Structural Engineering and Mechanics, *Vol. 66, No. 4 (2018) 495-504*
DOI: https://doi.org/10.12989/sem.2018.66.4.495

Propagation of elastic waves in thermally affected embedded carbon-nanotube-reinforced composite beams via various shear deformation plate theories

Farzad Ebrahimi* and Pooya Rostami

Department of Mechanical Engineering, Faculty of Engineering, Imam Khomeini International University, Qazvin, Iran

(Received February 1, 2018, Revised February 15, 2018, Accepted February 19, 2018)

Abstract. The current study is dedicated to study the thermal effects of wave propagation in beams, reinforced by carbon nanotubes (CNT). Beams, made up of carbon nanotube reinforced composite (CNTRC) are the future materials in various high tech industries. Herein a Winkler elastic foundation is assumed in order to make the model more realistic. Mostly, CNTs are pervaded in cross section of beam, in various models. So, it is tried to use four of the most profitable reconstructions. The homogenization of elastic and thermal properties such as density, Yong's module, Poisson's ratio and shear module of CNTRC beam, had been done by the demotic rule of mixture to homogenize, which gives appropriate traits in such settlements. To make this investigation, a perfect one, various shear deformation theories had been utilized to show the applicability of this theories, in contrast to their theoretical face. The reigning equation had been derived by extended Hamilton principle and the culminant equation solved analytically by scattering relations for propagation of wave in solid bodies. Results had been verified by preceding studies. It is anticipated that current results can be applicable in future studies.

Keywords: wave propagation; graphene; thermal effects; CNTRC; Winkler elastic foundation; rule of mixture; beam; CNT

1. Introduction

Recently great attempts done due to achieve new composite materials with novel qualities in aerospace and high tech industries. One of the most recent materials are the composites, reinforced by CNTs. The revolutionary properties of CNT, persuaded scientists and engineers to use them in various materials and components and many different distributions. One of these materials are the combination of high strength CNTs with various polymer matrixes. Such a material may have great application, not only in aerospace but also in automotive engineering for building the structure of supercars. One of the most favorable applications is the biomechanical and medical applications, in which high strength material are more favorable. In one hand, in all of these applications, wave propagation is the key features in the material properties. On the other hand, the composite sheets or beams may be situate in high temperature thermal environments. So, there is a lack of knowledge in study of wave propagation in CNTRC materials. By means of continuum mechanics, Sofiev and Alizada (2010), explained that the modified Young's moduli is acquired by considering the vacancies. And also, the triple components of this module are introduced as macroscopic value and factors of vacancy and scale effects. Besides, Alizada (2011) studied the beams with coatings of nano scale and their stability. Recent investigations are mostly focused on vibration of CNTRC structures. Yan *et al.* (2011) investigated the behaviour of a functionally graded (FG) beam which is edge cracked. They used rotational spring model to model the beam and crack and used Hamilton principle to gain the governing equation. Discrete singular convolution method had been used to solve vibration of skew laminated plates by Civalek (2009). By means of mapping, straight sided domains had been converted into circular field. The effect of skew angle had been shown in this study. Simsek (2010) studied the FG beams and their fundamental frequency by means of various high order theories of beams. Yas and Samadi (2012) studied CNTRC and made a complete investigation in vibration and buckling in beam structures. They used Timoshenko beam to model their CNTRC beams. Besides they used the regulation of mixture to homogenize the properties of CNTRC beams and utilized various distributions of CNTs in their matrix. Ebrahimi and Habibi (2017) used Halpin-Tsai model to homogenize the elastic properties of CNTRC plate imposed to low velocity impact. Both buckling influences and vibrational characteristic analysis of macro beams made up of FG-CNTRC had been done by Nejati and Eslampanah (2016), under axial load. They modeled a cantilever CNTRC beam under axial load and utilized Eshelby type, Mori-Tanaka method to find the impressive elastic modulus. The assumption of Euler Bernoulli beam with surrounded circular Winkler foundation had been used by Civalek (2016) due to model the buckling of micro tubes. Shen *et al.* (2017) examined the vibration influences in a post buckled CNTRC beam on thermal environment. They used Von Kármán nonlinear displacement relations of a CNTRC beam and extended rule of mixture to homogenize the elastic properties. Lin and

*Corresponding author, Associate Professor
E-mail: febrahimy@eng.ikiu.ac.ir

Copyright © 2018 Techno-Press, Ltd.

ISSN: 1225-4568 (Print), 1598-6217 (Online)

Xiang (2014) pondered the vibration effects in CNTRC beams by means of both first and third order shear plate theories. Besides the influence of volume fraction of CNT and various distributions had been tested. The nonlinear equation of large deformations of composite laminated plates had been investigated by Civalek (2010). It had been found that the regular Shannon kernel gives better results in such problems. Alibeigloo (2015) pondered FG-CNTRC beams under bending by means of a thin piezoelectric layer. To analyse, they used semi analytical method and differential quadrature method. Bending behaviour of single walled carbon nanotubes (SWCT) with higher order shear deformation theories had been done by Akgöz (2015). They had shown the dependency of bending characteristics to elastic foundation and also the accomplishment of shear deformation in small ratios of slenderness. Ghorbanpour Arani *et al.* (2016), used nonlocal piezoelectricity theory based on Eringen (1972) nonlocal theories to study wave propagation in FG-CNTRC micro plates. They used sinusoidal shear deformation theory and rule of mixture to homogenize the elastic properties. Another research about the beams with nano-coatings is Sofiev *et al.* (2012) in which the uniform expansion loads and the multilayer beam consists of elastic and nano material filled parts. Ebrahimi and Barati (2016) studied the both uniform and nonlinear temperature rise in vibration of nanobeams made of magneto-electro-elastic materials. Alibeigloo (2013) made a static analyse in FG-CNTRC beams with two layers of piezoelectric actuators. The dependency of flexoelasticity on nonlocality had been shown by Ebrahimi (2017) in investigation of buckling properties of magneto-electro-elastic made functionally graded nanobeams. Ghorbanpour Arani *et al.* (2017) used nonlocal piezoelectricity theory to investigate the CNTRC micro plates with magnetic field. They used both damping and spring foundations in their model. Energy method utilized in this investigation to achieve to governing equation. The assumption of Timoshenko beam for vibration of composite laminated composite beams had been done by Chen (2012). It had been shown that the length scale effects can be highlight better by means of Timoshenko beam theory. Wu *et al.* (2015) investigated both vibration of FG-CNTRC in which reinforcement had been done by CNT reinforced composite face sheets. Differential transform method combined with Eringen nonlocal elasticity theory, had been used in order to investigate the thermal effects of vibration in functionally graded nanobeams by Ebrahimi (2015). The structure contains a uniform homogeneous part embedded by CNTRC face sheets. And also, solution procedure done by differential quadrature method. Recently, Wattanasakulpong and Ungbhakorn (2013) made a complete research in bending and free vibration of FG-CNTRC beams by means of analytical solution. In one hand, in a new investigation by Ebrahimi and Salari (2015), both linear and nonlinear temperature effects are investigated. On the other hand, Ebrahimi and Barati (2015) investigated the effect of non-locality with higher order shear deformation beam theory to investigate the thermo vibrational analysis of functionally graded nano beams. Both static and dynamic examination of cylindrical panels, made up of FG-CNTRC had been done by Zhang *et al.* (2014). Eshelby type, Mori-Tanaka method had been used to find the effective elastic attributes of composite substances. The effectiveness of elastic properties of CNTRC plates by Mori-Tanaka theory had been derived by Sobhani Aragh *et al.* (2012) and used third order plate theory to reach governing equations. The rotation of nanobeams made of functionally graded materials had been investigated by Ebrahimi *et al.* (2016), in which nanobeams are subjected to thermal effects. Fantuzzi *et al.* (2017) concentrated on micro mechanical attributes of CNTs. Besides, first order shear deformation theory had been utilized to study the vibration of FG-CNTRC reinforced by sighted plates. They used mapping technique and domain decomposition method. Solution procedure governing equation had been solved using differential quadrature method. Janghorban and Nami (2017) studied wave propagation in FG-CNTRC plates. The effect of Silica aerogel foundation on bending analyse of FG porous sandwich nanoplates had been studied by Ghorbanpour Arani and Zamani (2017). Analysis of nano-structure's mechanical behaviors is one of recent interesting research topics. (Ebrahimi and Barati 2016f, g, h, i, j, k, l, m, n, Ebrahimi and Barati 2017). For instance, thermal buckling and free vibration analysis of FG nanobeams subjected to temperature distribution have been exactly investigated by Ebrahimi and Salari (2015a, b, c) and Ebrahimi *et al.* (2015 a, b). Ebrahimi and Barati (2016o, p, q) investigated buckling behavior of smart piezoelectrically actuated higher-order size-dependent graded nanoscale beams and plates in thermal environment.

More recently, Shen and Xiang (2013) made a nonlinear analyse in CNTRC beams in thermal environments. It should be mentioned that in this paper temperature dependent properties of SWCNT had been used.

As it is clear from previous researches, there is no study in wave propagation of CNTRC beam in thermal environments. Herein it is tried to make a complex investigation in wave propagation in thermal effects. Besides various shear deformation theories and different distributions of CNT in polymer matrix had been studied. Then the governing equation had been solved by means of extended Hamilton principle and solved analytically. And also, temperature dependent properties of CNT had been used to reach to realistic results.

2. Theory and formulation

2.1 Thermo mechanical attributes of CNTRC beams

Herein a combination of CNTRC beam, SWCTs and a polymer matrix is assumed which is subjected to heat flames or other thermal sources as exhibited in Fig. 1. Dimensions of CNTRC beam are as follows: (L) is introduced as the length of beam and thickness is expressed by (h). Herein, Beam is situated in shear layer and springs are the schematics of Winkler substructure. It is necessary to homogenize the elastic attributes in order to reach to equals. According to rule of mixture (Arani *et al.* 2017)

$$E_{11} = \eta_1 V_{CNT} E_{11}^{CNT} + V_m E^m \tag{1}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{CNT}}{E_{22}^{CNT}} + \frac{V_m}{E^m} \tag{2}$$

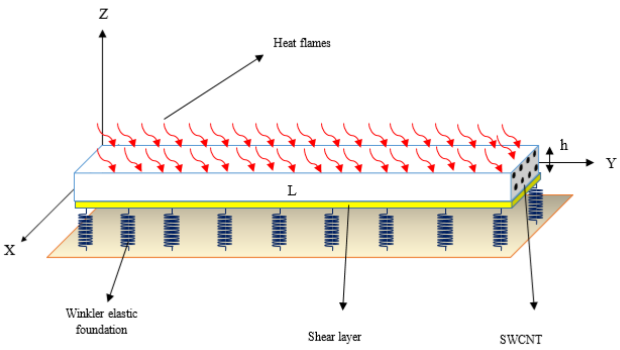


Fig. 1 Schematic effect of thermal effects in composite beam resting on Winkler substructure with a shear layer

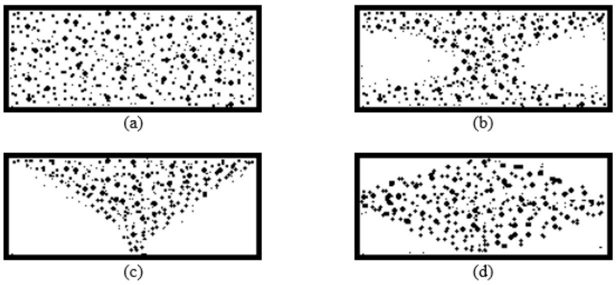


Fig. 2 Various distributions of CNT in matrix. (a)UD-Beam (b) X-Beam (c) V-Beam (d) O-Beam

Table 1 Various geometrical properties of CNTRC beams

Geometry of CNTRC beam	Volume fraction
Uniform distribution	V^{*}_{CNT}
V distribution	$(1+\frac{2z}{h})V^{*}_{CNT}$
X distribution	$4\frac{\lvert z\rvert}{h}V^{*}_{CNT}$
O distribution	$2(1-2\frac{\lvert z\rvert}{h})V^{*}_{CNT}$

$$\frac{\eta_3}{G_{22}}=\frac{V_{CNT}}{G_{12}^{CNT}}+\frac{V_m}{G^m} \tag{3}$$

$$\nu=V_{CNT}\nu^{CNT}+V_m\nu^m \tag{4}$$

$$\rho=V_{CNT}\rho^{CNT}+V_m\rho^m \tag{5}$$

$$V_{CNT}+V_m=1 \tag{6}$$

$$V^{*}_{CNT}=\frac{W_{CNT}}{W_{CNT}+(\rho^{CNT}/\rho^m)(1-W_{CNT})} \tag{7}$$

Where E_{11}, E_{22}, G_{22}, ρ, ν are the resultant properties of homogenized composite. Besides, η_1, η_2, η_3 are the manufacturing efficiency of CNTRC beam which shows the quality of mixture of CNT and polymeric matrix. Also, V_{CNT}, V_m are the volume fractions of carbon nanotube and matrix. $E_{11}^{CNT}, E_{22}^{CNT}, G_{12}^{CNT}$ are the Young's modulus and shear modulus of carbon nanotubes and E^m, G^m are the properties of matrix. In addition, ν^{CNT}, ρ^{CNT} are the Poisson's ratio and density of carbon nanotube. In contrast, ρ^m, ν^m are the attributes of polymer matrix. Eq. (7), explains the mass fraction of CNT. Fig. 2 shows various distributions of CNT in polymer matrix. Volume fractions of various CNTRC beams are available in Table 1.

2.2 Kinematic relations

Displacement relations of CNTRC beam can be illustrated by various shear deformation theories (Şimşek 2017)

$$u_x=u-z\frac{\partial w_b}{\partial x}-f\frac{\partial w_s}{\partial x} \tag{8}$$

$$u_z=w_b+w_s \tag{9}$$

in which (u) is the term of movements, and (w) is transverse displacement and w_b, w_s are the terms of bending and shear displacements of the beam. $f(z)$ explains various functions of shear deformation through the thickness. Various shear deformation theories employed in this paper are available in Table 2. Shear and simple strain narrations of the CNTRC beam can be elucidated by Hamilton precept as shown in Eqs. (10)-(13)

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}-z\frac{\partial^2 w_b}{\partial x^2}-f(z)\frac{\partial^2 w_s}{\partial x^2} \tag{10}$$

$$\gamma_{xz}=g\frac{\partial w_s}{\partial x} \tag{11}$$

$$\int_0^t \delta(U+V-K)dt=0 \tag{12}$$

$$\delta U=\int_v \sigma_{ij}\delta\varepsilon_{ij}\,dV=\int_v(\sigma_{xx}\delta\varepsilon_{xx}+\sigma_{xz}\delta\gamma_{xz})dV \tag{13}$$

Inserting Eqs. (10) and (11) into Eq. (13) yields

$$\delta U=\int_0^L\left(N\frac{d\delta u}{dx}-M_b\frac{d^2\delta w_b}{dx^2}-M_s\frac{d^2\delta w_s}{dx^2}+Q\frac{d\delta w_s}{dx}\right)dx \tag{14}$$

Where U is the strain term of energy, V illustrates the work done by exterior forces and K interoperates the kinetic energy. The strain term is as follows

$$\begin{aligned} N&=\int_A \sigma_{xx}\,dA\\ M_b&=\int_A \sigma_{xx}z\,dA\\ M_s&=\int_A f\,\sigma_{xx}\,dA\\ Q&=\int_A g\,\sigma_{xz}\,dA \end{aligned} \tag{15}$$

Differential of the work done by bending, shear and thermal effects can be explained in the following

Table 2 Various shear strain shape functions

Shear deformation theory	Reference	Abbreviation	Formula
Third order	Reddy (2000)	TSDT	$z\,(1-\frac{4z^3}{3h^2})$
Sinusoidal	Touratier (1991)	SSDT	$\frac{h}{\pi}\sin(\frac{\pi z}{h})$
Hyperbolic	Soldatos (1991)	HSDT	$h\sinh(\frac{z}{h})-z\cosh\frac{1}{2}$
First order	Reissner (1985)	FSDT	z

constitution

$$\delta V=\int_0^L (N^T(\frac{d(w_s+w_b)}{dx}\frac{d\delta(w_s+w_b)}{dx}))dx \tag{16}$$

where, N^T is applied thermal force since temperature changes

$$N^T=\int_{-h/2}^{h/2} E(z,T)\alpha(z,T)(T-T_0)dz \tag{17}$$

In which T_0 is the reference temperature which is mostly the room temperature. Following equation explain the variety of kinetic energy

$$\begin{aligned}\delta K=\int_0^L &(I_0[\frac{du}{dt}\frac{d\delta u}{dt}+(\frac{dw_b}{dt}+\frac{dw_s}{dt})(\frac{d\delta w_b}{dt}+\frac{d\delta w_s}{dt})]-\\ &I_1(\frac{du}{dt}\frac{d^2\delta w_b}{dxdt}+\frac{d^2w_b}{dxdt}\frac{d\delta u}{dt})+I_2(\frac{d^2w_b}{dxdt}\frac{d^2\delta w_b}{dxdt})\\ &-J_1(\frac{du}{dt}\frac{d^2\delta w_s}{dxdt}+\frac{d^2w_s}{dxdt}\frac{d\delta u}{dt})+K_2(\frac{d^2w_s}{dxdt}\frac{d^2\delta w_s}{dxdt})+\\ &J_2(\frac{d^2w_b}{dxdt}\frac{d^2\delta w_s}{dxdt}+\frac{d^2w_s}{dxdt}\frac{d^2\delta w_b}{dxdt}))dx\end{aligned} \tag{18}$$

where

$$(K_2,J_1,J_2,I_0,I_1,I_2)=\int \rho(z)(f^2,f,zf,1,z,z^2)dA \tag{19}$$

Next equations are available by inserting Eqs. (13)-(19) in Eq. (12) when δ_u, δw_b and δw_s corresponds to zero

$$\frac{\partial N}{\partial x}=I_0\frac{d^2u}{dt^2}-I_1\frac{d^3w_b}{dxdt^2}-J_1\frac{d^3w_s}{dxdt^2} \tag{20}$$

$$\begin{aligned}\frac{d^2M_b}{dx^2}=N^T\frac{d^2(w_b+w_s)}{dx^2}+I_0(\frac{d^2w_b}{dt^2}+\frac{d^2w_s}{dt^2})+\\ I_1\frac{d^3u}{dxdt^2}-I_2\frac{d^4w_b}{dx^2dt^2}-J_2\frac{d^4w_s}{dx^2dt^2}\end{aligned} \tag{21}$$

$$\begin{aligned}\frac{d^2M_s}{dx^2}+\frac{dQ}{dx}=N^T\frac{d^2(w_b+w_s)}{dx^2}+I_0(\frac{d^2w_b}{dt^2}+\\ \frac{d^2w_s}{dt^2})+J_1\frac{d^3u}{dxdt^2}-J_2\frac{d^4w_b}{dx^2dt^2}-K_2\frac{d^4w_s}{dx^2dt^2}\end{aligned} \tag{22}$$

The simple stress-strain relations to analyze the CNTRC macro beam are as follows

$$\sigma_{xx}=Q_{11}(z)\varepsilon_{xx} \tag{23}$$

$$\sigma_{xz}=Q_{55}(z)\gamma_{xz} \tag{24}$$

$$Q_{11}(z)=\frac{E_{11}(z)}{1-\nu^2} \tag{25}$$

$$Q_{55}(z)=G_{12}(z) \tag{26}$$

By inserting Eqs. (23)-(26) into (20)-(22)

$$N=A\frac{\partial u}{\partial x}-B\frac{\partial^2 w_b}{\partial x^2}-B_s\frac{\partial^2 w_s}{\partial x^2}-N_x^T \tag{27}$$

$$M_b=B\frac{\partial u}{\partial x}-D\frac{\partial^2 w_b}{\partial x^2}-D_s\frac{\partial^2 w_s}{\partial x^2}-M_b^T \tag{28}$$

$$M_s=B_s\frac{\partial u}{\partial x}-D_s\frac{\partial^2 w_b}{\partial x^2}-H_s\frac{\partial^2 w_s}{\partial x^2}-M_s^T \tag{29}$$

$$Q=A_s\frac{\partial w_s}{\partial x} \tag{30}$$

where cross sectional properties are

$$\begin{aligned}&(A,B,B_s,D,D_s,H_s)=\\ &\int_A E(z)(1,z,f,z^2,zf,f^2)dA\end{aligned} \tag{31}$$

$$A_s=\int_A g^2G(z)dA \tag{32}$$

The governing equations derived by inserting Eqs. (31)-(32) into Eqs. (27)-(30) can be simplified as follows

$$A\frac{\partial^2 u}{\partial x^2}-B\frac{\partial^3 w_b}{\partial x^3}-B_s\frac{\partial^3 w_s}{\partial x^3}-I_0\frac{\partial^2 u}{\partial t^2}+I_1\frac{\partial^3 w_b}{\partial x\partial t^2}+J_1\frac{\partial^3 w_s}{\partial x\partial t^2}=0 \tag{33}$$

$$\begin{aligned}&B\frac{\partial^3 u}{\partial x^3}-D\frac{\partial^4 w_b}{\partial x^4}-D_s(\frac{\partial^4 w_s}{\partial x^4})-(N^T)\frac{\partial^2(w_b+w_s)}{\partial x^2}-I_0(\frac{\partial^2 w_b}{\partial t^2}+\frac{\partial^2 w_s}{\partial t^2})-I_1\frac{\partial^3 u}{\partial t^2\partial x}\\ &+I_2\frac{\partial^4 w_b}{\partial t^2\partial x^2}+J_2\frac{\partial^4 w_s}{\partial x^2\partial t^2}=0\end{aligned} \tag{34}$$

$$\begin{aligned}&B_s\frac{\partial^3 u}{\partial x^3}-D_s\frac{\partial^4 w_b}{\partial x^4}-H_s\frac{\partial^4 w_s}{\partial x^4}+A_s\frac{\partial^2 w_s}{\partial x^2}-(N^T)\frac{\partial^2(w_b+w_s)}{\partial x^2}-I_0(\frac{\partial^2 w_b}{\partial t^2}+\frac{\partial^2 w_s}{\partial t^2})\\ &-J_1\frac{\partial^3 u}{\partial t^2\partial x}+J_2\frac{\partial^4 w_b}{\partial t^2\partial x^2}+K_2\frac{\partial^4 w_s}{\partial x^2\partial t^2}=0\end{aligned} \tag{35}$$

3. Solution procedure

Desperation relations of wave propagation in solid bodies can be used to solve the governing equation

$$u(x,t)=U_n\exp[i(\beta_1x+\beta_2y-\omega t)] \tag{36}$$

$$w_b(x,t)=W_{bn}\exp[i(\beta_1x+\beta_2y-\omega t)] \tag{37}$$

$$w_s(x,t)=W_{sn}\exp[i(\beta_1x+\beta_2y-\omega t)] \tag{38}$$

where β_1, β_2 are equal. In addition, U_n, U_{bn}, W_{sn} are the unknown parameter. By inserting Eqs. (36)-(38) into Eqs. (33)-(35) gives

$$\left\{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} - \omega^2 \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}\right\} \begin{Bmatrix} U_n \\ W_{bn} \\ W_{sn} \end{Bmatrix} = 0 \quad (39)$$

where

$$a_{11} = -A\beta^2, a_{12} = B_s\beta^2, a_{13} = -B_s\beta^2 i + B\beta^2 i$$
$$a_{21} = -B_s\beta^2, a_{22} = H_s\beta^2 + A_{55} + N^T\ \beta^2,$$
$$a_{23} = -H_s\beta^2 i - A_s\beta i + D_s\beta^2 i + N^T\ \beta^2$$
$$a_{31} = B\beta^3 i + B_s\beta^3 i, a_{32} = -E_{11}\beta^3 i + H_s\beta^3 i$$
$$+A_s k i + N^T\ \beta^2,$$
$$a_{33} = D_s\beta^4 + H_s\beta^4 + A_s\beta^2 + k_w + D\beta^4$$
$$-D_s\beta^4 + N^T\ \beta^2$$

and

$$b_{11} = I_0, b_{12} = -J_1, b_{13} = J_1\beta i + I_1\beta$$
$$b_{21} = J_1, b_{22} = -K_2, b_{23} = K_2\beta i + J_2\beta i$$
$$b_{31} = -I_1\beta i + J_1\beta i, b_{32} = -K_2\beta i + J_1 k i,$$
$$b_{33} = K_2\beta^2 + 2J_2\beta^2 + A_s\beta^2 - I_0 - I_2\beta^2$$

To achieve the critical frequency, it is necessary to take the determinant equal to zero. The roots of Eq. (39) are as follows

$$\omega_1 = M^0,\ \omega_2 = M^1,\ \omega_3 = M^2 \quad (40)$$

Phase velocity can be illustrated by

$$C_{p(i)} = \frac{M^j(\beta)}{\beta},\ j = 1,2,3 \quad (41)$$

In above equation C_p expresses the phase velocity, β is the symbol of wave number of a CNTRC beam.

4. Numerical results of eigenvalue problem

This part is dedicated to clarify the results of theories, explained in previous theories. First of all, it is necessary to explain material properties of matrix and CNT. The polymer matrix chosen in this investigation is Methyl Methacrylate. The attributes of proper SWCNT are available as follows.

Thermal relying attributes of CNT are listed in Table.4. (Shen *et al.* 2013).

The results of current investigation had been checked by previous investigation as shown in Table 5.

As it is shown in both Fig. 3, and Table 6, wave frequency increases by the rise of temperature. According to Fig. 3(a), for UD-Beam, wave frequency increases from T=300 to 400 but the value of wave frequency and similar behaviour of wave frequency observed from T=400 and T=500. Fig. 3(b) shows the increasement of wave frequency by wave number for O-Beam. Obviously the results are nearly close for the Temperatures T=400, 500 and 700 but results for T=300 is not close to others and V-Beam have similar behaviour. In contrast, in X-Beam, the difference of results between various temperatures is nearly the same according to Fig. 3(c). Fig. 4 shows that O-Beam and V-Beam have similar manner but X-Beam has lower responses than UD-Beam. And also the results of all distributions, get closer between the wave number of 40 to approximately 65. Fig. 5. Fig. 5 shows the increasement of wave frequency for various wave numbers in two different temperatures, T=300 and T=500. The logarithmic scale behaviour of increasement in wave frequency for various temperatures shown in Fig. 6. The effect of Winkler foundation in thermal enviroment shown in Fig. 7. It had been observed that in constant temperature, results have a high increase and then sudden downfall from wave numbers, 0 to 500, then they will be overlaped. Simply, increase of Winkler foundation, causes this effect in wave frequency. As a matter of fact, by increase of temperature the foregoing effect will be reduced. Fig. 8 shows the variation of phase velocity by means of temperature rise for various shear deformation theories. SSDT has higher values than TSDT and FSDT. Besides, phase velocity increases by the rise of temperature. The effect of thermal parameter, N^T shown in Fig. 9, in which V-Beam has higher values of phase velocity than X-Beam and UD-Beam. This is beacause of the difference of distribution of caron nanotube and polymer matrix in different geometries which causes higher module of Yound and shear, in which they are both effective in the eigenvalue problem. Herein, the results of O-Beam are neglected because they are completely the same as V-Beam. Fig. 10 shows the effects of temperature rise in phase velocity for different geometries of beams in which UD-Beam has a little higher values than X-Beam but

Table 3 Attributes of polymer matrix (Shen *et al.* 2013)

T	ρ^m	E^m	α^m
300	1150 kg/m^3	2.5 Gpa	4.5×10^{-6} / K

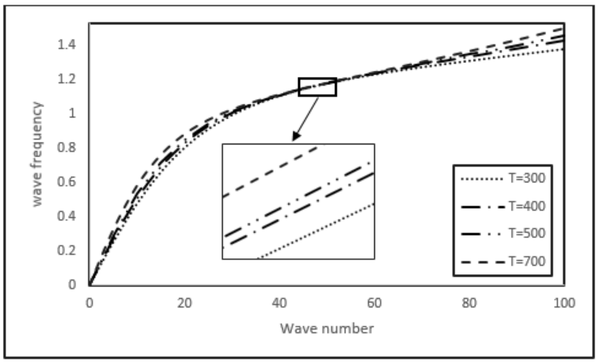
Table 4 Thermal characteristics of a (10, 10) SWCNT with length = 9.26 nm, radius = 0.68 nm, $\nu_{12}^{CNT} = 0.175$

T (k)	E_{11}^{CNT} (Tpa)	E_{22}^{CNT} (Tpa)	G_{12}^{CNT} (Tpa)	α_{11}^{CNT} ($\times10^{-6}$ / K)
300	5.6466	7.0800	1.9445	5.1682
400	5.5679	6.9814	1.9703	5.0905
500	5.5308	6.9348	1.9643	5.0189
700	5.4744	6.8641	1.9644	4.8943

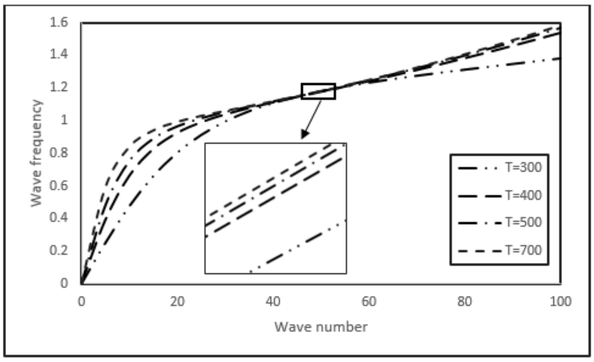
Table 5 Comparison of dimensionless frequency for various plate theories in UD-Beam

Shape function of plate theory	Dimensionless frequency of UD-Beam $\varpi = \omega L\sqrt{\frac{I_{00}}{A_{110}}}$	
	Present	Wattanasakulpong (2013)
FSDT	0.99761	0.997
TSDT	0.974607	0.9745
ESDT	0.97563	0.9756
HSDT	0.974537	0.9745
SSDT	0.9749101	0.9749

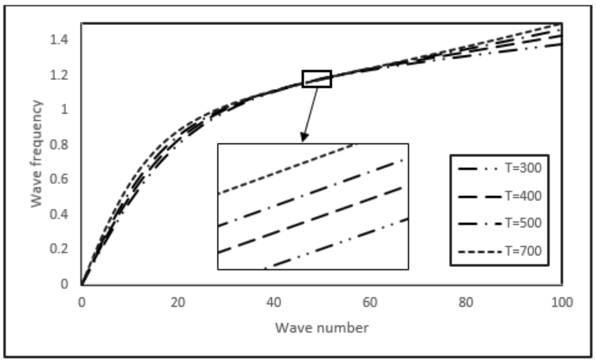
V-Beam is highly greater than the other two. Besides the results of UD-Beam and O-Beam are nearly the close so it is neglected as previous figure. Fig. 11 shows the temperature effect in increase of wave frequency for various



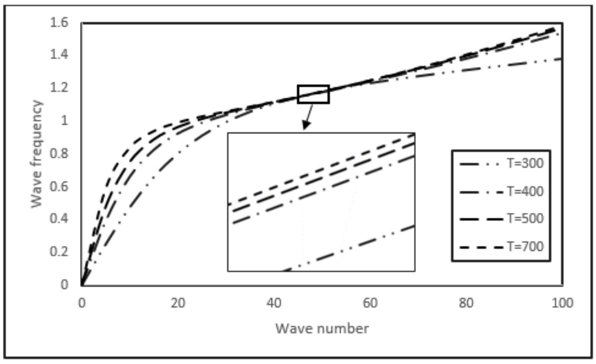
(a)



(b)



(c)



(d)

Fig. 3 Wave frequency vs. wave number for various temperatures in various distributions of CNT in composite by means of SSDT. (a) Uniform distribution (b) O-distribution (c) X-distribution (d) V-distribution

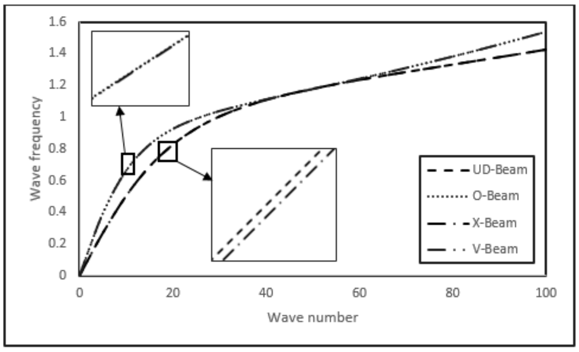
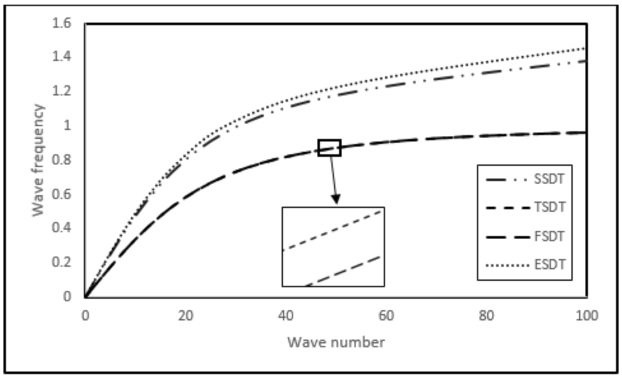
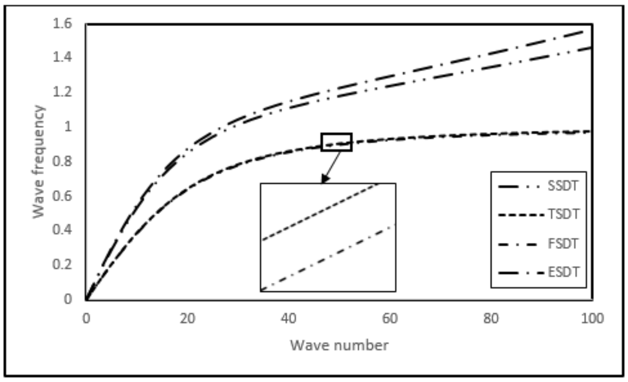


Fig. 4 Wave frequency vs wave number for various distributions in SSDT and T=400



(a)



(b)

Fig. 5 Variation of wave frequency .vs wave number for various shear deformation theories in different temperatures. (a) T=300 (b) T=500

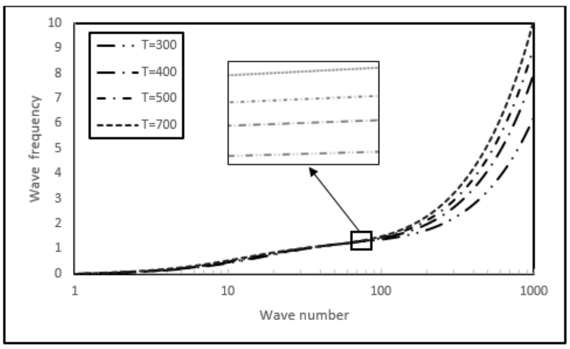
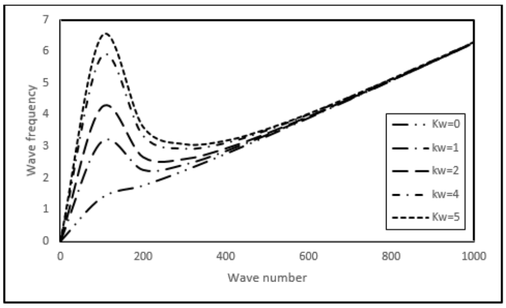
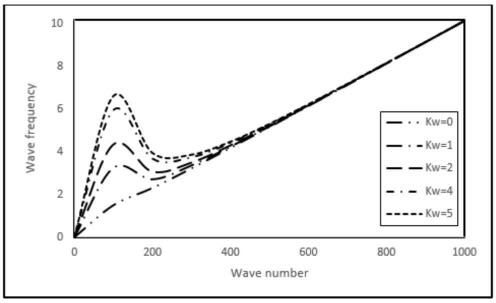


Fig. 6 Wave frequency vs. wave number in logarithmic form in UD-Beam and SSDT for various temperatures

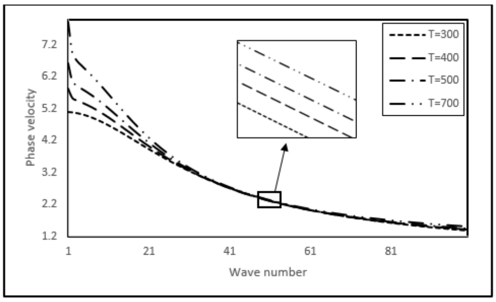


(a)

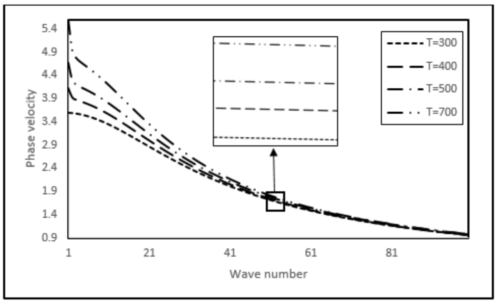


(b)

Fig. 7 Wave frequency for various wave numbers in different Winkler numbers. (a) T=300 (b) T=700



(a)



(b)

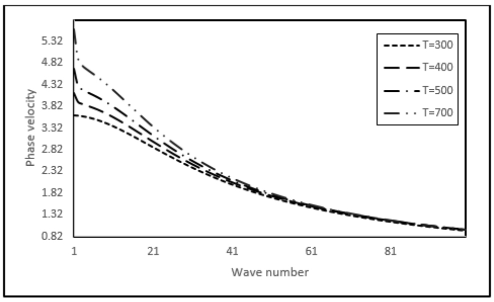


Fig. 8 Phase velocity vs. wave number in different temperatures by means of various shear deformation theories. (a) SSDT (b) TSDT (c) FSDT

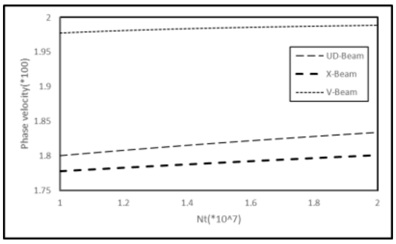


Fig. 9 Effect of thermal parameter in phase velocity for different geometries

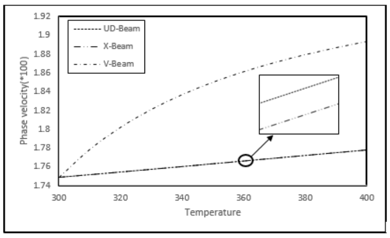
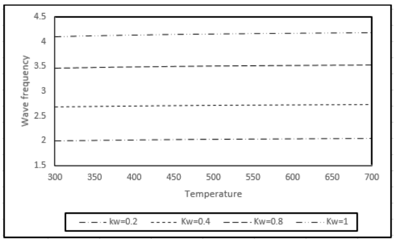
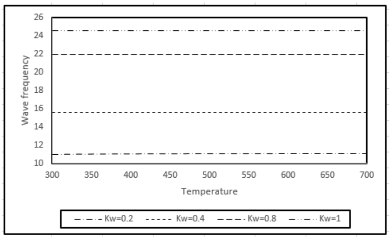


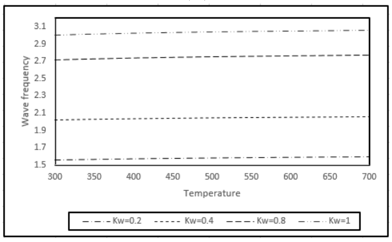
Fig. 10 Effect of temperature in phase velocity for various geometries of beam



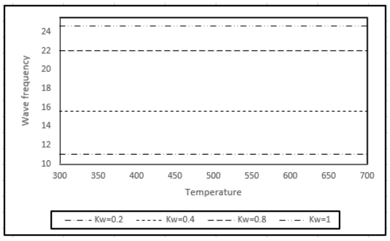
(a)



(b)



(c)



(d)

Fig. 11 Wave frequency vs. temperature for different beam geometries. (a) UD-Beam (b) O-Beam (c) X-Beam (d) V-Beam

Table 6 Effect of temperature rise in wave number for various CNT distributions

T	Wave no.	Uniform	O-distribution	X-distribution	V-distribution
300	20	0.80226379	0.80226379	0.80226379	0.80226379
	40	1.10569451	1.10569451	1.10569451	1.10569451
	60	1.2307596	1.2307596	1.2307596	1.2307596
	80	1.31019343	1.31019343	1.31019343	1.31019343
	100	1.37896114	1.37896114	1.37896114	1.37896114
400	20	0.8259355	0.92689826	0.82582074	0.92689826
	40	1.10774696	1.11429876	1.1077376	1.11429876
	60	1.23428795	1.24430374	1.23427232	1.24430374
	80	1.33059472	1.38460582	1.330506	1.38460582
	100	1.42494568	1.54055571	1.42474863	1.54055571
500	20	0.84823164	0.96391246	0.84808932	0.96391246
	40	1.10946896	1.11606642	1.10945855	1.11606642
	60	1.23709486	1.24672309	1.23707829	1.24672309
	80	1.34629125	1.39686425	1.34619993	1.39686425
	100	1.4594181	1.56568374	1.45921972	1.56568374
700	20	0.88022669	0.99220505	0.88004491	0.99220505
	40	1.1116407	1.1172538	1.11162925	1.1172538
	60	1.24045118	1.24828869	1.24043399	1.24828869
	80	1.36447492	1.40464593	1.36438333	1.40464593
	100	1.49843493	1.58143957	1.49824076	1.58143957

geometries of CNTRC beams. Whith rise of temperature while the wave number is 50. The wave frequency rises by the increasement of K_W. As it is obvious from the figure, O-Beam and V-eam have higher responses than the other two. For a UD-Beam, the differences between responses of different winkler parameters are nearly the same. Although for others, there is a higher difference between K_W=0.4, 0.8. This gap increases for a X-Beam and then decreases for a V-Beam. Table 7 presents the effects of tempereture rise in wave frequency of various shear deformation theories. In which SSDT and ESDT are nearly simillar and both TSDT and FSDT give simillar responses.

5. Conclusions

The accomplishment of thermal influences had been studied. The growth of wave frequency due to temperature rise shown in different plots and tables. The effects of different distributions of CNT in polymer matrix investigated combined by various shear deformation theories. Besides The effect of Winkler elastic foundation in wave frequency and the accomplishment of thermal surroundings observed. The Temperature affect in variation of phase velocity had been done due to reach different properties of wave propagation. It had been found that the effect of Winkler elastic foundation decreases by the increase of temperature. Phase velocity of CNTRC beams increases by the thermal effects. Besides, by increasing the wave number, wave frequency increases which had been discussed for various geometries of CNTRC beams. In

Table 7 Effect of temperature and various shear deformation theories in two different temperatures

T	Wave number	Wave frequency			
		SSDT	TSDT	FSDT	ESDT
300	5	0.25032258	0.17747	0.177555	0.258721
	10	0.47469854	0.339276	0.339422	0.490747
	15	0.65898974	0.475837	0.476011	0.681534
	20	0.80226379	0.585036	0.585206	0.830129
	25	0.91094539	0.669686	0.66983	0.95
	30	0.99321947	0.734452	0.734553	1.03
	35	1.05629373	0.783948	0.783997	1.095302
	40	1.10569451	0.822005	0.821995	1.14755
	45	1.1454151	0.851561	0.851485	1.189938
	50	1.17827544	0.874786	0.874639	1.225376
	55	1.20625729	0.893264	0.893041	1.255905
	60	1.2307596	0.908146	0.907841	1.282968
	65	1.25277876	0.920274	0.919883	1.307587
	70	1.27303218	0.930269	0.929786	1.3305
	75	1.29204224	0.938594	0.938013	1.352242
	80	1.31019343	0.945595	0.944911	1.373203
	85	1.32777155	0.951538	0.950745	1.393674
	90	1.34499086	0.956626	0.955718	1.413871
	95	1.36201313	0.961018	0.959988	1.433956
	100	1.37896114	0.964836	0.963679	1.454049
500	5	0.288656	0.205632	0.20572926	0.298168
	10	0.531721	0.386182	0.38634242	0.548668
	15	0.715936	0.53157	0.53174922	0.738109
	20	0.848232	0.64168	0.64184357	0.874266
	25	0.942942	0.722762	0.72288855	0.972304
	30	1.01264	0.782118	0.78219564	1.045287
	35	1.066207	0.825871	0.82589175	1.10234
	40	1.109469	0.858554	0.85851242	1.14939
	45	1.146168	0.883359	0.8832511	1.190215
	50	1.178713	0.9025	0.90232002	1.227227
	55	1.208676	0.917508	0.91725232	1.261986
	60	1.237095	0.929457	0.9291198	1.295507
	65	1.264665	0.939104	0.93868133	1.328463
	70	1.291858	0.946997	0.94648194	1.3613
	75	1.318994	0.953531	0.95291891	1.394315
	80	1.346291	0.959001	0.95828603	1.427703
	85	1.3739	0.963627	0.96280361	1.461589
	90	1.401918	0.967577	0.96663902	1.496053
	95	1.430411	0.97098	0.96992102	1.53114
	100	1.459418	0.973935	0.97274977	1.566872

propagation of elastic waves in CNTRC beams, it should be mentioned that the results of SSDT and ESDT are nearly converged. Although the responses of TSDT and FSDT are nearly similar. The temperature affected wave frequency has higher differences by increasing of Winkler elastic function coefficient, while the distribution changes from uniform distribution. For the future works in wave propagation of CNTRC beams, it is recommended to investigate the special boundary condition which are useful in industry such as clamped-hinged-clamped with assumption of Timoshenko beam theories.

References

Akgöz, B. and Civalek, Ö. (2016), "Bending analysis of embedded carbon nanotubes resting on an elastic foundation using strain gradient theory", *Acta Astronaut*., **119**, 1-12.

Alibeigloo, A. (2013), "Static analysis of functionally graded carbon nanotube-reinforced composite plate embedded in piezoelectric layers by using theory of elasticity", *Compos. Struct*., **95**, 612-622.

Alibeigloo, A. and Liew, K.M. (2015), "Elasticity solution of free vibration and bending behavior of functionally graded carbon nanotube-reinforced composite beam with thin piezoelectric layers using differential quadrature method", *Int. J. Appl. Mech*., **7**(1), 1550002.

Alizada, A.N. and Sofiyev, A.H. (2011), "Modified young's moduli of nano-materials taking into account the scale effects and vacancies", *Meccan*., **46**(5), 915-920.

Alizada, A.N. and Sofiyev, A.H. (2011), "On the mechanics of deformation and stability of the beam with a nanocoating", *J. Reinf. Plast. Compos*., **30**(18), 1583-1595.

Alizada, A.N., Sofiyev, A.H. and Kuruoglu, N. (2012), "Stress analysis of a substrate coated by nanomaterials with vacancies subjected to uniform extension load", *Acta Mech*., **223**(7), 1371-1383.

Aragh, B.S., Barati, A.N. and Hedayati, H. (2012), "Eshelby-Mori-Tanaka approach for vibrational behavior of continuously graded carbon nanotube-reinforced cylindrical panels", *Compos. Part B: Eng*., **43**(4), 1943-1954.

Arani, A.G., Jamali, M., Mosayyebi, M. and Kolahchi, R. (2016), "Wave propagation in FG-CNT-reinforced piezoelectric composite micro plates using viscoelastic quasi-3D sinusoidal shear deformation theory", *Compos. Part B: Eng*., **95**, 209-224.

Baltacıoglu, A.K., Akgöz, B. and Civalek, Ö. (2010), "Nonlinear static response of laminated composite plates by discrete singular convolution method", *Compos. Struct*., **93**(1), 153-161.

Chen, W.J. and Li, X.P. (2013), "Size-dependent free vibration analysis of composite laminated Timoshenko beam based on new modified couple stress theory", *Arch. Appl. Mech*., **83**(3), 431-444.

Civalek, Ö. and Demir, C. (2016), "A simple mathematical model of microtubules surrounded by an elastic matrix by nonlocal finite element method", *Appl. Math. Comput*., **289**, 335-352.

Ebrahimi, F. and Barati, M.R. (2016), "A nonlocal higher-order shear deformation beam theory for vibration analysis of size-dependent functionally graded nanobeams", *Arab. J. Sci. Eng*., **41**(5), 1679-1690.

Ebrahimi, F. and Barati, M.R. (2016), "Vibration analysis of smart piezoelectrically actuated nanobeams subjected to magneto-electrical field in thermal environment", *J. Vibr. Contr*., 1077546316646239.

Ebrahimi, F. and Barati, M.R. (2016f), "A nonlocal higher-order shear deformation beam theory for vibration analysis of size-dependent functionally graded nanobeams", *Arab. J. Sci. Eng*., **41**(5), 1679-1690.

Ebrahimi, F. and Barati, M.R. (2016g), "Vibration analysis of nonlocal beams made of functionally graded material in thermal environment", *Eur. Phys. J. Plus*, **131**(8), 279.

Ebrahimi, F. and Barati, M.R. (2016h), "Dynamic modeling of a thermo-piezo-electrically actuated nanosize beam subjected to a magnetic field", *Appl. Phys. A*, **122**(4), 1-18.

Ebrahimi, F. and Barati, M.R. (2016i), "A unified formulation for dynamic analysis of nonlocal heterogeneous nanobeams in hygro-thermal environment", *Appl. Phys. A*, **122**(9), 792.

Ebrahimi, F. and Barati, M.R. (2016j), "A nonlocal higher-order refined magneto-electro-viscoelastic beam model for dynamic analysis of smart nanostructures", *Int. J. Eng. Sci*., **107**, 183-196.

Ebrahimi, F. and Barati, M.R. (2016k), "Hygrothermal effects on vibration characteristics of viscoelastic FG nanobeams based on nonlocal strain gradient theory", *Compos. Struct*.

Ebrahimi, F. and Barati, M.R. (2016l), "Buckling analysis of nonlocal third-order shear deformable functionally graded piezoelectric nanobeams embedded in elastic medium", *J. Brazil. Soc. Mech. Sci. Eng*., 1-16.

Ebrahimi, F. and Barati, M.R. (2016m), "Magnetic field effects on buckling behavior of smart size-dependent graded nanoscale beams", *Eur. Phys. J. Plus*, **131**(7), 1-14.

Ebrahimi, F. and Barati, M.R. (2016n), "Buckling analysis of smart size-dependent higher order magneto-electro-thermo-elastic functionally graded nanosize beams", *J. Mech*., 1-11.

Ebrahimi, F. and Barati, M.R. (2016o), "An exact solution for buckling analysis of embedded piezoelectro-magnetically actuated nanoscale beams", *Adv. Nano Res*., **4**(2), 65-84.

Ebrahimi, F. and Barati, M.R. (2016p), "Electromechanical buckling behavior of smart piezoelectrically actuated higher-order size-dependent graded nanoscale beams in thermal environment", *Int. J. Smart Nano Mater*., 1-22.

Ebrahimi, F. and Barati, M.R. (2016q), "Small scale effects on hygro-thermo-mechanical vibration of temperature dependent nonhomogeneous nanoscale beams", *Mech. Adv. Mater. Struct*., Just Accepted.

Ebrahimi, F. and Barati, M.R. (2017), "A nonlocal strain gradient refined beam model for buckling analysis of size-dependent shear-deformable curved FG nanobeams", *Compos. Struct*., **159**, 174-182.

Ebrahimi, F. and Habibi, S. (2017), "Nonlinear eccentric low-velocity impact response of a polymer-carbon nanotube-fiber multiscale nanocomposite plate resting on elastic foundations in hygrothermal environments", *Mech. Adv. Mater. Struct*., 1-14.

Ebrahimi, F. and Karimiasl, M. (2017), "Nonlocal and surface effects on the buckling behavior of flexoelectric sandwich nanobeams", *Mech. Adv. Mater. Struct*., 1-10.

Ebrahimi, F. and Salari, E. (2015c), "Nonlocal thermo-mechanical vibration analysis of functionally graded nanobeams in thermal environment", *Acta Astronaut*., **113**, 29-50.

Ebrahimi, F. and Salari, E. (2015), "Nonlocal thermo-mechanical vibration analysis of functionally graded nanobeams in thermal environment", *Acta Astronaut*., **113**, 29-50.

Ebrahimi, F. and Salari, E. (2015), "Thermo-mechanical vibration analysis of nonlocal temperature-dependent FG nanobeams with various boundary conditions", *Compos. Part B: Eng*., **78**, 272-290.

Ebrahimi, F. and Salari, E. (2015a), "Thermal buckling and free vibration analysis of size dependent Timoshenko FG nanobeams in thermal environments", *Compos. Struct*., **128**, 363-380.

Ebrahimi, F. and Salari, E. (2015b), "Size-dependent free flexural vibrational behavior of functionally graded nanobeams using semi-analytical differential transform method", *Compos. Part B: Eng*., **79**, 156-169.

Ebrahimi, F., Barati, M.R. and Haghi, P. (2017), "Thermal effects on wave propagation characteristics of rotating strain gradient temperature-dependent functionally graded nanoscale beams", *J. Therm. Stress*., **40**(5), 535-547.

Ebrahimi, F., Ghadiri, M., Salari, E., Hoseini, S.A.H. and Shaghaghi, G.R. (2015a), "Application of the differential transformation method for nonlocal vibration analysis of functionally graded nanobeams", *J. Mech. Sci. Technol.*, **29**(3), 1207-1215.

Ebrahimi, F., Salari, E. and Hosseini, S.A.H. (2015b), "Thermomechanical vibration behavior of FG nanobeams subjected to linear and nonlinear temperature distributions", *J. Therm. Stress.*, **38**(12), 1360-1386.

Eringen, A.C. and Edelen, D.G.B. (1972), "On nonlocal elasticity", *Int. J. Eng. Sci.*, **10**(3), 233-248.

Fantuzzi, N., Tornabene, F., Bacciocchi, M. and Dimitri, R. (2017), "Free vibration analysis of arbitrarily shaped functionally graded carbon nanotube-reinforced plates", *Compos. Part B: Eng.*, **115**, 384-408.

Ghorbanpour Arani, A. and Zamani, M.H. (2017), "Investigation of electric field effect on size-dependent bending analysis of functionally graded porous shear and normal deformable sandwich nanoplate on silica aerogel foundation", *J. Sandw. Struct. Mater.*, 1099636217721405.

Ghorbanpour Arani, A., Jamali, M., Mosayyebi, M. and Kolahchi, R. (2017), "Analytical modeling of wave propagation in viscoelastic functionally graded carbon nanotubes reinforced piezoelectric microplate under electro-magnetic field", *J. Nanomater. Nanoeng. Nanosyst.*, **231**(1), 17-33.

Gürses, M., Civalek, Ö., Korkmaz, A.K. and Ersoy, H. (2009), "Free vibration analysis of symmetric laminated skew plates by discrete singular convolution technique based on first-order shear deformation theory", *Int. J. Numer. Meth. Eng.*, **79**(3), 290-313.

Janghorban, M. and Nami, M.R. (2017), "Wave propagation in functionally graded nanocomposites reinforced with carbon nanotubes based on second-order shear deformation theory", *Mech. Adv. Mater. Struct.*, **24**(6), 458-468.

Lin, F. and Xiang, Y. (2014), "Vibration of carbon nanotube reinforced composite beams based on the first and third order beam theories", *Appl. Math. Model.*, **38**(15), 3741-3754.

Nejati, M., Eslampanah, A. and Najafizadeh, M. (2016), "Buckling and vibration analysis of functionally graded carbon nanotube-reinforced beam under axial load", *Int. J. Appl. Mech.*, **8**(1), 1650008.

Reddy, J.N. (2000), "Analysis of functionally graded plates", *Int. J. Numer. Meth. Eng.*, **47**(1-3), 663-684.

Reissner, E. (1985), "Reflections on the theory of elastic plates", *Appl. Mech. Rev.*, **38**(11), 1453-1464.

Shen, H.S. and Xiang, Y. (2013), "Nonlinear analysis of nanotube-reinforced composite beams resting on elastic foundations in thermal environments", *Eng. Struct.*, **56**, 698-708.

Shen, H.S., He, X.Q. and Yang, D.Q. (2017), "Vibration of thermally postbuckled carbon nanotube-reinforced composite beams resting on elastic foundations", *Int. J. Non-Lin. Mech.*, **91**, 69-75.

Şimşek, M. (2010), "Fundamental frequency analysis of functionally graded beams by using different higher-order beam theories", *Nucl. Eng. Des.*, **240**(4), 697-705.

Soldatos, K.P. (1992), "A transverse shear deformation theory for homogeneous monoclinic plates", *Acta Mech.*, **94**(3), 195-220.

Touratier, M. (1991), "An efficient standard plate theory", *Int. J. Eng. Sci.*, **29**(8), 901-916.

Wattanasakulpong, N. and Ungbhakorn, V. (2013), "Analytical solutions for bending, buckling and vibration responses of carbon nanotube-reinforced composite beams resting on elastic foundation", *Comput. Mater. Sci.*, **71**, 201-208.

Wu, H., Kitipornchai, S. and Yang, J. (2015), "Free vibration and buckling analysis of sandwich beams with functionally graded carbon nanotube-reinforced composite face sheets", *Int. J. Struct. Stab. Dyn.*, **15**(7), 1540011.

Yan, T., Kitipornchai, S., Yang, J. and He, X.Q. (2011), "Dynamic behaviour of edge-cracked shear deformable functionally graded beams on an elastic foundation under a moving load", *Compos. Struct.*, **93**(11), 2992-3001.

Yas, M.H. and Samadi, N. (2012), "Free vibrations and buckling analysis of carbon nanotube-reinforced composite Timoshenko beams on elastic foundation", *Int. J. Press. Vess. Pip.*, **98**, 119-128.

Zhang, L.W., Lei, Z.X., Liew, K.M. and Yu, J.L. (2014), "Static and dynamic of carbon nanotube reinforced functionally graded cylindrical panels", *Compos. Struct.*, **111**, 205-212.

CC