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Numbers Cup Optimization: A new method for optimization problems

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Abstract. In this paper, a new meta-heuristic optimization method is presented. This new method is named "Numbers Cup Optimization" (NCO). The NCO algorithm is inspired by the sport competitions. In this method, the objective function and the design variables are defined as the team and the team members, respectively. Similar to all cups, teams are arranged in groups and the competitions are performed in each group, separately. The best team in each group is determined by the minimum or maximum value of the objective function. The best teams would be allowed to the next round of the cup, by accomplishing minor changes. These teams get grouped again. This process continues until two teams arrive the final and the champion of the Numbers Cup would be identified. In this algorithm, the next cups (same iterations) will be repeated by the improvement of players' performance. To illustrate the capabilities of the proposed method, some standard functions were selected to optimize. Also, size optimization of three benchmark trusses is performed to test the efficiency of the NCO approach. The results obtained from this study, well illustrate the ability of the NCO in solving the optimization problems.

Keywords: optimization; meta-heuristic; standard function; truss structure; size optimization

1. Introduction

Optimization of complex problems requires powerful tools. In order to resolve these problems, new optimization methods have been presented which are inspired by the natural or social phenomena and are known as "meta-heuristic methods". Meta-heuristic techniques usually have relatively the same process to achieve the optimal solution. In the most of these methods, during the algorithm process, a number of random responses are generated in the permissible area, and then it moves toward the optimum point during further processes. Most of these algorithms are the population-based ones and during the search process, in moving toward the optimal solution, random searches are also considered to be performed. Therefore, there is an ability to escape from the local optimum traps. Hence, a higher probability is provided to reach the global optimality (Prugel-Bennett 2010).

In the recent years, an increasing number of the meta-heuristic methods have been introduced, for example: Particle Swarm Optimization (PSO); which is inspired by the social behaviors of animals, such as birds and fishes (Kennedy and Eberhart 1995), Ant Colony Optimization (ACO); which uses the seeking behavior of the ants (Dorigo and Blum 2005), Firefly Algorithm (FA); which is modeled by observation of the flicker fireflies (Yang 2009), Ray Optimization (RO); in which each factor is considered as a beam of light and moves in the search space to find the optimum point (Kaveh and Khayatazad 2012), Colliding Bodies Optimization (CBO); which is based on one-dimensional collisions between bodies (Kaveh and Mahdavi 2014), Crow Search Algorithm (CSA); which works based on intelligent behaviors of crows (Askarzadeh 2016), Kidney-inspired Algorithm (KA); which uses the kidney process in the human body (Jaddi *et al.* 2017), and Optimal Foraging Algorithm (OFA); which is inspired by the animal Behavioral Ecology Theory (Zhu and Zhang 2017). As none of the mentioned algorithms claim to optimize all kinds of problems i.e., linear and/or non-linear, constrained and/or non-constrained problems, there are still many opportunities to explore new innovative methods. Hence, this article presents a new metaheuristic optimization method, named as the "Numbers Cup Optimization" (NCO), which is inspired by sports competitions. This method clearly depicts the concept of meta-heuristic optimization, regarding the competitions among random numbers, in order to reach the optimal response (champion).

In the last decades, these introduced algorithms have been used to solve the structural optimization. Structural optimization problems are generally divided into three classes (Klarbring 2008):

1- Size optimization: the cross sections of the members are considered as design variables.

2- Shape optimization: the coordinates of the nodes are considered as design variables.

3- Topology optimization: the connectivities of the members are selected as design variables.

Different types of structural optimization problems have been presented in the literature. Wang *et al.* (2002) presented a study for truss structure with combined size and shape optimization. Rahami *et al.* (2008) optimized truss structures using the genetic algorithm with sizing, geometry and topology design variables. Kaveh and Talatahari (2009a) made a study on size optimization of space trusses

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using a hybrid Big Bang-Big Crunch algorithm. Dede *et al.* (2011) minimized the weight of the truss structures by using adopted Genetic Algorithm. Sonmez (2011) studied on truss structures taking into account the size optimization with Artificial Bee Colony algorithm. Miguel and Miguel (2012) made a study on shape and size optimization of truss structures considering dynamic constraints through modern metaheuristic algorithms (Harmony Search and Firefly Algorithm). Sadollah *et al.* (2012) presented a study on size optimization with discrete design variables of truss structures using the Mine Blast Algorithm. Miguel *et al.* (2013) employ the Firefly Algorithm (FA) in the simultaneous optimization of size, shape, and topology in truss structures. Ahrari and Atai (2013) presented a novel truss optimizer based on the principles of Evolution Strategies by taking into account the size and shape optimization. Kaveh and Mahdavi (2015) studied the application of Colliding Bodies Optimization (CBO) method, for size and topology optimization of steel trusses. Dede and Togan (2015) used the Teaching Learning Based Optimization (TLBO) as an optimization engine in the size and shape optimization of the truss structures under frequency constraints. Kaveh and Mahdavi (2016) applied a new single-solution search optimization algorithm to the size optimization of truss structures. Dede (2018) presented a new and efficient optimization algorithm called Jaya for size optimization of steel grillage structure. In this paper, the NCO algorithm is applied for finding the optimal design of planar truss structures under some constraints. In this process, size optimization is taken into account while the topology of the truss structure is fixed.

The goal of the NCO method is to find the optimal response in less number of function evaluations (NFE). This method has two convergency procedures; one at the beginning of each course, and the other during each course. The optimization iterations are as same as the courses.

The remainder of this paper is organized as follows: In section 2, the new optimization method is introduced. In sections 3, standard functions, include unimodal and multimodal functions, are implemented by the proposed method. In section 4, the NCO method is applied for size optimization of truss structures. Conclusions are derived in section 5.

2. Methodology

As mentioned previously, the algorithm is inspired by sport cups procedures. The model of sport cups is close to the optimization concept in its exact meaning, and that is to find the champion (optimum response) after each course of the competition. In sport competitions, teams are first grouped, and then the members of each group compete with each other to become the group's best team. The best teams again compete until two teams find their way to the final. In the final round, the champion team which is the problem's optimum response is found. The sports cups may be held yearly, bi-yearly, etc. This subject in the NCO method is defined as cup courses. The courses are as same as the optimization iterations. Teams try to perform better in next courses of competitions, and this is sought after by boosting the performance of team members. The method procedure would be divided into the following steps:

1- Initialization, including:

• Determining the number of initial population of variables (the total number of teams, calculated by Eq. (1)),

• The maximum number of iterations, Max_it,

• The lower limit, X_{lower}, and the upper limit, X_{upper}, for design variables,

• Determining the parameters α,β,EN ,where $0<\alpha<1,\ 0<\beta<\alpha,\ EN<N_t$. α,β are integer numbers and EN is a natural number.

$$N_t = 2(N_g)^n \tag{1}$$

N_t Total number of teams (total number of points)

N_g Number of teams in each group

n Number of rounds of the cup, up to the final round ($n>0$)

Also, the total number of points is calculated by using Eq. (2). Subsequently, the number of groups is calculated by dividing $2(N_g)^n$ by N_g (Eq. (3)). All primary points are generated randomly, according to the minimum and maximum limits.

$$N_t = N_g \times G_n \tag{2}$$

$$G_n = 2(N_g)^{n-1} \tag{3}$$

G_n Number of primary groups

2. random points of the design variables are grouped in accordance with the parameters defined in the previous step, and the competition will begin in each group. Variables are the team members, and teams with more capable members could perform better. This is determined by calculating the objective function for each team, and comparing the obtained result with results of other teams.

3. After calculating the objective function for all teams of each group, the best team of the round, F_{Rbest}, with its optimum variables, X_{Rbest}, are determined among the best teams of each group, F_{gbest}, and their corresponding variables, X_{gbest}, respectively, according to resulting in minimum (maximum) value for the objective function.

4. The best team of each group enter the next round and get grouped again. The groupings are random. At this round, teams will try to do their best, because they are motivated to stay in the competition, and ascend to the next round. These efforts are modeled in NCO using Eq. (4). The formula consists of three parts. The second term on the right side indicates the efforts of a team to incorporate techniques which the best team of the round, X_{Rbest}, has utilized. The third term indicates cases such as injuries, bans by receiving cards, return of an injured player, etc., which are unpredictable.

$$\begin{aligned} X_{new} = X_{gbest} + \frac{(N_g - \frac{N_g}{n+1})N_r}{N_t}(X_{Rbest} - X_{gbest}) + \\ \frac{1}{(ii+1)N_g}(rand - 0.5) \end{aligned} \tag{4}$$

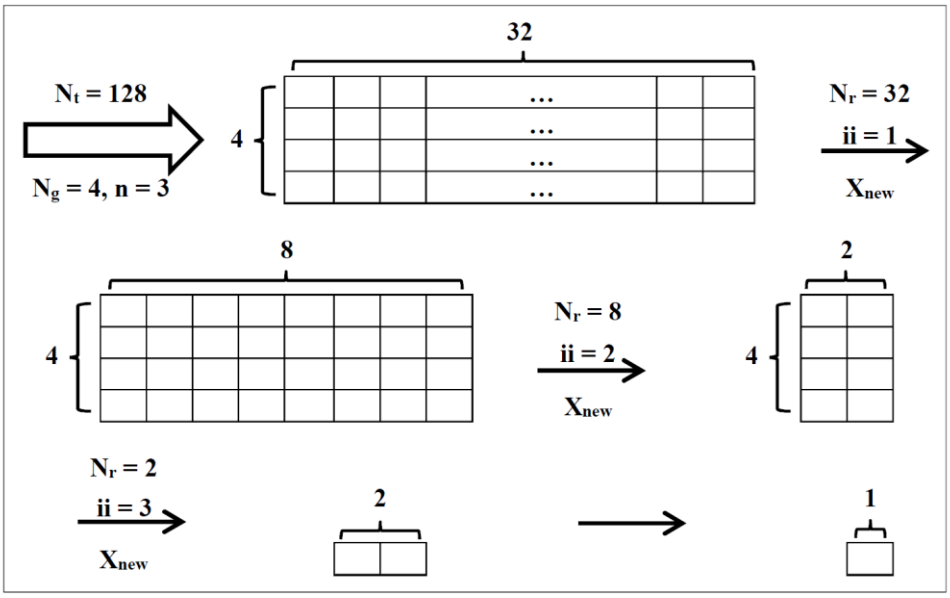


Fig. 1 The process of one course with an input of $2(4)^3$

Dimensions of all four terms of Eq. (4) are $nd\times1$, where nd represents the number of design variables.

X_{gbest} Design variables of the best team in each group $(X_{gbest_{nd\times1}})$

N_g Number of teams in each group

n Number of rounds, up to the final round

N_r Number of best teams in each round

N_t Total number of teams

X_{Rbest} Design variables of the best team in each round $(X_{Rbest_{nd\times1}})$

ii Counter of the cup rounds, from the first round up to the final

$rand$ A vector including random numbers between 0 and 1 $(rand_{nd\times1})$

X_{new} Design variables of the retrieved team $(X_{new_{nd\times1}})$

For instance, in a three variable function, nd=3, with an initial population of $N_t=2(4)^2$, in round $ii=1$, the number of best teams is $N_r=8$ and the value of the expression $\frac{(N_g-\frac{N_g}{n+1})N_r}{N_t}$ would be $\frac{(4-\frac{4}{2+1})8}{32}=0.67$. In this round, eight best teams (F_{gbest}) are selected from eight groups of four, and the variables of these eight best teams, X_{gbest}, create a matrix with 3×8 dimensions. Between these eight teams, the best team, F_{Rbest}, accompanied with its design variable vector, $X_{Rbest_{3\times1}}$, will be selected. Retrieving and updating columns of matrix X_{gbest} is performed by Eq. (4). For example, for one of the columns, Eq. (4) is applied as followed

$$[X_{new}]_{3\times1}=[X_{gbest}]_{3\times1}+0.67\times[X_{Rbest_{3\times1}}-X_{gbest_{3\times1}}]_{3\times1}+\frac{1}{(1+1)4}[rand_{3\times1}-0.5]_{3\times1}.$$

Finally, by retrieving all the columns of matrix X_{gbest}, matrix X_{New} would also have a 3×8 dimension.

5. Again, these retrieved members (X_{New}) are arranged in N_g-team groups and the previous process is repeated until they result in two N_g-team groups. The winners of these groups will reach the final, so that the team with the minimum (maximum) objective function will be the champion of that course of competitions.

At present, the process of one sample course is described. The expression $\frac{\left(N_g-\frac{N_g}{n+1}\right)N_r}{N_t}$ in Eq. (4), which is a number greater than zero and less than one, is explained according to Fig. 1. For example, for an initial population of N_t=2(4)^3 in the first round, ii=1, the number of best teams is N_r=32 and the value of the expression $\frac{\left(N_g-\frac{N_g}{n+1}\right)N_r}{N_t}$ would be $\frac{\left(4-\frac{4}{3+1}\right)32}{128}=0.75$. Similarly, in round two, $ii=2$, the number of best teams would be $N_r=8$ and the foregoing expression's value would be obtained equal to $\frac{\left(4-\frac{4}{3+1}\right)8}{128}=0.1875$. In the last round, $ii=3$, the number of best teams would be $N_r=2$ and the expression value would be $\frac{\left(4-\frac{4}{3+1}\right)2}{128}=0.0469$. The decrements in the value of the expression $\frac{(N_g-\frac{N_g}{n+1})N_r}{N_t}$ would be higher by moving toward the final, owing to the proximity of the team's powers in higher levels; so, less changes take place in design variables of these teams.

According to the description above, for instance, in round $ii = 1$, 32 best teams, F_{gbest}, are determined, and between these 32 teams, one best team, F_{Rbest}, would be selected. For a sample single variable function, if the optimum variable $X_{Rbest} = -1$ would be identified from $X_{gbest} = [4, -5, 2, 0, -1, 1, \ldots]$, due to the minimum of the objective function, the application of Eq. (4), for instance, for digit 4 from X_{gbest} would be as following

$$X_{new} = 4 + 0.75 \times (-1 - 4) + \frac{1}{(1+1)4}(rand - 0.5)$$

$$\xrightarrow{rand=0.9} X_{new} = 0.3.$$

Fig. 1 illustrates the process of one course of Numbers Cup with an input of $N_t = 2(4)^3$.

6. The figure above (mentioned steps) illustrates one course of Numbers Cup. For the next cups, the teams try to have a better performance, and they do this through strengthening their team members (i.e., variables).

Strengthening each variable is accomplished through inclination to its best response in the previous course. Herein, X_{Rbest} is the vector of champion variables of the previous course. According to the intervals modified by Eq. (5), at the start of the new course, EN[1] number of random points are created for each variable, which $EN<N_t$.

$$X_{new\_lower} = X_{Rbest} - \gamma \,,\, X_{new\_upper} = X_{Rbest} + \gamma \tag{5}$$

X_{Rbest} Champion variables of the previous course
γ Half of the new interval
X_{new_lower} New lower limit
X_{new_upper} New upper limit
γ in Eq. (5) is calculated using the equation below

$$\gamma = \frac{\sigma \times (X_{upper} - X_{lower})}{2} \tag{6}$$

X_{upper} Initial upper limit of variables
X_{lower} Initial lower limit of variables
σ Coefficient of γ
σ in Eq. (6) is calculated from the following equation

$$\sigma = \alpha - \frac{it}{\tau} \tag{7}$$

α Interval coefficient, $0 < \alpha < 1$
it Iteration counter
$\frac{1}{\tau}$ Iteration coefficient
τ in Eq. (7) is calculated from the equation below

$$\tau = \frac{Max\_it}{\alpha - \beta} \tag{8}$$

α, β Interval coefficients; $0 < \alpha < 1, 0 < \beta < \alpha$
Max_it Maximum number of iteration
For example, with $\alpha = 0.4, \beta = 0.1$ and maximum iteratiosn of $Max\_it = 100$, τ will be obtained equal to 333.3333. $(X_{upper} - X_{lower})$ is assumed equal to 3. Thus, at the second iteration $\sigma = 0.3940$ and $\gamma = 0.5910$ will be obtained which γ is the size of neighboring interval for X_{Rbest}. The size of this neighboring is decreased by increasing the number of iterations; at the last iteration, $it = 100$, the value of γ would be 0.15. In fact, the neighboring interval of the optimum response is reduced in each iteration compared by the previous one, and at the last iteration, it will reach its minimum. A number of EN points in the interval of $(X_{new\_lower}, X_{new\_upper})$ will be produced for each design variable at the start of each iteration. This will converge the design variables to the optimum variable. Thus, the parameters α, β are chosen regarding the objective function's complexities.

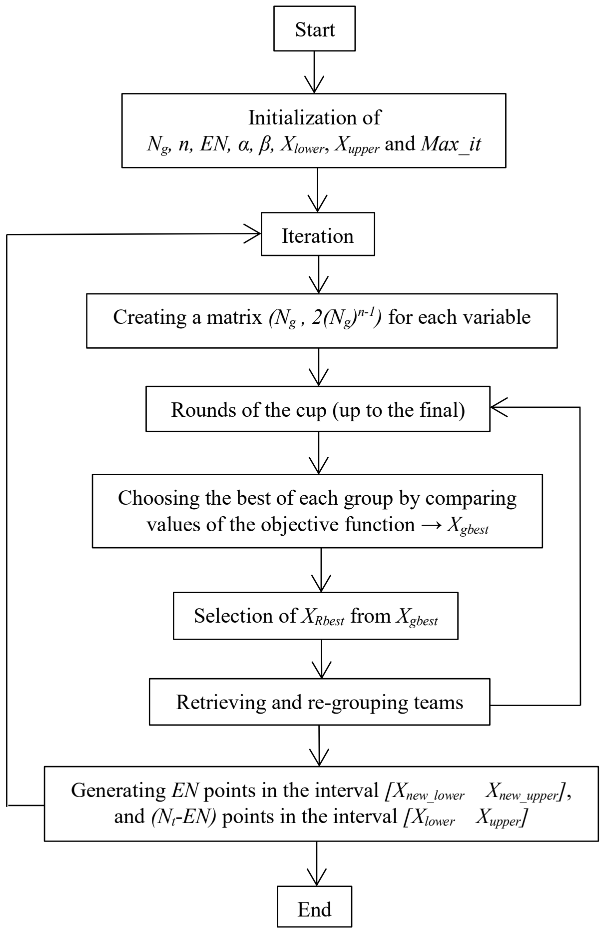


Fig. 2 Flowchart of method

7. Another innovation introduced in the NCO algorithm to avoid local optimums is the possibility of appearing new stars in teams. This is accomplished through generating $N_t - EN$ new random points in the primary interval at the beginning of each course (iteration).

Thus, the Numbers Cup Optimization algorithm induces convergency by generating EN number of points, at the start of each iteration, from the neighboring interval of the previous iteration's optimum response, and also escapes

[1]Extended Number

from the local optimums by the generation of $N_t - EN$ number of points at the start of each iteration, in the initial interval of each design variable.

Finally, the ultimate optimum response is obtained by ending the maximum number of iterations (courses) or satisfying the convergency criteria. The members of the champion team are the optimum values of the design variables. The flowchart of the proposed algorithm is illustrated in Fig. 2.

3. Testing optimization functions

In this section, some standard functions were chosen to verify the performance of the NCO method. Also, this section presents a comparison between NCO and the HS-OBL algorithm. The HS-OBL is a hybrid optimization approach, which the HS (harmony search) method is merged with the opposition-based learning (OBL) method (Gaoa *et al*. 2012). The range of some benchmark functions of the HS-OBL algorithm is bigger than other references. This condition is better illustrated the ability of the proposed method. The benchmark functions include both unimodal and multimodal functions. For all evaluations, computational procedures have been implemented by the MATLAB computer program.

3.1 Benchmark functions

Test functions used in simulation are as followed:

Beale function

$$f(x) = (1.5 - x_1 + x_1x_2)^2 + (2.25 - x_1 + x_1x_2^2)^2 + (2.625 - x_1 + x_1x_2^3)^2$$

Branin function

$$f(x) = (x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6)^2 + 10(1 - \frac{1}{8\pi})\cos(x_1) + 10$$

Colville function

$$f(x) = 100(x_1^2 - x_2)^2 + (x_1 - 1)^2 + (x_3 - 1)^2 + 90(x_3^2 - x_4)^2 + 10.1\left((x_2 - 1)^2 + (x_4 - 1)^2\right) + 19.8(x_2 - 1)(x_4 - 1)$$

Wood function

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 90(x_4 - x_3^2)^2 + (1 - x_3)^2 + 10.1\left[(x_2 - 1)^2 + (x_4 - 1)^2\right] + 19.8(x_2 - 1)(x_4 - 1)$$

Ackley function

$$f(x) = -20\exp\left(-0.2\sqrt{\frac{1}{d}\sum_{i=1}^{d}x_i^2}\right) - \exp\left(\frac{1}{d}\sum_{i=1}^{d}\cos(2\pi x_i)\right) + 20 + \exp(1)$$

Bohachevsky function

$$f(x) = \sum_{i=1}^{d-1}\left[x_i^2 + 2x_{i+1}^2 - 0.3\cos(3\pi x_i) - 0.4\cos(4\pi x_{i+1}) + 0.7\right]$$

Griewank function

$$f(x) = \sum_{i=1}^{d}\frac{x_i^2}{4000} - \prod_{i=1}^{d}\cos(\frac{x_i}{\sqrt{i}}) + 1$$

Powell function

$$f(x) = \sum_{i=1}^{d/4}\begin{bmatrix}(x_{4i-3} + 10x_{4i-2})^2 + 5(x_{4i-1} - x_{4i})^2 + \\ (x_{4i-2} - 2x_{4i-1})^4 + 10(x_{4i-3} - x_{4i})^4\end{bmatrix}$$

Rosenbrock function

$$f(x) = \sum_{i=1}^{d-1}\left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right]$$

Sphere function

$$f(x) = \sum_{i=1}^{d}x_i^2$$

3.2 Implementation and numerical results

In the HS-OBL algorithm, all benchmark functions have been run with a population of 100, and the number of function evaluation (NFE) of 10000. For an accurate comparison, it has been tried to set the NFE in the NCO method nearby 10000. In addition, the optimization results are based on the average of 1000 independent trials.

The value of parameters N_g, n, EN, α, β and Max_it, in the NCO are 4, 2, 24, 0.2, 0.0001 and 238, respectively. Thus, the value of the NFE is 9996. Optimized fitness results are summarized in Table 1.

The results in Table 1 indicate the ability of the NCO method to reach an average respond less than the HS-OBL method. The important feature shown in Table 1, is the good performance of the NCO method in high dimension functions. Among the obtained results given in Table 1, the average optimal solution of the Rosenbrock function has a considerable difference with the global minimum. This function is unimodal, and the global minimum lies in a narrow, parabolic valley so that convergence to the minimum is difficult.

As follows, the values of the objective functions are plotted in terms of the number of trials. In these figures, the red lines show the average of optimum responses in 1000 trials.

Figs. 3-6 belong to low dimension Functions.

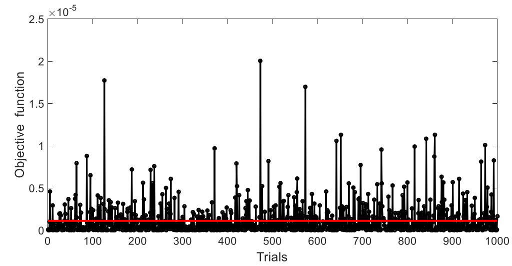


Fig. 3 Beale Function[2]

[2] The maximum and minimum values of the objective function are 2.0071e-5 and 1.6182e-10, respectively.

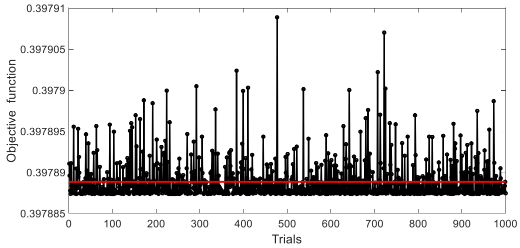


Fig. 4 Branin Function[3]

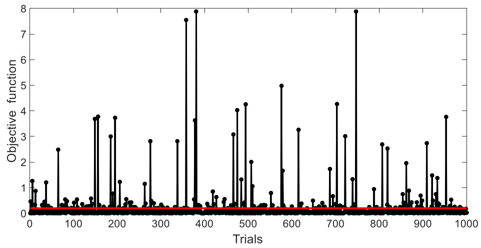


Fig. 5 Colville Function[4]

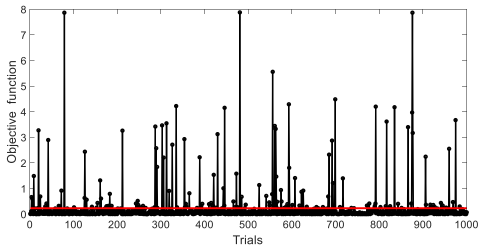


Fig. 6 Wood Function[5]

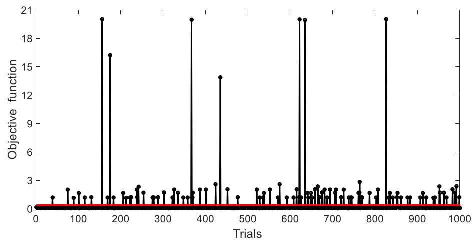


Fig. 7 Ackley Function[6]

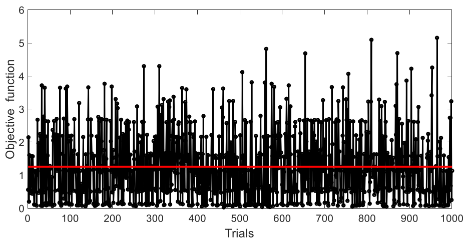


Fig. 8 Bohachevsky Function[7]

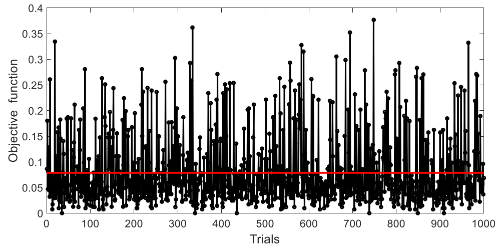


Fig. 9 Griewank Function[8]

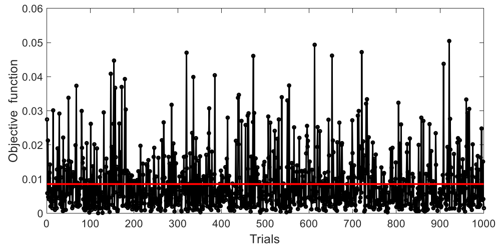


Fig. 10 Powell Function[9]

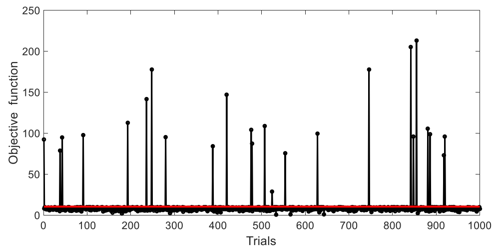


Fig. 11 Rosenbrock Function[10]

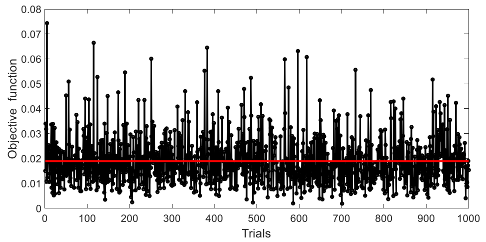


Fig. 12 Sphere Function[11]

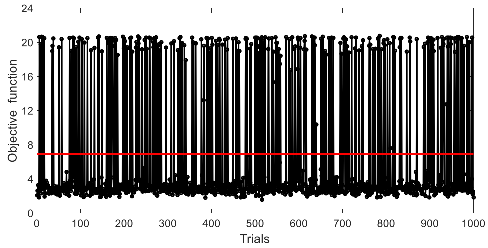


Fig. 13 Ackley Function[12]

[3] The maximum and minimum values of the objective function are 0.397909 and 0.397887, respectively.

[4] The maximum and minimum values of the objective function are 7.8810 and 5.4852e-5, respectively.

[5] The maximum and minimum values of the objective function are 7.8790 and 6.3459e-5, respectively.

[6] The maximum and minimum values of the objective function are 20.0138 and 0.0462, respectively.

[7] The maximum and minimum values of the objective function are 5.1514 and 0.0279, respectively.

[8] The maximum and minimum values of the objective function are 0.3767 and 5.0641e-5, respectively.

[9] The maximum and minimum values of the objective function are 0.0505 and 2.8580e-5, respectively.

[10] The maximum and minimum values of the objective function are 213.0283 and 0.5335, respectively.

[11] The maximum and minimum values of the objective function are 0.0743 and 0.0019, respectively.

[12] The maximum and minimum values of the objective function are 20.7527 and 1.5704, respectively.

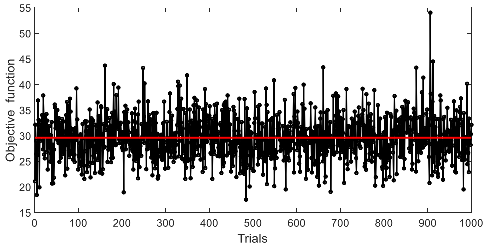


Fig. 14 Bohachevsky Function[13]

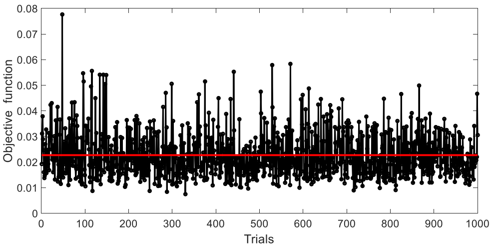


Fig. 15 Griewank Function[14]

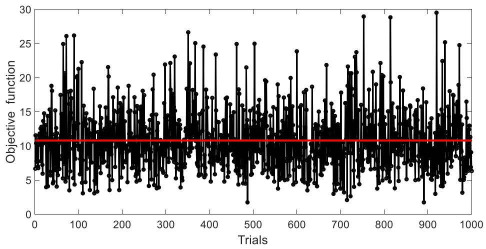


Fig. 16 Powell Function[15]

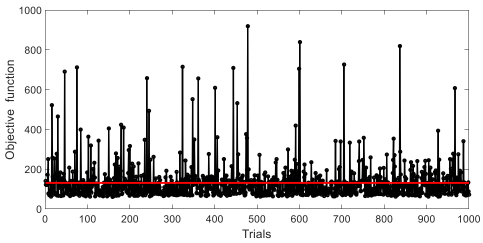


Fig. 17 Rosenbrock Function[16]

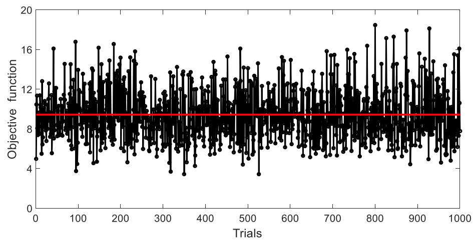


Fig. 18 Sphere Function[17]

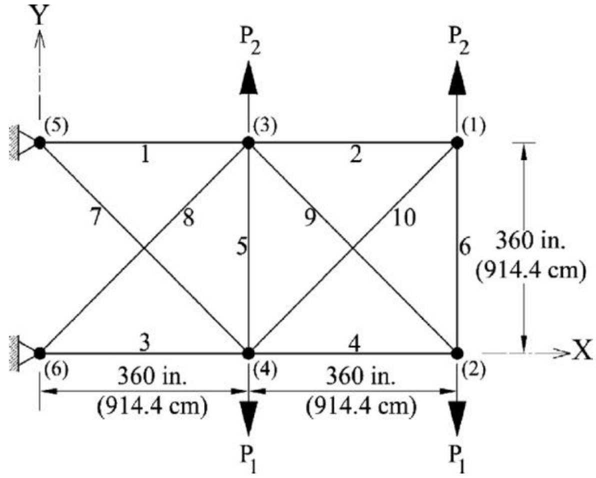


Fig. 19 A 10-bar planar truss

Figs. 7-12 belong to high dimension functions with d=10.

Figs. 13-18 belong to high dimension functions with d=50.

4. Size optimization of truss structure

For size optimization, the cross-sectional areas of the truss members are the design variables. Displacement and allowable stress are taken as the constraint. Three common truss examples as benchmark problems are used for size optimization using the proposed algorithm. This algorithm is applied to problem with both continuous and discrete variables. The final results are compared to the solutions of other methods to demonstrate the efficiency of the present approach.

4.1 A 10-bar planar truss structure

In this example, the 10-bar 2D truss structure is considered as given in Fig. 19. This truss structure is previously designed by Lee and Geem (2004), Li *et al.* (2007), Kaveh and Talatahari (2009b), and Kaveh *et al.* (2015). The material density is 0.1 lb/in^3 and the modulus of elasticity is 10,000 *ksi*. The members are subjected to the stress limits of ±25 *ksi* and all nodes in both vertical and horizontal directions are subjected to the displacement limits of ±2.0 *in*. The number of variables is 10 for cross-sectional areas. The design variables are continuous and their ranges are 0.1 to 35.0 in^2. For this problem, two cases are considered:

Case 1: $P1$=100 *kips* and $P2$=0,

Case 2: $P1$=150 *kips* and $P2$=50 *kips*.

The parameters value of N_g, n, EN, α and β, in the NCO are 4, 2, 20, 0.1 and 0.0001, respectively. For cases 1 and 2, the value of the Max_it is 200 and 155, respectively. The comparison of results with those of the other references is given in Tables 2-3.

As seen in the results of Table 2, the HS, PSO, PSOPC, HPSACO, MCSS and IMCSS algorithms obtain the best solutions after 20000, 150000, 150000, 10650, 8875 and

[13] The maximum and minimum values of the objective function are 54.0796 and 17.5675, respectively.

[14] The maximum and minimum values of the objective function are 0.0776 and 0.0075, respectively.

[15] The maximum and minimum values of the objective function are 29.5173 and 1.7180, respectively.

[16] The maximum and minimum values of the objective function are 919.7881 and 60.9763, respectively.

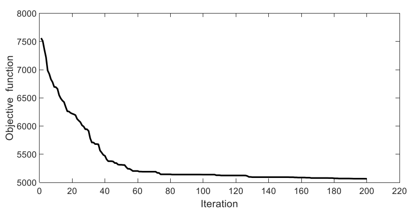
[17] The maximum and minimum values of the objective function are 18.4670 and 3.4469, respectively.

Table 1 Average optimal solutions within 1000 trials

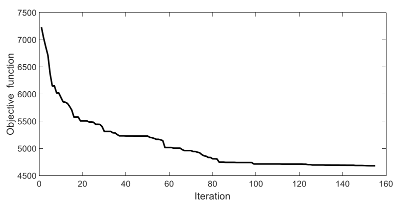
Function		Range	Global Minimum	HS-OBL	NCO
Beale		[-4.5,9]	0	1.2965e-6	1.1289e-6
Branin		[-5,15]	0.397887	0.3979	0.397889
Colville		[-10,20]	0	0.3900	0.2292
Wood		[-10,20]	0	0.390	0.2339
Ackley	d=10	[-32,64]	0	4.8698	0.3793
	d=50		0	16.5508	6.9409
Bohachevsky	d=10	[-15,30]	0	2.3355	1.2586
	d=50		0	216.1322	29.6484
Griewank	d=10	[-20,10]	0	21.2527	0.0789
	d=50		0	102.2558	0.0227
Powell	d=10	[-4,5]	0	0.0097	0.0086
	d=50		0	113.1235	10.7894
Rosenbrock	d=10	[-20,10]	0	13.5661	10.4147
	d=50		0	1.4064e4	131.0507
Sphere	d=10	[-200,100]	0	0.2854	0.0189
	d=50		0	1.1281e3	9.4423

Table 2 The 10-bar truss optimization result (case1)

Design variables	Lee and Geem	Li *et al.*		Kaveh and Talatahari	Kaveh *et al.*		
Area (in^2)	HS	PSO	PSOPC	HPSACO	MCSS	IMCSS	NCO
A_1	30.15	33.469	30.569	30.307	29.5766	30.0258	31.1567
A_2	0.102	0.11	0.1	0.1	0.1142	0.1	0.1004
A_3	22.71	23.177	22.974	23.434	23.8061	23.6277	22.3469
A_4	15.27	15.475	15.148	15.505	15.8875	15.9734	14.9622
A_5	0.102	3.649	0.1	0.1	0.1137	0.1	0.1011
A_6	0.544	0.116	0.547	0.5241	0.1003	0.5167	0.4386
A_7	7.541	8.328	7.493	7.4365	8.6049	7.4567	7.6323
A_8	21.56	23.34	21.159	21.079	21.6823	21.4374	21.6152
A_9	21.45	23.014	21.556	21.229	20.3033	20.7443	21.2733
A_{10}	0.1	0.19	0.1	0.1	0.1117	0.1	0.1
Weight (*lb*)	5057.88	5529.5	5061	5056.56	5086.9	5064.6	5064.9986
No. of analyses	20000	150000	150000	10650	8875	8475	8400



Case 1



Case 2

Fig. 20 Convergence history for the 10-bar truss

8475 analyses. The NCO algorithm achieves its best solution after 8400 analyses. The best weights of the HS, PSO, PSOPC, HPSACO, MCSS and IMCSS algorithms are 5057.88, 5529.5, 5061, 5056.56, 5086.9 and 5064.6 *lb*, respectively, while for the NCO is 5064.9986 *lb*. Although the NCO method can't obtain the minimum weight, but it obtains the less number of function evaluations (NFE) than other algorithms.

As seen in the results of Table 3, the PSO, PSOPC, HPSACO, MCSS and IMCSS algorithms obtain the best solutions after 150000, 150000, 9625, 7350 and 6625 analyses. The NCO algorithm achieves its best solution after 6510 analyses. The best weights of the HS, PSO, PSOPC, HPSACO, MCSS and IMCSS algorithms are 4668.81, 4679.47, 4677.7, 4675.78, 4686.47 and 4679.15 *lb*, respectively, while for the NCO is 4680.2270 *lb*. Although the NCO method doesn't obtain the minimum

Table 3 The 10-bar truss optimization result (case 2)

Design variables	Lee and Geem	Li *et al.*		Kaveh and Talatahari	Kaveh *et al.*		
Area (in^2)	HS	PSO	PSOPC	HPSACO	MCSS	IMCSS	NCO
A_1	23.25	22.935	23.473	23.194	22.863	23.299	24.0446
A_2	0.102	0.113	0.101	0.1	0.120	0.1	0.1026
A_3	25.73	25.355	25.287	24.585	25.719	25.682	25.5745
A_4	14.51	14.373	14.413	14.221	15.312	14.510	13.8881
A_5	0.1	0.1	0.1	0.1	0.101	0.1	0.1030
A_6	1.977	1.99	1.969	1.969	1.968	1.969	1.9771
A_7	12.21	12.346	12.362	12.489	12.310	12.149	12.3192
A_8	12.61	12.923	12.694	12.925	12.934	12.360	12.6078
A_9	20.36	20.678	20.323	20.952	19.906	20.869	20.4504
A_{10}	0.1	0.1	0.103	0.101	0.100	0.1	0.1012
Weight (lb)	4668.81	4679.47	4677.7	4675.78	4686.47	4679.15	4680.2270
No. of analyses	-	150000	150000	9625	7350	6625	6510

Table 4 The allowable cross sections

No.	Area (mm^2)	No.	Area (mm^2)	No.	Area (mm^2)
1	71.613	23	1690.319	45	5141.925
2	90.968	24	1696.771	46	5503.215
3	126.451	25	1858.061	47	5999.988
4	161.29	26	1890.319	48	6999.986
5	198.064	27	1993.544	49	7419.43
6	252.258	28	729.031	50	8709.66
7	285.161	29	2180.641	51	8967.724
8	363.225	30	2238.705	52	9161.272
9	388.386	31	2290.318	53	9999.98
10	494.193	32	2341.931	54	10322.56
11	506.451	33	2477.717	55	10903.2
12	641.289	34	2496.769	56	12129.01
13	645.16	35	2503.221	57	12838.68
14	792.256	36	2696.769	58	14193.52
15	816.773	37	2722.575	59	14774.16
16	939.998	38	2896.768	60	15806.42
17	1008.385	39	2961.284	61	17096.74
18	1045.159	40	3096.768	62	18064.48
19	1161.288	41	3206.445	63	19354.8
20	1283.868	42	3303.219	64	21612.86
21	1374.191	43	3703.218		
22	1535.481	44	4658.055		

weight, but it obtains the less NFE than other algorithms.

For the NCO method, the convergence history of both cases is given in Fig. 20.

4.2 A 52-bar planar truss structure

The 52-bar 2D truss structure is considered as given in Fig. 21. This truss structure is previously designed by Li *et al.* (2007), Kaveh *et al.* (2015), and Kaveh and Talatahari (2009c). The members of this structure are divided into 12 groups: (1) A_1-A_4, (2) A_5-A_{10}, (3) A_{11}-A_{13}, (4) A_{14}-A_{17}, (5) A_{18}-A_{23}, (6) A_{24}-A_{26}, (7) A_{27}-A_{30}, (8) A_{31}-A_{36}, (9) A_{37}-A_{39}, (10) A_{40}-A_{43}, (11) A_{44}-A_{49}, (12) A_{50}-A_{52}.

The material density is 7860.0 kg/m^3 and the modulus of elasticity is 2.07×10^5 Mpa. The members are subjected to the stress limits of ±180 Mpa. As seen in Fig. 21, loads of P_x=100 kN and P_y=200 kN, are applied to the structure. The design variables are discrete and are selected from Table 4. The parameters value of N_g, n, EN, α, β and Max_it, in the NCO are 3, 2, 13, 0.2, 0.0001 and 150, respectively. The comparison of results with those of the other references is given in Table 5.

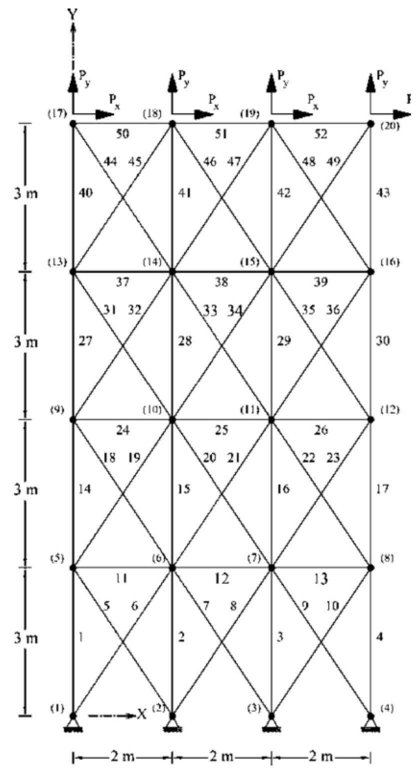


Fig. 21 A 52-bar planar truss

Table 5 The 52-bar truss optimization result

Design variables	Li *et al.*	Kaveh *et al.*		Kaveh and Talatahari	
Area (mm^2)	HPSO	MCSS	IMCSS	DHPSACO	NCO
A_1- A_4	4658.055	4658.055	4658.055	4658.055	4658.055
A_5- A_{10}	1161.288	1161.288	1161.288	1161.288	1161.288
A_{11}- A_{13}	363.255	363.225	494.193	494.193	494.193
A_{14}- A_{17}	3303.219	3303.219	3303.219	3303.219	3303.219
A_{18}- A_{23}	940	939.998	939.998	1008.385	939.998
A_{24}- A_{26}	494.193	506.451	494.193	285.161	494.193
A_{27}- A_{30}	2238.705	2238.705	2238.705	2290.318	2238.705
A_{31}- A_{36}	1008.38	1008.385	1008.385	1008.385	1008.385
A_{37}- A_{39}	388.386	388.386	494.193	388.386	494.193
A_{40}- A_{43}	1283.868	1283.868	1283.868	1283.868	1283.868
A_{44}- A_{49}	1161.288	1161.288	1161.288	1161.288	1161.288
A_{50}- A_{52}	792.256	729.031	494.193	506.451	494.193
Weight (*kg*)	1905.49	1904.05	1902.61	1904.83	1902.6055
No. of analyses	50000	4225	4075	5300	3900

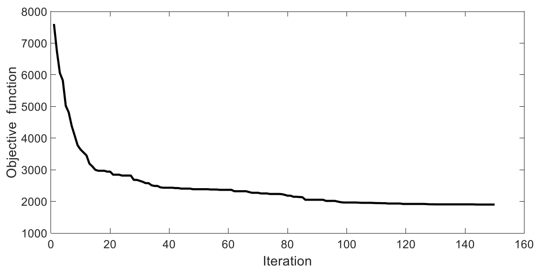


Fig. 22 Convergence history for the 52-bar truss

As seen in the results of Table 5, the HPSO, MCSS, IMCSS and DHPSACO algorithms obtain the best solutions after 50000, 4225, 4075 and 5300 analyses. The NCO algorithm achieves its best solution after 3900 analyses. The best weights of the HPSO, MCSS, IMCSS and DHPSACO algorithms are 1905.49, 1904.05, 1902.61 and 1904.83 *kg*, respectively, while for NCO is 1902.6055 *kg*. Thus, the NCO method obtains the minimum weight with the less NFE than other algorithms.

For the NCO method, the convergence history is given in Fig. 22.

4.3 A 200-bar planar truss structure

In this example, the 200-bar truss structure is considered as given in Fig. 23. This truss structure is previously designed by Sonmez (2011), Togan and Daloglu (2008), and Dede and Ayvaz (2015). The material density is 0.283 lb/in^3 and the modulus of elasticity is 30000 *ksi*. The members are subjected to the stress limits of ±10 *ksi*. There was no displacement limit but the minimum cross-section area was not allowed to be less than 0.1 in^2. The members of this structure are divided into 29 groups. This structure is subjected to three different load conditions:

case 1: 1 *kips* acting in the positive x-direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62, and 71,

case 2: 10 *kips* acting in the negative y-direction at nodes 1, 2, . . ., 6, 8, 10, 12, 14, 15, . . ., 20, 22, 24, 26, 28, 29, . . ., 73, 74, and 75,

case 3: Cases 1 and 2 are combined.

The parameters value of N_g, n, EN, α, β and Max_it, in the NCO are 3, 2, 13, 0.2, 0.00005 and 5000, respectively. The comparison of results with those of the other references is given in Table 6.

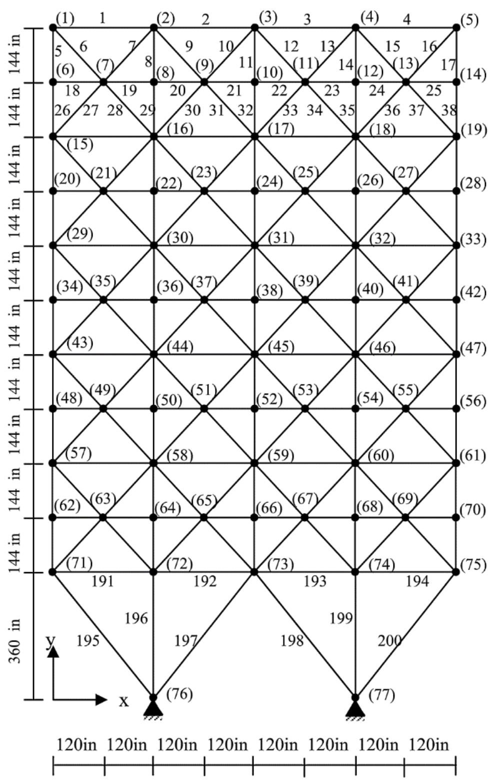


Fig. 23 A 200-bar planar truss

Table 6 The 200-bar truss optimization result under load case 3

Group	Design variables	Sonmez	Togan and Daloglu	Dede and Ayvaz	
	Area (in^2)	ABC-AP	GA	TLBO	NCO
1	1,2,3,4	0.1039	0.347	0.113546	0.1138
2	5,8,11,14,17	0.9463	1.081	0.948427	0.9415
3	19,20,21,22,23,24	0.1037	0.1	0.107798	0.1038
4	18,25,56,63,94,101,132,139,170,177	0.1126	0.1	0.100009	0.1026
5	26,29,32,35,38	1.9520	2.142	1.934462	1.9411
6	6,7,9,10,12,13,15,16,27,28,30,31,33,34,36,37	0.2930	0.347	0.288872	0.2988
7	39,40,41,42	0.1064	0.1	0.211586	0.1129
8	43,46,49,52,55	3.1249	3.565	3.090253	3.1135
9	57,58,59,60,61,62	0.1077	0.347	0.103114	0.1339
10	64,67,70,73,76	4.1286	4.805	4.090254	4.2153
11	44,45,47,48,50,51,53,54,65,66,68,69,71,72,74,75	0.4250	0.44	0.451050	0.4288
12	77,78,79,80	0.1046	0.44	0.100707	0.1319
13	81,84,87,90,93	5.4803	5.952	5.479848	5.4758
14	95,96,97,98,99,100	0.1060	0.347	0.101144	0.1586
15	102,105,108,111,114	6.4853	6.572	6.479849	6.4610
16	82,83,85,86,88,89,91,92,103,104,106,107,109,110,112,113	0.5600	0.954	0.532949	0.6077
17	115,116,117,118	0.1825	0.347	0.132492	0.1780
18	119,122,125,128,131	8.0445	8.525	7.944450	8.1164
19	133,134,135,136,137,138	0.1026	0.1	0.100486	0.2341
20	140,143,146,149,152	9.0334	9.3	8.944437	9.2933
21	120,121,123,124,126,127,129,130,141,142,144,145,147,148,150,151	0.7844	0.954	0.701077	0.8631
22	153, 154, 155,156	0.7506	1.764	1.377693	0.1518
23	157, 160, 163, 166, 169	11.3057	13.3	11.239401	11.3145
24	171, 172, 173, 174, 175, 176	0.2208	0.347	0.228718	0.2689
25	178, 181, 184, 187, 190	12.2730	13.3	12.239392	12.2479
26	158, 159, 161, 162, 164, 165, 167, 168, 179, 180, 182,183, 185, 186, 188, 189	1.4055	2.142	1.684935	1.0949
27	191, 192, 193, 194	5.1600	4.805	4.913586	5.7098
28	195, 197, 198, 200	9.9930	9.3	9.718956	10.3528
29	196, 199	14.70144	17.17	15.021916	14.3023
	Weight (*lb*)	25533.79	28544.014	25664.0023	25597.7688
	No. of analyses	1450000	51,360	-	130000

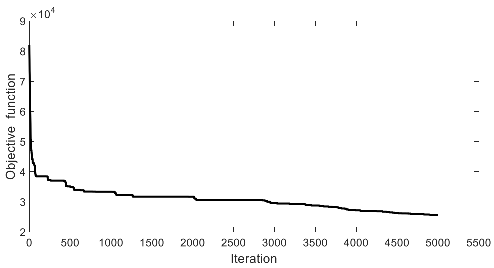


Fig. 24 Convergence history for the 200-bar truss

As seen in the results of Table 6, the ABC-AP and GA algorithms obtain the best solutions after 1450000 and 51,360 analyses. The NCO algorithm achieves its best solution after 130000 analyses. The best weights of the ABC-AP, GA and TLBO algorithms are 25533.79, 28544.014 and 25664.0023 *lb*, respectively, while for NCO is 25597.7688 *lb*. Thus, ABC-AP and NCO methods lead to the minimum weights, while the NCO obtains the response with the less NFE.

For the NCO method, the convergence history is given in Fig. 24.

5. Conclusions

In this article, a new optimization algorithm, so called the "Numbers Cup Optimization" (NCO), is introduced. The NCO is designed based upon the Sport Cups' procedure. In order to evaluate the algorithm, it was examined on a set of standard benchmark functions. The obtained results are compared with the intended reference results and global optimum to demonstrate the ability of the proposed method. Also, in order to verify the method performance, the planar truss structures taken from the literature are considered. This method is implemented for the size optimization of 2D trusses. The optimization results are compared with the previous studies to demonstrate the efficiency of the NCO method.

As seen in the results, the NCO algorithm has found optimum solutions within a lower number of analysis, particularly in high dimensional problems. Finally, the NCO method has an interesting algorithm and is an effective and reliable method in terms of efficiency.

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