Structural Engineering and Mechanics, *Vol. 67, No. 1 (2018) 53-67*
DOI: https://doi.org/10.12989/sem.2018.67.1.053

On the elastic parameters of the strained media

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(Received January 16, 2018, Revised March 28, 2018, Accepted March 29, 2018)

Abstract. The changes of parameters of pressure and velocity of propagation of elastic pressure and shear waves in uniformly deformed solid compressible media are studied within the nonclassically linearized approach (NLA) of nonlinear elastodynamics to create a new theoretical basis of the geomechanical interpretation of various groups of geophysical observational and experimental data. The cases of small and large deformations are considered while their describing by various elastic potentials, i.e., problems considering the physical and geometric nonlinearity. Convenient analytical formulae are obtained to calculate the indicated parameters in the deformed isotropic media within the nonclassical linear and nonlinear solution in the NLA.

Specific numerical experiments are conducted in case of overall compression of various materials. It is shown that the method (generally accepted in the studies of mechanics of standard constructional materials) of additional linearization (relative to the pressure parameter) in the basic correlations of the NLA introduces substantial quantitative and qualitative errors into the results at significant preliminary deformations. The influences of the physical and geometric nonlinearity on the studied characteristics of the medium are large in various materials and differ qualitatively. The contribution of nonlinear components to the values of the considered parameters prevails over linear components at large deformations. When certain critical values of compression deformations in the medium are achieved, elastic waves with actual velocity cannot propagate in it. The values of the critical deformations for pressure and shear waves differ within different elastic potentials and variants of the theory of initial deformations.

Keywords: earth’s core; high pressure; instability; elastic waves with actual velocity

1. Introduction

Data of various geophysical observations and numerous geomechanical, geochemical, petrological, petrophysical, etc. laboratory experimental results constitute the initial database while constructing the theoretical models of the structure and material composition of the Earth. Processing and interpretation of all this data set on special programs allowed coming to a definite opinion regarding the internal structure and distribution of elastic and physical parameters of the medium of the Earth’s depths (Dziewonski and Anderson 1981, Kennet *et al.* 1995, Sumita and Bergman 2007, Litasov and Shatskiy 2016, www.sciencedirect.com/..., https://ds.iris.edu/...). The implementation of integral criteria is considered a measure of the reliability of the adopted distributions. The distributions of fundamental elastic parameters, density of the medium and pressure are carried out in such a way that the calculated mass of the simulated composite ball corresponds to the known mass of the Earth, the calculated inertia regarding the axis of the rotation - to the measured moment of inertia of the Earth, the calculated spectrum of the periods of free-oscillations - the observed period of the Earth's free-oscillations caused by strong earthquakes. Some seismotomographic corrections have been made in recent years (Van der Hilst and Karason 1999, Montelli *et al.* 2004, Trampert *et al.* 2004, Ritsema 2005). The implementation of integral criteria is necessary but insufficient. A variety of differential criteria have been put forward within various disciplines which should be met while solving the problem of distribution of the above mentioned parameters (Anderson 1995, Anderson 2007). Theoretically, differential criteria should play the role of sufficient conditions while solving the considered problems. There occurred a difficult situation and differences between separate scientific opinions. It is assumed that the main geospheres: the crust, mantle and inner core of the Earth are deformable solids. In this connection, first of all, the distribution of elastic and physical parameters of the medium along with integral and other differential criteria should also correspond to the fundamental requirements of the mechanics of a deformable solid body (MDSB) (Lyav 1935, Eringen and Suhubi 1975, Truesdell 1975). The latest results published in (Guliyev 2016, Guliyev 2017a, b, c, Guliyev *et al.* 2017, Khazan 2017) indicate the necessity to develop differential criteria that follow from MDSB. It was established in these works that the generally accepted distribution of pressure and elastic parameters of the medium of the internal solid core of the Earth in geophysics (Dziewonski and Anderson 1981, www.sciencedirect.com/..., https://ds.iris.edu/...) consistent with the integral criteria do not meet the requirements of MDSB.

As a usual, elastic parameters of the medium (elasticity

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ISSN: 1225-4568 (Print), 1598-6217 (Online)

moduli, velocities of propagation of elastic waves, etc.) are determined in laboratory experiments using specially made, pre-undeformed samples. Linear theories of elasticity and propagation of elastic waves in the undeformed media makes the theoretical basis of the physico-mechanical interpretation of experimental data. The parameters determined only in such experimental and theoretical studies are called elastic. They being the nominal data also have a representativeness (i.e., experimentally determined data in samples describe the behavior of the material or the medium as a whole). The geological medium of the Earth' depths is under the influence of geodynamic evolution all the time. The experiments related to the physico-mechanical studies including elastic properties of geological media of large (mantle) and super-large (inner core) depths on several positions have a particular specificity. Great progress has been achieved in the field of technique and technology to carry out unique experiments (Altshuler *et al.* 2004, Tateno *et al.* 2010, Mao *et al.* 2012b, Lu *et al.* 2013, Litasov and Shatskiy 2016). The experiments are transient, and geometric dimensions of the tested artificially created samples (these samples are created in the experiment (Mao *et al.* 2004, 2012a, 2012b), their representativeness to real conditions on the scales of geological objects is hardly attainable) are very small. In this connection, experimental technologies allow studying only too simplified situations. Therefore, the results have limit values essentially. Important frontiers have been achieved in the field of theoretical substantiation of formulation of experiments and understanding of the processed and interpreted experimental data. It is necessary to be able to distinguish the effects (complex, even immense complexity) of geodynamic (historical and modern) impacts in the process of synthesis of data of physico-mechanical properties from experimental results. In particular, experiments using shock-wave compression are conducted on the basis of the rapid kinetics of shear deformations of substance (Altshuler *et al.* 2004). Deformations are caused during the experiment to record the results of the measurement. Therefore, it is very problematic to bring them into correlation with deformations occurred in the Earth's long-term natural development and can be a source of significant errors in the results. Such kind of different ways of appearance of errors and uncertainties also exist in other experimental technologies of studies of the geological medium under high thermobaric conditions.

Experiments show that the thermobaric impact on the nature of the change of the physico-mechanical parameters makes a different influence in different levels of deformation. This influence for the same medium differs at different stages of deformation both quantitatively and qualitatively. Moreover, these influences for various media (e.g., silicate and basalt glasses, iron alloys, etc.) also differ on the character. The determination of any single trends of this influence is not succeeded in the experiments. The arisen situation is explained for each medium by the experimenters in different ways. For example, the behavior of physico-mechanical parameters is explained by the softening regime in aluminosilicate networks and by changes of locally-arranged structures. The peculiarities in changes of velocities of pressure and shear waves in these media are used as analogs to understand the properties of melts in the Earth's interior. The current status of this problem can be found in more detail in publications (Karki and Stixrude 2010, Sanchez-Valle and Bass 2010, Murakami and Bass 2011, Nomura *et al.* 2011, Sato *et al.* 2011, Shen *et al.* 2011, Kono *et al.* 2012, Weigel *et al.* 2012, Cormier and Cuello 2013, Sakamaki *et al.* 2013, Sanloup *et al.* 2013, Ghosh *et al.* 2014, Liu and Lin 2014, Prescher *et al.* 2014, Wang *et al.* 2014).The situation is also analogous relating to the experimental studies of the assumed medium of inner core (Bullen 1978, Dziewonski and Anderson 1981, Anderson 1995, Deuss 2014, Kennett *et al.* 1995, Anderson 2007, Sumita and Bergman 2007, Heiffrich and Kaneshima 2010, Tateno *et al.* 2010, Mao *et al.* 2012a, Hirose *et al.* 2013, Ohtani *et al.* 2013, Badro *et al.* 2014, Chen *et al.* 2014, Decremps *et al.* 2014, Li and Fei 2014, Nimmo 2015, Souriau and Calvet 2015, Antonangeli and Ohtani 2015, Prescher *et al.* 2015, Litasov and Shatskiy 2016). Therefore, it creates a certain scientific and practical interest of development of a single theoretical basis of physico-mechanical interpretations of various complexes of geophysical data taking into account high thermobaric conditions and nonlinearity of deformation. Beginning with Murnaghan and Birch’s works, nonlinear theory of deformations has been used in a number of the above mentioned and other works to describe the results of numerous laboratory experimental studies of various minerals, rocks and materials, and also in the study of the problem of distribution of physico-mechanical parameters of the medium of the Earth's depths. Separate forms of equations of state for the assumed media of different depths of the Earth are proposed. Theories of hyperelastic materials (Green’s) (an expression for the potential energy of elastic deformation is given) or the theory of the general elastic body (Cauchy) of one-to-one correlations connecting the components of stress and strain tensors with observance of the tensor dimension are used as the initial theoretical bases. A complete return of energy is provided at the stage of elastic deformation in the first case, and full recoverability of the initial geometric shape of the body after removal of loads is provided in the second case. The processes of deformation are described either by the Eulerian or Lagrangian method.

The characteristics of distribution of parameters of pressure, velocities of propagation of elastic waves in nonlinearly deformed isotropic media are studied in this paper based on the NLA of nonlinear elastodynamics related with the above-mentioned. The deformations are described by various elastic potentials within the Lagrangian method. The application of NLA to the considered range of problems has a certain methodological advantage in comparison with the direct use of the general nonlinear theory, including Birch-Murnaghan’s theory of finite deformations (Birch 1952, Bullen 1978, Anderson 1995), etc. First of all, the nonlinear problem is reduced to linear (nonclassical) and well-studied mathematical problems. The problems of strength, stability (both on geometric shape changes and on “internal” instability) and propagation of elastic waves in elastically deformable solid media are studied from a single theoretical base. In the case of homogeneous deformed states, simple analytical equations are obtained in the structure of which the contributions of linear (just these components should be named elastic properties of the medium) and nonlinear impacts on the quantitative values of parameters of elastic properties are distinguished explicitly. On the other hand, it is possible to study the above-mentioned problems in elastic-plastic and viscoelastic, etc. statements in a similar simplicity using the concept of "continuing loading" and a quasi-static approach (Guz 1999). Moreover, it is possible to determine the areas of applicability (areas in which the determination of physico-mechanical properties including other parameters is realized within those limits of deformations where the initial fundamental restrictions of the mechanics of a deformable solid are observed) of the obtained results theoretically. It is necessary to conduct additional experimental studies to solve this problem in case of direct application of general nonlinear theories. Such circumstance reduces the generality of theoretical results and creates additional channels of inclusion of errors and uncertainties into the results. The problem of changing the density parameter of isotropic media depending on the growth of nonuniform homogeneous deformation is studied in (Guliyev 2013, Guliyev and Askerov 2007) within the NLA of nonlinear elastodynamics.

Nonlinear and non-classically linearized deformation theory is now widely used in many sections of Earth science. Such integration of sciences makes it possible to study specific scientific practical problems and to reveal the role of deformations in geological, geophysical, seismological and mining-mechanical processes (Alexandrov *et al.* 2001, Prodaivoda *et al.* 2004, Vyzhva *et al.* 2005, Guliyev 2010).

2. The principal correlations of the nonclassical-linearized theory

Problems of the stressed-deformed state related with traditional (metallic, composite, glassy, ceramic, wooden, etc.) materials, structures and constructions are usually studied in the mechanics of deformable solids. Their initial non-stressed state is known with sufficient accuracy. The scales of the objects under study usually allow carrying out direct observation and controlling of ongoing processes. It essentially facilitates the acceptance of reasonable simplifying assumptions both at the stage of formulation of problems, statement of problems and theoretical modeling, and at the stage of development of algorithms and solution methods. Both exact and approximate methods of solution are used; linearization is usually carried out in the vicinity of the initially undeformed state. The above-mentioned makes the theoretical basis of the classical models of the mechanics of deformable solids.

The situation in geology, geophysics and mining is much more complicated. Sedimentary, metamorphic and magmatic rocks were formed, evolved, destructed during the intervals of time determined in millions and billions of years. There is no an unstress state in the entrails of the Earth which could be taken in the form of an initial one. Rocks are constantly under the influence of a huge amount of arbitrarily changing stress of a diverse nature. The nature of manifestation and impact of stress is different in various situations. Periodicity and duration of their action are counted in seconds, hours, days, years, centuries, ages, etc. Geological objects on scales are also diverse. Their linear dimensions can be measured in millimeters and thousands of kilometers. At the same time, many phenomena and processes occur at various depths of the Earth's interior which are mainly inaccessible for direct observation. Naturally, the ability of deformation of various rocks significantly differs both in magnitude and in nature, i.e. there is a huge diversity in the physical and mechanical properties of geomaterials and conditions of deformation. Due to these enormous temporal and geometric scales and inaccessibility of geological objects, it is not possible to directly observe the processes occurring in them and to make appropriate simplifications in mathematical problems. All these factors and the accumulated experience stipulate the use of the nonclassical method of linearization of the complete system of equations, correlations, boundary and initial conditions of a general physically and geometrically nonlinear theory. The essence of this method lies in the fact that the linearization is carried out sequentially in a small vicinity of the actual (point M in the graph of Fig. 1) in other words, really deformed state. At the same time, any natural or artificial process is considered in the form of two qualitatively different states-undisturbed (which, in its turn, consists of the natural and the initial state) and disturbed. It is considered that all the values and parameters of the process under study are known, or at least methods of their determination are known in the undisturbed state. At the same time, the values of these parameters can be arbitrary, and the problem on their determination is nonlinear. Similar values and parameters (including new ones) in the disturbed state are unknown. The required parameters, in comparison with the undisturbed ones are too small (but not infinitely small), which allows carrying out the linearization with a sufficient degree of accuracy. The obtained linear differential equations with respect to disturbances as coefficients contain parameters of linear and nonlinear physico-mechanical properties of the medium, force and geometric parameters of initially stressed state. The total displacement consists of the sum of displacements of the initial state and disturbances. The basic systems of the classical linear theories, where the linearization is carried out in the vicinity of natural undeformed state are written with respect to the displacements of the initial state. Only parameters characterizing the linear physico-mechanical properties of the medium are included in them as coefficients.

Two stages are distinguished in the initial state: natural (undeformed) and initially deformed. At the same time, it is possible to consider the problems with small linear and nonlinear (the point M is too close to the point A in Figure 1) and large nonlinear deformations (the point M is too far from the point A), to correlate all the measured values per unit of area or natural, or initially deformed states. It is convenient to use the Lagrangian coordinates while compiling the basic equations and correlations of the NLA

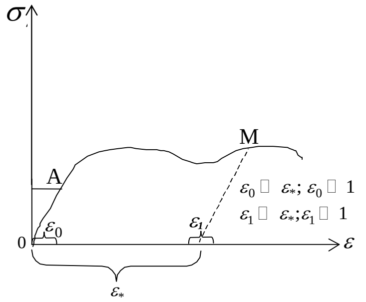


Fig. 1 The method of linearization. σ - stress, ε - strain, ε_0 - values of deformations in the initially deformed state, ε_* - values of deformation in the actual state, ε_1 - values of disturbances of deformations

in the indicated two cases, and the Eulerian coordinates are used while measuring the values on the area of the disturbed state. The reference and the associated coordinate systems (Guz 1986, 2004) also differ. All these differences relate to mathematical formalizations. Nevertheless, their consideration is fundamentally important as it is necessary to compare the parameters and values determined and modeled under the same conditions and assumptions while comparing theoretical results with experimental ones.

The basic idea of the nonclassical linearization of equations and correlations of the nonlinear elastodynamics is shown in Fig. 1. In the classical method, the linearization is carried out in a small vicinity of the beginning of deformation (point A). In the nonclassical-in a small vicinity of the actual state (point M). Different variants of the theory of initial deformations are considered in studies (Guz 1986, 1999, 2004) depending on the value of the strain up to the point M. Within the NLA, while determining the components (see Eq. (4)), three different variants are distinguished (Guz 1986) depending on the values of deformations in the initial state: a) the theory of large (finite) initial strains (t.o.l.i.s.) ; b) the first variant of the theory of small initial strains (shearings and elongations are small in comparison with unit) (f.v.o.t.o.s.i.s.); c) the second varinat of the theory of small initial strains (in addition to the first variant, it is assumed that the components of stress and strain tensors are related to Hooke's law) (s.v.o.t.o.s.i.s.). Two cases of representation of plane harmonic wave also differ. In the first case, the changes of distances between the material particles are not taken into account due to the initial deformation and the velocity of wave propagation is called the "natural" velocity (Thurston and Brugger 1964). In the second case, these changes are taken into account and the velocity is called "true".

Let's consider an unbounded elastic space subject to homogeneous deformation in the Lagrangian coordinate system $x_n = x_n^1$ coinciding with the Cartesian coordinates in the natural state

$$u_m^0 = \left(\lambda_m - 1\right)\lambda_m\,;\ \lambda_m = const, m = 1,2,3 \tag{1}$$

where u_m^0 are the components of vector of displacements of initially deformed state, and λ_m are the coefficients of elongation (shortening) along the coordinate axes. The components of the Green's deformation tensor ε_{ij}^0 in the case of large and first variant of the theory of small and second variant of the theory of small initial deformations are accordingly determined in the form (Guz 1986)

$$2\varepsilon_{ij}^0 = \delta_{ij}(\lambda_i^2 - 1) \tag{2}$$

$$\varepsilon_{ij}^0 = \delta_{ij}(\lambda_j - 1) \tag{3}$$

where δ_{ij}-is Kronecker symbol.

The problems of propagation of elastic waves in the deformed bodies are studied in detail (Truesdell 1975, Biot 1965, Guz 1986, 2004). These studies are also widely applied in various fields (Kuliev and Jabbarov 1998, 2000, Aleksandrov *et al.* 2001, Ritsema 2005, Helffrich and Kaneshima 2010, Karki and Stixrude 2010, Kono *et al.* 2012, Mao *et al.* 2012b, Ohtani *et al.* 2013, Decremps *et al.* 2014, Liu and Lin 2014, Akbarov 2015, Hadji *et al.* 2015, Li and Tao 2015, Kakar and Kakar 2016, Tao *et al.* 2016, Teachavorasinskun and Pongvithayapanu 2016). In case of compressible models of media, the basic systems of equations of motion for the theory of large and various variants of the theory of small initial deformations in a single form take the form (Guz 1986, 2004)

$$\omega_{ij\alpha\beta}\frac{\partial^2 u_\alpha}{\partial x_i\,\partial x_\beta} - \rho\frac{\partial^2 u_j}{\partial t^2} = 0\,,\quad \omega_{ij\alpha\beta} = const\,. \tag{4}$$

In case of the theory of large initial deformations

$$\omega_{ij\alpha\beta} = \lambda_j\lambda_\alpha\left[\delta_{ij}\delta_{\alpha\beta}A_{i\beta} + \left(1-\delta_{ij}\right)\left(\delta_{i\alpha}\delta_{j\beta} + \delta_{i\beta}\delta_{j\alpha}\right)\mu_{ij}\right] + +\delta_{i\beta}\delta_{j\alpha}S_{\beta\beta}^0\,. \tag{5}$$

For the theory of small initial deformations according to the first and second variants of the theory

$$\omega_{ij\alpha\beta} = \lambda_j\lambda_\alpha\left[\delta_{ij}\delta_{\alpha\beta}A_{i\beta} + (1-\delta_{ij})(\delta_{i\alpha}\delta_{j\beta} + \delta_{i\beta}\delta_{j\alpha})\mu_{ij}\right] + \\ +\delta_{i\beta}\delta_{j\alpha}\sigma_{\beta\beta}^0 \tag{6}$$

$$\omega_{ij\alpha\beta} = \delta_{ij}\delta_{\alpha\beta}A_{i\beta} + (1-\delta_{ij})(\delta_{i\alpha}\delta_{j\beta} + \delta_{i\beta}\delta_{j\alpha})\mu_{ij} + \\ +\delta_{i\beta}\delta_{j\alpha}\sigma_{\beta\beta}^0 \tag{7}$$

It is obtained considering the elastic waves propagating along the axis according to Eq. (4) in case of the theory of large initial deformations (Guz 1986, 2004)

$$\begin{aligned} \rho C_{lx_1}^2 &= \lambda_1^4 A_{11} + \lambda_1^2 S_{11}^0; \\ \rho C_{sx_2}^2 &= \lambda_1^2\lambda_2^2\mu_{12} + \lambda_1^2 S_{11}^0; \\ \rho C_{sx_3}^2 &= \lambda_1^2\lambda_3^2\mu_{13} + \lambda_1^2 S_{11}^0 \end{aligned} \tag{8}$$

In case of the second variant of the theory of small initial deformations

$$\rho V_{lx_1}^2 = \lambda_1^2(A_{11} + \sigma_{11}^0);\ \rho V_{sx_2}^2 = \lambda_1^2(\mu_{12} + \sigma_{11}^0);\ \rho V_{sx_3}^2 = \\ = \lambda_1^2(\mu_{13} + \sigma_{11}^0) \tag{9}$$

In correlations (5)-(7), the structure of values $A_{i\beta}$, μ_{ij} and $S^0_{\beta\beta}$ (or $\sigma^0_{\beta\beta}$) for each variant of problem statements is concretized with the definition of the elasticity correlations of nonlinear theory (elastic potentials). Concrete expressions of these values for the simplest elastic potentials describing compressible and incompressible media are given in (Guz 1986, 1999, 2004). In the expressions (4)-(9), u_i(i=1, 2, 3) are the components of disturbance of displacements, $S^0_{\beta\beta}$ are the components of stress tensor per unit area in the initially deformed state, σ^0_{ii} is the component of stress tensor per unit area in the natural state, σ^0_{ii} is the density of the medium. It is easy to obtain similar equations in the propagation of elastic waves along other coordinate axes using cyclic permutations of indices in (8) and (9).

The experimentally observed acoustoelastic effect (of a different series of reactions, velocities of the polarized shear waves on the actions of initial deformations) in comparatively solid compressible media is described using elastic potentials in the structure of which along with the first two, the third algebraic invariant of Green's deformation tensor is also taken into account. The simplest elastic potential corresponding to this requirement is a potential of the Murnaghan type. In this connection, the results concerning the Murnaghan-type potential is given below.

In this case, for all variants of the theory of initial strains (Guz 1986, 2004)

$$A_{i\beta} = \lambda + 2a\varepsilon^0_{nn} + 2b\left(\varepsilon^0_{ii} + \varepsilon^0_{\beta\beta}\right) + \\ +2\delta_{i\beta}\left(\mu + b\varepsilon^0_{nn} + c\varepsilon^0_{ii}\right) \tag{10}$$

$$\mu_{ij} = \mu + b\varepsilon^0_{nn} + \frac{1}{2}c\left(\varepsilon^0_{ii} + \varepsilon^0_{jj}\right), \\ \lambda, \mu, a, b, c = const \tag{11}$$

For the theory of large initial strains

$$S^0_{\beta\beta} = \lambda\varepsilon^0_{nn} + 2\mu\varepsilon^0_{\beta\beta} + a(\varepsilon^0_{nn})^2 + 2b\varepsilon^0_{nn}\varepsilon^0_{\beta\beta} + \\ +b\left(\varepsilon^0_{nn}\right)^2 + c\left(\varepsilon^0_{\beta\beta}\right)^2 \tag{12}$$

where λ, μ- are the Lame's elasticity moduli, a, b, c-are the elasticity moduli of the third order.

There are correlations for the first and the second variant of the theory of small initial deformations accordingly

$$\sigma^0_{\beta\beta} = \lambda\varepsilon^0_{nn} + 2\mu\varepsilon^0_{\beta\beta} + a(\varepsilon^0_{nn})^2 + 2b\varepsilon^0_{nn}\varepsilon^0_{\beta\beta} + \\ +b\left(\varepsilon^0_{nn}\right)^2 + c\left(\varepsilon^0_{\beta\beta}\right)^2, \ 2\varepsilon^0_{ij} = \delta_{ij}\left(\lambda_j^2 - 1\right), \tag{13}$$

$$\sigma^0_{\beta\beta} = \lambda\varepsilon^0_{nn} + 2\mu\varepsilon^0_{\beta\beta} + a(\varepsilon^0_{nn})^2 + 2b\varepsilon^0_{nn}\varepsilon^0_{\beta\beta} + \\ +b\left(\varepsilon^0_{nn}\right)^2 + c\left(\varepsilon^0_{\beta\beta}\right)^2, \ \varepsilon^0_{ij} = \delta_{ij}\left(\lambda_j - 1\right). \tag{14}$$

Eqs. (8) and (9) considering the correlations (10)-(14) allow studying the behavior of parameters of velocities of propagation of elastic waves in isotropic compressible comparatively solid media depending on the nature of the change of an arbitrary value of homogeneous deformations within the theory of large and two variants of the theory of small initial deformations. These equations and correlations are obtained and applied to solve various concrete problems of mechanics in (Guz 1986, 1999, 2004). An additional linearization is carried out while studying the concrete problems based on practical mechanical considerations in the expressions of Eqs. (12)-(14). It is assumed that it is possible to accept for all variants of the theory in the linear approximation for the purposes of obtaining simple analytical and concrete results in the standard problems of mechanics in the subsequent calculations

$$\lambda_\beta = 1 + \left(\frac{\mathrm{P}^0_\alpha}{\mu}\right)k_\alpha \tag{15}$$

Where the coefficients k_α(α=1, 2, 3) are calculated for each variant of the theory involving expressions (12)-(14) taking into account the following correlations for the theory of large, the first and the second variants of small initial strains accordingly

$$P^0_\beta = \lambda_\beta S^0_{\beta\beta}; \quad P^0_\beta = \lambda_\beta\sigma^0_{\beta\beta}; \quad P^0_\beta = \sigma^0_{\beta\beta}. \tag{16}$$

The given approach means that it is possible to assume that in the linear approximation (with respect to the parameter $\frac{P^o}{\mu}$) for the theory of finite (large) and the first variant of initial deformations

$$2\varepsilon^0_{ij} = \lambda_j^2 - 1 = \frac{1}{\mu}\left(\sigma^0_{jj} - \frac{\lambda}{3K_0}\sigma^0_{\alpha\alpha}\right); K_0 = \lambda + \frac{2}{3}\mu, \tag{17}$$

but for the second variant of the theory of small initial deformations

$$\varepsilon^0_{ij} = \lambda_j - 1 = \frac{1}{2\mu}\left(\sigma^0_{ij} - \frac{\lambda}{3K_0}\sigma^0_{\alpha\alpha}\right); K_0 = \lambda + \frac{2}{3}\mu, \tag{18}$$

where K_0-is modulus of volumetric compression of linear elastic isotropic body.

Such an approximation, i.e., expression (15) cannot always be applied in the problems of geomechanics as the smallness of the parameter $\frac{P_0}{\mu}$ is not always preserved (Dziewonski and Anderson 1981, Anderson 1995, Anderson 2007). Therefore, the Eqs. (8) and (9) will be used later considering complete structures of correlations (12)-(14) without additional simplifications.

3. Nonlinear character of the dependence of the basic parameters of the medium on the change of strains

The normal components of stress tensor are determined according to Eqs. (12)-(14) within the theory of large and small initial strains within the NLA.

Let's consider the case of overall strain to achieve clarity and simplicity of presentation, moreover, in the problems of geomechanics of large depths of overall strain, i.e., $\varepsilon_{11}^0 = \varepsilon_{22}^0 = \varepsilon_{33}^0 = \varepsilon_0$ is the most important case. At the same time, we obtain from the correlation (12)-(14) considering (16) to determine the pressure:
for the theory of large and the first variant of small initially deformed states

$$P_0 = (1+2\varepsilon_0)^{\frac{1}{2}}\left[3K_0\,\varepsilon_0 + (9a+15b+c)\,\varepsilon_0^2\right]; \quad K_0 = \lambda + \frac{2}{3}\mu, \tag{19}$$

for the second variant of the theory of small initially deformed states

$$P_0 = 3K_0\,\varepsilon_0 + (9a+15b+c)\,\varepsilon_0^2 \tag{20}$$

We obtain in the case of the theory of large and the first variant of small initial deformations using the correlation (8), (9) and (10)-(14) in case of overall homogeneous deformation to determine the dependencies of quasi velocities of propagation of elastic waves on deformations

$$\begin{aligned} \rho C_{l\,x_1}^2 &= \lambda + 2\mu + (7\lambda + 10\mu + 6a + 10b + 2c)\times \\ &\times \varepsilon_0 (33a + 55b + 9c + 10\lambda + 12\mu)\,\varepsilon_0^2 + \\ &+ (24a + 40b + 4c + 3\lambda + 2\mu)\,\varepsilon_0^3 + \\ &+ (9a + 15b + c)\,\varepsilon_0^4, \end{aligned} \tag{21}$$

$$\begin{aligned} \rho C_{s\,x_2}^2 = \rho C_{sx_3}^2 &= \mu + (3\lambda + 6\mu + 3b + c)\,\varepsilon_0 + \\ &+ (6\lambda + 8\mu + 9a + 27b + 5c)\,\varepsilon_0^2 + \\ &+ (18a + 42b + 6c)\,\varepsilon_0^3, \end{aligned} \tag{22}$$

in case of the second variant of the theory of small initial deformations

$$\begin{aligned} \rho V_{l\,x_1}^2 &= \lambda + 2\mu + (5\lambda + 6\mu + 6a + 10b + 2c)\,\varepsilon_0 + \\ &+ (21a + 35b + 5c + 7\lambda + 6\mu)\,\varepsilon_0^2 + \\ &+ (24a + 40b + 4c + 3\lambda + 2\mu)\,\varepsilon_0^3 + \\ &+ (9a + 15b + c)\,\varepsilon_0^4, \end{aligned} \tag{23}$$

$$\begin{aligned} \rho V_{s\,x_2}^2 = \rho V_{s\,x_3}^2 &= \mu + (3\lambda + 4\mu + 3b + c)\,\varepsilon_0 + \\ &+ (9a + 21b + 3c + 6\lambda + 5\mu)\,\varepsilon_0^2 + \\ &+ (3\lambda + 2\mu + 18a + 33b + 3c)\,\varepsilon_0^3 + \\ &+ (9a + 15b + c)\,\varepsilon_0^4. \end{aligned} \tag{24}$$

Multiplying left and right sides of correlations (21) and (23) by $(\lambda + 2\mu)^{-1}$, (22) and (24) by μ^{-1} and considering that

$$\rho C_{l0}^2 = \lambda + 2\mu, \quad \rho C_{S0}^2 = \mu \tag{25}$$

$$\rho V_{l0}^2 = \lambda + 2\mu, \quad \rho V_{S0}^2 = \mu \tag{26}$$

we obtain for the calculation of values of parameters of velocities of propagation of elastic pressure and shear waves

$$\begin{aligned} \left(\frac{C_{lx_1}}{C_{l_0}}\right)^2 &= 1 + \left[\frac{5-3\nu}{1-\nu} + (\lambda+2\mu)^{-1}(6a+10b+2c)\right]\varepsilon_0 + \\ &+ \left[\frac{2(3-\nu)}{(1-\nu)} + (33a+55b+9c)(\lambda+2\mu)^{-1}\right]\varepsilon_0^2 + \\ &+ (\lambda+2\mu)^{-1}(42a+70b+10c)\,\varepsilon_0^3, \end{aligned} \tag{27}$$

$$\begin{aligned} \left(\frac{C_{sx_2}}{C_{S_0}}\right)^2 = \left(\frac{C_{sx_3}}{C_{S_0}}\right)^2 &= 1 + \left[\frac{6(1-\nu)}{1-2\nu} + \frac{1}{\mu}(3b+c)\right]\varepsilon_0 + \\ &+ \left[\frac{4(2-\nu)}{1-2\nu} + \frac{1}{\mu}(9a+27b+5c)\right]\varepsilon_0^2 + \\ &+ \frac{1}{\mu}(18a+42b+6c)\,\varepsilon_0^3, \end{aligned} \tag{28}$$

$$\begin{aligned} \left(\frac{V_{lx_1}}{V_{l_0}}\right)^2 &= 1 + \left[\frac{3-\nu}{1-\nu} + (\lambda+2\mu)^{-1}(6a+10b+2c)\right]\varepsilon_0 + \\ &+ \left[\frac{3+\nu}{1-\nu} + (\lambda+2\mu)^{-1}(21a+35b+5c)\right]\varepsilon_0^2 + \\ &+ \left[\frac{1+\nu}{1-\nu} + (\lambda+2\mu)^{-1}(24a+40b+4c)\right]\varepsilon_0^3 + \\ &+ (\lambda+2\mu)^{-1}(9a+15b+c)\,\varepsilon_0^4, \end{aligned} \tag{29}$$

$$\begin{aligned} \left(\frac{V_{Sx_2}}{V_{S_0}}\right)^2 = \left(\frac{V_{Sx_3}}{V_{S_0}}\right)^2 &= 1 + \left[\frac{2(2-\nu)}{1-2\nu} + \mu^{-1}(3b+c)\right]\varepsilon_0 + \\ &+ \left[\frac{5+2\nu}{1-2\nu} + \mu^{-1}(9a+21b+3c)\right]\varepsilon_0^2 + \\ &+ \left[\frac{2(1+\nu)}{1-2\nu} + \mu^{-1}(18a+33b+3c)\right]\varepsilon_0^3 + \\ &+ \mu^{-1}(9a+15b+c)\varepsilon_0^4. \end{aligned} \tag{30}$$

Where C_{l_0} and C_{S_0} are the velocities of pressure and shear waves in the medium without initial deformations within the theory of large initial deformations; V_{l_0} and V_{S_0} are just the same within the second variant of the theory of small initial deformations. ν is Poisson's ratio of the material in correlations (27)-(30). The parameters of pressure and shear waves are expressed as $\left(\frac{C_{lx_1}}{C_{l_0}}\right)^2$, $\left(\frac{C_{sx_2}}{C_{S_0}}\right)^2$ and $\left(\frac{V_{lx_1}}{V_{l_0}}\right)^2$, $\left(\frac{V_{sx_2}}{V_{S_0}}\right)^2$ in the text of the manuscript within the theory of large, the first variant of the theory of small and the second variant of the theory of small initial deformations accordingly.

In case of the quadratic elastic potential, Eqs. (19) and

(20) take more simple form accordingly

$$\frac{P_0}{\mu}=\frac{3K_0}{\mu}\varepsilon_0\left(1+2\varepsilon_0\right)^{\frac{1}{2}}=\frac{2(1+\nu)}{(1-2\nu)}\varepsilon_0\left(1+2\varepsilon_0\right)^{\frac{1}{2}}, \quad (31)$$

$$\frac{P_0}{\mu}=\frac{3K_0}{\mu}\varepsilon_0=\frac{2(1+\nu)}{1-2\nu}\varepsilon_0, \quad (32)$$

and Eqs. (21)-(24) take the following form accordingly

$$\rho C_{lx_1}^2=\lambda+2\mu+\left(7\lambda+10\mu\right)\varepsilon_0+2\left(5\lambda+6\mu\right)\varepsilon_0^2, \quad (33)$$

$$\rho C_{sx_2}^2=\rho C_{sx_3}^2=\mu+3\left(\lambda+2\mu\right)\varepsilon_0+2\left(3\lambda+4\mu\right)\varepsilon_0^2, \quad (34)$$

$$\rho V_{lx_1}^2=\lambda+2\mu+\left(5\lambda+6\mu\right)\varepsilon_0+\left(7\lambda+6\mu\right)\varepsilon_0^2+ +\left(3\lambda+2\mu\right)\varepsilon_0^3, \quad (35)$$

$$\rho V_{sx_2}^2=\rho V_{sx_3}^2=\mu+\left(3\lambda+4\mu\right)\varepsilon_0+\left(6\lambda+5\mu\right)\varepsilon_0^2+ +\left(3\lambda+2\mu\right)\varepsilon_0^3, \quad (36)$$

or

$$\left(\frac{C_{lx_1}}{C_{l_0}}\right)^2=1+\frac{\varepsilon_0}{1-\nu}\left[5-3\nu+2(3-\nu)\varepsilon_0\right], \quad (37)$$

$$\left(\frac{C_{Sx_2}}{C_{S_0}}\right)^2=\left(\frac{C_{Sx_3}}{C_{S_0}}\right)^2=1+\frac{\varepsilon_0}{1-2\nu}\times \quad (38)$$

$$\left(\frac{V_{lx_1}}{V_{l_0}}\right)^2=1+\frac{\varepsilon_0}{1-\nu}\times \times\left\{3-\nu+\varepsilon_0\left[3+\nu+(1+\nu)\varepsilon_0\right]\right\}, \quad (39)$$

$$\left(\frac{V_{Sx_2}}{V_{S_0}}\right)^2=\left(\frac{V_{Sx_3}}{V_{S_0}}\right)^2=1+\frac{\varepsilon_0}{1-2\nu}\times \times\left\{2(2-\nu)+\varepsilon_0\left[5+2\nu+2(1+\nu)\varepsilon_0\right]\right\}. \quad (40)$$

The structure of equations for calculations of parameters of velocities significantly depends on the shape of elastic oscillations (Guz 2004). The changes of distances between the fixed points of the medium due to initial deformations are taken into account during their modeling in this manuscript.

4. Initial data for numerical experiments

The structures of Eqs. (31), (32) and (37)-(40) allow carrying out calculations for the entire interval of variation of the Poisson's ratio $0<\nu<0.5$ in case of using the quadratic elastic potential. It follows from the structure of Eqs. (19), (20) and (27)-(30) that it is also necessary to have information on the elasticity moduli of the second (λ, μ) and the third (a, b, c) orders to carry out concrete calculations in case of using an elastic potential of the Murnaghan type.

Table 1

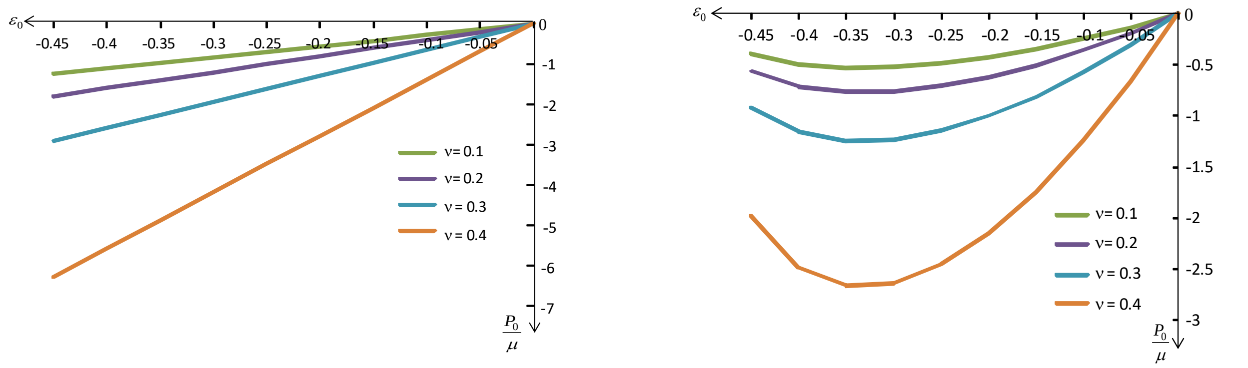
Medium parameters	$10^{-3}a$, MPa	$10^{-3}b$, MPa	$10^{-3}c$, MPa	$10^{-3}\lambda$, MPa	$10^{-3}\mu$, MPa	ν
Plexiglass	$\frac{-3{,}99}{0{,}268}$	$\frac{-7{,}16}{-3{,}12}$	$\frac{-14{,}4}{-6{,}77}$	4,04	1,9	0,3401
Steel	$\frac{-325}{-269}$	$\frac{-309}{-214}$	$\frac{-799}{-483}$	94,4	79,0	0,2722
Plagiogranite	$\frac{-3{,}87}{-}$	$\frac{-1{,}99}{-}$	$\frac{-6{,}24}{-}$	39,95	26,63	0,1999

The necessary data for a number of materials are given in Table 1. Data for plexiglas and steel 092c are taken from the study (Guz 1986), and data for plagiogranite are taken from the study (Prodaivoda *et al.* 2012). The data relating to the t.o.l.i.s. are given in the numerator and the data relating to the s.v.o.t.o.s.i.s are given in the denominator. The dashes in the column of plagiogranite indicate a lack of data for this case.

5. Numerical results and their discussions

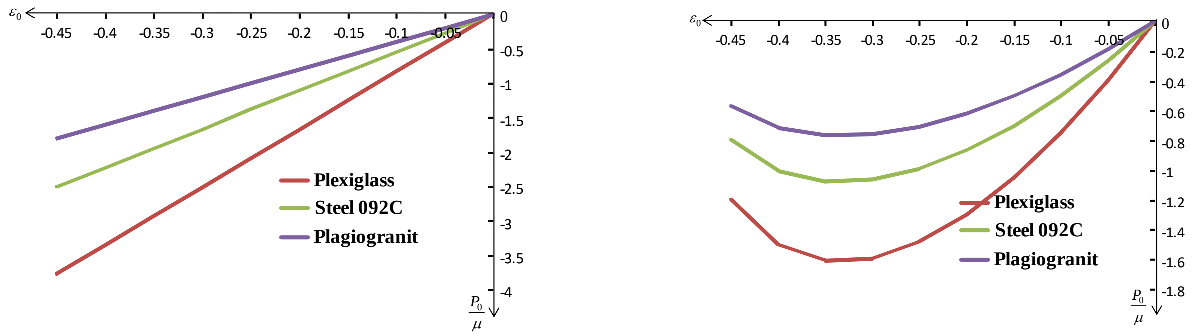
The graphs of the change of the parameter $\frac{P_0}{\mu}$ depending on the value of increase of compression strains are given in Fig. 2 while using the quadratic elastic potential. The results of Fig. 2(a) relate to the linear approximation (Eq. (32) within the s.v.t.s.i.s.). The results of Fig. 2(b) relate to the nonlinear solution within the t.o.l.i.s. and the f.v.o.t.o.s.i.s. (Eq. (31)). The results related to plexiglas, steel 092c and plagiogranite within the quadratic elastic potential are shown in Fig. 3.

The results of nonlinear solutions for plexiglas, steel 092C and plagiogranite are reflected in Fig. 4 within various variants of the theory of initial deformations in case of using elastic potential of the Murnagan type. The results of the linear approximation are shown in Fig. 3(a). It follows from the comparisons of graphs shown in Fig. 4(a) and 4(b) that the results of different variants of the theory of initial deformations differ essentially. These differences are numbered at times. Comparisons of Figs. 3 and 4 show that the results relating to different elastic potentials also differ among themselves essentially. For example, the value of the parameter $\frac{P_0}{\mu}$ for steel under deformation $\varepsilon_0=-0.3$ within the elastic potential of the Murnaghan is almost larger 6 times in comparison with the quadratic elastic potential, although such values unlikely have any physical significance. The graphs in Figs. 3-5 and in the following figures reflect the results of theoretical calculations within the mathematical modeling of the process of overall deformation of bodies under isotropic approximation. The physical interpretation of these results according to experimental studies should be carried out within $\frac{P}{\mu}<1$



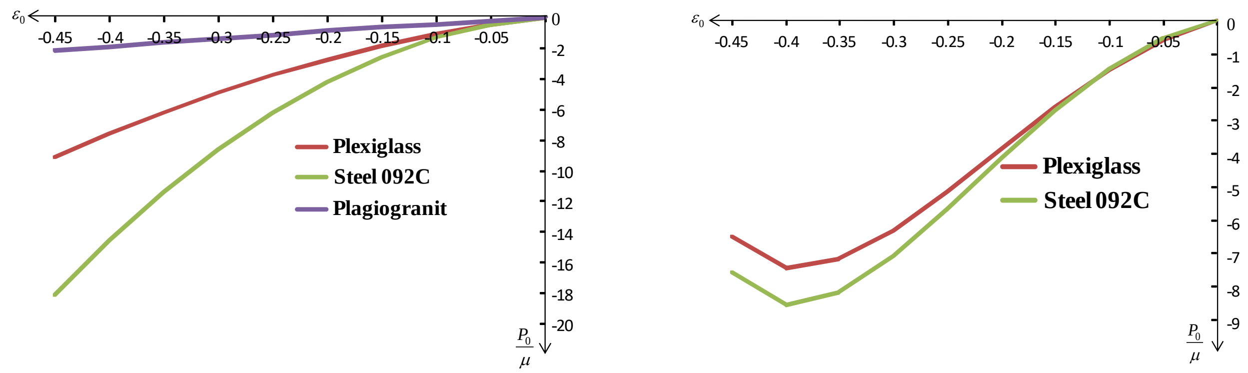
(a) (b)

Fig. 2 Parameter $\frac{P_0}{\mu}$ depending on compression deformation: (a) linear approximation (32), (b) nonlinear solution, Eq. (31)



(a) (b)

Fig. 3 Parameter $\frac{P_0}{\mu}$ depending on compression deformation: (a) linear approximation within the second variant of the theory of small initial strains (32), (b) nonlinear solution within the theory of large and the first variant of the theory of small initial strains (31). Violet color - plagiogranit, red - plexiglass, green - steel 092C



(a) (b)

Fig. 4 Parameter $\frac{P_0}{\mu}$ depending on compression deformation: (a) nonlinear solution within the second variant of the theory of small initial strains (20), (b) nonlinear solution within the theory of large and the first variant of the theory of small initial strains (19)

and ε_0<0.02 applying to standard structural materials. It is difficult to put forward similar restrictive conditions applied to the geological media of deep horizons of the Earth's interior. If there are any known conditions regarding strength, stability, etc. then they must be observed. Such studies are the subject of separate studies and they are not considered in this manuscript. The results on clarification the nature of quantitative and qualitative effects of linear, nonlinear, small and large elastic deformations on elastic parameters of the media under study are provided in this

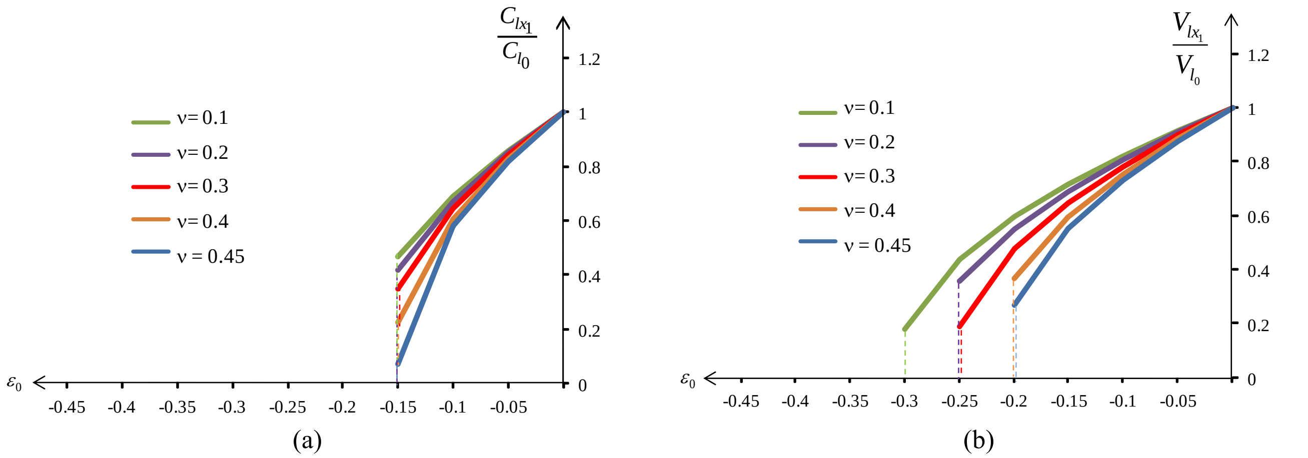


Fig. 5 Parameters $\frac{C_{lx_1}}{C_{l0}}$ (a) и $\frac{V_{lx_1}}{V_{l0}}$ (b) depending on compression deformation while using the quadratic elastic potential ((37) and (39) only considering linear constituents)

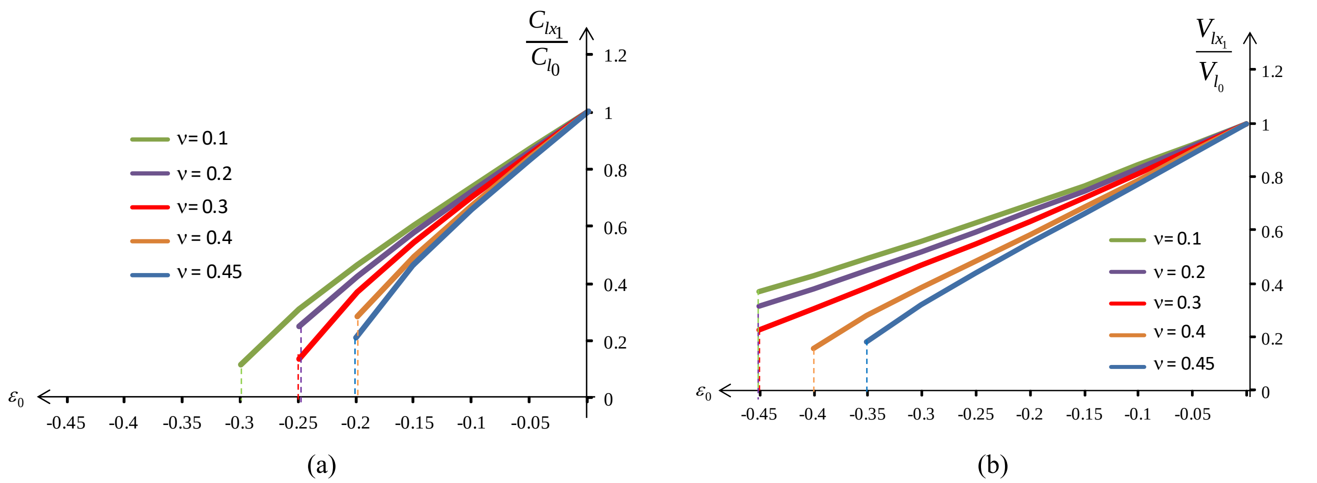


Fig. 6 Parameters $\frac{C_{lx_1}}{C_{l0}}$ (a) and $\frac{V_{lx_1}}{V_{l0}}$ (b) depending on compression deformation while using the quadratic elastic potential ((37) and (39) considering all constituents)

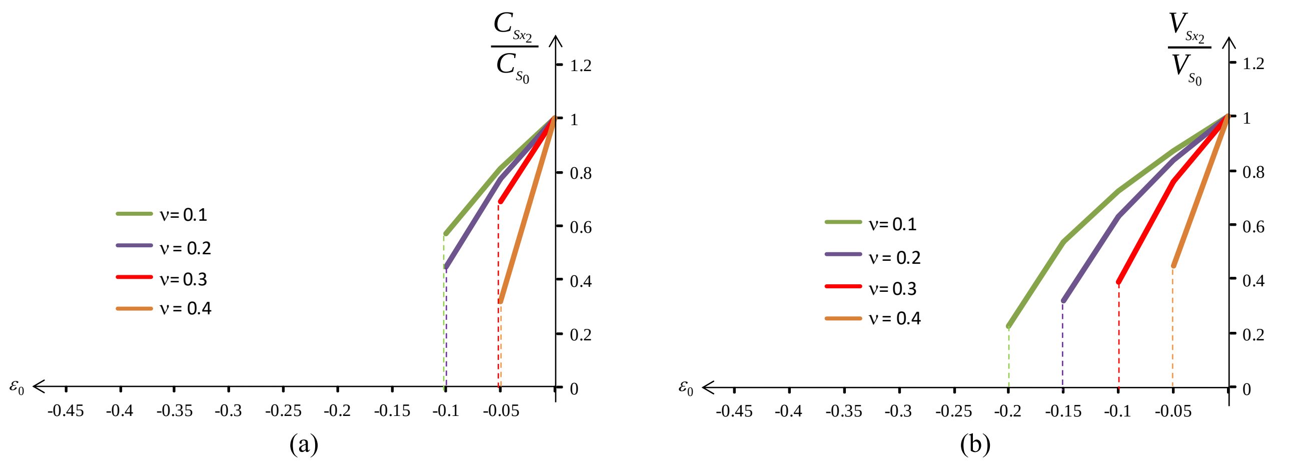


Fig. 7 Parameters $\frac{C_{Sx_2}}{C_{S0}}$ (a) and $\frac{V_{Sx_2}}{V_{S0}}$ (b) depending on compression deformation while using the quadratic elastic potential ((38) and (40) only considering linear constituents)

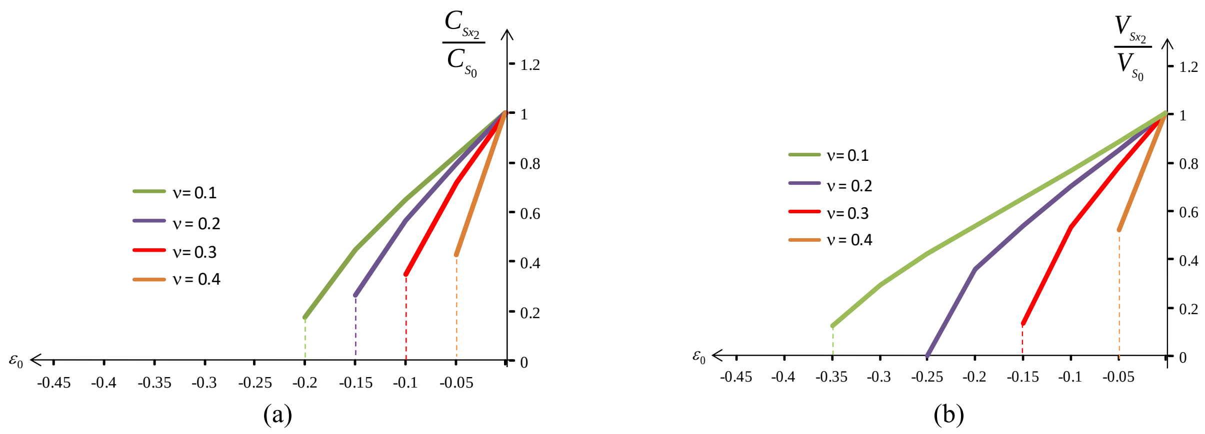


Fig. 8 Parameters $\dfrac{C_{Sx_2}}{C_{S0}}$ (a) and $\dfrac{V_{Sx_2}}{V_{S0}}$ (b) depending on compression deformation while using the quadratic elastic potential ((38) and (40) considering all constituents)

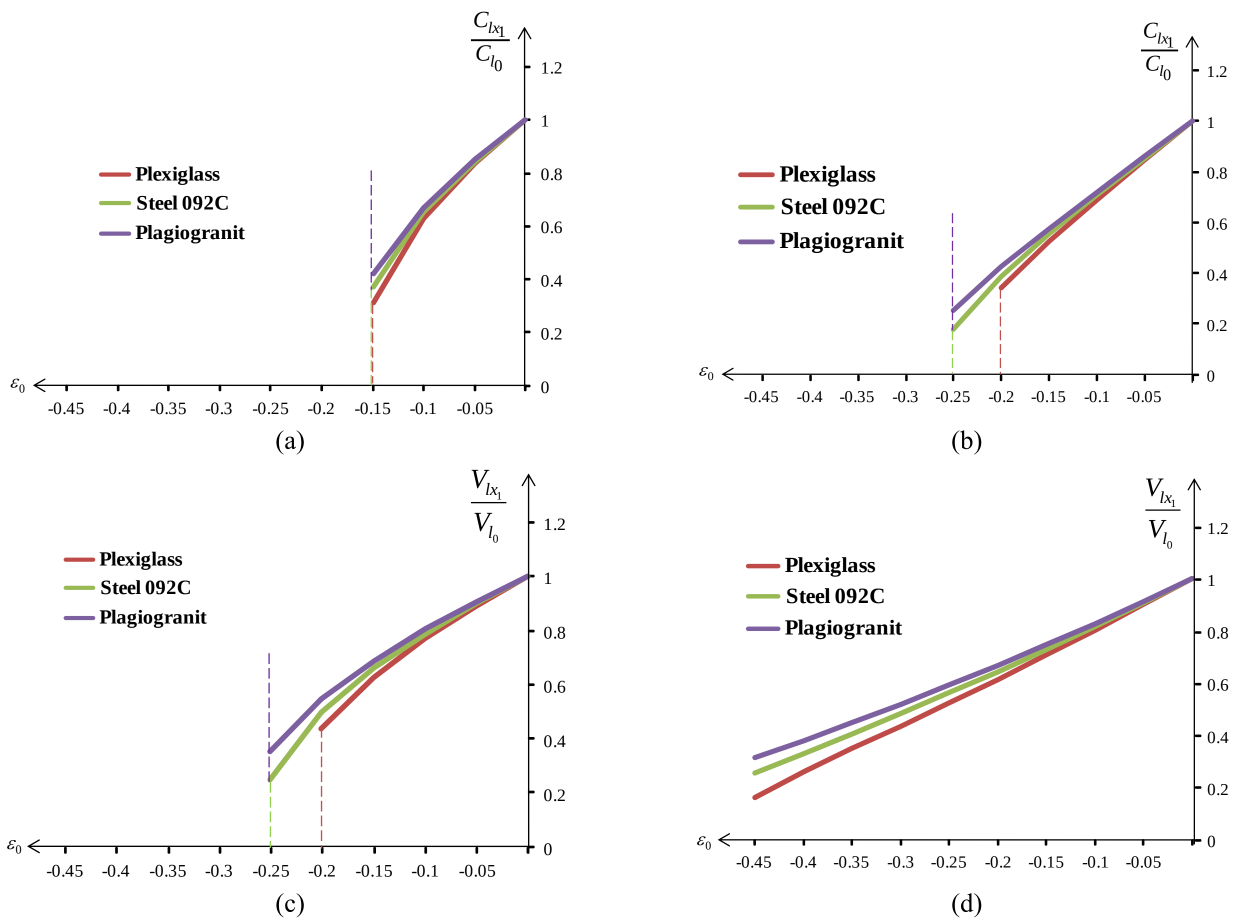


Fig. 9 Parameter of pressure elastic waves depending on compression deformation: (a) linear approximation, (37) considering linear constituents, (b) nonlinear solution within the theory of large and the first variant of small initial strains, (37) considering all constituents, (c) linear approximation, (39) considering linear constituents, (d) nonlinear solution within the second variant of the theory of small initial strains, (39) considering all constituents

manuscript. The graphs characterizing the dependences of the change of parameters of velocities on the increase of values of compression deformation are accordingly shown in Figs. 5 and 6 within the linear approximation and nonlinear solution. The calculations are carried out for different values of Poisson's ratio ν within the quadratic elastic potential. It is seen that elastic pressure waves with actual velocity cannot propagate in the medium while achieving particular values of compression deformation. The numerical values of the critical values of compression

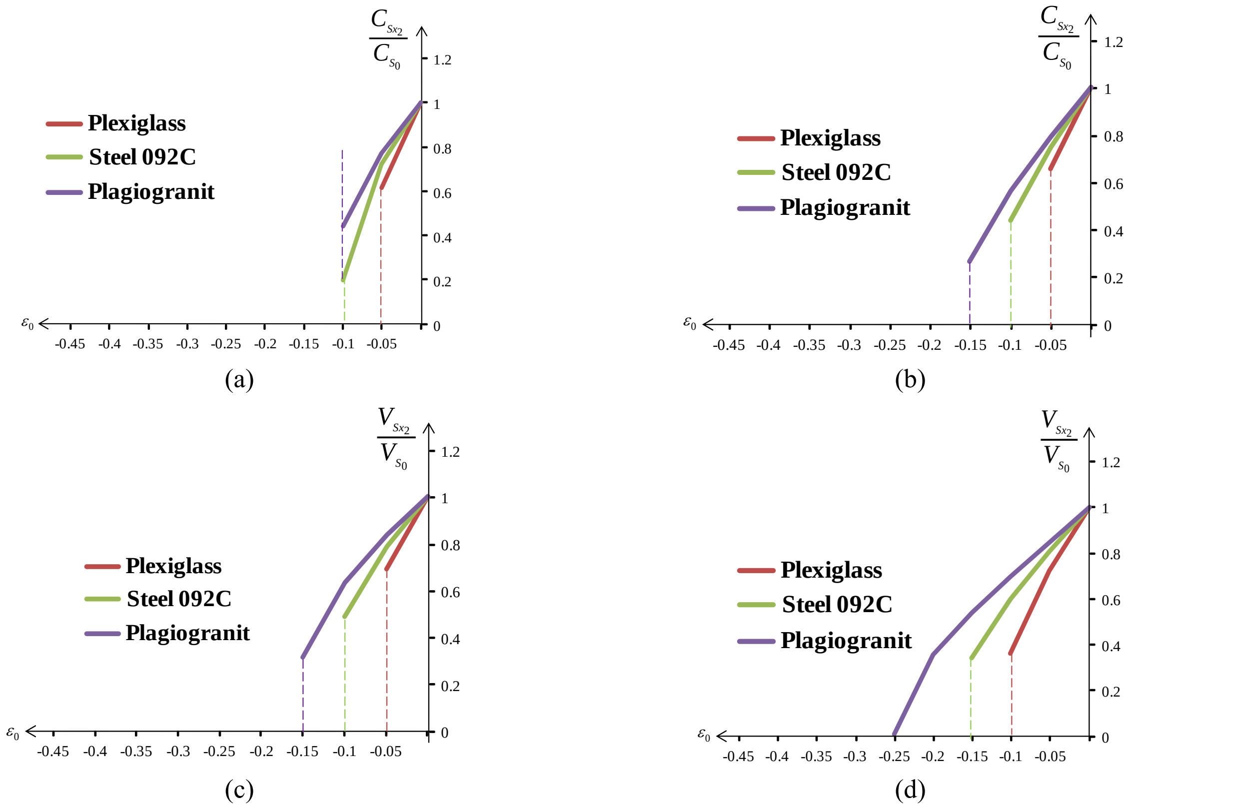


Fig. 10 Parameter of shear elastic waves depending on compression deformation: (a) linear approximation, (38) only considering linear constituents, (b) nonlinear solution within the theory of large and the first variant of small initial strains, (38) considering all constituents, (c) linear approximation, (40) only considering linear constituents, (d) nonlinear solution within the second variant of the theory of small initial strains, (40) considering all constituents

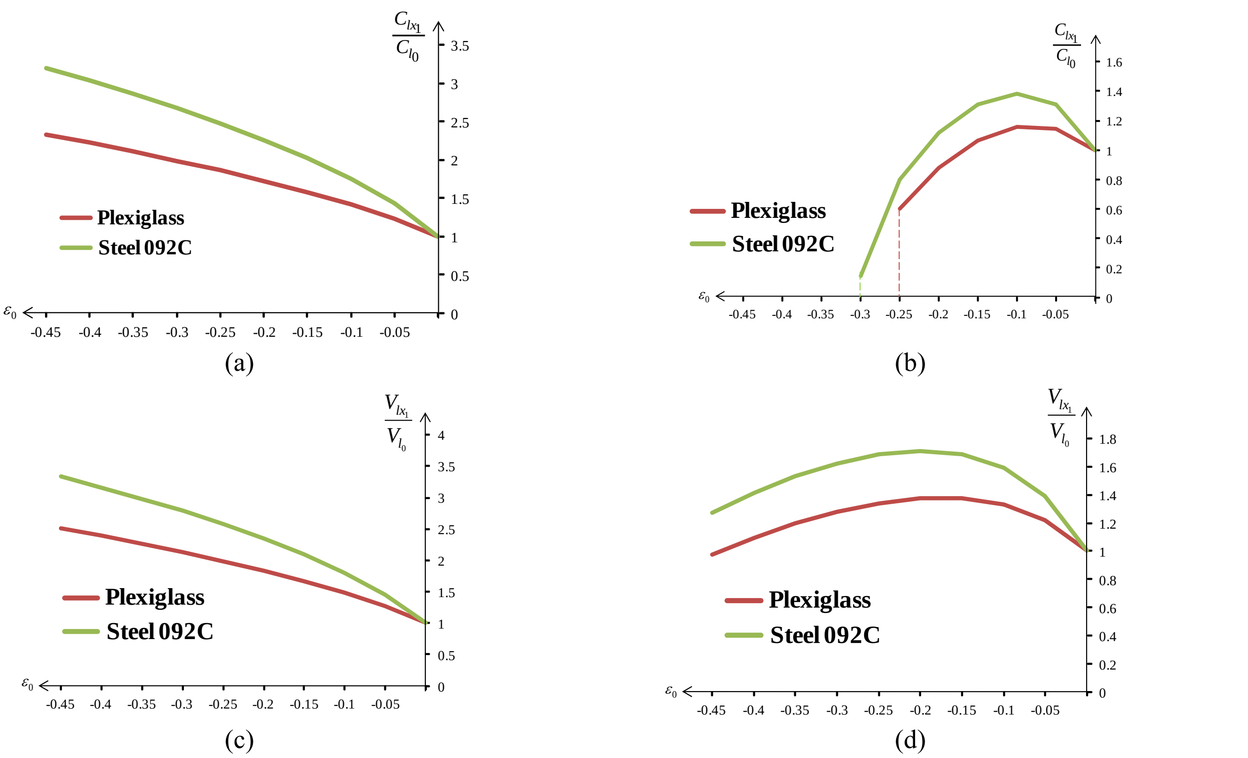


Fig. 11 Parameter of pressure elastic waves depending on compression deformation: (a) linear approximation, (27) only considering linear constituents, (b) nonlinear solution within the theory of large and the first variant of small initial strains, (27) considering all constituents, (c) linear approximation, (29) only considering linear constituents, (d) nonlinear solution within the second variant of the theory of small initial strains, (29) considering all constituents

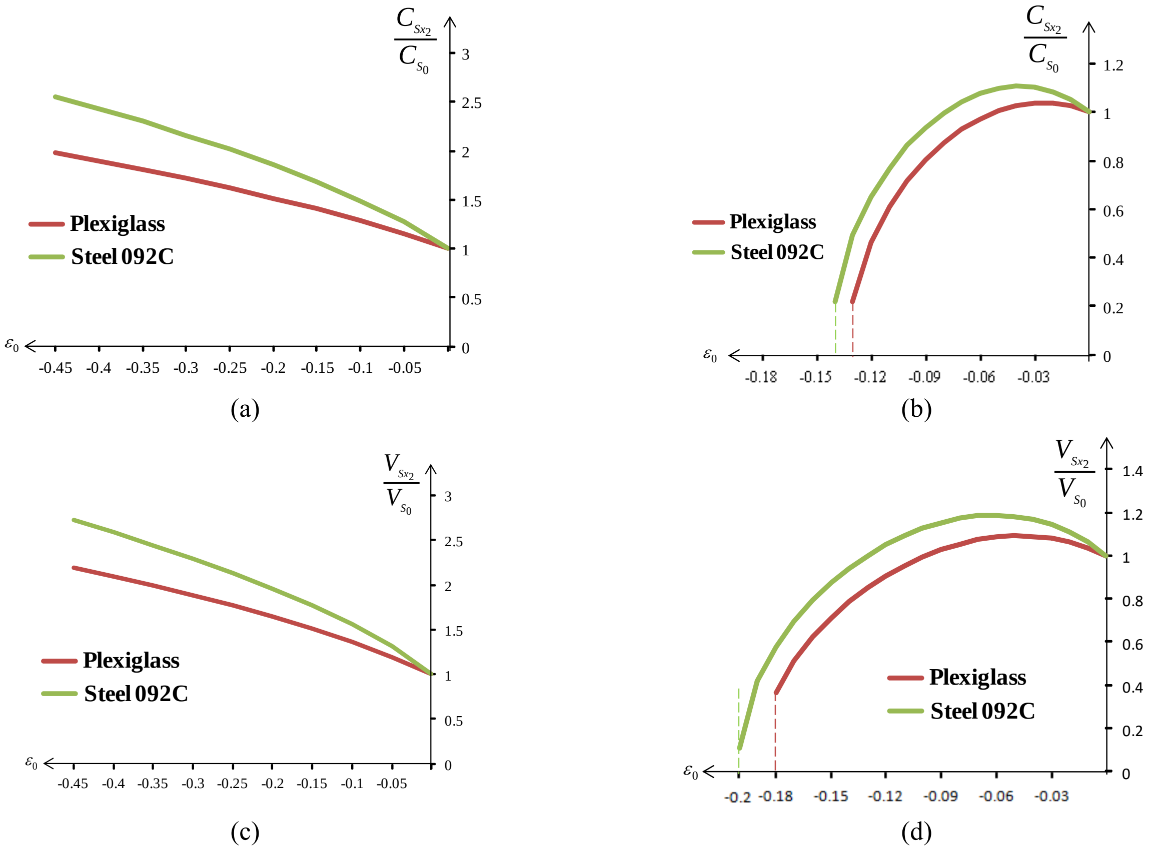


Fig. 12 Parameter of shear elastic waves depending on compression deformation: (a) linear approximation, (28) only considering linear constituents, (b) nonlinear solution within the theory of large and the first variant of small initial strains, (28) considering all constituents, (c) linear approximation, (30) only considering linear constituents, (d) nonlinear solution within the second variant of the theory of small initial strains, (30) considering all constituents

deformations in linear and nonlinear solutions of t.o.l.i.s. and s.v.o.t.o.s.i.s. corresponding to this phenomenon differ among themselves. Nevertheless, there are such critical values of compression deformations for all values of Poisson's ratio in all variants of the theory. Similar results are also obtained for shear waves (Figs. 7 and 8). For ν=0.45 the parameters of velocities take the critical value at $\varepsilon_0 \approx -0.03$ in the considered cases. Therefore, the results are not reflected in the figures in the accepted scales relative to deformations.

Graphs of the change of the parameters of pressure and shear elastic waves in plexiglass (red), steels 092C (green) and plagiogranite (violet) depending on the increase of compression deformations are shown in Figs. 9-10. The calculations are carried out using the quadratic elastic potential. Similar results for these materials calculated using the elastic potential of the Murnaghan type are shown in Figs. 11-12. Comparisons of the results of Figs. 8-9 with Figs. 11-12 show that the results which are obtained through involving various elastic potentials differ qualitatively and quantitatively. In this case, it is necessary to give preference to results corresponding to the potential of the Murnaghan type as it is known that (Guz 1986) this potential correctly describes the acoustoelastic effect. In this example, behaviors of the change of parameters of pressure and shear waves fundamentally differ due to the increase of the value of compression deformation within the linear and nonlinear approximations.

It follows from the results of Figs. 5-10 that in case of the quadratic elastic potentials, the values of parameters of velocities of pressure and shear waves decrease both at small (considering both versions) and large initial deformations due to the increase of compression deformation. Such results don't coincide with available (Guz 1986) experimental data. Similar results obtained within the elastic potential of the Murnaghan type for a number of materials show a fundamentally different nature of the change. The parameters of velocities of pressure and shear waves up to the determined value of the compression deformations increase within all variants of the theory of initial deformations. Further, the parameters of velocities of both types of waves continue to increase due to the increase of values of compression deformations within the linear approximations but they decrease within the nonlinear solutions. There are critical values of compression deformation for both types of waves within the nonlinear solutions. Elastic volume waves with actual velocity cannot propagate in the medium while achieving them. Despite the quantitative difference in the increase of parameters of velocities in the initial stage of increase of values of compression deformation, the critical values of compression deformations are revealed within the considered variants of the theory and at quadratic potential in the further stages of deformations (at large deformations).

Based on the obtained theoretical equations and numerical results, the inability to propagate elastic waves with real velocity in the deformed media can be explained as follows. Dimensions of stresses and the elasticity moduli are the same. The quantitative values of velocities of propagation of elastic waves are determined in the form of certain ratios of the elasticity moduli to the density of the medium. In case of the deformed media, ratios of values of components of stress tensor to the density cause nonlinear actions of the medium by the velocity dimension. The levels of normal components of stress tensor reach values comparable to values of the elasticity moduli of medium of propagation of elastic waves under high and superhigh baric conditions. Therefore, depending on the nature of an influence and stress level, the parameters of velocity of elastic waves can increase, decrease and turn to zero during the deformation process.

6. Conclusions

The dependences of pressure change and parameters characterizing the velocities of pressure and shear elastic waves on the increase of compression deformation in elastic isotropic media are analyzed within the linear and nonlinear approximations of the NLA involving various elastic potentials. It is shown that the results of linear and nonlinear solutions obtained within various elastic potentials differ quantitatively and qualitatively between themselves. The consideration of large and nonlinear deformations of compression allowed determining their critical values while achieving of which elastic pressure and shear waves with real velocities cannot propagate in elastic isotropic media. These results have important theoretical and applied values in the development of structural and substantial models of the Earth.

Acknowledgments

The research described in this paper was financially supported by the scientific-research program of the National Academy of Sciences of Azerbaijan (NASA) on a subject "Complex of theoretical and experimental studies of interdisciplinary problems of geomechanics" approved by the decision of the Presidium of the NAS of Azerbaijan № 5/3 dated on February 11, 2015 (2015-2017).

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