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Structural damage detection using a damage probability index based on frequency response function and strain energy concept

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Abstract. In this study, an efficient damage index is proposed to identify multiple damage cases in structural systems using the concepts of frequency response function (FRF) matrix and strain energy of a structure. The index is defined based on the change of strain energy of an element due to damage. For obtaining the strain energy stored in elements, the columnar coefficients of the FRF matrix is used. The new indicator is named here as frequency response function strain energy based index (FRFSEBI). In order to assess the performance of the proposed index for structural damage detection, some benchmark structures having a number of damage scenarios are considered. Numerical results demonstrate that the proposed index even with considering noise can accurately identify the actual location and approximate severity of the damage. In order to demonstrate the high efficiency of the proposed damage index, its performance is also compared with that of the flexibility strain energy based index (FSEBI) provided in the literature.

Keywords: damage identification; damage index; strain energy; frequency response function

1. Introduction

Many structural systems may suffer some local damage during their lifetime. If the damage is not monitored and timely fixed, it can affect the performance of the structure, increase the cost of maintenance and in a terrible event may lead to the overall collapse of the structure. Accordingly, the subject of detecting the location and the severity of the damage has been considered as an important topic in the structural engineering. The damage identification methods have been classified into two groups: destructive method and non-destructive method. Destructive methods due to their high cost and inefficiency are not attractive. Moreover, Damage detection techniques based on the non-destructive tests due to their lower cost and high efficiency can be more appropriate. Non-destructive methods classify into two categories including dynamic and static identification methods. The dynamic identification methods have shown their superior accuracy and more popularity in comparison with the static ones. Therefore, in recent years many procedures based on the use of dynamic characteristic change have been proposed. Among them, damage identification methods based on a damage index is more interesting.

In the context of structural damage based on the damage index method, some research has been reported. A method for damage detection using a damage index was proposed by Barroso and Rodriguez (2004). In the study, the focus of the problem was on a four-story model of an existing physical model at the University of British Columbia where simulated data were used for the system identification and a new method based on ratios between stiffness and mass values from the eigenvalue problem was presented to identify the undamaged state of the structure. A new index based on the change of strain energy in each element before and after the occurrence of damage was presented by Sharifi and Banan (2008). In the paper, mode shape vectors were used to obtain damage index. The influence of statistical errors on damage detection based on structural flexibility and mode shape curvature has been investigated by Tomaszewska (2010). A new damage index called CPI for detecting the damage severity in structural elements by combining modal parameters was proposed by Fayyadh *et al*. (2011). The index was based on the combined effect of both the natural frequencies and mode shapes when a change in stiffness of the structural element occurs. A two stage method for structural damage detection using a modal strain energy based index and particle swarm optimization was presented by Seyedpoor (2012). A new indicator based on the concepts of flexibility matrix and strain energy of a structure was presented by Seyedpoor and Nobahari (2013). In the paper, a relative change of strain energy of an element before and after damage with using the concept of flexibility matrix has been utilized to introduce the index. A flexibility based damage index has been proposed for structural damage detection by Zhang *et al*. (2013). Numerical results have shown the efficiency of the proposed index for successfully detecting damage locations in both the single and multiple damage cases. A new damage index based on the change of strain energy computing by the flexibility matrix was presented by Seyedpoor and Montazer (2014). Numerical results indicated that the method can provide a reliable tool to accurately identify the multiple structural damage. An indicator for structural damage localization using the

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change of strain energy based on static noisy data was presented by Seyedpoor and Yazdanpanah (2014). A damage index based on the auto correlation function under white noise excitation was presented by Zhang and Schmidt (2015). The maximum values of the auto correlation function of the vibration response signals (displacement, velocity and acceleration) were collected and formulated as a vector called Auto Correlation Function at Maximum Point Value Vector (AMV). The relative change of the normalized AMV before and after damage occurrence in the structure was adopted as the damage index. A damage identification method for truss structures using a flexibility based damage probability index and differential evolution algorithm was proposed by Seyedpoor and Montazer (2016). A new method for structural damage identification using a damage index was presented by Zareh Hosseinzadeh *et al*. (2016). In the paper, first, the damaged structure was excited by short duration impact acceleration and then, the displacement time history responses under free vibration conditions were analyzed by continuous wavelet transform (CWT) and wavelet residual force (WRF) was calculated. Finally, an effective damage-sensitive index was proposed.

In this paper, a new damage index named here as frequency response function strain energy based index (FRFSEBI) is introduced. The FRF matrix of the structure is estimated from the mode shapes and natural frequencies. The columnar coefficients of the FRF matrix are used to obtain the strain energy of structural elements. Then, a relative change of strain energy of elements has been utilized to introduce the new index. Numerical results demonstrate the high efficiency of the proposed index for detecting the actual location and approximate severity of both single and multiple damages.

2. Frequency response function

A frequency response function expresses the structural response to an applied force as a function of frequency. This function is formed from the measured data or analytic functions. The response can be displacement, velocity, or acceleration. Based on the principles of structural dynamics, differential equation governing the dynamic behavior of multiple degrees of freedom structures is a second order equation and can be given by Eq. (1) (Chopra 2001)

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = F(t) \tag{1}$$

where M, C and K are mass, damping and stiffness matrices of the structure, respectively; $\ddot{X}(t)$, $\dot{X}(t)$, $X(t)$ are acceleration, velocity and displacement vectors of structures at time t. Also, $F(t)$ is the vector of externally applied forces in degrees of freedom of the structure.

If the vector of externally applied force is considered to be harmonic, the force and the displacement of structures using the Fourier transform can be represented by Eqs. (2)-(3) (Craig 1981)

$$F(t) = F(\Omega)e^{-i\Omega t} \tag{2}$$

$$X(t) = X(\Omega)e^{-i\Omega t} \tag{3}$$

where Ω is the frequency of harmonic excitation, and $X(\Omega)$, $F(\Omega)$ are the displacement and externally applied force on the frequency domain. Substituting Eqs. (2)-(3) into Eq. (1) is led to Eq. (4) as

$$X(\Omega) = H(\Omega)F(\Omega) \tag{4}$$

where $H(\Omega)$ is structural response in the frequency domain and is named as the frequency response function (FRF). Frequency response function is given as Eq. (5)

$$H(\Omega) = [K - i\Omega C - \Omega^2 M]^{-1} \tag{5}$$

The computational cost incurred in obtaining the $H(\Omega)$ matrix by Eq. (5) is too expensive, as for each Ω value, a large complex matrix needs to be inverted. Using the technique of modal decomposition and assuming Rayleigh damping for the system, the following $H(\Omega)$ matrix in terms of modal parameters can be obtained (Begambre and Laier 2009)

$$H(\Omega) = \phi \ \mathrm{diag}\left(\frac{1}{\omega_j^2 - 2i\,\Omega\omega_j\varepsilon_j - \Omega^2} \right) \phi^{\mathrm{T}} \tag{6}$$

where ω_j is jth circular frequency of the structure, ε_j is the modal damping ratio for the jth mode, and i equals to $\sqrt{-1}$. Also, φ is the mode shape matrix and Ω is the frequency of harmonic excitation.

3. The frequency response function strain energy based index

In this study, a damage index is proposed to identify the multiple damage cases of structures, including trusses and frames, based on the change of strain energy of the structural elements due to damage. In order to calculate the strain energy stored in the elements, modal analysis data and the elements of FRF matrix are used.

For constructing the index, first the modal data of healthy and damaged structures including natural frequencies and mode shapes are needed. For this, a modal analysis (Golizadeh and Barzegar 2012; Golizadeh *et al*. 2008) is performed according to Eq. (7)

$$\left(K - \omega_i^2 M\right)\phi_i = 0, \quad i = 1,2,...,ndf \tag{7}$$

where M and K are the mass and stiffness matrices of the structure, respectively; ω_i and φ_i are the ith circular frequency and mode shape vector of the structure, respectively and ndf is the total number of active degrees of freedom.

Secondly, the frequency response function (FRF) matrix of healthy and damaged structures is obtained through the mode shapes and natural frequencies using the Eq. (6). The presence of the frequency of excitation load is led to forming a three-dimensional FRF matrix. In this case, if the frequency of excitation load is equal to zero (Ω=0), the FRF matrix will transform to two-dimensional flexibility matrix.

Then, frequency response change matrix FCM, is defined as the difference of the FRF matrices of healthy and damaged structure as

$$FCM = FRFH - FRFD \tag{8}$$

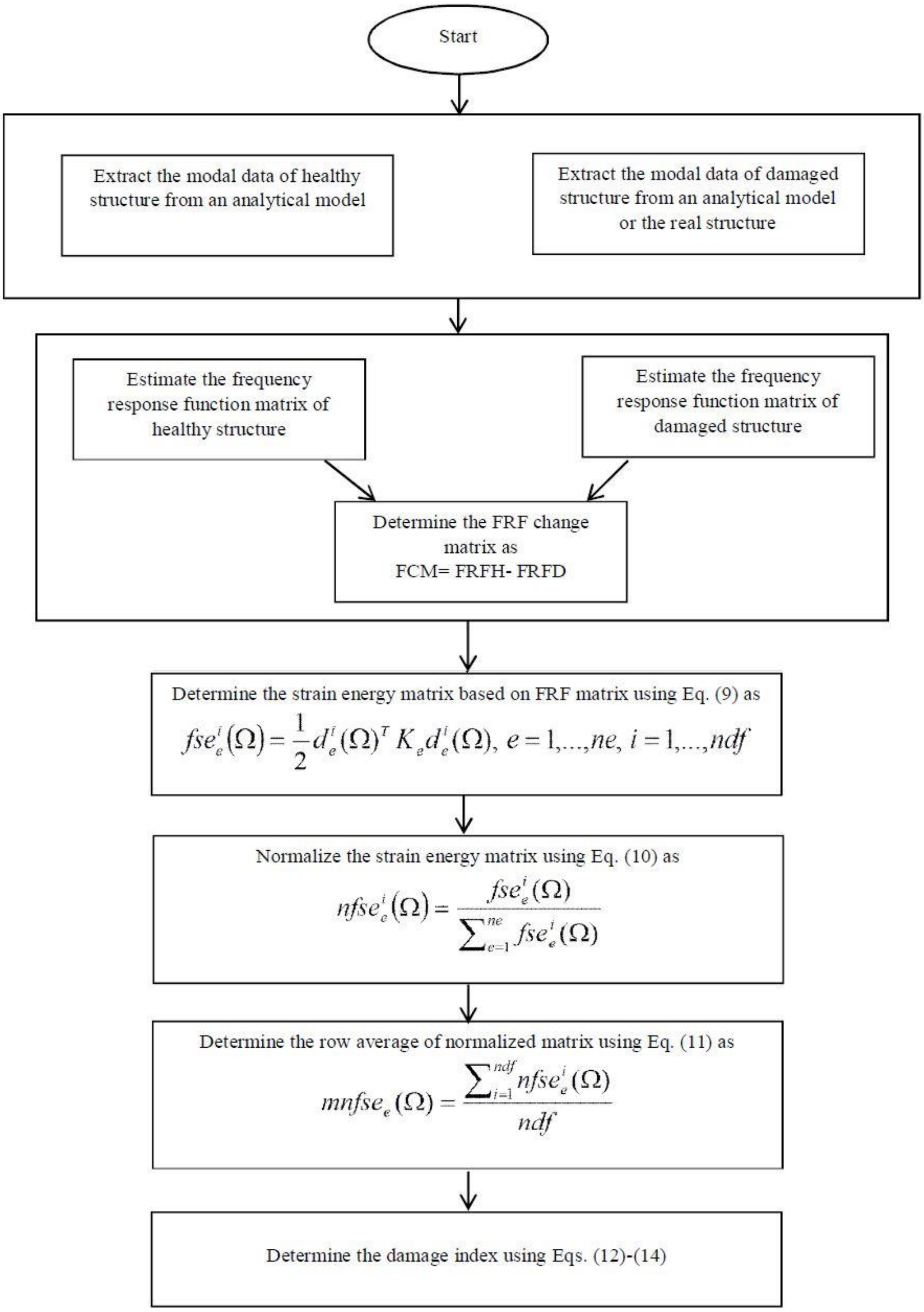


Fig. 1 The main steps of the proposed FRFSEBI

Each column of the FRF matrix represents the nodal displacement pattern of the structure when a harmonic force is applied to the degree of freedom corresponding to that column. Therefore, the columnar coefficients of the *FCM* matrix is the change of nodal displacement pattern that can be utilized to obtain the change of strain energy stored in structural elements as

$$fse_e^i(\Omega) = \frac{1}{2} d_e^i(\Omega)^T K_e d_e^i(\Omega),$$

$$e = 1,\dots,ne,\ i = 1,\dots,ndf \tag{9}$$

where K_e is the element stiffness matrix of healthy structure; *ne* is the total number of structural elements; $d_e^i(\Omega)$ is the vector of nodal displacements corresponds to *e*th element and *i*th column of *FCM* matrix. The size of this vector for a two-dimensional truss element is 4×1 and for a two-dimensional frame element is 6×1. It should be noted that, as the damage locations are unknown for a real-world damaged structure, therefore for this case the element stiffness matrix of the healthy structure is used for estimating the strain energy.

For computational purpose, it is better to normalize the energy of elements with respect to the total energy of the structure as

$$nfse_e^i(\Omega) = \frac{fse_e^i(\Omega)}{\sum_{e=1}^{ne} fse_e^i(\Omega)} \tag{10}$$

In order to form a more efficient damage index, row average of the normalized matrix of Eq. (10) is calculated.

$$mnfse_e(\Omega) = \frac{\sum_{i=1}^{ndf} nfse_e^i(\Omega)}{ndf} \tag{11}$$

where *ndf* is the total number of columns in the *FCM* matrix.

In this step, for the third dimension dependent to the frequency of excitation load, the average of $mnfse_e\ (\Omega)$ is calculated for different values of excitation frequency. The frequency of excitation load can be considered according to the frequencies of structures. Moreover, in order to avoid working with a complex value, the magnitude of a complex value is used.

$$mnkfse_e = abs(mean_\Omega(mnfse_e(\Omega))) \tag{12}$$

where *abs*() is a symbol to represent the magnitude of a complex value and *mean*() stands for the average of some values.

In order to create a normal distribution for the damage index of Eq. (12), it is also normalized as

$$nIndex_e = \max\left(\frac{mnkfse_e - mean_e(mnkfse_e)}{std_e(mnkfse_e)}, 0\right) \tag{13}$$

where *std*() is the standard diversion of the components of a vector.

Finally, the frequency response function strain energy based index (FRFSEBI) is defined by

$$FRFSEBI_e = \frac{1}{3} \times \frac{nIndex_e}{\sqrt{\sum_{e=1}^{ne} nIndex_e^2}}, \quad e = 1, \ldots, ne \tag{14}$$

According to the Eq. (14), for a healthy element the index will be equal to zero (FRFSEBI=0) while for a damaged element the index will be greater than zero (FRFSEBI > 0). The main steps of the proposed index FRFSEBI is shown in Fig. 1.

4. Test examples

In order to show the capabilities of the proposed index for structural damage detection, two illustrative test examples are considered. The first example is a 45-bar planar truss and the second one is a 15-element planar frame. In the second example, the efficiency of the indicator FRFSEBI, compared to the damage indicator FSEBI proposed by Nobahari and Seyedpoor (2013) is assessed. A parametric study is also made and the effects of parameters such as the number of damaged elements, the number of modes and the measurement noise effect on the effectiveness of the method are studied. Results are presented in two sections, without considering the noise and with considering noise 3%.

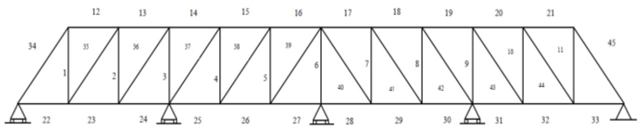


Fig. 2 A planar truss having 45 elements

Table 1 Three different damage cases induced in 45-bar planar truss

Case 1		Case 2		Case 3	
Element no.	Damage ratio	Element no.	Damage ratio	Element no.	Damage ratio
26	0.18	38	0.41	7	0.34
-	-	-	-	35	0.20
-	-	-	-	36	0.25

4.1 Forty five-bar planar truss

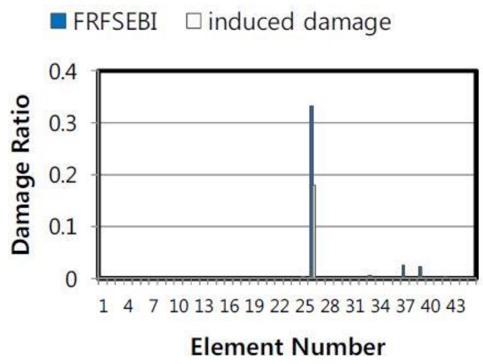
The first example is a 2D truss. The 45-bar planar truss (Villalba and Laier 2012) shown in Fig. 2 is modeled using the finite element method. The modulus of elasticity, cross sectional area and material density are 200 GPa, 0.001 m^2 and 7800 kg/m^3, respectively. Damage in the structure is simulated as a relative reduction in the elasticity modulus of individual bars. Three damage cases given in Table 1 are induced in the structure.

4.1.1 The effect of number of modes

In order to investigate the effect of number of vibrating modes on the performance of the method, the proposed index FRFSEBI is evaluated when 5, 8 and 11 modes are considered to estimate the FRF matrix. Figs. 3 to 5 show the performance of proposed index for damage cases 1 to 3, respectively when the number of different modes are considered. It can be observed that for accurately locating the damage cases 1 and 2, five mode shapes of the structure, are required to be considered and for case 3, eleven mode shapes are needed. The results demonstrate the efficiency of the proposed index for detecting the location and approximate severity of the both single and multiple damage cases.

4.1.2 The effect of measurement noise

In order to assess the performance of the proposed damage index, the effect of measurement noise on the performance of the method is investigated. The measurement noise is considered here by a standard error of 3% affecting mode shapes. Figs. 6-8 show the mean values of FRFSEBI for 100 independent runs for damage scenarios 1 to 3, respectively when the first 5 and 10 mode shapes are considered. Here, those elements whose indexes exceed 0.1 are selected as suspected damaged elements. As shown in the figures, the damaged elements identified by FRFSEBI for damage scenario 1 by considering 5 and 10 modes is element 26; for damage scenario 2 using 5 and 10 modes is



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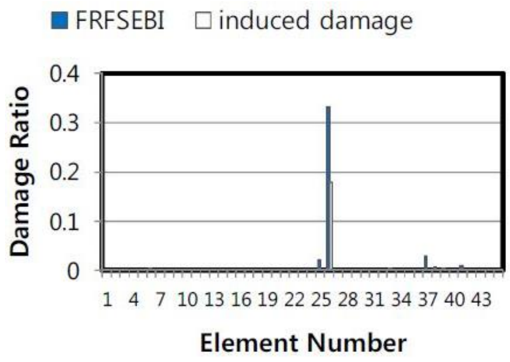

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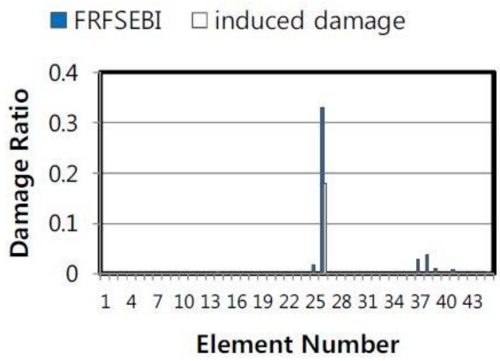
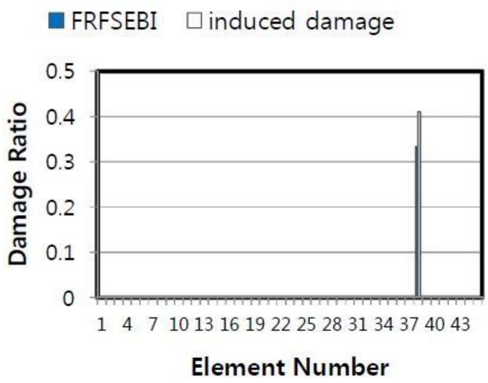


(c)

Fig. 3 Damaged identification by FRFSEBI for damage case 1 of planar truss considering: (a) five modes (b) eight modes, (c) eleven modes



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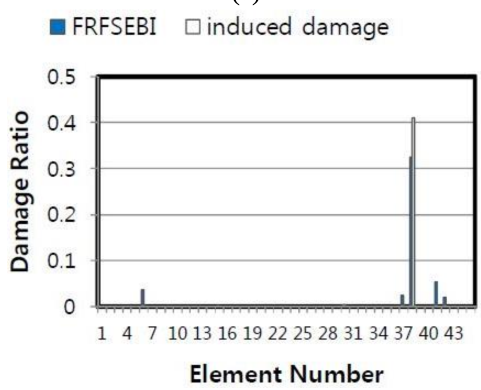

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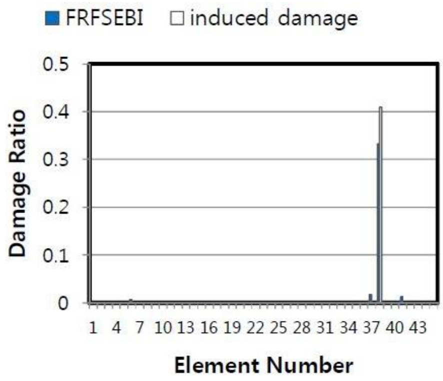


(c)

Fig. 4 Damaged identification by FRFSEBI for damage case 2 of planar truss considering: (a) five modes (b) eight modes, (c) eleven modes

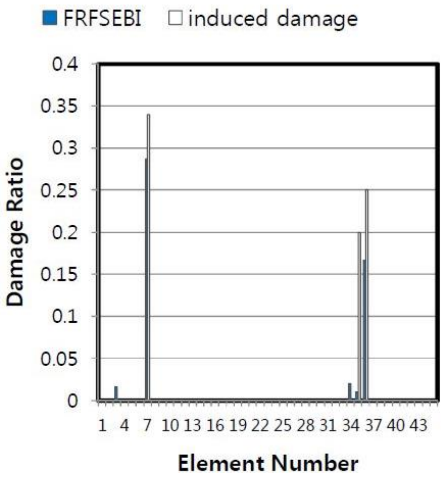
element 38; and for damage scenario 3 using 5 mode shapes are elements 7 and 36, and using 10 mode shapes are elements 7,35 and 36. It can be observed that for accurately locating the damage cases 1 to 3, five, five and ten mode shapes of the structure, respectively, are required to be considered. The results demonstrate that the index has a good performance even in the presence of noise and it can correctly detect the location and approximate severity of the damage. As shown in the figures, noise has a slight influence on identifying damage of 2D-truss.

4.2 Fifteen-element planar frame

The 15-element planar frame (Esfandiari *et al*. 2013) shown in Fig. 9 is modeled using the finite element method. The modulus of elasticity, moment of inertia and material density are 200 GPa, 0.2644 m^4 and 7800 kg/m^3, respectively. Three damage cases given in Table 2, apply to the structure and the performance of the proposed index is compared with the damage index FSEBI (Seyedpoor and Nobahari 2013).

4.2.1 The efficiency of FRFSEBI compared to FSEBI

In order to assess the competence of the proposed index for structural damage detection, the efficiency of FRFSEBI is compared with that of the flexibility strain energy based index (FSEBI). Figs. 10 to 12 show the FRFSEBI value for damage cases 1 to 3, with considering 5 and 7 mode shapes, and compares it with the FSEBI. As shown in the figures, the potentially damaged elements identified by FRFSEBI for considering both 5 and 7 modes in damage scenario 1 is element 13; in damage scenario 2 are elements 5 and 13;



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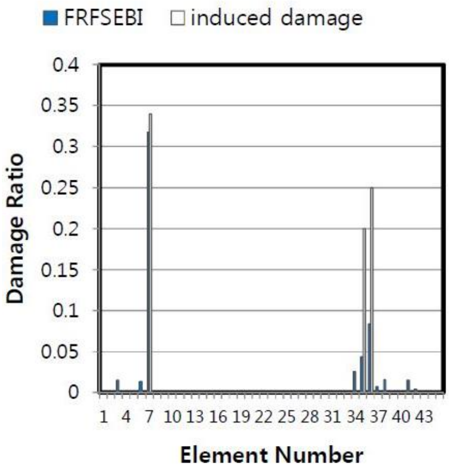

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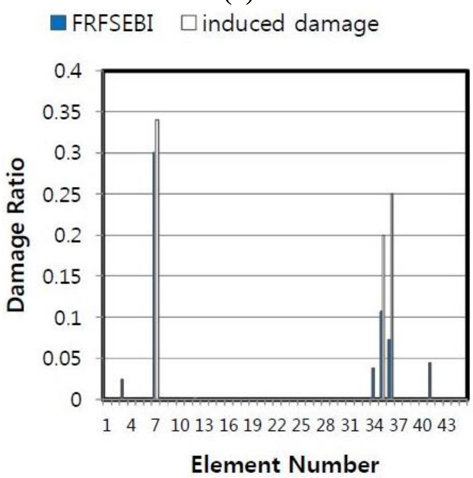
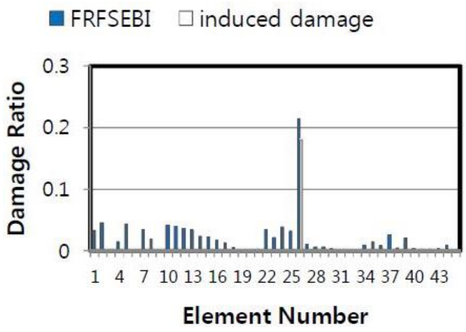
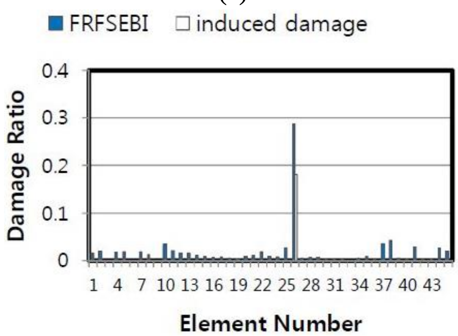


(c)

Fig. 5 Damaged identification by FRFSEBI for damage case 3 of planar truss considering: (a) five modes (b) eight modes, (c) eleven modes

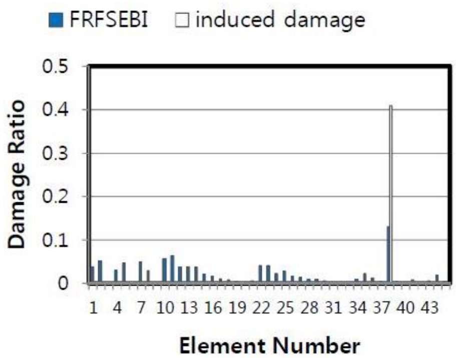


(a)

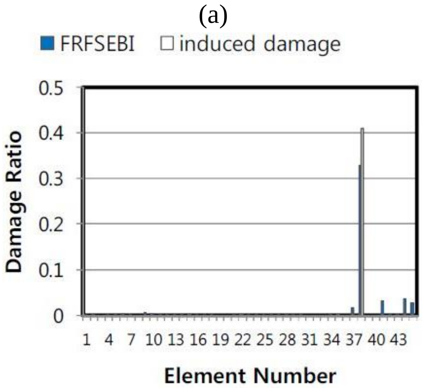


(b)

Fig. 6 Damage identification of FRFSEBI for damage case 1 of planar truss considering noise 3% (a) five modes (b) ten modes



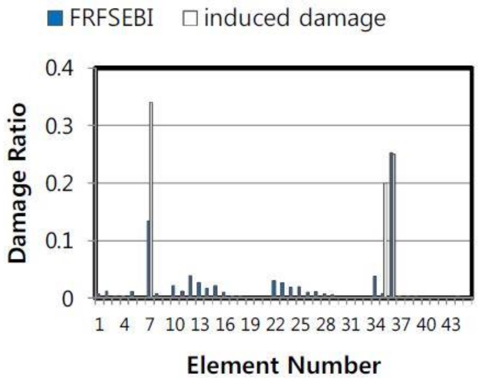
(a)



(b)

Fig. 7 Damage identification of FRFSEBI for damage case 2 of planar truss considering noise 3% (a) five modes (b) ten modes

and for damage scenario 3 using 5 mode shapes are elements 6,7,9 and 11, and using 7 mode shapes are elements 6,7,9,10 and 11. Those elements whose indexes exceed 0.05 are selected here as suspected damaged elements. It can be observed that for accurately locating the damage cases 1 to 3, five, five and seven mode shapes of the structure, respectively, are required to be considered. Also, the potentially damaged elements identified by FSEBI



(a)

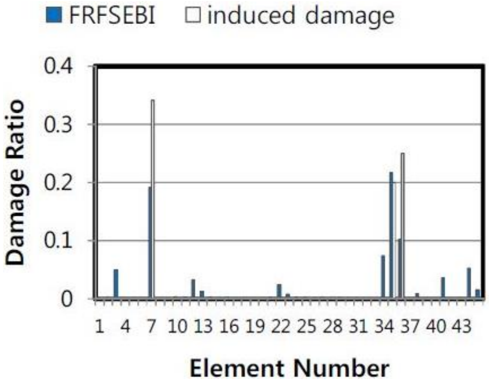


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Fig. 8 Damage identification of FRFSEBI for damage case 3 of planar truss considering noise 3% (a) five modes (b) ten modes

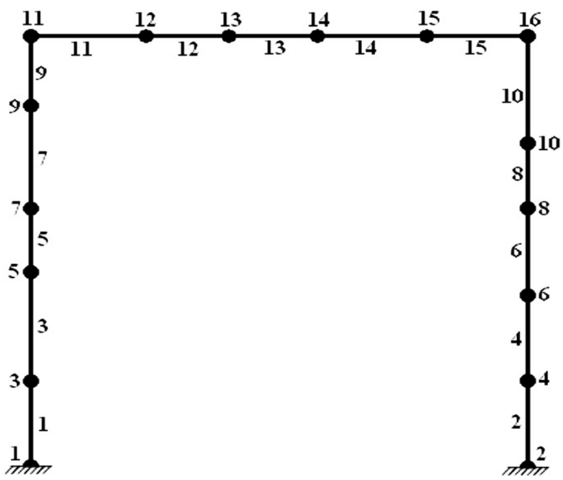
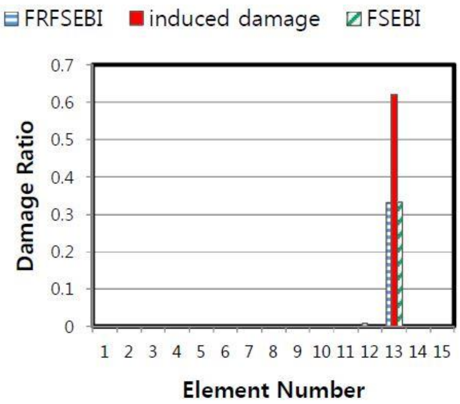


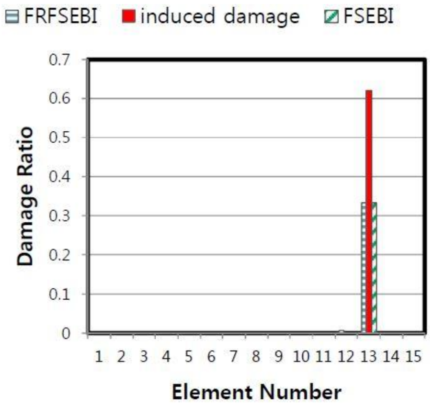
Fig. 9 A portal frame having 15 elements

Table 2 Three different damage cases induced in 15-element planar frame

Case 1		Case 2		Case 3	
Element no.	Damage ratio	Element no.	Damage ratio	Element no.	Damage ratio
13	0.62	5	0.30	6	0.25
-	-	13	0.30	7	0.20
-	-	-	-	9	0.15
-	-	-	-	10	0.25
-	-	-	-	11	0.30

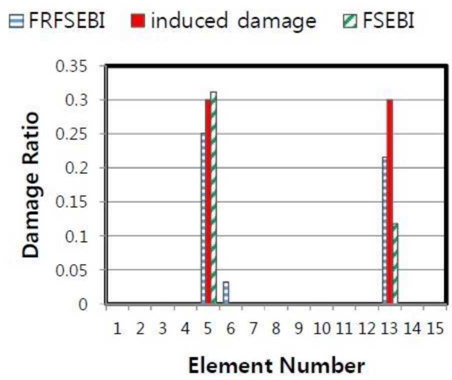


(a)



(b)

Fig. 10 Identification results of FRFSEBI and FSEBI for damage case 1 considering: (a) five modes (b) seven modes



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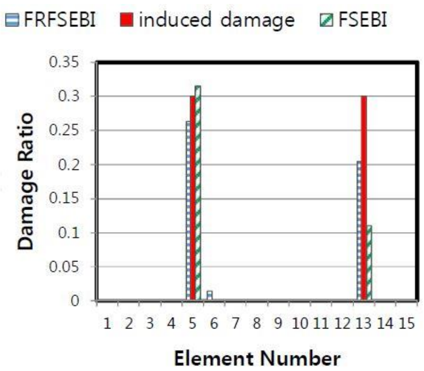

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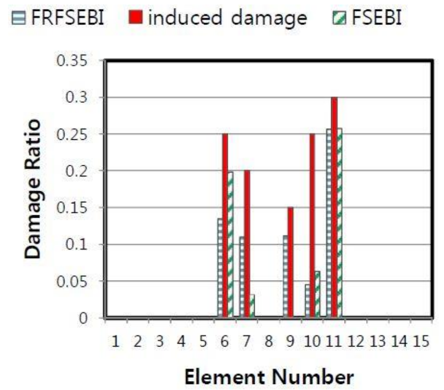
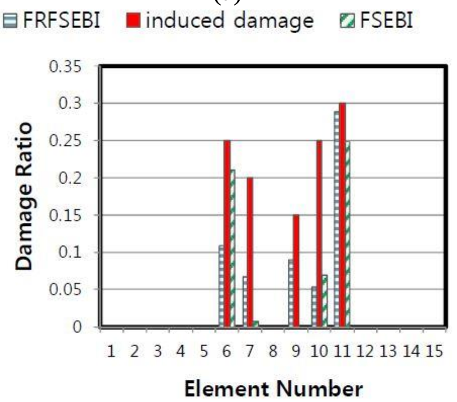
Fig. 11 Identification results of FRFSEBI and FSEBI for damage case 2 considering: (a) five modes (b) seven modes



(a)



(b)

Fig. 12 Identification results of FRFSEBI and FSEBI for damage case 3 considering: (a) five modes (b) seven modes

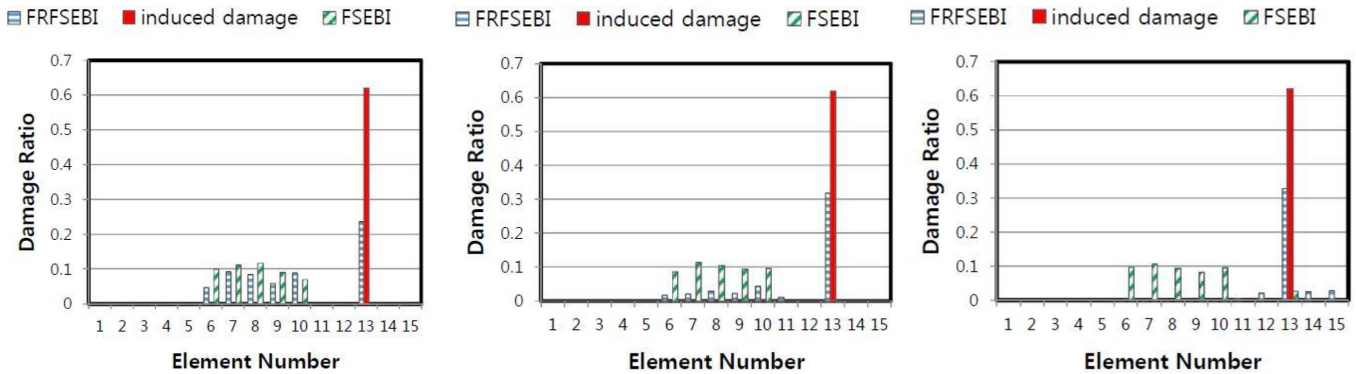
in damage scenario 1 for 5 and 7 modes is element 13; in damage scenario 2 for 5 and 7 modes are elements 5 and 13; and in damage scenario 3 for 5 and 7 mode shapes are elements 6,10 and 11. It is revealed that the FRFSEBI for detecting the exact location and approximate severity of the damage is more efficient than FSEBI while the number of vibrating modes considered for FRFSEBI and FSEBI, is equal.

4.2.2 The effect of measurement noise

The measurement noise is considered here by a standard error of 3% affected mode shapes. Figs. 13 to 15 show the mean values of FRFSEBI and FSEBI for 100 independent runs for damage scenarios 1 to 3, when 5, 10 and 15 mode shapes are considered. Here, those elements whose indices exceed 0.10 are selected as suspected damage elements. As shown in the figures, the potentially damaged elements identified by FRFSEBI for damage scenario 1 by considering 5,10 and 15 modes is element is 13; for damage scenario 2 by considering 5 modes are elements 6,7,8 and 10, using 10 mode shapes are elements 6,8 and 13, and using 15 mode shapes are elements 5 and 13; for damage scenario 3 by considering 5 modes are elements 6,7,9 and 10, using 10 mode shapes are elements 6,9,10 and 11, and using 15 mode shapes are elements 6,7,9,10 and 11. It can be observed that for accurately locating the damage cases 1, five mode shapes of the structure are required to be considered and for case 2 and 3 of damage scenarios 15 mode shapes are needed. Also, the potentially damaged elements identified by FSEBI for damage scenario 1 by considering 5 modes are elements 6, 7 and 8, using 10 modes are elements 7 and 8, and using 15 modes is elements 7; for damage scenario 2 by 5 and 15 modes are elements 7 and 8 and using 10 modes is element 8; for damage scenario 3 by considering 5 and 10 modes are elements 7 and 8, and using 15 modes are elements 7, 8 and 10. Therefore, the index FRFSEBI for identifying the exact location and approximate severity of the damages is more efficient than FSEBI while the number of vibrating modes considered for both FRFSEBI and FSEBI is equal. As a result, for this example, the index FSEBI has a poor performance in the presence of noise.

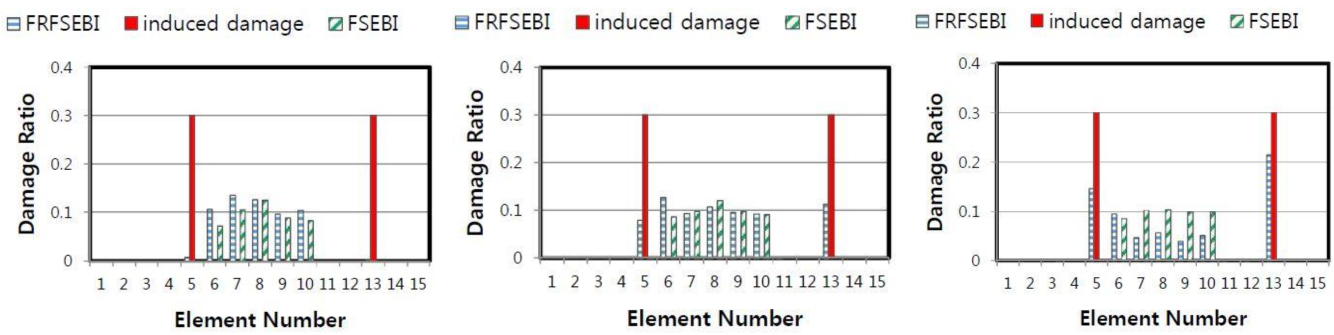
5. Conclusions

An efficient method for structural damage identification using frequency response function (FRF) matrix and the concept of strain energy has been proposed. The frequency response function matrix estimated using modal analysis data. Each column of the FRF matrix represents the nodal displacement pattern of the structure when a harmonic force is applied to the degree of freedom corresponding to that column. Therefore, the columnar coefficients of the FRF



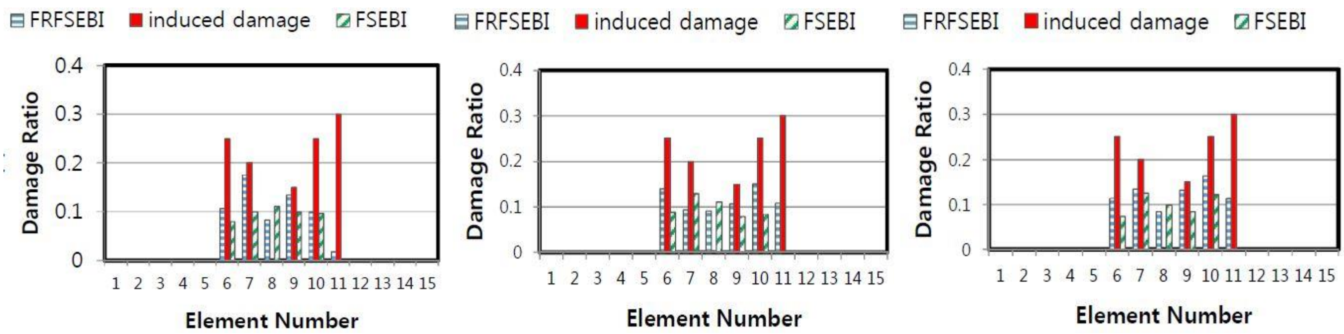
(a) (b) (c)

Fig. 13 Identification results of FRFSEBI and FSEBI for damage case 1 with considering 3 % noise: (a) 5 modes (b) 10 modes (c) 15 modes



(a) (b) (c)

Fig. 14 Identification results of FRFSEBI and FSEBI for damage case 2 with considering 3 % noise: (a) 5 modes (b) 10 modes (c) 15 modes



(a) (b) (c)

Fig. 15 Identification results of FRFSEBI and FSEBI for damage case 3 with considering 3 % noise: (a) 5 modes (b) 10 modes (c) 15 modes

matrix can be utilized to obtain the strain energy stored in structural elements. The proposed indicator is named here as frequency response function strain energy based index (FRFSEBI). In order to assess the performance of the proposed index for structural damage detection, two illustrative test examples are considered. The results demonstrate that the index even with considering noise can accurately identify the location and approximate severity of the damaged element of 2D-truss and 2D-frame, by considering only the first few modes of the structures. Also, the proposed method is more efficient when compared with the index FSEBI provided in the literature.

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