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Wave propagation analysis of smart strain gradient piezo-magneto-elastic nonlocal beams

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Abstract. This study presents the investigation of wave dispersion characteristics of a magneto-electro-elastic functionally graded (MEE-FG) nanosize beam utilizing nonlocal strain gradient theory (NSGT). In this theory, a material length scale parameter is propounded to show the influence of strain gradient stress field, and likewise, a nonlocal parameter is nominated to emphasize on the importance of elastic stress field effects. The material properties of heterogeneous nanobeam are supposed to vary smoothly through the thickness direction based on power-law form. Applying Hamilton's principle, the nonlocal governing equations of MEE-FG nanobeam are derived. Furthermore, to derive the wave frequency, phase velocity and escape frequency of MEE-FG nanobeam, an analytical solution is employed. The validation procedure is performed by comparing the results of present model with results exhibited by previous papers. Results are rendered in the framework of an exact parametric study by changing various parameters such as wave number, nonlocal parameter, length scale parameter, gradient index, magnetic potential and electric voltage to show their influence on the wave frequency, phase velocity and escape frequency of MEE-FG nanobeams.

Keywords: wave propagation; functionally graded nanobeam; magneto-electro-elastic materials; nonlocal strain gradient theory

1. Introduction

In recent years, a new invention in material science have gained an enormous attention of many researchers. This novel type of materials, named functionally graded materials (FGMs), are of giant importance in various engineering applications, particularly cases which involve heavy thermo-mechanical loadings. Indeed, FGMs are consist of two mainly different layers with various mechanical properties and this new feature helps to attaining flexible material properties through the thickness of structures. Even though, in lots of engineering fields it is important to utilize materials which have the ability of energy transformations. In many applications, structural elements are subjected to electric, magnetic or electro-magnetic fields. In one types of smart materials this issue is considered by mixing both electro-magnetic and mechanical properties of structures. These materials are a combination of piezo-electric and piezo-magnetic materials and this mixture makes it possible to a large diversity of energy conversions from each of electric, magnetic and elastic to the other one (Ebrahimi and Barati 2016a, Barati *et al.* 2016). These materials are called magneto-electro-elastic materials (MEEMs) which are able to exchange a magnetic field to mechanical one and vice versa. Hence, it is crucial to investigate the mechanical behaviors of MEE-FG structures (Huang *et al.* 2007, Li *et al.* 2008, Kattimani *et al.* 2015).

Lately, nanotechnology and nanostructures have gained an unbelievable role in the modern engineering and the rate of nanostructures' employment in various micro/nano electro-mechanical-systems (MEMS/NEMS) is rising with a high speed. Therefore, such structures must be analyzed properly in different mechanical aspects and this process cannot be performed using classical continuum theory. The mechanical responses of nanosize structures are completely different from those in the macro scale and this is the main issue of developing size-dependent continuum theories. In these theories, small scale effects are considered by means of defining scale parameters. It is common to use the nonlocal elasticity of Eringen (Eringen 1972, 1983) in the most of articles to capture the size influences on bending, buckling and vibration analysis of nanostructures (Reddy 2007, Thai *et al.* 2012, Li *et al.* 2010, Akgoz and Civalek 2016, Civalek and Demir 2016). Moreover, lots of researches are performed by different scientists on wave propagation problem analysis of nanostructures based on nonlocal elasticity theory. For example, Narendar and Gopalakrishnan (2009) showed the effect of nonlocal scale parameter on the wave propagation of multi-walled carbon nanotubes. Investigation of size effects on the flexural wave propagation behavior of nanoplates resting on elastic medium is performed by Wang *et al.* (2010). Narendar *et al.* (2012) investigated the effect of longitudinal magnetic field on wave propagation behaviors of equivalent continuum structure of single-walled carbon nanotubes rested in elastic medium using nonlocal elasticity. The electro-magneto wave propagation analysis of viscoelastic sandwich nanoplates is examined by Arani *et al.* (2016) considering surface effects.

With the rapid development of technology, it is now common to use FG beams and plates in

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micro/nanoelectromechanical systems (MEMS/NEMS), such as the components in the form of shape memory alloy thin films with a global thickness in micro-or nano-scale (Fu *et al.* 2003, Witvrouw and Mehta 2005, Lü *et al.* 2009), electrically actuated MEMS devices (Hasanyan *et al.* 2008), and atomic force microscopes (AFMs) (Rahaeifard *et al.* 2009). Eltaher *et al.* (2012) studied free vibration analysis of size-dependent FG nanobeams employing finite element method. Likewise, the size-dependent static buckling characteristics of FG nanobeams is investigated based on the nonlocal elasticity theory by Eltaher *et al.* (2013). Thermal vibration characteristics of FG nanobeams exposed to different thermal loadings is studied by Ebrahimi *et al.* (2015 b). Ebrahimi and Barati (2016a) performed a new analysis on the free vibration of FG nanobeams using a higher-order beam theory. Ebrahimi and Barati (2016b) examined nonlocal effects on hygro-thermal vibration of FG nanoscale beams. In another work, Ebrahimi and Barati (2016c) presented a unified formulation for dynamic analysis of nonlocal FG nanobeams in hygro-thermal environment. Barati *et al.* (2016) discussed thermal buckling behavior of an embedded FG nanoplate in different types of thermal environments via a refined plate model. Also, Ghadiri *et al.* (2016) examined the surface effects on vibration response of rotating FG nanobeams based on Eringen's nonlocal elasticity.

Moreover, it is worth mentioning that magneto-electro-mechanical behavior of nanostructures with piezomagnetic properties must be exactly analyzed in order to increase the quality and reliability of the modern engineering designs in MEMS/NEMS. Ke *et al.* (2014) explored free vibration characteristics of homogenous MEE nanoplates based on the nonlocal elasticity and Kirchhoff plate theory. Also, Ansari *et al.* (2015) examined nonlinear forced vibration behavior of magneto-electro-thermo-elastic Timoshenko nanobeams according to the nonlocal elasticity theory. Recently, Ebrahimi and Barati (2016d, e, f) investigated vibration and buckling behavior of nonlocal MEE-FG beams under magneto-electrical field. Based on the text, just a few works can be mentioned related to the wave dispersion analysis of piezoelectric and magneto-electro-elastic nanobeams and nanoplates.

Investigation of wave propagation in a homogeneous piezoelectric nanoplate with consideration of the surface piezoelectricity and small-scale effects is done by Zhang *et al.* (2014). The propagation characteristics of the longitudinal waves in a piezoelectric nanoplate is studied by Zang *et al.* (2014). Zhang *et al.* (2015) investigated the propagation behaviors of the flexural wave of piezoelectric FG nanobeam with surface and thermal effects. Narendar (2016) has recently shown the dispersion of elastic waves in functionally graded magneto-electro-elastic rods based on nonlocal elasticity. The wave propagation analysis of a functionally graded nano-rod made of magneto-electro-elastic material subjected to an electric and magnetic potential is discussed by Arefi (2016).

Although the nonlocal elasticity theory can predict the mechanical responses of nanostructures, in this theory it is only paid attention to the stiffness-softening effect on such structures. In other words, some researchers proved a stiffness enhancement on mechanical behavior of nanostructures which is neglected in all papers in which nonlocal elasticity theory is employed (Lim *et al.* 2009). Recently, nonlocal strain gradient theory (NLSGT) is introduced to obtain more accurate responses of nanostructures, while both stiffness-softening and stiffness-hardening effects are involved (Lim *et al.* 2015). In fact, only one scale parameter is introduced in nonlocal elasticity theory to describe size-dependency of nanostructures. But, in nonlocal strain gradient theory two scale parameters are introduced related to nonlocal stress field and strain gradients stress field to provide more accurate prediction of mechanical behavior of nanostructures. Recently, a number of authors have been applied the nonlocal strain gradient theory in their researches.

Buckling analysis of Euler-Bernoulli nanobeams using nonlocal strain gradient theory is performed by Li and Hu (2015). Li *et al.* (2015) examined the flexural wave propagation behaviors of FG nanobeams based on nonlocal strain gradient theory. They reported that if strain gradient effect is neglected in wave propagation analysis of nanobeams, the wave frequencies become underestimated. Farajpour *et al.* (2016) has recently used the NSGT to study the buckling characteristics of nanoplates in thermal environment. Free vibration analysis of nonlocal strain gradient nanobeams constructed from FGMs is investigated by Li *et al.* (2016a). Nonlinear bending and free vibration analysis of nonlocal strain gradient FG nanobeams is performed by Li *et al.* (2016b). Most recently, Ebrahimi *et al.* (2016) analyzed the wave dispersion characteristics of a temperature-dependent heterogeneous nanoplates employing nonlocal strain gradient theory. Therefore, it can be concluded that no effort has been allocated to the wave propagation analysis of a magneto-electro-elastic FG nanobeam via the nonlocal strain gradient theory.

Herein, nonlocal strain gradient theory is utilized to analyze the wave propagation behavior of MEE-FG nanobeams under magneto-electrical field based on the Euler-Bernoulli beam model. The present theory contains two scale parameters to describe size-dependency of MEE-FG nanobeam much accurately. The power-law function is applied to describe the material property distribution of nanobeam across the thickness. Hamilton's principle is used to develop the nonlocal governing equations of MEE-FG nanobeam. These equations are solved analytically to find wave frequency, escape frequency and phase velocity as functions of wave number. Finally, by changing different parameters including nonlocal parameter, length scale parameter, material graduation, magnetic and electric potentials, the wave propagation characteristics of MEE-FG nanobeam affected by these parameters are discussed in detail.

2. Theory and formulation

2.1 The material properties of MEE-FG nanobeam

A magneto-electro-elastic functionally graded (MEE-FG) nanobeam with length L and thickness of h, shown in Fig.1 is assumed to be subjected to a magnetic potential $\Upsilon(x,z,t)$ and an electric potential $\Phi(x,z,t)$ here. The MEE-FG nanobeam is made of $BaTiO_3$ and $CoFe_2O_4$

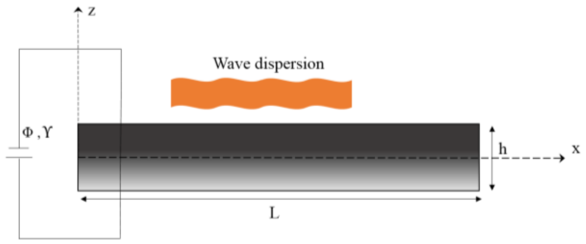


Fig. 1 Geometry of FG nanobeam under applied magneto-electric field

Table 1 Magneto-electro-elastic coefficients of material properties (Ramirez *et al.* 2006)

Properties	$BaTiO_3$	$CoFe_2O_4$
c_{11} (GPa)	166	286
c_{55}	43	45.3
e_{31} (Cm^{-2})	-4.4	0
e_{15}	11.6	0
q_{31} (N/Am)	0	580.3
q_{15}	0	550
s_{11} ($10^{-9}\ C^2m^{-2}N^{-1}$)	11.2	0.08
s_{13}	12.6	0.093
χ_{11} ($Ns^2C^{-2}/2$)	5	-590
χ_{33}	10	157
$d_{11}=d_{33}$	0	0
ρ (kgm^{-3})	5800	5300

with the properties established in Table 1. The material properties of MEE-FG nanobeam is considered to vary gradually in the thickness direction via modified power-law distribution. So, the material properties can be expressed by

$$P_cV_c + P_mV_m = 1 \tag{1}$$

In above equation P_m and P_c denote metal and ceramic material properties, respectively and their volume fractions are related to each other as follows

$$V_c + V_m = 1 \tag{2}$$

The volume fraction of ceramic can be calculated in a desired thickness as follows

$$V_c = (\frac{z}{h} + \frac{1}{2})^p \tag{3}$$

Here p is gradient index which controls the smooth distribution of material through the thickness of the beam and z is the distance from neutral plane of the FG nanobeam. Now, substituting Eqs. (2) and (3) inti Eq. (1) results in an equation for equivalent material properties of FG beam

$$P(z) = (P_c - P_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + P_m \tag{4}$$

It is worth mentioning that the top surface of MEE-FG nanosize beam ($z = +{}^{h}/_{2}$), is fully $CoFe_2O_4$, whereas the bottom surface ($z = -{}^{h}/_{2}$), is fully $BaTiO_3$.

2.2 Kinematic relations

Employing Euler-Bernoulli beam model, the displacement field at each point of FG beam can be written as follows

$$u_x(x,z) = u(x) - z\frac{\partial w}{\partial x} \tag{5a}$$

$$u_z(x,z) = w(x) \tag{5b}$$

In above equations, u and w are the displacement components in the mid-plane along the coordinates x and z, respectively. In order to convince Maxwell's equation in the quasi-static estimation, a combination of a cosine and linear variation is considered to simulate the electric and magnetic potential distributions along the thickness direction as follows (Ebrahimi and Barati 2016f)

$$\Phi(x,z,t) = -cos(\xi z)\phi(x,z,t) + \frac{2z}{h}V \tag{6}$$

$$\Upsilon(x,z,t) = -cos(\xi z)\gamma(x,z,t) + \frac{2z}{h}\Omega \tag{7}$$

where $\xi = \pi/h$. In above equations, V and Ω are the external applied electric voltage and magnetic potential, respectively. Based on the Euler-Bernoulli beam theory, the only nonzero strain of the nanobeam can be shown as follows

$$\varepsilon_{xx} = \varepsilon_{xx}^{(0)} + z\varepsilon_{xx}^{(1)} \tag{8}$$

where

$$\varepsilon_{xx}^{(0)} = \frac{\partial u}{\partial x} \quad , \quad \varepsilon_{xx}^{(1)} = -\frac{\partial^2 w}{\partial x^2} \tag{9}$$

Now, based on Eq. (6), the relation between electric field (E_x, E_z) and electric potential (Φ), can be stated as (Ebrahimi and Barati 2016f)

$$E_x = -\Phi_{,x} = cos(\xi z)\frac{\partial \phi}{\partial x} \tag{10}$$

$$E_z = -\Phi_{,z} = -\xi\, sin(\xi z)\phi - \frac{2V}{h} \tag{11}$$

Also, according to Eq. (7), the relation between magnetic field (H_x, H_z) and magnetic potential (ϒ), can be noted as

$$H_x = -\Upsilon_{,x} = cos(\xi z)\frac{\partial \gamma}{\partial x} \tag{12}$$

$$H_z = -\Upsilon_{,z} = -\xi\, sin(\xi z)\phi - \frac{2\Omega}{h} \tag{13}$$

Now according to the Hamilton's principle it is tried to find the Euler-Lagrange equations of the MEE-FG nanobeam

$$\int_0^t \delta(\Pi_S - \Pi_K + \Pi_W)dt = 0 \tag{14}$$

where, Π_S is strain energy, Π_K is kinetic energy and Π_W is the work done by external energy. The variation of strain energy is expressed as

$$\delta\Pi_S = \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{xz}\delta\gamma_{xz} - D_x\delta E_x - D_z\delta E_z - B_x\delta H_x - B_z\delta H_z)dzdx \tag{15}$$

Substituting Eq. (8) in Eq. (15) yields

$$\delta\Pi_S = \int_0^L (N\delta\varepsilon_{xx}^{(0)} + M\delta\varepsilon_{xx}^{(1)})dx + \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \Big(-D_x cos(\xi z)\delta\left(\frac{\partial\phi}{\partial x}\right) + D_z\xi\, sin(\xi z)\delta\phi - B_x cos(\xi z)\delta\left(\frac{\partial\gamma}{\partial x}\right) + B_z\xi\, sin(\xi z)\delta\gamma\Big) dzdx \tag{16}$$

in which N and M are axial force and bending moment, respectively, and can be expressed as

$$N = \int \sigma_{xx} dA\ , \quad M = \int \sigma_{xx} z dA \tag{17}$$

The variation of virtual work done by external applied forces can be expressed in the following form

$$\delta\Pi_W = \int_0^L \left([N_E + N_H]\frac{\partial w}{\partial x}\frac{\partial\delta w}{\partial x}\right) dx \tag{18}$$

where N^E and N^H are normal in-plane forces generated due to electric voltage and magnetic potential, respectively, and can be defined in the following form

$$N^E = -\int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{e}_{31}\frac{2V}{h}dz\ , \quad N^H = -\int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{q}_{31}\frac{2\Omega}{h}dz \tag{19}$$

The first variation of kinetic energy can be expressed as

$$\delta\Pi_K = \int [\dot{u_x}\delta\dot{u_x} + \dot{u_z}\delta\dot{u_z}]\rho(z)dV = \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(I_0[\dot{u}\delta\dot{u} + \dot{w}\delta\dot{w}] - I_1\left[\dot{u}\frac{\partial\delta\dot{w}}{\partial x} + \dot{w}\frac{\partial\delta\dot{u}}{\partial x}\right] + I_2\left[\frac{\partial\dot{w}}{\partial x}\frac{\partial\delta\dot{w}}{\partial x}\right]\right) \tag{20}$$

In all of the equations the dot-superscript denotes the differentiation with respect to time; and the mass inertias used in above equations are supposed to be in the following form

$$(I_0, I_1, I_2) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, z^2)\rho(z)dz \tag{21}$$

By substituting Eqs. (16), (18), and (20) into Eq. (14) and setting the coefficients of δu, δw, $\delta\phi$ and $\delta\gamma$ to zero, the Euler-Lagrange equations of MEE-FG nanobeam can be written as

$$\frac{\partial N}{\partial x} = I_0\ddot{u} - I_1\frac{\partial\ddot{w}}{\partial x} \tag{22}$$

$$\frac{\partial^2 M}{\partial x^2} - (N^E + N^H)\frac{\partial^2 w}{\partial x^2} = I_0\ddot{w} + I_1\frac{\partial\ddot{u}}{\partial x} - I_2\frac{\partial^2\ddot{w}}{\partial x^2} \tag{23}$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \left(cos(\xi z)\frac{\partial D_x}{\partial x} + \xi sin(\xi z)D_z\right) dz = 0 \tag{24}$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \left(cos(\xi z)\frac{\partial B_x}{\partial x} + \xi sin(\xi z)B_z\right) dz = 0 \tag{25}$$

2.2 The nonlocal strain gradient theory for MEE materials

According to the nonlocal strain gradient theory, the stress field takes into consider the effects of nonlocal elastic stress field besides strain gradient stress field. So, the theory can be expressed as follows for magneto-electro-elastic solids (Ebrahimi *et al.* 2016, Li *et al.* 2016)

$$[1-(e_1a)^2\nabla^2][1-(e_0a)^2\nabla^2]\sigma_{ij} = [1-(e_1a)^2\nabla^2]\big(C_{ijkl}\varepsilon_{kl} - e_{mij}E_m - q_{nij}H_n\big) - l^2[1-(e_0a)^2\nabla^2]\nabla^2\big(C_{ijkl}\varepsilon_{kl} - e_{mij}E_m - q_{nij}H_n\big) \tag{26}$$

$$[1-(e_1a)^2\nabla^2][1-(e_0a)^2\nabla^2]D_i = [1-(e_1a)^2\nabla^2](e_{ikl}\varepsilon_{kl} + s_{im}E_m + d_{in}H_n) - l^2[1-(e_0a)^2\nabla^2]\nabla^2(e_{ikl}\varepsilon_{kl} + s_{im}E_m + d_{in}H_n) \tag{27}$$

$$[1-(e_1a)^2\nabla^2][1-(e_0a)^2\nabla^2]B_i = [1-(e_1a)^2\nabla^2]\big(q_{ijk}\varepsilon_{kl} + d_{im}E_m + \chi_{in}H_n\big) - l^2[1-(e_0a)^2\nabla^2]\nabla^2\big(q_{ijk}\varepsilon_{kl} + d_{im}E_m + \chi_{in}H_n\big) \tag{28}$$

By discarding the terms of order $O(\nabla^2)$ and also assuming $e = e_0 = e_1$, and defining the Laplacian operator as $\nabla^2 = \frac{\partial^2}{\partial x^2}$, the simplified constitutive relation can be written as follows

$$\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)\sigma_{ij} = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\big(C_{ijkl}\varepsilon_{kl} - e_{mij}E_m - q_{nij}H_n\big) \tag{29}$$

$$\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)D_i = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)(e_{ikl}\varepsilon_{kl} + s_{im}E_m + d_{in}H_n) \tag{30}$$

$$\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)B_i = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\big(q_{ijk}\varepsilon_{kl} + d_{im}E_m + \chi_{in}H_n\big) \tag{31}$$

Therefore, the stress-strain equations can be defined as follows

$$\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)\sigma_{xx} = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)(\tilde{c}_{11}\varepsilon_{xx}-\tilde{e}_{31}E_z - \tilde{q}_{31}H_z) \tag{32}$$

$$\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)D_x = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)(\tilde{s}_{11}E_x+\tilde{d}_{11}H_x) \tag{33}$$

$$\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)D_z = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\left(\tilde{e}_{31}\varepsilon_x+\tilde{s}_{33}E_z + \tilde{d}_{33}H_z\right) \tag{34}$$

$$\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)B_x = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\left(\tilde{d}_{11}E_x+\tilde{\chi}_{11}H_x\right) \tag{35}$$

$$\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)B_z = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\left(\tilde{q}_{31}\varepsilon_x+\tilde{d}_{33}E_z + \tilde{\chi}_{33}H_z\right) \tag{36}$$

where $\mu=ea$, $\lambda=l$ are nonlocal and length scale parameters, respectively. Also, $\tilde{c}_{ij}$, $\tilde{d}_{ij}$, $\tilde{e}_{ij}$, $\tilde{s}_{ij}$, $\tilde{q}_{ij}$ and $\tilde{\chi}_{ij}$ are reduced coefficients of FG nanobeam when it is subjected to a plane stress state (Ke *et al.* 2014)

$$\tilde{c}_{11}=c_{11}-\frac{{c_{13}}^2}{c_{33}} \quad , \quad \tilde{s}_{11}=s_{11}$$

$$\tilde{e}_{31}=e_{31}-\frac{c_{13}e_{33}}{c_{33}} \quad , \quad \tilde{q}_{31}=q_{31}-\frac{c_{13}q_{33}}{c_{33}} \quad , \quad \tilde{d}_{11}=d_{11} \quad , \quad \tilde{\chi}_{11}=\chi_{11} \tag{37}$$

$$\tilde{d}_{33}=d_{33}-\frac{e_{33}q_{33}}{c_{33}} \quad , \quad \tilde{s}_{33}=s_{33}-\frac{{e_{33}}^2}{c_{33}} \quad , \quad \tilde{\chi}_{33}=\chi_{33}-\frac{{q_{33}}^2}{c_{33}}$$

Now, the force-strain and moment-strain equations of the nonlocal FG beam can be developed by integrating from Eqs. (32)-(36) across the cross-section area of the beam

$$\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)N = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\left(A_{xx}\frac{\partial u}{\partial x}+A_{31}^e\phi + A_{31}^m\gamma\right)-(N^E+N^H) \tag{38}$$

$$\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)M = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\left(B_{xx}\frac{\partial u}{\partial x}+E_{31}^e\phi + E_{31}^m\gamma\right)-(M^E+M^H) \tag{39}$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)D_x\cos(\xi z)\,dz = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\left(E_{15}^e\frac{\partial w}{\partial x}+F_{11}^e\frac{\partial \phi}{\partial x} + F_{11}^m\frac{\partial \gamma}{\partial x}\right) \tag{40}$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)D_z\xi\sin(\xi z)\,dz = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\left(A_{31}^e\frac{\partial u}{\partial x}-F_{33}^e\phi - F_{33}^m\gamma\right) \tag{41}$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)B_x\cos(\xi z)\,dz = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\left(E_{15}^m\frac{\partial w}{\partial x} + F_{11}^m\frac{\partial \phi}{\partial x}+X_{11}^m\frac{\partial \gamma}{\partial x}\right) \tag{42}$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)B_z\xi\sin(\xi z)\,dz = \left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\left(A_{31}^m\frac{\partial u}{\partial x}-F_{33}^m\phi - X_{33}^m\gamma\right) \tag{43}$$

in which

$$\{A_{xx},B_{xx},D_{xx}\}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\tilde{c}_{11}\{1,z,z^2\}dz \tag{44}$$

$$\{A_{31}^e,E_{31}^e,F_{31}^e\}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\tilde{e}_{31}\xi\sin(\xi z)\{1,z,z^3\}dz \tag{45}$$

$$\{F_{11}^e,F_{33}^e\}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\{\tilde{s}_{11}cos^2(\xi z),\tilde{s}_{33}\xi^2 sin^2(\xi z)\}dz \tag{46}$$

$$\{A_{31}^m,E_{31}^m,F_{31}^m\}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\tilde{q}_{31}\xi\sin(\xi z)\{1,z,z^3\}dz \tag{47}$$

$$\{F_{11}^m,F_{33}^m\}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\{\tilde{d}_{11}cos^2(\xi z),\tilde{d}_{33}\xi^2 sin^2(\xi z)\}dz \tag{48}$$

$$\{X_{11}^m,X_{33}^m\}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\{\tilde{\chi}_{11}cos^2(\xi z),\tilde{\chi}_{33}\xi^2 sin^2(\xi z)\}dz \tag{49}$$

Moreover, the normal moments, made by electro-magnetic field, used in Eq. (39) can be developed by following equations

$$M^E=-\int_{-\frac{h}{2}}^{\frac{h}{2}}\tilde{e}_{31}\frac{2V}{h}zdz \quad , \quad M^H=-\int_{-\frac{h}{2}}^{\frac{h}{2}}\tilde{q}_{31}\frac{2\Omega}{h}zdz \tag{50}$$

The nonlocal governing equations of a MEE-FG nanobeam can be attained by substituting Eqs. (38)-(43) into Eqs. (22)-(25) as follows

$$\left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\left(A_{xx}\frac{\partial^2 u}{\partial x^2}-B_{xx}\frac{\partial^3 w}{\partial x^3}+A_{31}^e\frac{\partial \phi}{\partial x}+A_{31}^m\frac{\partial \gamma}{\partial x}\right)+\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)\left(-I_0\ddot{u}+I_1\frac{\partial \ddot{w}}{\partial x}\right)=0 \tag{51}$$

$$\left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\left(B_{xx}\frac{\partial^3 u}{\partial x^3}-D_{xx}\frac{\partial^4 w}{\partial x^4}+E_{31}^e\frac{\partial^2 \phi}{\partial x^2}+E_{31}^m\frac{\partial^2 \gamma}{\partial x^2}\right)+\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)\left(-I_0\ddot{w}-I_1\frac{\partial \ddot{u}}{\partial x}+I_2\frac{\partial^2 \ddot{w}}{\partial x^2}-(N^H+N^E)\frac{\partial^2 w}{\partial x^2}\right)=0 \tag{52}$$

$$\left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\left(A_{31}^e\left(\frac{\partial u}{\partial x}\right)-E_{31}^e\frac{\partial^2 w}{\partial x^2}+F_{11}^e\frac{\partial^2 \phi}{\partial x^2}+F_{11}^m\frac{\partial^2 \gamma}{\partial x^2}-F_{33}^e\phi-F_{33}^m\gamma\right)=0 \tag{53}$$

$$\left(1-\lambda^2\frac{\partial^2}{\partial x^2}\right)\left(A_{31}^m\left(\frac{\partial u}{\partial x}\right)-E_{31}^m\frac{\partial^2 w}{\partial x^2}+F_{11}^m\frac{\partial^2 \phi}{\partial x^2}+X_{11}^m\frac{\partial^2 \gamma}{\partial x^2}-F_{33}^m\phi-X_{33}^m\gamma\right)=0 \tag{54}$$

3. Solution procedure

The displacement fields of the waves propagating in the x-z plane are supposed to be in the following form:

$$\begin{Bmatrix} u(x,z,t) \\ w(x,z,t) \\ \phi(x,z,t) \\ \gamma(x,z,t) \end{Bmatrix}=\begin{Bmatrix} U\,exp[i(kx-\omega t)] \\ W\,exp[i(kx-\omega t)] \\ \Phi\,exp[i(kx-\omega t)] \\ \Upsilon\,exp[i(kx-\omega t)] \end{Bmatrix} \tag{55}$$

where U, W, Φ and Υ are the unknown coefficients, k is the wave number of propagated waves along x direction respectively, and finally ω is frequency of propagated waves. Substituting Eq. (55) to Eqs. (51)-(54) yields

$$([K]-\omega^2[M])\{\Delta\}=\{0\} \tag{56}$$

In Eq. (56), the unknown parameters can be expressed as

$$\{\Delta\}=\{U,W\Phi,\Upsilon\}^T \tag{57}$$

Table 2 Comparison of frequencies of MEE-FG nanobeam for different nonlocal parameters

μ	p=0.2		p=1		p=5	
	Ebrahimi and Barati 2016a	present	Ebrahimi and Barati 2016a	present	Ebrahimi and Barati 2016a	present
0	9.30465	9.37202	8.44476	8.49624	7.86815	7.91150
1	8.8769	8.94118	8.05654	8.10565	7.50644	7.54651
2	8.50319	8.56476	7.71737	7.76441	7.19042	7.22881
3	8.17302	8.23220	7.41771	7.46293	6.91123	6.94812

$$[K]=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \quad [M]=\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \tag{58}$$

where a_{ij}, m_{ij} are as given in appendix. The matrix given by Eq. (56) stands for the coefficient matrix and the wave frequency can be developed by solving the following eigenvalue equation

$$|[K]-\omega^2[M]|=0 \tag{59}$$

Now, the wave frequency can be derived by solving Eq. (59) for ω. This derived wave frequency can be showed as a function of wave number as follows

$$\omega=M(k) \tag{60}$$

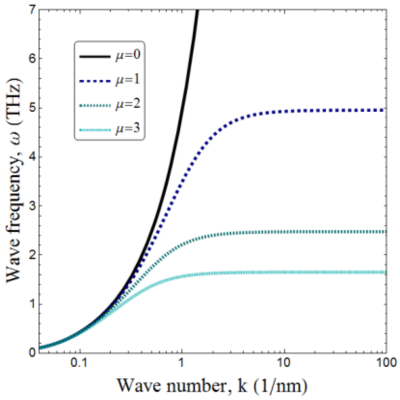
The expressed frequency shows the wave frequency of MEE-FG nanobeam. Also, phase velocity of nanobeam, which is wave frequency divided by wave number, can be showed in the following form as a function of wave number

$$c_p=\frac{\omega}{k} \tag{61}$$

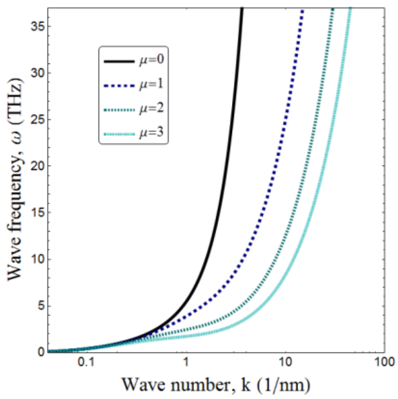
whenever wave number is tended to infinity, the escape frequency of FG nanobeam can be developed. As a predictable phenomenon, if the frequency of waves propagating through a nanobeam reaches to the escape frequency, the flexural waves do not spread anymore.

4. Results and discussion

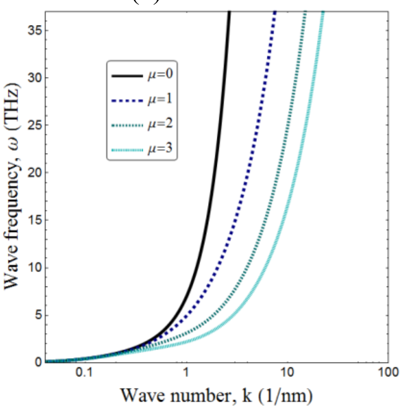
In this part, nonlocal strain gradient theory is employed for wave dispersion analysis of an MEE-FG nanobeam made of $BaTiO_3$ and $CoFe_2O_4$ subjected to an external electric voltage and magnetic potential. In the presented results, the variation of wave frequency and phase velocity and also escape frequency of MEE-FG nanobeam under different wave numbers, nonlocal parameters, length scale parameters, magnetic potentials, electric voltages and gradient indices are shown in detail. The thickness of nanobeam is h=50 nm. The frequencies of MEE-FG nanobeam are validated with those presented by Ebrahimi and Barati (2016a) for various nonlocal parameters and a good agreement can be seen as tabulated in Table 2. In this table it is considered that V=Ω=0.



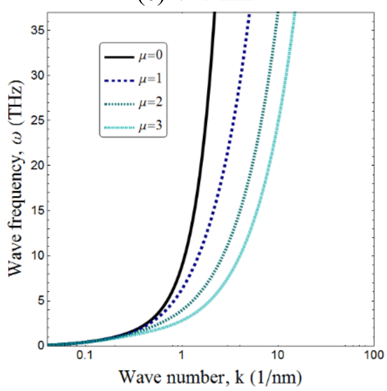
(a) λ=0 nm



(b) λ=0.5 nm

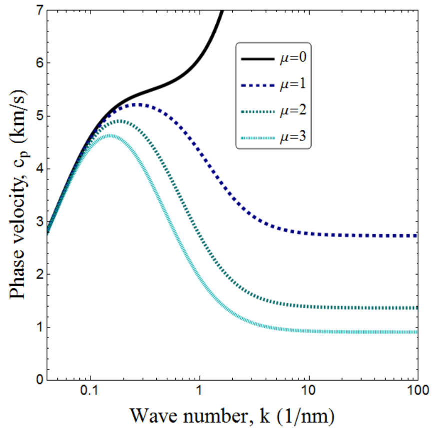


(c) λ=1 nm

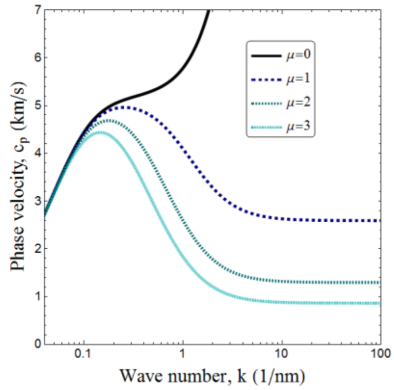


(d) λ=1.5 nm

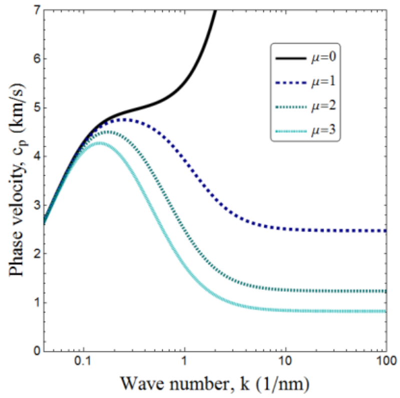
Fig. 2 Variation of wave frequency of MEE-FG nanobeam versus wave number for various nonlocal and length scale parameter (p=1, V=Ω=0)



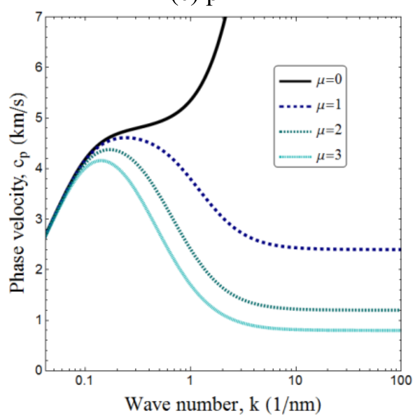
(a) p=0.2



(b) p=0.5



(c) p=1



(d) p=2

Fig. 3 Variation of phase velocity of MEE-FG nanobeam versus wave number for various nonlocal parameters and gradient indices (λ=0.5, V=Ω=0)

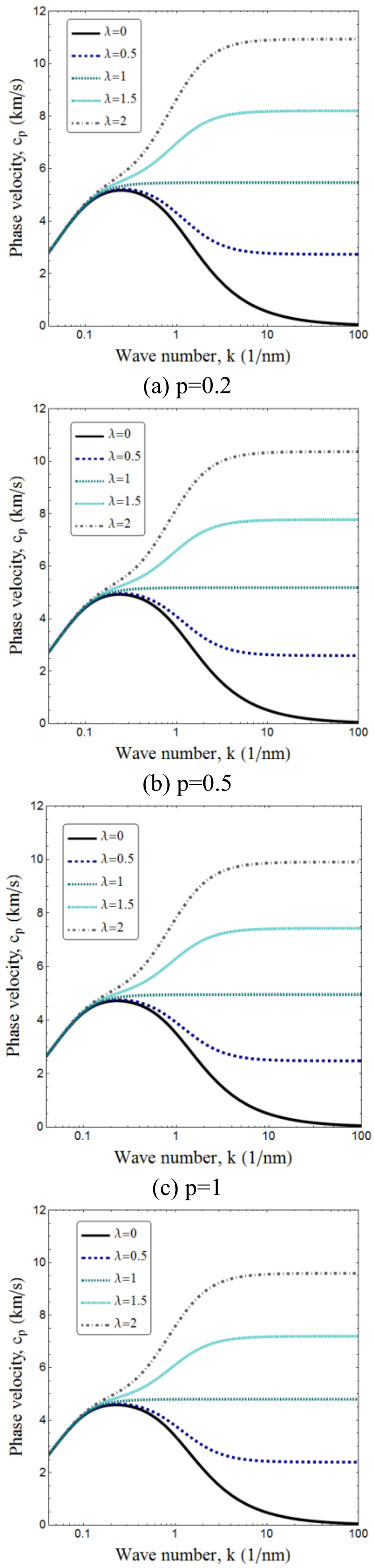
Small scale influence on the wave frequency of MEE-FG nanobeam versus wave number (k) is investigated in the framework of nonlocal strain gradient theory in Fig. 2 for various length scale parameters at p=1, V=Ω=0. It is observed that the value of wave frequency doesn't change if wave number reaches to 10×10^9 once the length scale parameter is set into zero and the nonlocal parameter has a nonzero value.

In spite of mentioned situation, the wave frequency enormously tends to infinity when the strain gradient effect is considered ($\lambda \neq 0$). Such observation is not reported in all previous works on wave propagation analysis of smart nanobeams based on nonlocal elasticity theory. Also, influence of nonlocal parameter on wave frequency is not sensible at small wave numbers. However, nonlocal parameter has a decreasing effect on the wave frequency of MEE-FG nanobeam, particularly in wave numbers larger than 0.3×10^9. This is due to stiffness-softening effect introduced by nonlocal elasticity. The influence of length scale parameter is different from the nonlocal parameter.

In fact, by increasing the amount of length scale parameter, the wave frequency rises. This is related to stiffness-hardening effect introduced by strain gradient theory. Moreover, the more the length scale parameter is, the more the wave frequency slope is; that emphasizes on the increasing effect of this parameter on the wave frequency of MEE-FG nanobeams. So, it can be concluded that a change in the value of nonlocal and length scale parameters affects significantly the wave frequency of MEE-FG nanobeams, especially at larger wave numbers. So, it is crucial to consider both nonlocal and length scale parameters for more accurate analysis of wave propagation behavior of smart nanobeams.

The variation of phase velocity of MEE-FG nanobeams versus wave number for various nonlocal parameters and gradient indices (p) is illustrated in Fig. 3 when the length scale parameter is set into 0.5 nm. It can be interpreted that once the nonlocal parameter is zero, the curve has a turning point and will tends to infinity for all gradient indices. At first, the amount of phase velocity rises with an increase in the wave number value and then it starts to diminish once arrived to its peak. It is worth mentioning that there is not a unique wave number available in which phase velocity arrives to its peak amount in it for various nonlocal parameter values. This peak wave number is moving to the left if the nonlocality increases. Also, gradient index has a reducing impact on phase velocity of MEE-FG nanobeams. Because, by increase of gradient index the portion of metal phase in structure increases. Moreover, increase of gradient index decreases the maximum value of phase velocity for every value of nonlocal parameter.

In addition, the distribution of phase velocity versus wave number for various length scale parameters and gradient indices is plotted in Fig. 4 when μ=1 nm, V=Ω=0. In this figure the effect of length scale parameter is divided into three forms. Whenever the length scale parameter is equal to nonlocal parameter, the phase velocity rises to its maximum and after that remains constant. On the other hand, this trend is completely different whether the length scale parameter is larger or smaller than nonlocal parameter. If this parameter is smaller than nonlocal parameter, after



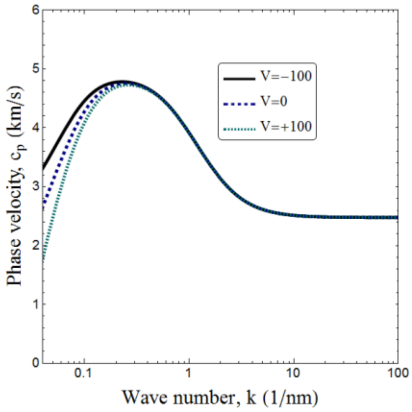
(a) p=0.2

(b) p=0.5

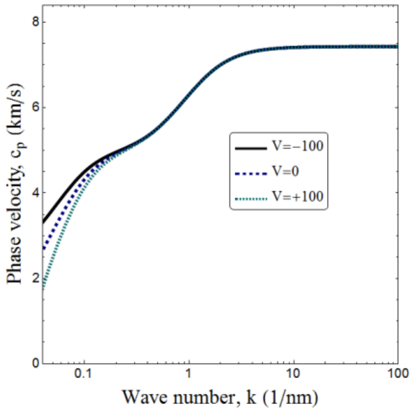
(c) p=1

(d) p=2

Fig. 4 Variation of phase velocity of MEE-FG nanobeam versus wave number for various length scale parameters and gradient indices (μ=1 nm, V=Ω=0)



(a) λ=0.5 nm

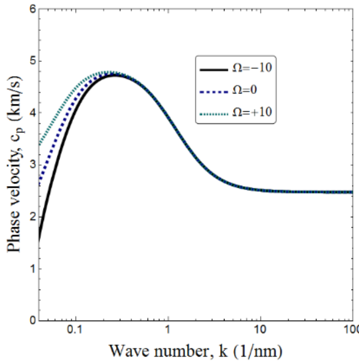


(b) λ=1.5 nm

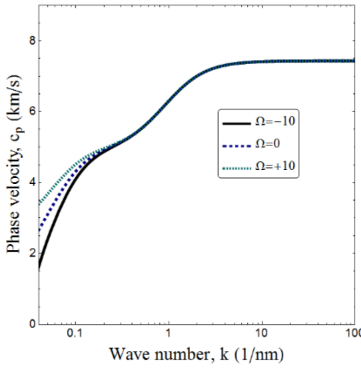
Fig. 5 Variation of phase velocity of MEE-FG nanobeam versus wave number for various electric voltages (p=1, μ=1 nm)

achieving the peak value, phase velocity decreases smoothly; and if the length scale parameter is larger than nonlocal parameter, the trend is increasingly, means, an increase in the value of wave number results in a raise in the magnitude of phase velocity. For all these case, phase velocity is not affected by larger values of wave number. Once again, it can be clearly seen that gradient index has a decreasing effect on the phase velocity of MEE-FG nanosize beams for every value of length scale parameter.

The influences of electric voltage and magnetic potential are on phase velocity of MEE-FG nanobeams are respectively plotted in Figs. 5 and 6 when μ=1 nm and p=1. In these figures to cases are considered as $\lambda>\mu$ and $\lambda<\mu$. If the length scale parameter is set into a value smaller than nonlocal parameter, the magnitude of phase velocity increases to its peak and then starts to be damped for every value of magnetic potential and electric voltage. But, when the length scale parameter is larger than nonlocal parameter, the phase velocity increases significantly with the rise of wave number until it becomes constant at large wave numbers. It can be figured out that increasing the applied voltage/ magnetic potential from their negative to positive values results in decreasing/increasing the amount of phase velocity. Such phenomena are only important at smaller wave numbers. So, if the wave number becomes greater than 0.5×10^9, phase velocity remains constant for all values of electric voltage and magnetic potential.



(a) λ=0.5 nm



(b) λ=1.5 nm

Fig. 6 Variation of phase velocity of MEE-FG nanobeam versus wave number for various magnetic potentials (p=1, μ=1 nm)

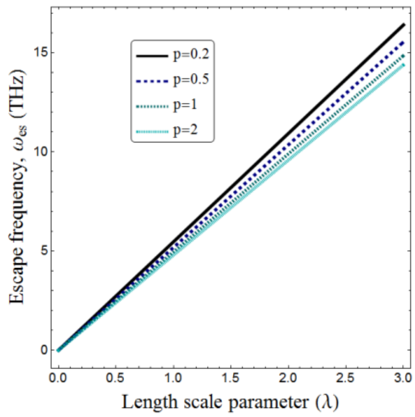


Fig. 7 Variation of escape frequency of MEE-FG nanobeam versus length scale parameter for various gradient indices (μ=1 nm)

Also, variation of escape frequency versus length scale parameter for various gradient indices is illustrated in Fig. 7 when μ=1 nm. In this figure, escape frequency is obtained by setting the wave number to infinity. It is observed that when the length scale parameter increases, the escape frequency rises significantly for every value of gradient index. The gradient index is assumed to be changed and it is clear that at a constant value of length scale parameter, the escape frequency value is greater at smaller gradient indices which shows the decreasing influence of gradient index on escape frequency.

Effect of electric voltage on the escape frequency of

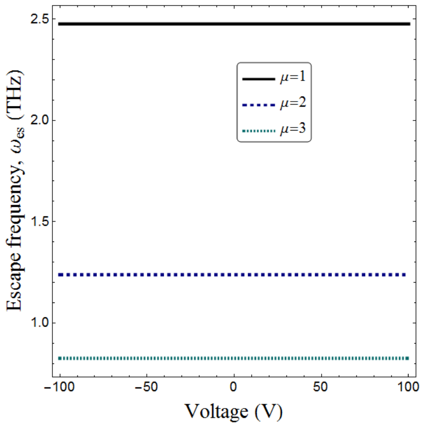


Fig. 8 Variation of escape frequency of MEE-FG nanobeam versus applied voltage for various nonlocal parameters (λ=0.5 nm, p=1)

MEE-FG nanobeam for different nonlocal parameters is depicted in Fig. 8 at λ=0.5 nm, p=1. A decreasing effect of nonlocality on the escape frequency of MEE-FG nanobeams is observed, regardless of the value of applied voltage. A usual outcome of this figure is the independency of escape frequency from the applied electric voltage. Because, escape frequencies are derived when wave number is set to infinity and in this region electric voltage has no effect on wave frequencies.

5. Conclusions

Present paper develops a nonlocal strain gradient-based magneto-electro-elastic functionally graded (MEE-FG) beam model to study the flexural waves propagating in nanobeams. To capture size-dependency of such nanobeam, two scale parameters related to nonlocal stress field and strain gradients are considered in one theory. Material properties are distributed along the thickness according to the power-law rule of mixture. Exploiting the Hamilton's principle, the nonlocal governing equations are derived and solved by implementing and analytical solution. It is deduced that effects of nonlocal and length scale parameters are significant at larger wave umbers. However, nonlocal and length scale parameters introduce stiffness-softening and stiffness-hardening effects, therefore, respectively reduces and increases the wave frequencies and phase velocities of MEE-FG nanobeam. Also, it is found that effect of electric voltage and magnetic potential is only important at smaller wave numbers. In fact, negative voltages and magnetic potentials respectively give larger and smaller phase velocities. Also, it is concluded that gradient index has a reducing impact on phase velocity and wave frequencies. Furthermore, it is noticed that escape frequency of MEE-FG nanobeams is independent of electric voltage and magnetic potential.

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Appendix

In Eqs. (57) an (58) a_{ij} and m_{ij} , $(i,j = 1,2,3,4)$ are defined as follows

$$
\begin{aligned}
a_{11} &= -(1+\lambda^2k^2)A_{xx}k^2 \\
a_{12} &= iB_{xx}(1+\lambda^2k^2)k^3 \\
a_{13} &= ik(1+\lambda^2k^2)A_{31}^e \\
a_{14} &= ik(1+\lambda^2k^2)A_{31}^m \\
a_{21} &= -iB_{xx}(1+\lambda^2k^2)k^3 \\
a_{22} &= -(1+\lambda^2k^2)D_{xx}k^4 + (1+\mu^2k^2)[(N^E+N^H)k^2] \\
a_{23} &= -E_{31}^e(1+\lambda^2k^2)k^2 \\
a_{24} &= -E_{31}^m(1+\lambda^2k^2)k^2 \\
a_{31} &= ik(1+\lambda^2k^2)A_{31}^e \\
a_{32} &= E_{31}^e(1+\lambda^2k^2)k^2 \\
a_{33} &= -(1+\lambda^2k^2)(F_{33}^e + F_{11}^ek^2) \\
a_{34} &= -(1+\lambda^2k^2)(F_{33}^m + F_{11}^mk^2) \\
a_{41} &= ik(1+\lambda^2k^2)A_{31}^m \\
a_{42} &= E_{31}^m(1+\lambda^2k^2)k^2 \\
a_{43} &= -(1+\lambda^2k^2)(F_{33}^m + F_{11}^mk^2) \\
a_{44} &= -(1+\lambda^2k^2)(X_{33}^m + X_{11}^mk^2)
\end{aligned}
\tag{A.1}
$$

and

$$
\begin{aligned}
m_{11} &= -I_0(1+\mu^2k^2) \\
m_{12} &= ikI_1(1+\mu^2k^2) \\
m_{13} &= m_{14} = 0 \\
m_{21} &= iI_1k(1+\mu^2k^2) \\
m_{22} &= -(I_0+iI_2k)(1+\mu^2k^2) \\
m_{23} &= m_{24} = 0 \\
m_{31} &= m_{32} = m_{33} = m_{34} = 0 \\
m_{41} &= m_{42} = m_{43} = m_{44} = 0
\end{aligned}
\tag{A.2}
$$