Structural Engineering and Mechanics, *Vol. 67, No. 3 (2018) 267-275*
DOI: https://doi.org/10.12989/sem.2018.67.3.267

Simplified model for analysis of soil-foundation system under cyclic pushover loading

Ouassila Kada*[1,2], Ahmed Benamar[2a] and Abdelkader Tahakourt[3b]

[1]Civil Engineering Department, Faculty of Technology, University of Bejaia, Targua Ouzemour, 06000, Algeria
[2]Normandie University, UNIHAVRE, CNRS, LOMC, 53 rue Prony, CS 80540, 76058 Le Havre Cedex, France
[3]Laboratoire de Génie de Construction et Architecture (LGCA), Faculty of Technology, University of Bejaia, Targua Ouzemour, 06000, Algeria

(Received October 20, 2017, Revised April 24, 2018, Accepted May 18, 2018)

Abstract. A numerical study of soil-foundation system under monotonic and cyclic pushover loading is conducted, taking into account both material and geometric nonlinearities. A complete and refined 3D finite element (FE) model, using contact condition and allowing separation between soil and foundation, is implemented and used in order to evaluate the nonlinear relationship between applied vertical forces and induced settlements. Based on the obtained curve, a simplified model is proposed, in which the soil inelasticity is satisfactorily represented by two vertical springs with trilinear behavior law, and the foundation uplifting is insured by gap elements. Results from modeling soil-foundation system supporting a bridge pier have shown that the simplified model is able to capture irreversible settlements induced by cyclic rocking, due to soil inelasticity and vertical loading, as well as large rotations due to foundation uplifting.

Keywords: soil-foundation interaction; foundation uplifting; soil inelasticity; pushover analysis

1. Introduction

Several research studies demonstrated the importance of taking into account nonlinear Soil Structure Interaction (SSI) when modelling seismic structural response, considering both soil inelasticity and foundation nonlinearity (Allotey and El Naggar 2008, Anastasopoulos and Kontoroupi 2014, Apostolou *et al*. 2007, Gajan *et al*. 2010, El Ganainy and El Naggar 2009, Gazetas *et al*. 2010, Grange *et al*. 2009, Loli *et al*. 2014). Implementation of the experiment through static, cyclic, and dynamic tests on vibrating and centrifugal tables (Allotey and El Naggar 2008, Amorosi *et al*. 2017, Espinoza *et al*. 2006, Harden and Hutchinson 2009, Hung and Liu 2014, Loli *et al*. 2014) allowed accounting for the effects of the nonlinear soil-foundation-structure interaction. Obtained results have highlighted the favourable effect of such a phenomenon because the coupled mechanisms of the foundation uplifting and the soil yielding tend to isolate the structure from the incident movement. This has an effect of limiting efforts in structures. However, these nonlinearities generate relatively high displacements, which may lead in some cases to the structural disorders, either by rupture of the foundations or by the inability of the structures to withstand such displacements. When some research studies demonstrated the beneficial effects of foundation uplift in computing the earthquake response of structures (Anastasopoulos *et al*. 2010, Smith-Pardo 2011, Faramarz *et al*. 2012), others (Jafarieh and Ghannad 2014, Psycharis and Jennings 1983, Xu and Spyrakos 1996) pointed that this is not necessarily always correct; it depend on the parameters of the system and the characteristics of the excitation.

It is well recognized that the best way to represent all aspects of nonlinear SSI problems is the rigorous global finite element method, but it is time consuming and requires a large computational effort (Allotey and El Naggar 2003). This incited the development of simplified methods. Some researchers take into account the SSI effect by re-evaluating the characteristics of fixed base model (Perez-Rocha and Viles 2004). A simplified method, widely applied in SSI analyses and commonly known as Winkler model, consists on replacing the soil-foundation system by uncoupled translational and rotational springs. It was used in (Jafarieh and Ghannad 2014, Psycharis and Jennings 1983, Song and Lee 1993) to examine the effects of rocking and uplift of linear structures by establishing equivalence relations between the Winkler model and the simplest two-spring model. Recently, it was also used by Nguyen *et al*. (2016) in their proposed foundation model, for the dynamic analysis of plates on foundation subjected to a moving oscillator. Later a nonlinear approach, denoted as "Beam-on-nonlinear-Winkler-foundation" model (BNWF), was introduced in the Winkler model to take into account both materiel and geometric nonlinearities. It was even recommended by FEMA356 (2000) and was used by Chen and Lai (2003) to analyze a response of a rigid pier with shallow foundation. The authors concluded that the nonlinear effect is very remarkable when the uplifting of the

*Corresponding author, Ph.D. Student
E-mail: was.kada@gmail.com
[a]Associate Professor
E-mail: ahmed.benamar@univ-lehavre.fr
[b]Professor
E-mail : atahakourt@yahoo.fr

http://www.techno-press.com/journals/sem&subpage=7 ISSN: 1225-4568 (Print), 1598-6217 (Online)

foundation and the yielding of the supporting soil are considered. It has also been used by Harden and Hutchinson (2009) in order to investigate its applicability to predict the benefits and consequences of shallow foundations allowed to rock, slide, and settle under large amplitude lateral seismic loading. Comparing to experimental results under static and dynamic loading, the BNWF model provide reasonable estimates of maximum moment and total settlement for the rocking-dominated systems. However, maximum sliding is not captured, which constitutes the major limitation of the model as mentioned in FEMA356 (2000) and confirmed by Allotey and El Naggar (2008). More recently, Raychowdhury (2011) used BNWF and concluded that the foundation compliance have a significant effect on the structural response. Looking at numerous studies, it is established by Allotey and El Naggar (2003) that BNWF provides a simplicity and ability to incorporate different nonlinear aspects of the behavior at a reduced computational effort compared to rigorous approaches. Nevertheless, besides the fact that it does not capture maximum horizontal displacement (Allotey and El Naggar 2008, Gajan *et al*. 2010, Harden and Hutchinson 2009), it is criticized for requiring a large number of springs which induces a significant number of degrees of freedom (El-Ganainy and El-Naggar 2009).

Otherwise, several researchers have developed nonlinear SSI models based on the macro-element approach, in which the soil-foundation system is replaced by a single point at the center of the foundation to which the macroelement is affected (Cremer *et al*. 2001, Faccioli *et al*. 2001, Grange *et al*. 2009). The macro-element approach is considered as to be able to give satisfactory predictions of the complete foundation response since it accounts for nonlinear behavior and coupling between the responses in the different directions (Allotey and El Naggar 2008). However, this approach often requires several parameters with too much specificity which makes it difficult to be generalized and implemented in the most common software tools (El-Ganainy and El-Naggar 2009).

In their work, Anastasopoulos and Kontoroupi (2014) proposed a simplified method to take into account soil inelasticity and foundation uplifting in order to analyze seismic performance of rocking systems. The soil-foundation system is replaced by horizontal and vertical elastic springs and dashpots associated to a nonlinear rotational spring. Compared to a rigorous 3D finite element model, their simplified model presents a good agreement, except the fact that, settlement cannot be captured with details since the model does not account for the uplifting during loading.

In the present paper, a simplified two-spring model is proposed in order to reproduce the effect of both material inelasticity (soil yielding) and geometric nonlinearity (foundation uplifting) on the nonlinear behavior of soil-foundation system. A typical bridge pier structure is used as an example to examine the performance of the proposed model. As the work is focused on the effect of the geometric nonlinearity of the foundation and the inelasticity of the soil, the pier and the foundation are considered rigid. A detailed 3D finite element model (3D-FE) is also implemented and utilized to calibrate the stiffness of the springs and their positions. A cyclic pushover analysis is conducted to examine the effectiveness of the simplified model by comparing the results obtained with those of the 3D-FE model in terms of moment, rotation, and settlement. The two-spring model has been shown to be an accurate approximation to the rigorous detailed 3D finite element model. Furthermore, when possible and when data are available, the stiffness of the two springs and their positions may also be derived following the methodology proposed here, either from experimental results or from empirical or analytical solutions. From literature, it can be noted that works dealing with simplified two-spring model do not reproduce the variation of the settlement due to cyclic rotations with taking into account for both soil plasticity and foundation uplifting. Capturing the permanent settlement is generally not included in such simplified models; it is rather reproduced by more sophisticated models.

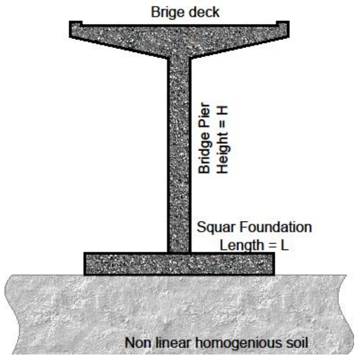

Fig. 1 Bridge pier on homogenous half space soil

Table 1 Soil characteristics

Parameter	Value
Undrained cohesion	$C_u = 130\ KPa$
Friction angle	$\phi = 10°$
Poisson's ratio	$\nu = 0.33$
Shear velocity	$V_s = 235.7\ m/s$
Masse density	$\rho = 1.8\ t/m^3$

2. Problem definition

A case study analysis conducted in this work concern a typical bridge pier supported by a square foundation on a homogenous half space soil as shown in Fig. 1.

The pier is of height $H = 13.2\ m$, the length of the square foundation is $L = 7\ m$ and the total masse supported by the pier is $m = 2400\ tonnes$. Table 1 summarizes the soil characteristics.

The bearing capacity BC of rigid footings depends on the stress distribution under both working and ultimate load conditions, respectively (Allotey and El Naggar 2008). FEMA 356 (2000) recommends the estimation of the BC under concentric vertical load with standard formulae. Faccioli *et al*. (2001) used analytical developed formulas. Harden and Hutchinson (2009) incorporated functions for a parabolic distributions and end tip resistance into the equation for the conventional bearing capacity of a footing.

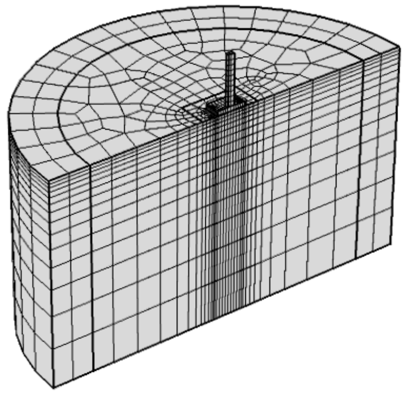

Fig. 2 3D-FE mesh model

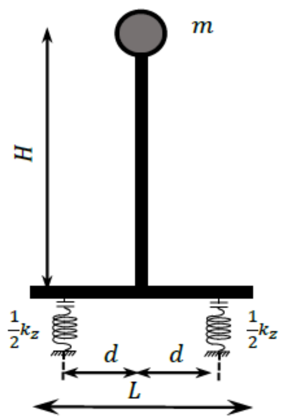

Fig. 3 Two-spring model characteristics

Table 2 Elastic vertical and rocking stiffness

Method	Vertical stiffness K_z (MN/mm)	Rocking stiffness K_θ $(MN.mm)$
Gazetas	2.37	23.82
FEMA	2.45	26.11
Wolf	3.36	25.52
3D-FE model	2.29	23.97

However, for a homogeneous soil, the formulas proposed by Terzaghi (1943) and modified later by several researchers, remains usual. In the case study of this work, the BC calculated from Terzaghi is $q_u = 1562.16\ KN/m^2$ which has led to a safety factor $F_S = 3.2$. To focus the analysis on the soil-foundation nonlinear behavior, the superstructure (pier-foundation) is considered rigid.

3. Finite element model

A detailed 3D finite element model is constructed using COMSOL (2012), a finite element software for solving Multiphysics problems. The mesh of the soil, the pier and the foundation consists of quadratic hexahedral elements with 20 nodes and 60 DOF per element. Exploiting the symmetry of the problem, only half of the geometric domain is considered. The pier and the foundation, assumed rigid, are represented by elastic elements with high Young's modulus and the contact condition between the foundation and the soil is reproduced by special contact interface allowing separation between the two domains. The soil domain, assumed fixed at its base, is divided into three zones (Fig. 2):

- A zone close to the foundation, extending over twice the surface of the foundation, which is finely refined to better take into account the nonlinearity and the conditions of contact between the soil and the foundation.

- An area extending up to seven times the length of the foundation is constituted of coarser mesh.

- A third zone with thickness equals to 20% of the radius of the soil geometry is defined with a regular mesh, and contains mapped infinite elements in order to take into account the effects of geometric truncation.

A meshing study was carried out by comparing, in the elastic range, the vertical and rotational stiffness of the soil-foundation system with those given by Gazetas (1991), those of FEMA356 (2000) for square foundation, and those from Wolf (1994) assuming an equivalent circular foundation.

It is found that a mesh constituted by 4715 quadratic hexahedral elements with a total of 68855 DOFs leads to satisfactory results. This mesh is shown Fig. 2 and the obtained values of vertical and rocking stiffness are reported in Table 2 from which, it can be seen that the obtained numerical results are close to that of empirical expressions especially to those of Gazetas.

The vertical stiffness of the soil is evaluated by computing the mean settlements of the foundation due to monotonic increasing vertical force applied as a distributed load on the rigid foundation surface.

The rocking stiffness is evaluated by applying an increasing horizontal force at the top of the pier and measuring the corresponding rotation of the foundation.

The mean values $\bar{w}$ and $\bar{\theta}$ of the settlement and the rotation, respectively, can be estimated by the following expressions in which S refers to the foundation surface.

$$\bar{w} = \frac{1}{S}\int_s w(x,y,0)\mathrm{d}s \ , \ \bar{\theta} = \frac{1}{S}\int_s \frac{w(x,y,0)}{x}\mathrm{d}s \tag{1}$$

For a rigid foundation, one can use the values at the extremities

$$\begin{aligned} \bar{w} &= \frac{1}{2}\big(w(L/2,0,0) + w(-L/2,0,0)\big) \\ \bar{\theta} &= \frac{1}{L}\big(w(L/2,0,0) - w(-L/2,0,0)\big) \end{aligned} \tag{2}$$

where L is the foundation length and w is the vertical displacement at a point defined by coordinates (x,y,z).

4. Simplified two-spring model

A methodology to reproduce a simplified model of soil-foundation system taking into account material and geometric nonlinearities is proposed.

As shown in Fig. 3, it consists on replacing the soil by two vertical nonlinear springs to reproduce the soil plasticity associated with gap elements to allow foundation

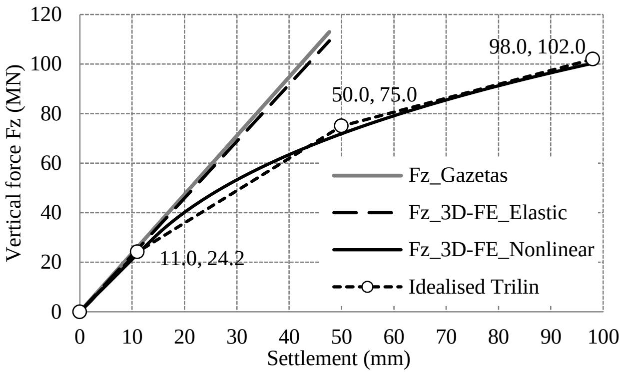

Fig. 4 Vertical force-settlement relation

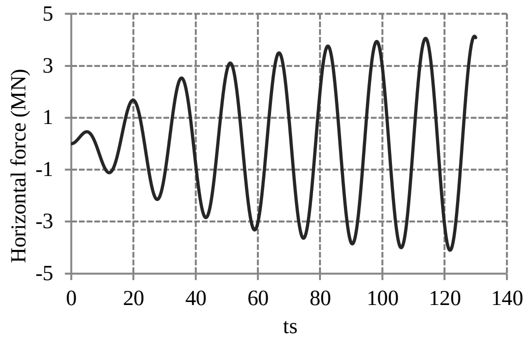

Fig. 5 Horizontal cyclic pushover load with increasing amplitude

uplifting. Calibration of the simplified model with a 3D finite element one (3D-FE) leads to reproduce adequate behavior law for the springs and estimate the distance d from the foundation center to the springs positions allowing a well capture of foundation uplifting.

4.1 Determination of the spring's nonlinear stiffness

A monotonic vertical pushover analysis is performed using the 3D-FE model in which the soil nonlinear behavior is represented by the Drucker-Prager constitutive model matched to Mohr Coulomb, mainly used to reproduce the plasticity failure in soils (Chegenizadeh *et al*. 2014). The main model parameters are the soil cohesion c and friction angle ϕ. The obtained vertical force-settlement (F_Z -w) curve is shown in Fig. 4 together with the elastic response from 3D-FE model and Gazetas expression. This curve is idealized with a trilinear elasto-plastic law by ensuring the total energy balance. The idealization provides a first yield force of 24 MN and a plastic force of 75 MN which correspond to settlements of 11 mm and 50 mm, respectively. These values are used to define the compression stiffness of the two springs. Each spring supports one half of the total force and undergoes the same total displacement. Reactions to tension forces are eliminated by imposing zero tension stiffness, via gap element, so that foundation uplift is captured by the model.

Analysis of the two-spring simplified model (2S) is performed using SeismoSoft (2016), a finite element software package distributed freely for no commercial use by SeismoSoft.

4.2 Determination of the spring's position

The distance, denoted here by d, between the foundation center and the spring's positions must be estimated so that the reaction moments will be correctly reproduced. Psycharis and Jennings (1983) proposed expressions to estimate d by establishing equivalence between a two-spring model and the winkler foundation model. They found, after analytical developments, $d = L\sqrt{3}/6$ for the case of full soil-foundation contact, and $d = L/2 - \tilde{S}/3$ for the case of uplifting, where $\tilde{S}$ is the average length of the soil-foundation contact after lift-off. Song and Lee (1993) adjust the d distance in such a manner that will result in the same rotational stiffness for the two-spring model as that for the Winkler foundation model, they obtain $d = B/2\sqrt{3}$. Later, Xu and Spyrakos (1996) evaluated the d expression in the elastic range for a circular shallow foundation of radius R as: $d = \sqrt{2/3}\,R$. It can be easily shown that writing the equilibrium between applied and reaction moments leads to $d = \sqrt{K_\theta/K_z}$, where K_z and K_θ are the vertical and the rocking stiffnesses, respectively, of a foundation laying on elastic half space. This expression is more general than the expression given Xu and Spyrakos and can be applied to the case of rectangular foundation.

The position d becomes variable when considering soil inelasticity and foundation uplifting due to variations of the contact surface and the soil mechanical properties. It is therefore very difficult to fix its value when dealing with both geometric and material nonlinearities, such variations of d can be obtained by performing pseudo-static horizontal cyclic pushover analysis with the 3D-FE model. In this study, a horizontal cyclic pushover analysis is conducted by applying at the top of the pier, a varying horizontal sinusoidal force with increasing amplitude as plotted in Fig. 5 with respect to time steps t_s. A vertical force of fixed value of $24\,MN$ is applied to represent weight.

The distance d is evaluated by extracting, from 3D-FE analysis, the vertical total reactions of the right half and the left half sides of the foundation surface F_{ZR} and F_{ZL}, respectively, and then equalising there moments to the total reaction moment M_T.

$$(F_{ZR} - F_{ZL})\, d = M_T \tag{3}$$

with

$$F_{ZR} = \int_S \sigma_z(\mathrm{x} \geq 0, \mathrm{y}, 0)\mathrm{d}s \tag{4}$$

$$F_{ZL} = \int_S \sigma_z(\mathrm{x} \leq 0, \mathrm{y}, 0)\, \mathrm{d}s \tag{5}$$

$$M_T = \int_S \sigma_z \cdot x \,\mathrm{d}s \tag{6}$$

σ_z is the vertical component of the stress tensor acting

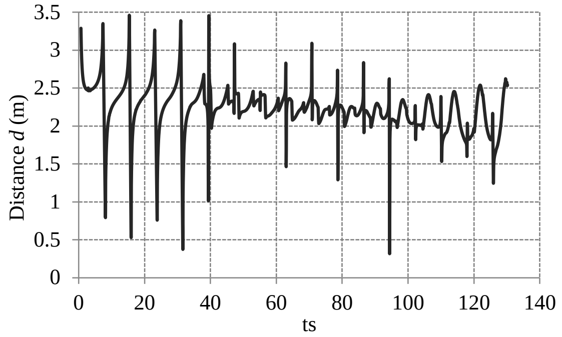

Fig. 6 Variation of the spring's position from increasing cyclic horizontal pushover analysis

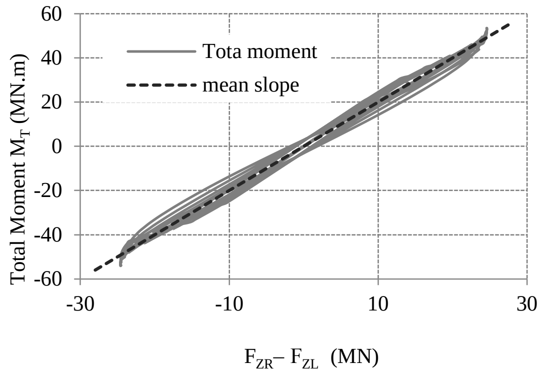

Fig. 7 Distance d between the spring position and the foundation center

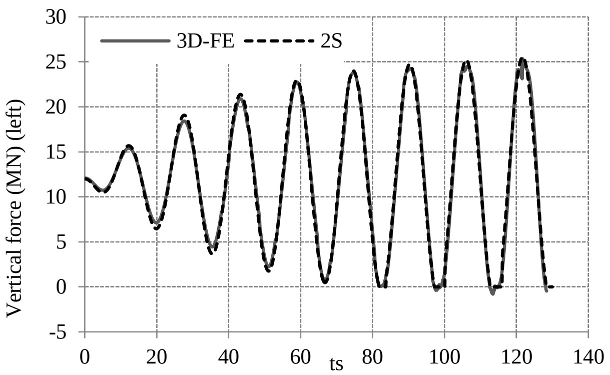

Fig. 8 Vertical force on the left side of the foundation

on the contact surface S. The integrals are performed using numeric quadrature of the 4th order, and the stress tensor is computed from the FE-solution obtained using a time step of $\Delta t_s = 0.01$ which is small enough to avoid rapid variations that are detrimental to numerical modelling involving contact problem. This costly solution required 3GB memory consumed more than 9 hours CPU-time on a quad-core computer.

Variations of the distance d according to time steps t_s are presented in Fig. 6. The high fluctuations that can be seen in this figure show that the values of d computed from Eq. (3) are very sensitive to the values of M_T, F_{ZR} and F_{ZL} evaluated from Eqs. (4)-(6). Indeed, numerical

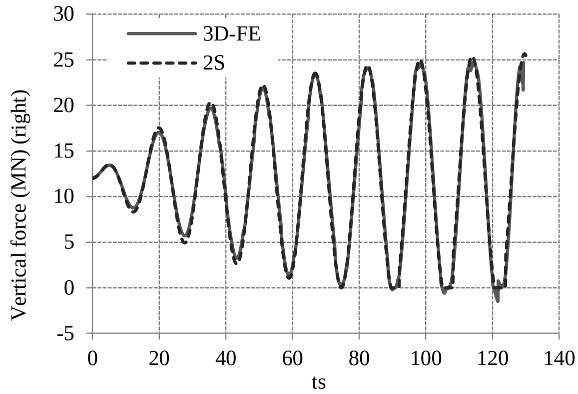

Fig. 9 Vertical force on the right side of the foundation

Table 3 Uplifting initiation time step

Model	Time step of uplifting initiation	
	right side	left side
3D-FE	74.4	82.1
2S	74.6	81.9

and rounding errors may provide slight different values of F_{ZR} and F_{ZL} when M_T is very small, so that the ratio $M_T/(F_{ZR} - F_{ZL})$ can take very large and unaccurate values. Nevertheless, the overall trend of the plotted curve gives $d = 2m$, and its initial value, which correspond to the elastic domain, is very close to that provided by $d = \sqrt{K_\theta/K_z} = 3.18\,m$.

In order to get a representative value for d, it is more convenient to plot, as shown in Fig. 7, the total moment M_T as a function of the difference between the reaction forces: $(F_{ZR} - F_{ZL})$. The distance d can then be interpreted as the mean slop of the obtained loops. It is represented by the dotted line in Fig. 7 and its value is $d = 2m$.

5. Validation of the simplified model

The capability of the proposed simplified two-spring model to reproduce the response of soil structure system taking into account both material and geometric nonlinearities is examined throw a comparison with results obtained from the 3D-FE model in terms of settlement, foundation rotation and moment base reaction.

Pushover analysis is considered as a method capable of identifying the soil inelasticity and local failure mechanisms in the Soil-Foundation-Structure systems (Falamarz-Sheikhabadi and Zerva 2016). Nowadays, the pushover analysis method is certified and found its way to seismic guidelines, it is included in some codes such as FEMA356 (2000), and ATC40 (1996). It's gaining more popularity, and it's already used to perform analyses for a large range type of structures. Liping *et al.* (2008) employed it to study the nonlinear SSI effect. From (Gazetas *et al.* 2010, Panagiotidou *et al.* 2012), it was concluded that for the majority of structures that have safety factor F_S greater than 2, the monotonic pushover curves are representative of the moment capacity of the system even under dynamic

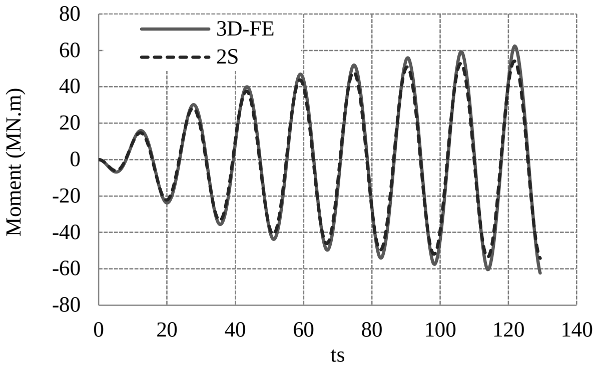

Fig. 10 Moment-Time step variation

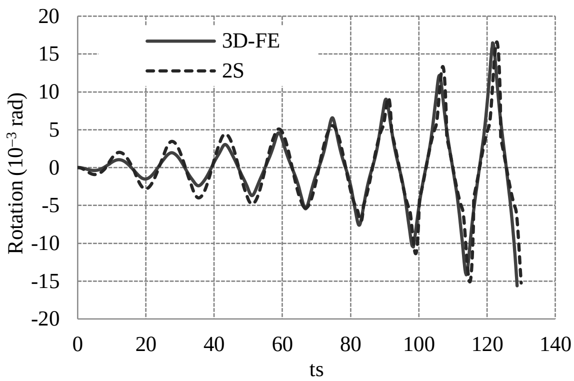

Fig. 11 Rotation-Time step variation

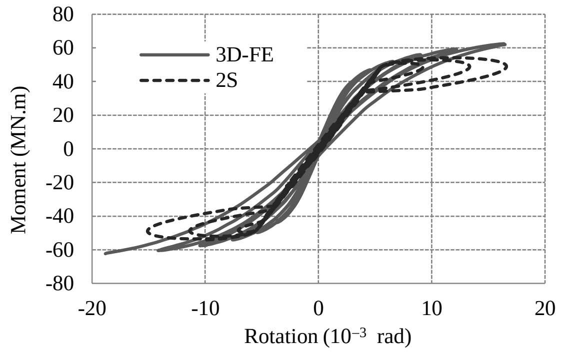

Fig. 12 Moment-Rotation curves

loads. The work of Panagiotidou *et al*. (2012) presents a numerical study for developing the pushover curves of a surface foundation carrying a relatively tall, slender structure in order to correlate static, monotonic, and cyclic loading with the seismic response. Recently, Falamarz-Sheikhabadi and Zerva (2016) used the pushover method to examine the sensitivity of the nonlinear resistance of a bridge pier in the collapse prevention limit state to numerical modelling effects. In the present study, cyclic pushover analysis is performed by applying at the top of the pier a horizontal sinusoidal varying force with progressively increasing amplitude (see Fig. 5).

5.1 Foundation uplifting

Initiation of the uplift can be detected by examining the forces developed on the two sides of the foundation (right and left) as plotted in Figs. 8 and 9.

In the 3D-FE model, these forces are calculated by integrating the vertical stresses at the base of each half of the foundation, according to Eqs. (4) and (5). Whereas, in the simplified model, these forces represent the vertical reaction developed in each spring, computed from $k_{ZR}(w_R).w_R$ and $k_{ZL}(w_L).w_L$. Where k_{ZR} and k_{ZL} are the right and left spring's stiffness, respectively, and w_R and w_L are the corresponding settlements. Values of the stiffness depend on the adopted trilinear law.

It can first easily be seen, from Figs. 8 and 9, a perfect matching between results obtained by the 2S simplified model and those of the 3D-FE model. It can also be deduced from the clipped pics that the reaction forces are bounded by $0.0\,MN$ and $24\,MN$ that correspond to not loaded and full loaded cases. A half-foundation is unloaded when it loses contact with the soil (uplifting).

The first occurrence of the uplifting is detected on the right side of the foundation, observed from Fig. 9 when $F_{ZR} = 0$ at $t_s \cong 74$. Table 3 summarizes the exact values of uplifting initiation on the right and the left sides extracted from the two models.

5.2 Moment and rotation variations

Variations of moment reaction and corresponding foundation rotation are plotted versus time steps in Figs. 10 and 11. Doted-line curves represent result obtained from 2S model and solid-line curves represent those of 3D-FE model. These results show that the moments are practically identical for all time steps, while the rotations seems to be slightly overestimated by the 2S model for first cycles (before uplifting). In the remaining steps, after uplifting, foundation rotations are well captured by the 2S model. It is worth noting that after uplifting, the rotations not only increase greatly as pointed in (Xu and Spyrakos 1996) but also they produce pics with elongated shapes.

Moment-rotation relationship $(M - \theta)$ is one of the most important parameters describing soil-foundation systems. The energy dissipation associated with the area inside the $(M - \theta)$ loops is a measure of the degree of nonlinearity (Gajan *et al*. 2010). As outlined in (Anastasopoulos and Kontoroupi 2014, Panagiotidou *et al*. 2012, Raychowdhury 2011), the prevalence of uplifting versus bearing capacity mechanisms is mainly controlled by Fs. High values of F_S (lightly loaded foundations) undergo mainly alternating uplifting. FEMA356 (2000) stipulates that a value of Fs greater or less than 2 indicates whether the foundation uplifting or the soil yielding occurs first. Gazetas (2010) denoted that structure with $F_S > 2$ undergoes predominantly uplifting. In their study, Allotey and El Naggar (2003) concluded that the moment-rotation responses can be grouped into three categories: uplift-dominant, uplift-yield and yield dominant. From a nonlinear time-history analyses, Smith-Pardo (2011) concluded that uplifting and reaching the bearing capacity of the supporting soil can occur before yielding.

The $(M - \theta)$ hysteretic curves obtained from the 3D-

Table 4 Uplifting initiation condition

Model		Rotation (10^{-3} rad)	Moment (MN.m)
3D-FE	right side	6.4	50.8
	left side	7.1	52.2
2S	right side	5.5	48.0
	left side	5.6	48.2

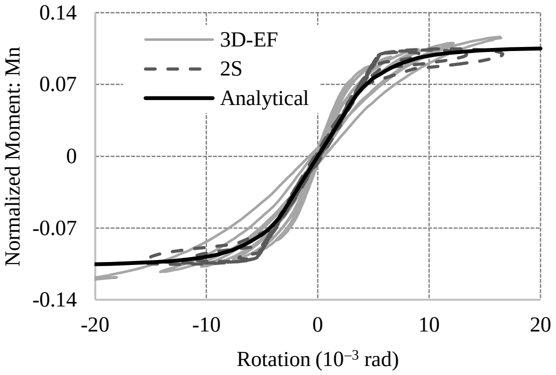

Fig. 13 Normalized Moment-Rotation relation from 3D-FE model, 2S model and from analytical solution for $\chi = 0.31$ *and* $\psi = 209.42$ (Allotey and El Naggar 2003)

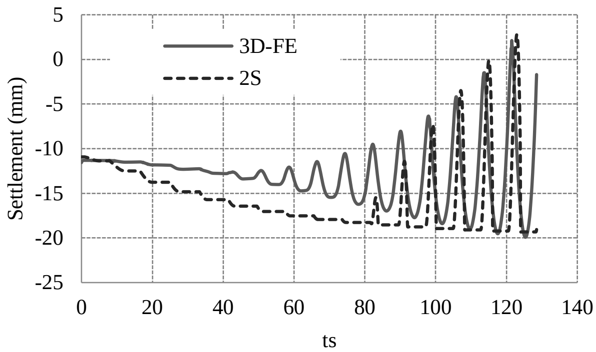

Fig. 14 Settlement-Time step variation

FE and the 2S models are compared in Fig. 12. The results show, from the S-shaped obtained loops, that the behavior of the analyzed soil-foundation system is more predominated by uplift. As outlined in (Chen and Lai 2003), S-shaped moment-rotation response characterizes the uplifting effect of the foundation. In present study, this is an expected result since the safety factor of the foundation is greater than 2 $(F_S > 2)$.

As mentioned by Allotey and El Naggar (2003), the backbone curve of the pseudo-static cyclic moment-rotation response forms an important part of the cyclic response of a footing. As an extension of the solution provided by Siddharthan *et al*. (1992), they developed an analytical solution for moment-rotation response of foundations in which the state of stress of combined soil yield and foundation uplift is considered. In addition, to enable a comparison between different footings under different response conditions, the moment M is normalized and expressed as a function of rotation through two

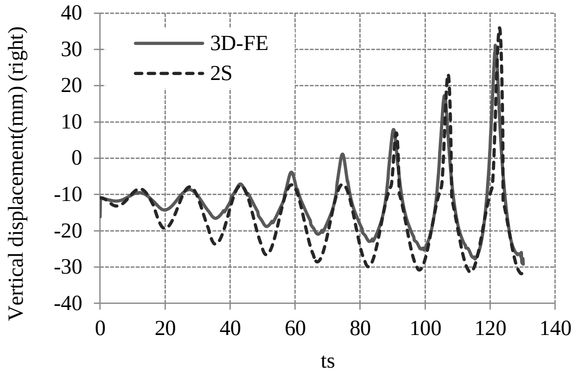

Fig. 15 Vertical displacement on the right side of the foundation

nondimensional parameters (χ, ψ).

$$\chi = \frac{1}{F_s} \quad ; \quad \psi = \frac{k_z}{q_u L^3}$$

For $\chi \leq 0.5$, the normalized moment M_n is expressed by

$$M_n = \frac{M}{q_u L^3} = \begin{cases} \frac{\psi\theta}{12} & 0 \leq \theta \leq \frac{2\chi}{\psi} \\ \frac{\chi}{6}\left(3 - 2\sqrt{\frac{2\chi}{\psi\theta}}\right) & \frac{2\chi}{\psi} \leq \theta \leq \frac{1}{2\phi\chi} \\ \frac{1}{2}(\chi - \chi^2) - \frac{1}{24\psi^2\theta^2} & \theta \geq \frac{1}{2\psi\chi} \end{cases}$$

where F_s is the vertical safety factor, k_z is the vertical stiffness of the soil, and L, q_u are, respectively, the length and the bearing capacity of the foundation.

The $M - \theta$ curves obtained from both 3D-FE and 2S models are normalized according to the above expressions ($q_u L^3 = 535.85$ MN.m, $\chi = 0.31$ and $\psi = 209.4$). They are plotted, together with the analytical curve, in Fig. 13 which clearly shows that the predicted numerical results are very close to the analytical solution for all range of rotation values. In addition, the computed rotations and moments values corresponding to the foundation uplift initiation at time steps reported in Table 3 are found as listed in Table 4.

These values are very close to that provided by the analytical solution (Allotey and El Naggar 2003, Siddharthan *et al.* 1992) given as follows

$$M = \frac{PL}{2} - \frac{2P^2}{3q_u L} = 48.8\ \mathrm{MN.m}$$

$$\theta = \frac{q_u^2 L^3}{2Pk_v} = 7.6 \times 10^{-3}\ \mathrm{rad}$$

5.3 Settlement variation

Fig. 14 presents the settlement–time step variation. The first value corresponds to the initial settlement which can easily be calculated by $P/K_Z = 10.9\ mm$, where P is the weight of the superstructure. Settlement is first dominated by the soil inelasticity characterized by increasing values comparing to the initial value. It is noted from Fig. 14 that the permanent settlement is followed by a few cycles of transient movement, cyclic pics appearing on the time-

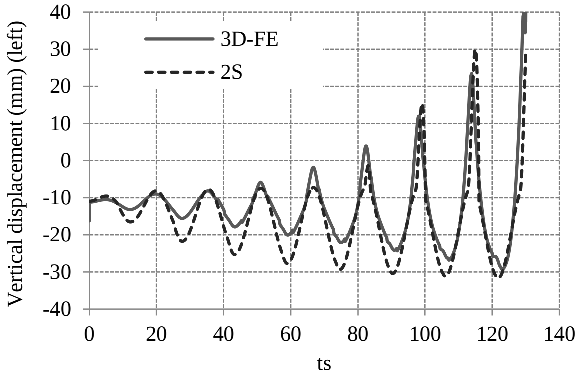

Fig. 16 Vertical displacement on the left side of the foundation

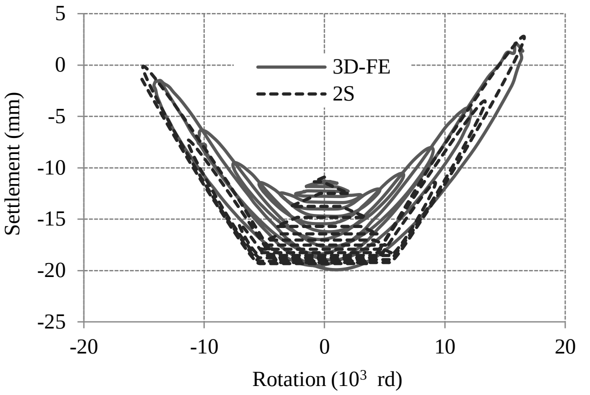

Fig. 17 Settlement-Rotation curves

settlement curve are an indication of the uplift mechanism as outlined by (Grange *et al*. 2009, Raychowdhury 2011). As mentioned in (Allotey and El Naggar 2003, Gazetas *et al*. 2010), soil yielding would occur before foundation uplift when $F_S > 2$ even if the behavior is uplifting predominant.

It is also noticed from Fig. 14 that during the first cycles, the 2S model provides higher settlements than those given by the 3D-FE model (before uplifting) however the two curves become closer until providing practically the same value of the permanent settlement: $19.92\,mm$ from the 3D-FE model and $19.35\,mm$ from the 2S model. The difference between the two curves, at time steps ranging broadly from 10 to 90, can be explained by the fact that in the 3D-EF model, the settlement represents an average value of the vertical displacement calculated under the foundation while in the 2S model, it is a nodal value of the vertical displacement of the foundation center. In addition, the choice of the trilinear idealization of the spring's law behavior, especially elastic and yielding limits, significantly affects the shape of the 2S model resulting curves and consequently the degree of equivalence between the two models.

Furthermore, vertical displacements in each side of the foundation are plotted in Fig. 15 for the right side and in Fig. 16 for the left side. For the 3D-FE model, the mean value of the vertical displacement in each side is computed according to Eq. (1) by integrating over half foundation surface. For the 2S model, vertical displacements are the computed deformations of each spring. Here also, it can be seen from these figures that, for the same reasons given above, vertical displacements provided by 2S model remain greater than those predicted by 3D-FE model until response becomes uplifting predominated.

To get more insight comparison between the results of the two models, the rocking effect on the settlement can be examined by plotting the settlement as a function of the foundation rotation as shown in Fig. 17. Rocking being the predominant mode of movement in the system, so when one side of the foundation uplift, in the other side, the soil yields and thus generating a digging in the ground up to achieve a permanent settlement of the foundation (Allotey and El Naggar 2003, El Ganainy and El Naggar 2009). This is clearly visible in Fig. 17, from increasing values of settlement at zero rotation.

It can also be seen that the uplift reaches the center of the foundation when the rotations correspond to settlements values lower than the initial value under the effect of weight load only. The difference between rotations and settlements computed by the two models, noted in Figs. 11 and 14, is also visible here and remain acceptable. The 2S model can thus provide results close to those of the more sophisticated 3D-FE model.

6. Conclusions

When subjected to strong motions, structures for which the predominant mode is rocking, can undergo large rotations of the foundation causing their uplifting besides soil yielding. This phenomenon must be taken into account, it can even be expected to serve as rocking isolation for soil-foundation systems, but in turn, it must be well controlled. If the non-linear effects at the foundation level are controlled adequately, this may be an effective method for reducing seismic actions on the superstructure. In this work, a simplified two-spring model is proposed to study the nonlinear behavior of soil-foundation system. A rigid pier laying on a homogeneous soil is used as an example for which parameters of the two springs are defined in such a way that results of a rigorous 3D finite element model are reproduced taking into account soil yielding and foundation uplifting. Characteristics of the springs are obtained by carrying out static and cyclic nonlinear pushover analyses. A trilinear idealization of the obtained curve capacity is proposed as law behavior of springs. A great attention has been paid to the position of the two springs in order to conveniently account for rocking behavior. The effectiveness of the proposed model is demonstrated by comparing its results with those obtained by a detailed 3D-FE model. The simplified model reasonably captures the most important parameters of rocking foundation under cyclic loading. The overall behavior of the soil-foundation system inducing material and geometric nonlinearities is well reproduced. The model remarkably captured the response given by the 3D-FE in terms of shape, maximum and permanent values of settlement, rotation and moment. This simplified modelling may constitute an attractive alternative to construct fast models easy to implement in traditional calculation codes. It will be very interesting to extend this approach to dynamic time history analyses considering more structures with different soil types.

References

Allotey, N. and El Naggar, M.H. (2003), "Analytical moment-rotation curves for rigid foundations based on a Winkler model", *Soil Dyn. Earthq. Eng.*, **23**(5), 367-381.

Allotey, N. and El Naggar, M.H. (2008), "An investigation into the Winkler modeling of the cyclic response of rigid footings", *Soil Dyn. Earthq. Eng.*, **28**(1), 44-57.

Amorosi, A., Boldini, D. and Di Lernia, A. (2017), "Dynamic soil-structure interaction: A three-dimensional numerical approach and its application to the Lotung case study", *Comput. Geotech.*, **90**, 34-54.

Anastasopoulos, I., Gazetas, G., Loli, M., Apostolou, M. and Gerolymos, N. (2010), "Soil failure can be used for seismic protection of structures", *Bullet. Earthq. Eng.*, **8**(2), 309-326.

Anastasopoulos, I. and Kontoroupi, T. (2014), "Simplified approximate method for analysis of rocking systems accounting for soil inelasticity and foundation uplifting", *Soil Dyn. Earthq. Eng.*, **56**, 28-43.

Apostolou, M., Gazetas, G. and Garini, E. (2007), "Seismic response of slender rigid structures with foundation uplifting", *Soil Dyn. Earthq. Eng.*, **27**(7), 642-654.

ATC40 (1996), "Seismic evaluation and retrofit of concrete buildings", *Appl. Technol. Council*.

Chegenizadeh, A., Ghadimi, B., Nikraz, H. and Simsek, M. (2014), "A novel two-dimensional approach to modelling functionally graded beams resting on a soil medium", *Struct. Eng. Mech.*, **51**(5), 727-741.

Chen, X.C. and Lai, Y.M. (2003), "Seismic response of bridge piers on elasto-plastic Winkler foundation allowed to uplift", *J. Sound Vibr.*, **266**(5), 957-965.

COMSOL (2012), *COMSOL Multiphysics*, Stockholm, Sweden.

Cremer, C., Pecker, A. and Davenne, L. (2001), "Cyclic macro-element for soil-structure interaction: Material and geometrical non-linearities", *Int. J. Numer. Analy. Meth. Geomech.*, **25**(13), 1257-1284.

El-Ganainy, H. and El-Naggar, M.H. (2009), "Efficient 3D nonlinear Winkler model for shallow foundations", *Soil Dyn. Earthq. Eng.*, **29**(8), 1236-1248.

Espinoza, A., Mahin, S.A., Jeremić, B., Kutter, B.L. and Ugalde, J. (2006), "Rocking of bridge piers subjected to multi-directional earthquake loading", *Proceedings of the 8th National Conference on Earthquake Engineering*, San Fransisco, U.S.A.

Faccioli, E., Roberto, P. and Vivero, G. (2001), "Investigation of seismic soil-footing interaction by large scale cyclic tests and analytical models", *Proceedings of the 4th International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*.

Falamarz-Sheikhabadi, M.R. and Zerva, A. (2016), "Effect of numerical soil-foundation-structure modeling on the seismic response of a tall bridge pier via pushover analysis", *Soil Dyn. Earthq. Eng.*, **90**, 52-73.

Faramarz, K., Mehdi, S. and Farzane, P. (2012), "P-delta effects on earthquake response of structures with foundation uplift", *Soil Dyn. Earthq. Eng.*, **34**(1), 25-36.

FEMA356 (ASCE) (2000), *Prestandard and Commentary for the Seismic Rehabilitation of Buildings*.

Gajan, S., Raychowdhury, P., Hutchinson, T.C., Kutter, B.L. and Stewart, J.P. (2010), "Application and validation of practical tools for nonlinear soil-foundation interaction analysis", *Earthq. Spectr.*, **26**(1), 111-129.

Gazetas, G. (1991), *Foundations Vibrations*, Nostrand Reinhold.

Gazetas, G., Panagiotidou, A.I. and Gerolymos, N. (2010), "Pushover and inelastic-seismic response of shallow foundations supporting a slender structure", *Proceedings of the 5th International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*.

Grange, S., Kotronis, P. and Mazars, J. (2009), "A macro-element to simulate 3D soil-structure interaction considering plasticity and uplift", *Int. J. Sol. Struct.*, **46**(20), 3651-3663.

Harden, C.W. and Hutchinson, T.C. (2009), "Beam-on-nonlinear-winkler-foundation modeling of shallow, rocking-dominated footings", *Earthq. Spectr.*, **25**(2), 277-300.

Hung, H. and Liu, K. (2014), "Rocking behavior of bridge piers with spread footings under cyclic loading and earthquake excitation", **7**(6), 1001-1024.

Jafarieh, A.H. and Ghannad, M.A. (2014), "The effect of foundation uplift on elastic response of soil-structure systems", *Int. J. Civil Eng.*, **12**(2 A), 244-256.

Liping, L., Kui, X. and Xuan, C. (2008), "The research of applied pushover method in the earthquake resistance analysis of soil-structure interaction system", *Proceedings of the 14th World Conference on Earthquake Engineering*.

Loli, M., Knappett, J.A., Brown, M.J., Anastasopoulos, I. and Gazetas, G. (2014), "Centrifuge modeling of rocking-isolated inelastic RC bridge piers", *Earthq. Eng. Struct. Dyn.*

Nguyen, Q., Van, Fatahi, B. and Hokmabadi, A.S. (2016), "The effects of foundation size on the seismic performance of buildings considering the soil-foundation-structure interaction", *Struct. Eng. Mech.*, **58**(6), 1045-1075.

Panagiotidou, A.I., Gazetas, G. and Gerolymos, N. (2012), "Pushover and seismic response of foundations on stiff clay: Analysis with P-delta effects", *Earthq. Spectr.*, **28**(4), 1589-1618.

Perez-Rocha, and Viles, J.A. (2004), "Site- and interaction-dependent strength reduction factors", *Proceedings of the 13th World Conference on Earthquake Engineering*.

Psycharis, I.N. and Jennings, P.C. (1983), "Rocking of slender rigid bodies allowed to uplift", *Earthq. Eng. Struct. Dyn.*, **11**(1), 57-76.

Raychowdhury, P. (2011), "Seismic response of low-rise steel moment-resisting frame (SMRF) buildings incorporating nonlinear soil-structure interaction (SSI)", *Eng. Struct.*, **33**(3), 958-967.

SeismoSoft (2016), *SeismoStruct V.7, A computer Program for Static and Dynamic Non-linear Analysis of Framed Structures*.

Siddharthan, B.R., Ara, S. and Norris, G.M. (1992), "Simple rigid plastic model for seismic tilting of rigid walls", *J. Struct. Eng.*, **118**(2), 469-487.

Smith-Pardo, J.P. (2011), "Performance-based framework for soil-structure systems using simplified rocking foundation models", *Struct. Eng. Mech.*, **40**(6), 763-782.

Song, Y.H. and Lee, D.G. (1993), "An improved two-spring model for foundation uplift analysis", *Comput. Struct.*, **46**(5), 791-805.

Terzaghi, K. (1943), *Theoretical Soil Mechanics*, John Wiley & Sons, Inc., New York, U.S.A., London, U.K.

Wolf, J.P. (1994), *Foundation Vibration Analysis Using Simple Physical Model*.

Xu, C. and Spyrakos, C.C. (1996), "Seismic analysis of towers including foundation uplift", *Eng. Struct.*, **18**(4), 271-278.

PL