

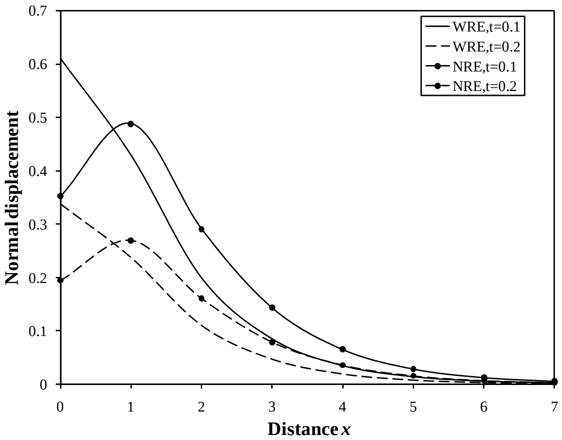

Fig. 12 Effect of reinforcement and time on normal displacement

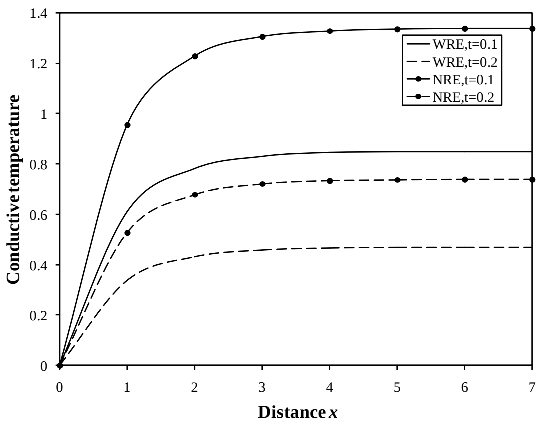

Fig. 13 Effect of reinforcement and time on conductive temperature

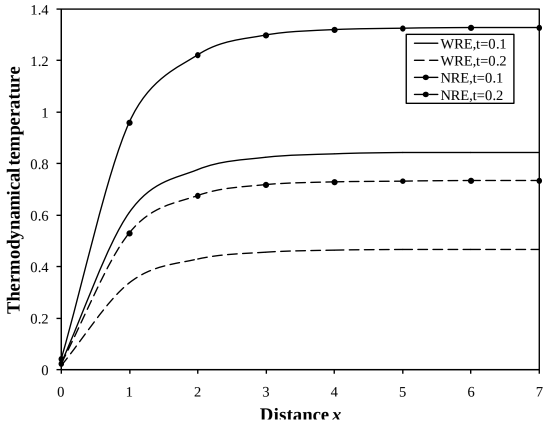

Fig. 14 Effect of reinforcement and time on thermodynamical temperature

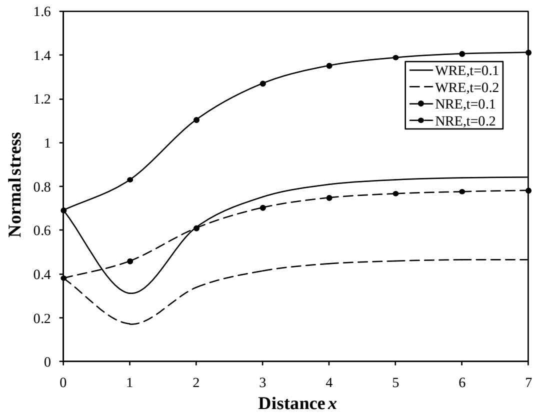

Fig. 15 Effect of reinforcement and time on normal stress

$x > 2$. It attains it minimum value at $x = 1$. The increment in the value of fractional parameter causes very less effect in the numerical value of tangential stress. We can see from the profile that removal of fractional parameter exhibits negligible effect on tangential stress while the numerical value of normal stress without fractional order case is less in the range $0 \leq x \leq 2$ and greater for $x > 2$ as compared to the fractional order case.

Group III

Figs. 12-16 represent the values of considered physical variables for two values of time which are taken to be 0.1 and 0.2 in the presence and absence of reinforcement. It is observed from the figures that increment in the value of time is making the magnitudes of field variables small for both the cases (WRE and NRE). Thus, time has a decreasing effect on all the field quantities. Also, the effect of reinforcement is quite pertinent on all the fields. Reinforcement exhibits increasing effect before $x = 1$ and

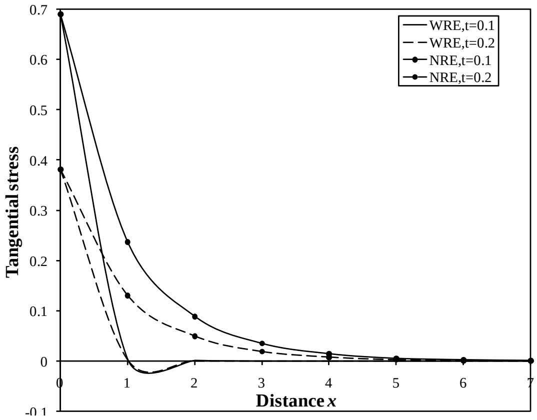

Fig. 16 Effect of reinforcement and time on tangential stress

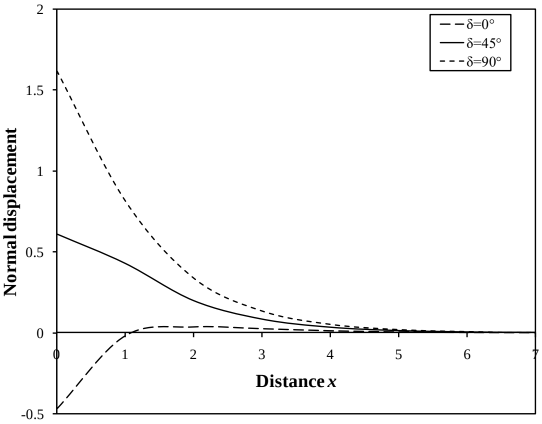

Fig. 17 Effect of inclination of load on normal displacement

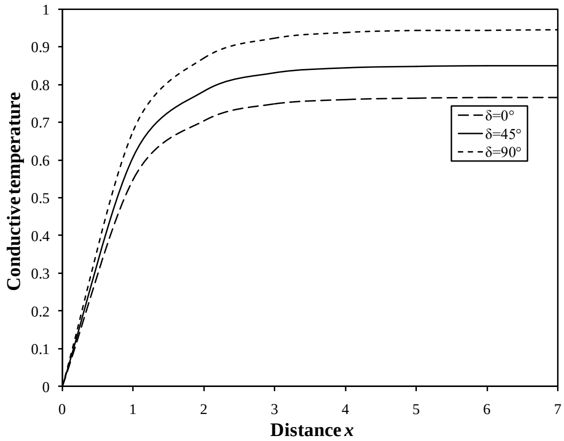

Fig. 18 Effect of inclination of load on conductive temperature

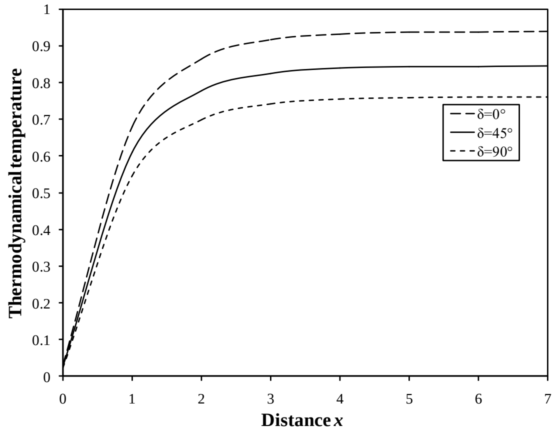

Fig. 19 Effect of inclination of load on thermodynamical temperature

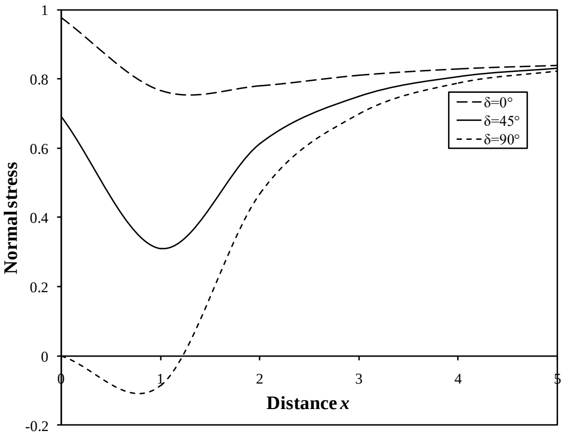

Fig. 20 Effect of inclination of load on normal stress

decreasing effect after x = 1 on the magnitude of normal displacement, which can be easily noticed from the Fig. 12. Figs. 13-16 exhibit the decreasing effect of reinforcement on all the other field variables as the numerical values of all the field quantities in WRE case are less than those of case NRE for both the values of time.

Group IV

The dynamic effect of inclination of load $\left(\delta = 0°, 45°, 90°\right)$ on normal displacement has been studied in Fig. 17. The profile begins with positive value for $\delta = 45°, 90°$ and with negative value for δ = 0°. Magnitude of displacement increases as the angle of inclination increases. So, inclination causes increasing effect on displacement.

Fig. 18 shows that the profile of conductive temperature is same for all the values of δ and starts with zero value satisfying the boundary condition. Inclination of angle causes increasing effect on conductive temperature. It is clear from Fig. 19 that inclination has decreasing effect on thermodynamical temperature. However, the profile is similar for all the three values of δ and begins with non-zero value.

Normal stress and tangential stress are plotted in Fig. 20 and Fig. 21 respectively for considered values of δ. It is clear from the figures that increment in the angle of inclination decreases the magnitude of normal stess. Normal stress vanishes initially for δ = 90◦ while tangential stress vanishes initially for δ = 0◦. The numerical values of tangential stress have been magnified by 10^1 for δ = 0, to depict the effect of inclination simultaneously on all the curves.

9. Conclusions

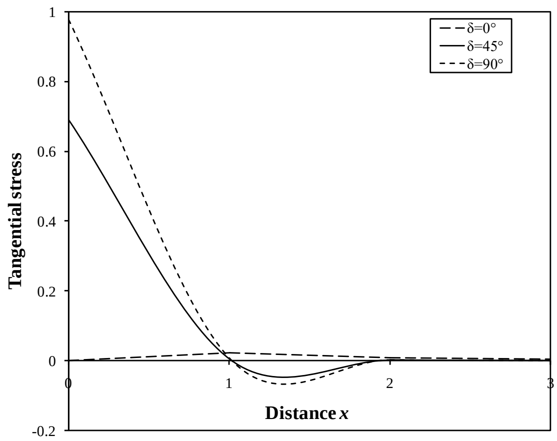

Fig. 21 Effect of inclination of load on tangential stress

This article presents an in-depth analysis of inclined load problem in the context of fractional order two temperature theory of generalized thermoelasticity in fiber reinforced thermoelastic medium with internal heat source and gravity. The main conclusions due to influence of different parameters can be summarized as follows:

• All the considered fields are found to be sensitive towards heat source effect. In the absence of heat source, all the field variables tend to zero.

• Presence of gravity acts to decrease the magnitude of all the physical fields except normal displacement and tangential stress.

• The fractional parameter causes slight impact over all the considered field variables.

• Reinforcement parameter causes decreasing effect on all the physical fields except normal displacement.

• Time factor has decreasing effect on all the field quantities.

• Angle of inclination of load has also affected the studied fields significantly. As the angle increases from δ = 0° to 90°, normal displacement, conductive temperature and tangential stress increase while thermodynamical temperature and normal stress decrease numerically.

Acknowledgments

One of the authors, Kavita Jain, is thankful to University Grants Commission, New Delhi, for the financial support as Junior Research Fellowship.

References

Abbas, I.A., Abd-Alla, A.N. and Othman, M.I.A. (2011), "Generalized magneto-thermoelasticity in a fiber reinforced anisotropic half-space", *Int. J. Thermophys.*, **32**(5), 1071-1085.

Abd-Alla, A.N., Abo-Dahab, S.M. and Alotaibi, H.A. (2017), "Propagation of a thermoelastic wave in a half space of a homogeneous isotropic material subjected to the effect of gravity field", *Arch. Civil Mech. Eng.*, **17**(3), 564-573.

Ailawalia, P. and Narah, N.S. (2009), "Effect of rotation in generalized thermoelastic solid under the influence of gravity with an overlying infinite thermoelastic fluid", *Appl. Math. Mech.*, **30**(12), 1505-1518.

Ailawalia, P. and Singla, S. (2015), "Disturbance due to internal heat source in thermoelastic solid using dual phase lag model", *Struct. Eng. Mech.*, **56**(3), 341-354.

Ailawalia, P., Khurana, G. and Kumar, S. (2009), "Effect of rotation in a generalized thermoelastic medium with two temperature under the influence of gravity", *Int. J. Appl. Math. Mech.*, **5**(5), 99-116.

Baksi, A., Roy, B.K. and Bera, R.K. (2008), "Study of two dimensional viscoelastic problems in generalized thermoelastic medium with heat source", *Struct. Eng. Mech.*, **29**(6), 673-687.

Belfield, A.J., Rogers, T.G. and Spencer, A.J.M. (1983), "Stress in elastic plates reinforced by fibers lying in concentric circles", *J. Mech. Phys. Sol.*, **31**(1), 25-54.

Bromwich, T.J.J.A. (1898), "On the influence of gravity on elastic waves and in particular on the vibrations of an elastic globe", *Proc. Lond. Math. Soc.*, **30**, 98-120.

Chen, P.J. and Gurtin, M.E. (1968), "On a theory of heat conduction involving two temperatures", *Z. Angew. Math. Phys.*, **19**(4), 614-627.

Chen, P.J., Gurtin, M.E. and Williams, W.O. (1968), "A note on non-simple heat conduction", *Z. Angew. Math. Phys.*, **19**(6), 969-970.

Chen, P.J., Gurtin, M.E. and Williams, W.O. (1969), "On the thermodynamics of non-simple elastic material with two temperatures", *Z. Angew. Math. Phys.*, **20**(1), 107-112.

Danilovskaya, V.I. (1950), "Thermal stresses in an elastic semi-space due to a sudden heating of its boundary", *Prikl. Mat. Mech.*, **14**, 316-318.

Deswal, S. and Kalkal, K.K. (2013), "Fractional order heat conduction law in micropolar thermo-viscoelasticity with two temperature", *Int. J. Heat Mass Transf.*, **66**, 451-460.

Deswal, S., Kalkal, K.K. and Sheoran S.S. (2017), "A magneto-thermo-viscoelastic problem with fractional order strain under GN-II model", *Struct. Eng. Mech.*, **63**(1), 89-102.

Eason, G. and Sneddon, I.N. (1959), "The dynamic stress produced in elastic body by uneven heating", *Proc. Roy. Soc. Edin. Soc.*, **65**, 143-176.

El-Karamany, A.S. and Ezzat, M.A. (2011), "Convolutional variational principle, reciprocal and uniqueness theorems in linear fractional two-temperature thermoelasticity", *J. Therm. Stress.*, **34**(3), 264-284.

Ezzat, M.A. (2010), "Thermoelectric MHD non-Newtonian fluid with fractional derivative heat transfer", *Phys. B*, **405**(19), 4188-4194.

Honig, G. and Hirdes, U. (1984), "A method for the numerical inversion of Laplace transforms", *J. Comp. Appl. Math.*, **10**(1), 113-132.

Jumarie, G. (2010), "Derivation and solutions of some fractional Black-Scholes equations in coarse-grained space and time. Application to Merton's optimal portfolio", *Comput. Math. Appl.*, **59**(3), 1142-1164.

Love, A.E.H. (1911), *Some Problems of Geodynamics*, Cambridge University Press, Cambridge.

Nowacki, W. (1959), "Some dynamic problems of thermoelasticity", *Arch. Mech. Stos.*, **9**, 325-334.

Povstenko, Y.Z. (2005), "Fractional heat conduction equation and associated thermal stress", *J. Therm. Stress.*, **28**(1), 83-102.

Rakshit, M. and Mukhopadhyay, B. (2007), "A two dimensional thermoviscoelastic problem due to instantaneous point heat source", *Math. Comput. Modell.*, **46**(11-12), 1388-1397.

Said, S.M. and Othman, M.I.A. (2016), "Wave propagation in a two temperature fiber reinforced magnetothermoelastic medium with three phase lag model", *Struct. Eng. Mech.*, **57**(2), 201-220.

Sarkar, N., Atwa, S.Y. and Othman, M.I.A. (2016), "The effect of hydrostatic initial stress on the plane waves in a fiber reinforced magneto-thermoelastic medium with fractional derivative heat transfer", *Int. Appl. Mech.*, **52**(2), 203-216.

Sengupta, P.R. and Nath, S. (2001), "Surface waves in fiber reinforced anisotropic elastic media", *Sadhana*, **26**(4), 363-370.

Sherief, H.H., El-Sayed, A.M. and El-Latief, A.M. (2010), "Fractional order theory of thermoelasticity", *Int. J. Sol. Struct.*, **47**(2), 269-275.

Singh, B. and Singh, S.J. (2004), "Surface waves at the free surface of a fiber reinforced elastic half-space", *Sadhana*, **29**(3), 249-257.

Yadav, R., Kalkal, K.K. and Deswal, S. (2017), "Two temperature theory of initially stressed electromicrostretch medium without energy dissipation", *Microsyst. Technol.*, **23**(10), 4931-4940.

Youssef, H. (2013), "State-space approach to fractional order two-temperature generalized thermoelastic medium subjected to moving heat source", *Mech. Adv. Mater. Struct.*, **20**(1), 47-60.

CC