# Single variable shear deformation model for bending analysis of thick beams

Salima Abdelbari<sup>1,2</sup>, Lemya Hanifi Hachemi Amar<sup>\*3,4</sup>, Abdelhakim Kaci<sup>2,3</sup> and Abdelouahed Tounsi<sup>2,5</sup>

<sup>1</sup>Département de Génie Civil, Institut des Sciences et de la Technologie, Centre Universitaire de Ain Témouchent, Algeria <sup>2</sup>Civil Engineering Department, Faculty of Technology, Material and Hydrology Laboratory, University of Sidi Bel Abbes, Algeria <sup>3</sup>Département de Génie Civil et Hydraulique, Faculté de Technologie, Université Dr Tahar Moulay, BP 138 Cité En-Nasr 20000 Saida, Algérie <sup>4</sup>Laboratoire des Ressources Hydriques et Environnement, Université Dr Tahar Moulay, BP 138 Cité En-Nasr 20000 Saida, Algérie <sup>5</sup>Department of Civil and Environmental Engineering, King Fahd University of Petroleum & Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia

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**Abstract.** In this work, a new trigonometry theory of shear deformation is developed for the static analysis of thick isotropic beams. The number of variables used in this theory is identical to that required in the theory of Euler-Bernoulli, sine function is used in the displacement field in terms of the coordinates of the thickness to represent the effects of shear deformation. The advantage of this theory is that shear stresses can be obtained directly from the relationships constitute, while respecting the boundary conditions at the free surface level of the beam. Therefore, this theory avoids the use of shear correction coefficients. The differential equilibrium equations are obtained using the principle of virtual works. A thick isotropic beam is considered, whose numerical study to show the effectiveness of this theory.

Keywords: thick beam; high order theory; virtual working principle; bending

## 1. Introduction

It is well known that the basic theory of beam bending based on assumptions Euler-Bernoulli neglects shear deformation and stress concentration. This theory is applicable for the slender beams and is not applicable for thick or short beams since it is based on the hypothesis that the normal to the neutral axis remains perpendicular to the same axis during and after bending. Therefore, the stress and the shear distortion are void. Since the Euler-Bernoulli theory neglected the transverse shear deformation, it overestimates the arrows in the case of thick beams or the effects of shear deformation are significant.

Bress (1859) and Timoshenko (1921) are the investigative pioneers to include the refined effects such as rotational inertia and shear deformation in beam theory. Timoshenko (1921) showed that the shear effects and greater than this of the rotational inertia for the transversal vibration of the beams. Noted by the beam theory of Timoshenko or the theory of first order shear deformations. (FSDT) in the literature of this theory, the distributions of the transverse shear deformation are assumed to be constant through the thickness of the beam and therefore coefficients of shear corrections are necessary to be determined. Cowper (1968) gave an enriched expression for the shear correction factor for different cross-section of the beams.

The precision of the Timoshenko beam theory for transverse vibrations of a simply supported beam for the

E-mail: lamiacci@hotmail.com

Copyright © 2018 Techno-Press, Ltd. http://www.techno-press.com/journals/sem&subpage=7 fundamental frequency is verified by Cowper (1968), using the planar stress elasticity solution. Several authors (Al-Basyouni et al. 2015, Arani and Kolahchi 2016, Kolahchi et al. 2016a, b, Bilouei et al. 2016, Madani et al. 2016, Bouderba et al. 2016, Bellifa et al. 2016, Zamanian et al. 2017, Zarei et al. 2017, Shokravi 2017a, b, Youcef et al. 2018) have employed also FSDT and classical theory to study beam/plates structures. To remove the gaps in the classical beam theory and the shear deformation theory of the first order, the theories of refining or high-order shear deformation are developed and are available in the literature for static and dynamic analysis (vibration) of beams. Several researchers (Levinso 1981, Baluch et al. 1984, Relfied and Murty 1982, Krishnu and Murty 1984, Di Sciuva et al. 1984, Bhinardd and Chandras 1993) have presented parabolic shear-deformation theories, that use a nonlinear variation of axial displacement in terms of the coordinate of the thickness. These theories respect the conditions of the zero shear stress at the upper and lower sides of the beam and therefore the shear correction factor becomes more necessary. Irretier (1986) studied the dynamic effects in homogeneous beams using refined theories that exceed the limits of the Euler-Bernoulli beam theory. These studied effects are rotational inertia, shear deformation and coupling between bending and torsion. Kant and Gupta (1988) and Heyliger and Reddy (1988) presented finite element models based on the high-order shear deformation theory for rectangular beams. However, these shifts based on finite element models do not respect the shear stress conditions at the upper and lower surface of the beam (April and Reddy 1992, Reddy 1997). There is another class of refined theories, which included

<sup>\*</sup>Corresponding author, Ph.D.



Fig. 1 Bending beam in the X-Z plane

trigonometric functions to represent the effects of shear deformation through the thickness. Levinson (1981) and Stein (1989) developed theories of shear deformation for thick beams by including a sine function in terms of the thickness coordinates in the field of motion. Ghorbanpour Arani et al. (2016) also used a sinusoidal shear deformation theory for viscoelastic nano-plates resting on orthotropic elastic medium. A synthesis study carried out by Ghugal and Shimpi (2001) indicated that the research work dealing with the analysis of the bending of thick beams using hyperbolic and trigonometric shear deformation theories is very rare and remains to developed. Kreja (2011) presented a literature review on computational models for laminated composite and sandwich panels. Recently, a number of high shear deformation theories (HSDTs) are also developed for analyzing beams and plates (Aldousari 2017, Baseri et al. 2016, Kar et al. 2016, Akavci 2015, Attia et al. 2015, Ahmed 2014, Ait Amar Meziane et al. 2014, Swaminathan and Naveenkumar 2014, Zehra and Shinde 2012a, Tounsi et al. 2013, Bouderba et al. 2013, Kar et al. 2015, Belkorissat et al. 2015, Mahapatra et al. 2016, Kolahchi and Moniri Bidgoli 2016, Ahouel et al. 2016, Sahoo et al. 2016, Bounouara et al. 2016, Boukhari et al. 2016, Kolahchi et al. 2017a, b, c, Mehar et al. 2017, Hirwani et al. 2017, Hajmohammad et al. 2017, Shokravi 2017c, d, Beldjelili et al. 2016, Bousahla et al. 2016, Mehar and Panda 2016, 2017a, b, Kolahchi and Cheraghbak 2017, Abdelaziz et al. 2017, Besseghier et al. 2017, Kolahchi 2017, Bellifa et al. 2017a, b). Recently, new beam/plate theories are developed with lower number of variables to study mechanical behavior of different structures (Houari et al. 2016, Khetir et al. 2017, Mouffoki et al. 2017, Zidi et al. 2017, Klouche et al. 2017, Hachemi et al. 2017, Kaci et al. 2018, Belabed et al. 2018, Mokhtar et al. 2018, Fourn et al. 2018, Yazid et al. 2018).

In this work a shear deformation theory that uses a single variable is developed. This theory is applied to the thick isotropic beam. The significant feature of this formulation is that, in addition to including the shear deformation effect, it deals with only one unknown as the Euler-Bernoulli. The effects of shear deformations are considered through a sinus function in terms of the coordinate of the thickness in the axial displacement. Numerical results are presented to validate the present theory.

#### 2. Theory and formulation

The beam in consideration is shown in the Fig. 1 and the Cartesian coordinates system (x, y, z) is adopted with the following areas of space

$$0 \le x \le L \quad ; \quad -\frac{b_2}{2} \le y \le \frac{b_2}{2} \quad ; \quad -\frac{b_2}{2} \le z \le \frac{b_2}{2} \tag{1}$$

Such as x, y and z are the Cartesian coordinates, L and b are the length and width of the beam in the directions x, and y respectively, and h is the thickness of the beam according to the z direction.

The beam is homogeneous, isotropic and linearly elastic.

## 2.1 Kinematic relations

The new proposed displacement field of the present theory is given as follows

$$u(x,z) = -z\frac{dw_0}{dx} - \alpha f(z)\frac{d^3w_0}{dx^3}$$
(2a)

$$w(x,z) = w_0(x) \tag{2b}$$

$$f(z) = \frac{h}{\pi} \sin \frac{\pi z}{h}$$
(3)

Such as

Where u is the axial displacement according to the direction x and  $w_0$  is the transverse displacement according to the direction z of the beam,  $\alpha$  is an unknown parameter to be determined.

Normal strain and transverse shear strain for beam are given by

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z \frac{d^2 w_0}{dx^2} - \alpha f(z) \frac{d^4 w_0}{dx^4}$$
(4a)

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = -\alpha f'(z) \frac{d^3 w_0}{dx^3}$$
(4b)

The stress-strain relationships used are as follows

#### 2.2 Governing equations

Governing equations are obtained using the principle of virtual work as follows (Zidi *et al.* 2014, Ait Atmane *et al.* 2015, Ait Yahia *et al.* 2015, Zemri *et al.* 2015, Benadouda *et al.* 2017, Menasria *et al.* 2017, Meksi *et al.* 2018, Attia *et al.* 2018, Bakhadda *et al.* 2018)

$$\left[\left(\delta U + \delta V\right) = 0\tag{6}$$

Where  $\delta U$  is the virtual variation of the energy of deformation and  $\delta V$  is the virtual variation of the work of the external forces.

The variation of the beam's deformation energy is given by

$$\delta U = \int_{0}^{L} \int_{A} (\sigma_x \delta \varepsilon_x + \tau_{xz} \delta \gamma_{xz}) dA dx$$

$$= \int_{0}^{L} \left( -M \frac{d^2 \delta w}{dx^2} - P \frac{d^4 \delta w}{dx^4} - Q \frac{d^3 \delta w}{dx^3} \right)$$
(7)

Where  $(\sigma_x, \tau_{xz})$  and  $(\varepsilon_x, \gamma_{xz})$  are the components of stress and strain, respectively, *A* is the top surface of the beam, *M*, *P* and *Q* are the stress results defined by

$$(M, P) = \int_{A} (z, \alpha f) \sigma_x dA$$

$$Q = \int_{A} \alpha f'(z) \tau_{xz} dA$$
(8)

The variation of the work of the external forces is expressed by

$$\delta V = -\int_{0}^{L} q \delta w dx \tag{9}$$

Where q is the transverse load.

Substitute the expressions of  $\delta U$  and  $\delta V$  From the Eqs. (7) and (9) in the Eq. (6), and after integrations by party.

The following equation of motion is determined

$$-\frac{d^2M}{dx^2} - \frac{d^4P}{dx^4} + \frac{d^3Q}{dx^3} - q = 0$$
(10)

Replace the Eqs. (8) in the Eq. (10) and after overrides the expressions found in the Eq. (10), the effects are obtained as follows

$$M = -D\frac{d^2w}{dx^2} - D_s \frac{d^4w}{dx^4}$$
(11a)

$$P = -D_s \frac{d^2 w}{dx^2} - H_s \frac{d^4 w}{dx^4}$$
(11b)

$$Q = -A_s \frac{d^3 w}{dx^3} \tag{11c}$$

Where D,  $D_s$ ,  $H_s$  and  $A_s$  are the stiffness coefficients' given as follows

$$A_{s} = \int_{A} G(\alpha f')^{2} dA$$

$$(D, D_{s}, H_{s}) = \int_{A} E(z^{2}, z \alpha f, \alpha^{2} f^{2}) dA$$
(12)

The governing equation in terms of displacement variables are obtained as follows

$$D\frac{d^4w}{dx^4} + (2D_s - A_s)\frac{d^6w}{dx^6} + H_s\frac{d^8w}{dx^8} - q = 0$$
(13)

#### 2.3 Analytical solutions

In this section, analytical solutions for bending are presented for anisotropic simply supported beam.

According to the Navier solution, the transverse displacements are expanded in Fourier series as given below

$$w = \sum_{n=1}^{\infty} W_n \sin(\beta x)$$
(14)

Where  $\beta = n \pi/L$ 

 $W_n$  is an arbitrary parameter to be determined, the following numerical examples are considered

Example 01

$$q(x) = q_0 \sin\left(\frac{\pi x}{L}\right) \tag{15}$$

The beam is subjected to a sine load according to the z direction, the load is expressed as

Where  $q_0$  is the intensity of the sine load in the middle of the beam.

Example 02:

The beam is subjected to a uniform load q(x) according to the direction z

$$q(x) = \sum_{n=1}^{\infty} q_n \sin\left(\frac{n\pi x}{L}\right) = q_0$$
(16)

$$q_n = \frac{4q_0}{n\pi}$$
 for  $n = 1,3,5.....$  (17a)

$$q_n = 0$$
 for  $n = 2, 4, 6$ ..... (17b)

Where  $q_n$  are the coefficients of Fourier expansion of load which are given by

Example 03

$$q(x) = \sum_{n=1}^{\infty} q_n \sin\left(\frac{n\,\pi\,x}{L}\right) = \frac{q_0 x}{L} \tag{18}$$

The beam is loaded by a load distributed linearly according to the direction z

The FOURIER coefficients for this loading case are given by

$$q_n = \frac{2q_0}{n\pi} \cos(n\pi)$$
 (19a)  
for  $n = 1, 3, 5.....$ 

$$q_n = 0$$
 for  $n = 2, 4, 6$ ..... (19b)

$$sw_n = q_n \tag{20}$$

Substitution the expressions of w and q(x) from in the following algebraic equation:

With

(

$$s = H_s \beta^8 + A_s \beta^6 + D\beta^4 - 2D_s \beta^6$$
(21)

The displacements and stresses are given by

$$w(x) = \sum_{n=1}^{\infty} \frac{q_n}{s} \sin(\beta x)$$
(22a)

$$u(x) = -z \sum_{n=1}^{\infty} \beta \frac{q_n}{s} \cos(\beta x) + \alpha f(z) \sum_{n=1}^{\infty} \beta^3 \frac{q_n}{s} \cos(\beta x)$$
(22b)

$$\sigma_x = E\left(z\sum_{n=1}^{\infty}\beta^2 \frac{q_n}{s}\sin(\beta x) - \alpha f(z)\sum_{n=1}^{\infty}\beta^4 \frac{q_n}{s}\sin(\beta x)\right)$$
(22c)

Table 1 Comparison of axial displacement  $(\overline{u})_{at} (x = 0, z = \pm \frac{h}{2})$ , transverse displacement  $(\overline{w})_{at} (x = \frac{L}{2}, z = 0)$ , axial stress  $(\overline{\sigma}_x)_{at} (x = \frac{L}{2}, z = \pm \frac{h}{2})$ , and transverse shear stress  $\overline{\tau}_{zx}$  at (x = 0, z = 0) for isotropic beam subjected to sinusoidal loading

S	Theory	Model	$\overline{u}$	$\overline{W}$	$\overline{\sigma}_{x}$	$\overline{ au}_{zx}$
	Present Theory	HSDT	12,715	1,429	9,986	1,906
	Zehra and Shinde (2012b)	SVSDT	12,311	1,414	9,95	2,631
4	Bernoulli-Euler	ETB	12,385	1,232	9,727	
	Timoshenko	FSDT	12,385	1,397	9,727	1,273
	Reddy	HSDT	12,715	1,429	9,986	1,906
	Ghugal and Shinpi (2001)	Exact	12,297	1,411	9,958	1,9
	Present Theory	HSDT	194,3365	1,263	61,052	4,773
	Zehra and Shinde (2012b)	SVSDT	202,142	1,242	55,709	8,711
10	Bernoulli-Euler	ETB	193,509	1,232	60,793	
	Timoshenko	FSDT	193,509	1,258	60,793	3,183
	Reddy	HSDT	193,337	1,264	61,053	4,779
	Ghugal and Shinpi (2001)	Exact	192,95	1,261	60,917	4,771

Table 2 Comparison of axial displacement  $(\overline{u})_{at} (x = 0, z = \pm \frac{h}{2})$ , transverse displacement  $(\overline{w})_{at} (x = \frac{L}{2}, z = 0)$ , axial stress  $(\overline{\sigma}_x)_{at} (x = \frac{L}{2}, z = \pm \frac{h}{2})$ , and transverse shear stress  $\overline{\tau}_{zx}_{at} (x = 0, z = 0)$  for isotropic beam subjected to uniformly distributed loading

S	Theory	Model	ū	$\overline{W}$	$\overline{\sigma}_x$	$\overline{ au}_{zx}$
	Present Theory	HSDT	16.177	1,814	12,711	2,640
	Zehra and Shinde (2012b)	SVSDT	15,753	1,808	12,444	2,980
4	Bernoulli-Euler	ETB	16 ,000	1,5630	12,000	-
	Timoshenko	FSDT	16,000	1,8063	12,000	2,400
	Reddy	HSDT	16,506	1,8060	12,260	2,917
	Ghugal and Shinpi (2001)	Exact	15,800	1,7852	12,200	3,000
	Present Theory	HSDT	250.682	1,601	75,277	7,324
	Zehra and Shinde (2012b)	SVSDT	250,516	1,6015	75,238	7,4875
10	Bernoulli-Euler	ETB	249,998	1,5630	75,000	-
	Timoshenko	FSDT	250,000	1,6015	75,000	6,0000
	Reddy	HSDT	251,285	1,6010	75,246	7,4160
	Ghugal and Shinpi (2001)	Exact	249,500	1,5981	75,200	7,5000

$$\tau_{xz} = G\alpha f'(z) \sum_{n=1}^{\infty} \beta^3 \frac{q_n}{s} \cos(\beta x)$$
(22d)

### 3. Numerical results and discussion

## 3.1 Verification studies

In this part, the results of axial displacement (u), transverse displacement (w), the axial bending stress  $(\sigma_x)$ , and transverse shear stress  $(\tau_{xz})$  are presented in the following non dimensional form

$$\overline{u} = \frac{E b u}{q h}, \quad \overline{w} = \frac{10 E b w h^3}{q L^4}, \quad \overline{\sigma}_x = \frac{b \sigma_x}{q}, \quad \overline{\tau}_{xz} = \frac{b \tau_{xz}}{q}, \quad S = L/h$$

The parameter  $\alpha$  is expressed as

$$\alpha = \frac{D_s}{A_s + (\pi/L)^2 H_s}$$

Example 01: sinusoidal Loading case

Table 1 shows a comparison of displacements and stresses for a simply supported isotropic beam submitted to a sinusoidal load. The comparison study is carried out with other theories such Zehra and Shinde (2012b), Ghugal and Shinpi (2001) and Reddy's theory.

The maximum axial displacement predicted by present theory is in good agreement with Reddy's solution (see Fig. 2).

Table 3 Comparison of axial displacement  $(\overline{u})_{\text{at}} (x = 0, z = \pm \frac{h}{2})$ , transverse displacement  $(\overline{w})_{\text{at}} (x = \frac{L}{2}, z = 0)$ , axial stress  $(\overline{\sigma}_x)_{\text{at}} (x = \frac{L}{2}, z = \pm \frac{h}{2})$ , and transverse shear stress  $\overline{\tau}_{zx}$  at (x = 0, z = 0) for isotropic beam subjected to linearly varying load

S	Theory	Model	ū	$\overline{W}$	$\overline{\sigma}_{x}$	$ar{ au}_{zx}$
	Present Theory	HSDT	8,088	0,907	6 ,355	1,320
	Zehra and Shinde (2012b)	SVSDT	7,773	0,8923	6,141	1,386
4	Bernoulli-Euler	ETB	8,000	0,7815	6,000	-
	Timoshenko	FSDT	8,000	0,9032	6,000	1,200
	Reddy	HSDT	8,253	0,9030	6,130	1,458
	Ghugal and Shinpi (2001)	Exact	7,900	0,8926	6,100	1,500
	Present Theory	HSDT	125,341	0,800	37 ,638	3,662
	Zehra and Shinde (2012b)	SVSDT	123,620	0,7903	37,129	3,0769
10	Bernoulli-Euler	ETB	124,999	0,7815	37,500	-
	Timoshenko	FSDT	125,000	0,8008	37,500	3,0000
	Reddy	HSDT	125,643	0,8005	37,623	3,7080
	Ghugal and Shinpi (2001)	Exact	124,750	0,7991	37,600	3,7500



Fig. 2 Variations of axial displacement  $\overline{u}$  through the thickness of a beam simply supports at  $\left(x = 0, z = \pm \frac{h}{2}\right)$  in the case of sine loading with S=4



Fig. 3 Variations of axial Normal stress  $\overline{\sigma}_x$  through the thickness of a beam simply supports at  $\left(x = 0.5L, z = \pm \frac{h}{2}\right)$  in the case of sine loading with S=4



Fig. 4 Variations of the transverse shear stress  $\overline{\tau}_{zx}$  through the thickness of a beam simply supports (x = 0, z = 0) in the case of sine loading with S=4



Fig. 5 Variations of axial displacement  $\overline{u}$  through the thickness of a beam simply supports at  $\left(x = 0, z = \pm \frac{h}{2}\right)$  in the case of uniformly distributed loading with S=4



 $(x = 0.5L, z = \pm \frac{h}{2})$  in Fig. 6 Variations of axial Normal stress  $\overline{\sigma}_x$  through the thickness of a beam simply supports at the case of uniformly distributed loading with S=4



Fig. 7 Variations of axial displacement  $\overline{u}$  through the thickness of a beam simply supports at  $\left(x=0, z=\pm \frac{h}{2}\right)$  when subjected to linearly variate load for example, i.e. subjected to linearly varying load for aspect ratio 4



Fig. 8 Variations of axial Normal stress  $\overline{\sigma}_x$  through the thickness of a beam simply supports at  $\left(x = 0.5L, z = \pm \frac{h}{2}\right)$  when subjected to linearly varying load for exact order to the thickness of a beam simply supports at when subjected to linearly varying load for aspect ratio 4

The maximal transverse displacement  $\overline{W}$  predicted by this theory and also comparable to that of Reddy's theory

for all dimensional ratios *S* (are 4 or 10).

Fig. 3 shows the distribution of the axial stress through

the thickness of the beam. From Fig. 3, it can be concluded that the results computed by this theory are in excellent agreement with those of Reddy.

From Table 1, it is found that axial stresses determined by Timoshenko theory (FSDT) and the Euler-Bernoulli (ETB) theory are identical.

The transverse shear stress predicted by this theory and also in very good agreement with Reddy's theory for all dimension ratios used. Fig. 4 confirms also this finding.

Example 02: uniformly distributed Loading case

Table 2 shows comparison of displacements and stresses for the simply supported isotropic beam subjected to uniformly distributed load. The axial and vertical displacement obtained by present theory are in good agreement with Reddy's theory. The bending stress  $\overline{\sigma}_x$ calculated by present theory is in excellent agreement with the theory of Reddy, while the theory of Timoshenko (FSDT) and Euler-Bernoulli theory (ETB) underestimate this stress compared to the present theory and the theory of Reddy for all aspect ratios. The variation of axial displacement and axial stress through the thickness of isotropic beam subjected to uniformly distributed loading are shown in Figs. 5 and 6, respectively, and a good agreement between the present results and those of Reddy's theory is observed.

Example 03: linearly varying Loading case

A comparison between displacement and stresses for a simply supported isotropic beam subjected to linearly varying load are shown in Table 3. The maximum axial displacement and transverse displacement predicted by present theory are in good agreement with Reddy's theory. Fig. 7 demonstrates also this remark. Fig. 8 shows that the axial stress predicted by present theory is in close agreement with Reddy's theory, whereas FSDT and ETB underestimate this constraint for all dimension ratios.

#### 5. Conclusions

This work presents a refined shear deformation theory with only a single variable for the investigation of the static behavior of thick isotropic beams. The equations of equilibrium are determined using the principle of virtual work. Analytical solutions for static flexure problems are obtained for a simply supported thick beam. Through this study, the following conclusions were drawn:

Present theory is variationally consistent and requires no shear correction factor.

The present theory gives good results compared to the other theory of shear deformation which uses more variable.

Finally, the current study provides a good foundation for extension to more general computational simulation for more complex geometrical configurations such as shells structures (Zine *et al.* 2017, Karami *et al.* 2018a, b) and other type of materials such as functionally graded (Belabed *et al.* 2014, Hebali *et al.* 2014, Bousahla *et al.* 2014, Larbi Chaht *et al.* 2015, Bourada *et al.* 2015, Hamidi *et al.* 2015, Bennoun *et al.* 2016, Sekkal *et al.* 2017a, b, Bouafia *et al.* 2017, El-Haina *et al.* 2017, Fahsi *et al.* 2017, Abualnour *et al.* 2018, Younsi *et al.* 2018, Bouhadra *et al.* 2018,

Benchohra *et al.* 2018, Karami *et al.* 2018c) and composite materials (Mahi *et al.* 2015, Draiche *et al.* 2016, Chikh *et al.* 2017).

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