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Curvature ductility prediction of high strength concrete beams

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Abstract. From the structural safety point of view, ductility is an important parameter, a relatively high level of curvature ductility would provide to the structure an increased chance of survival against accidental impact and seismic attack. The ductility of reinforced concrete beams is very important, because it is the property that allows structures to dissipate energy in seismic zone. This paper presents a revision of an earlier formula for predicting the curvature ductility factor of unconfined HSC beams to make it simpler in the use. The new formula is compared with the earlier formula and other numerical and experimental results. The new formula regroups all parameters can affecting the curvature ductility of unconfined HSC beams and it has the same domain of application as the earlier formula.

Keywords: Eurocode 2; curvature ductility; high strength concrete; reinforcement; reinforced concrete beams

1. Introduction

High strength concrete (HSC) provides several advantages to reinforced concrete structural elements, but; it makes these elements more fragile. However, in structural elements, this type of concrete is not found alone. It should be accompanied at least with an amount of reinforcement, depending on the design code used. From here, it appears that the behavior of HSC associated with the reinforcements differs completely than the normal strength concrete (NSC). Although HSC is more fragile than NSC, HSC structural elements present more curvature ductility compared to NSC elements due to the reduced depth of the neutral axis.

In principle, a structure response to an earthquake must have a class of ductility more than normal, because the seismic energy absorption capacity of the reinforced concrete structure depends on the level of the curvature ductility of the elements (Beams, Columns, …) (Arslan and Cihanli 2010). From here it comes the particular importance attached to the curvature ductility in seismic design. Seismic codes, such as: American code (ACI-318 2014), Canadian code (CSA-A23.3 2004) and Eurocode 8 (EN 1998-1 2003) recommend a relationship between the curvature ductility and the longitudinal reinforcements in the structural elements, by the requirement of a minimum and a maximum of reinforcement percentage. Recently, Baji and Ronagh (2015) and Baji *et al*. (2016) developed a probabilistic model to compare between the different design codes such as the American (ACI-318 2011), Canadian (CSA-A23.3 2004), European (EN 1992 2004), Australian (AS-3600 2009), New Zealand (NZS-3101 2006) and the fib Model Code (MC 2010) with regards to provide the minimum curvature ductility for reinforced concrete beams. The reliability analysis results show that the considered design codes are in good agreement, when compared to each other.

Ductility in reinforced concrete beams is an important factor in their design because it allows large deflections and rotations to occur without collapse of the beam. Ductility also allows redistribution of load and bending moments in multibeam deck systems and in continuous beams. It is also important in seismic design for dissipation of energy under hysteretic loadings (Barker and Puckett 2013). The curvature ductility factor μ_φ is defined as the ratio between the ultimate curvature φ_u and the curvature at first yield φ_y (Park and Ruitong 1988)

$$\mu_\varphi = \frac{\varphi_u}{\varphi_y} \tag{1}$$

There have been numerous experimental and numerical studies performed on the curvature ductility of unconfined HSC beams. Concerning the experimental studies, Maghsoudi and Bengar (2006), Maghsoudi and Sharifi (2009), Shohana *et al.* (2012) and Mohammad *et al.* (2013) tested singly and doubly unconfined reinforced beams in order to calculate the curvature and displacement ductility factors from the moment-curvature and moment-displacement diagrams respectively. Regarding the new numerical studies, Arslan and Cihanli (2010) proposed a simplified formula with the variation of concrete strength up to 110 MPa; it takes into account the effect of the concrete strength (f_{ck}), the yield strength of steel (f_{yk}), the tension steel ratio (ρ) and the balanced steel ratio (ρ_b), the proposed formula is given as follows

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$$\mu_\varphi = 40\left(f_{ck}\right)^{-0.17}\left(f_{yk}\right)^{-0.42}\left(\frac{\rho}{\rho_b}\right)^{-1.18} \tag{2}$$

In another numerical study, Ho *et al.* (2012) proposed a formula depended on the concrete strength (f_{ck}), the yield strength of tension and compression reinforcement (f_{yt} and f_{yc}, respectively), the tension and compression steel ratio (ρ_t and ρ_c, respectively) and the degree of reinforcement (λ). The degree of reinforcement λ equals to $[(f_{yt}\ \rho_t - f_{yc}\ \rho_c)/\ f_{yt}\ \rho_{bo}]$, where ρ_{bo} is the balanced steel ratio for singly reinforced beam section. The formula of Ho *et al.* (2012) is given by the following expression

$$\mu_\varphi = 10.7\left(f_{ck}\right)^{-0.45}(\lambda)^{-1.25} \times \left(1+95.2\left(f_{ck}\right)^{-1.1}\left(\frac{f_{yc}\rho_c}{f_{yt}\rho_t}\right)^{3}\right)\left(\frac{f_{yt}}{460}\right)^{-0.25} \tag{3}$$

In 2013, Lee (2013a, b) studied the curvature ductility of unconfined reinforced HSC beams. In the first research the proposed formula contains a new parameter, which is the stress of the compression reinforcement in the ultimate state (f_{sc}), this formula is given in Eq. (4). In the second study Lee (2013b) analysed the relation (moment-curvature) and the curvature ductility factor, the obtained results are compared with the experimental results of (Jang *et al*. 2008, Hong 2011, Rashid and Mansur 2005), the results were in good agreement with the tests.

$$\mu_\varphi = \left[\left(\rho_t - \rho_c\frac{f_{sc}}{f_{yc}}\right)/\,\rho_b\right]^{-1.279}\left(f_{yt}\right)^{-0.215} \times \left[-0.6\left(f_{ck}\right)^2 + 88\,f_{ck} + 2.285\right]10^{-3} \tag{4}$$

Recently, based on the Eurocode 2 (EN 1992, 2004) Bouzid and Kassoul (2016) proposed a new formula to predict the curvature ductility factor, this formula regroups all parameters that can affect the curvature ductility of unconfined HSC beams. The proposed formula has been compared with the formula of Lee (2013a) and the experimental results of Maghsoudi and Bengar (2006), Maghsoudi and Sharifi (2009) and Rashid and Mansour (2005), this formula is given as follows

$$\mu_\varphi = \left(\frac{132997.261}{-0.0003\,f_{ck}^{\ 2} + 0.0476\,f_{ck} - 0.367}\right) \times \left(36\rho\left(\frac{\rho'}{\rho}-\frac{1}{2}\right) - \frac{1}{3}\left(\frac{\rho'}{\rho}-\frac{7}{2}\right)\right) f_{yk}^{\ -2.268}\rho^{-0.93} \tag{5}$$

Although, the above formula Eq. (5) regroups all parameters that can influence the curvature ductility of unconfined HSC beams, the application of this formula is a bit difficult due to its length and its exponents. From here it comes the idea to revise the formula and make it more convenient to use. In this study, an adjustment to the exponents of ρ and f_{yk} and a reduction to the formula given in Eq. (5) have been considered.

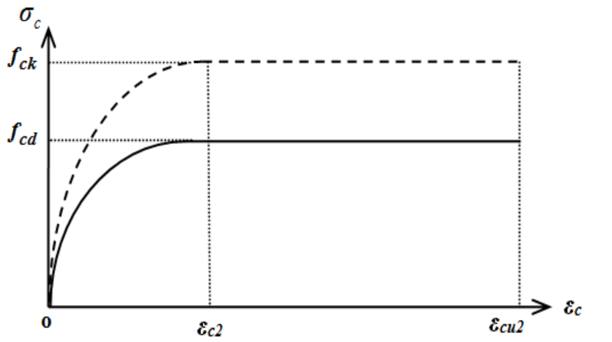

Fig. 1 Parabola-rectangle diagram for unconfined concrete under compression (EN 1992, 2004)

2. Evaluation method of the curvature ductility factor

2.1 Constitutive laws of materials

2.1.1 Concrete

The model of concrete used by Bouzid and Kassoul (2016) is the Parabola-rectangle model of the Eurocode 2 (EN 1992, 2004), (Fig. 1). The design value of the compressive strength of a cylindrical concrete specimens f_{cd} is defined by

$$f_{cd} = \frac{\alpha_{cc}\ f_{ck}}{\gamma_c} \tag{6}$$

Where, γ_c is the partial safety factor for concrete, equal to 1.5 for durable situations and 1.2 for accident situations. α_{cc} is the coefficient taking account of long term effects on the compressive strength and of unfavorable effects resulting from the way the load is applied, its value varies between 0.8 and 1.

The stress σ_c in the concrete is defined by

$$\sigma_c = \begin{cases} f_{cd}\left[1-\left(1-\dfrac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] & 0 \le \varepsilon_c \le \varepsilon_{c2} \\ f_{cd} & \varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2} \end{cases} \tag{7}$$

Where, ε_c is the compressive strain in the concrete and ε_{c2} is the strain at the maximum strength f_{cd}, and is expressed by

$$\varepsilon_{c2}(‰) = \begin{cases} 2 & f_{ck} \le 50\,MPa \\ 2.0 + 0.085\left(f_{ck} - 50\right)^{0.53} & f_{ck} > 50\,MPa \end{cases} \tag{8}$$

And, ε_{cu2} is ultimate compressive strain in the concrete, defined as

$$\varepsilon_{cu2}(‰) = \begin{cases} 3.5 & f_{ck} \le 50\,MPa \\ 2.6 + 35\left(\dfrac{90 - f_{ck}}{100}\right)^4 & f_{ck} > 50\,MPa \end{cases} \tag{9}$$

The exponent n takes the following values

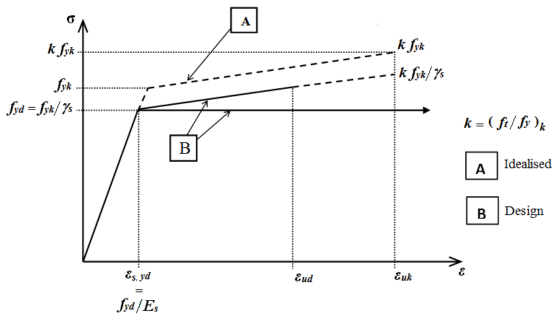

Fig. 2 Idealized and design stress-strain diagrams for reinforcing steel (EN 1992, 2004)

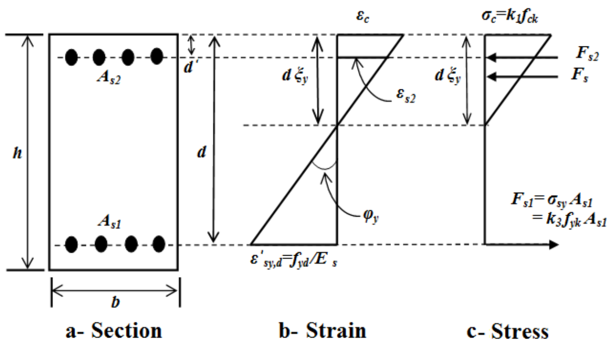

Fig. 3 Behavior of reinforced concrete beam section in flexure at first yield

$$n = \begin{cases} 2 & f_{ck} \le 50\,MPa \\ 1.4 + 23.4\left(\dfrac{90 - f_{ck}}{100}\right)^4 & f_{ck} > 50\,MPa \end{cases} \tag{10}$$

2.1.2 Steel

The stress-strain model of reinforcing steel used in the previous research of Bouzid and Kassoul (2016) is also the model of the Eurocode 2 (EN 1992, 2004), (Fig. 2).

The stress f_{yd} in reinforcing steel is equal to

$$f_{yd} = \frac{f_{yk}}{\gamma_s} \tag{11}$$

Where:

γ_s: The partial safety factor for reinforcing steel, equal to 1.15 for durable situations and 1.0 for accident situations.

$\varepsilon_{sy,d} = f_{yd}/E_s$: Elastic elongation of reinforcing steel at maximum load.

E_s: Modulus of elasticity of reinforcing steel, it equals to 200000 MPa.

$k = (f_t/f_y)_k$: Ratio of tensile strength to the yield stress, its recommended value is 10%.

ε_{uk}: Characteristic strain of reinforcement or prestressing steel at maximum load, this ultimate strain is limited by 5% for class B and 7.5% for class C.

ε_{ud}: The strain limit in reinforcing steel, its recommended value is $0{,}9\varepsilon_{uk}$.

The properties of reinforcing steel for different classes (A, B and C) can be found in the Annex C of the Eurocode 2 (EN 1992, 2004).

2.2 Evaluation method of curvature ductility factor

The study of the behavior of reinforced concrete beams in flexure requires a study in limit states (at first yield and at ultimate). In the following, a section of reinforced concrete beam in flexure is presented at these two limit states.

2.2.1 Curvature at first yield

To avoid some micro cracks in compression concrete and unacceptable deformations in tension reinforcement, the Eurocode 2 (EN 1992, 2004) limited the stress in concrete and reinforcement by $k_1 f_{ck}$ and $k_3 f_{yk}$, where; k_1 = 0.6 and k_3 = 0.8, respectively. At first yield, the cross section of a doubly reinforced concrete beam is shown in Fig. 3.

From Fig. 3(b), the curvature at first yield is expressed by

$$\varphi_y = \frac{\varepsilon_{sy,d}}{d(1-\xi_y)} \tag{12}$$

Where, ξ_y represents the height factor of the compression zone at first yield, d is the effective depth of the section and d' is the distance from extreme compression fiber to the centroid of the compression reinforcements.

From the same Fig. 3(b), the strain in the compression reinforcement ε_{s2}, is written as

$$\varepsilon_{s2} = \frac{(\xi_y d - d')}{d(1-\xi_y)}\frac{k_3 f_{yk}}{E_s} \tag{13}$$

Where, $k_3 = 0.8$ and $\gamma_s = 1$.

From the previous study of Bouzid and Kassoul (2016), the static equilibrium equation of the internal forces presented in Fig. 3(c) leads to a second order polynomial function with the variable ξ_y. If the strain in the compression reinforcement $\varepsilon_{s2} \le f_{yk}/E_s$, the acceptable solution is given by

$$\xi_y = \left(\frac{1}{2} + \frac{k_3 f_{yk}}{k_1 f_{ck}}(\rho + \rho')\right) - \sqrt{\left(\frac{1}{2} + \frac{k_3 f_{yk}}{k_1 f_{ck}}(\rho + \rho')\right)^2 - \frac{2 k_3 f_{yk}}{k_1 f_{ck}}\left(\rho + \frac{d'}{d}\rho'\right)} \tag{14}$$

Where $\rho = A_{s1}/bd$ is the ratio of tension reinforcement, $\rho' = A_{s2}/bd$ is the ratio of compression reinforcement and k_1 = 0.6.

Otherwise, if the strain in the compression reinforcement $\varepsilon_{s2} \ge f_{yk}/E_s$, the compression steel has yielded in compression, in this case the height factor of the compressed zone is given by

$$\xi_y = \frac{2 k_3 f_{yk}}{k_1 f_{ck}}(\rho - \rho') \tag{15}$$

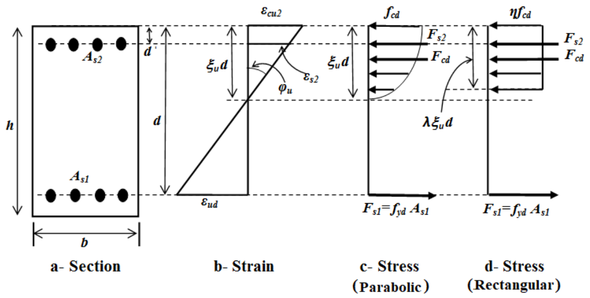

Fig. 4 Behavior of reinforced concrete beam section in flexure at the ultimate limit state

2.2.2 Curvature at the ultimate limit state

At the ultimate state, the cross section of a doubly reinforced concrete beam is shown in Fig. 4. From Fig. 4(b), the curvature at the ultimate state is expressed by

$$\varphi_u = \frac{\varepsilon_{cu2}}{\xi_u d} \tag{16}$$

Where, ξ_u is the height factor of the compression zone at the ultimate state.

As the height factor at first yield, the static equilibrium equation of the internal forces presented in Fig. 4(c) leads to a second order polynomial function with the variable ξ_u. In this case, the acceptable solution proposed by Park and Ruitong (1988) is given by the following expression

$$\xi_u = \frac{\left(f_{yd}\,\rho - \varepsilon_{cu2} E_s \rho'\right)}{2\lambda\eta\, f_{cd}} + \frac{\sqrt{\left(f_{yd}\,\rho - \varepsilon_{cu2} E_s \rho'\right)^2 + 4\lambda\eta\, f_{cd}\,\varepsilon_{cu2}\, E_s \rho' \frac{d'}{d}}}{2\lambda\eta\, f_{cd}} \tag{17}$$

According to the Eurocode 2 (EN 1992, 2004), the factors λ and η are expressed by the following expressions

$$\lambda = \begin{cases} 0.8 & f_{ck} \le 50\ MPa \\ 0.8 - \dfrac{f_{ck} - 50}{400} & 50\ MPa < f_{ck} \le 90\ MPa \end{cases} \tag{18}$$

And

$$\eta = \begin{cases} 1.0 & f_{ck} \le 50\ MPa \\ 1.0 - \dfrac{f_{ck} - 50}{200} & 50\ MPa < f_{ck} \le 90\ MPa \end{cases} \tag{19}$$

3. Proposed Formula of Bouzid and Kassoul (2016)

The parametric study conducted by Bouzid and Kassoul (2016) showed that the curvature ductility factor can be expressed by the following function

$$\mu_\varphi = A\rho^B \tag{20}$$

Where, A and B are coefficients can be determined based on the parameters f_{ck}, ρ, ρ'/ρ and f_{yk}.

Table 1 Modification of the exponent of ρ

ρ	$\rho^{-0.93}$	ρ^{-1}	$\rho^{-0.93} / \rho^{-1}$	$0.777\,\rho^{-1}$	Errors ($\rho^{-0.93}$; $0.777\rho^{-1}$) (%)
0.01	72.44	100.00	0.72	77.66	**6.72**
0.015	49.69	66.67	0.75	51.78	**4.04**
0.02	38.02	50.00	0.76	38.83	**2.08**
0.025	30.90	40.00	0.77	31.07	**0.54**
0.03	26.08	33.33	0.78	25.89	**0.73**
0.035	22.60	28.57	0.79	22.19	**1.83**
0.04	19.96	25.00	0.80	19.42	**2.78**
0.045	17.89	22.22	0.80	17.26	**3.64**
0.05	16.22	20.00	0.81	15.53	**4.40**
Average ($\rho^{-0.93}$ / ρ^{-1})			**0.777**		

To facilitate the determination of a general formula, the coefficient B is fixed by the value -0.93 and the coefficient A is written according to the parameters studied, so

$$A = ?\ f_{ck}, (\rho'/\rho), f_{yk}\ ) \tag{21}$$

Or

$$A = \alpha(f_{ck}) \times \beta(f_{yk}) \times \gamma(\rho'/\rho) \tag{22}$$

Where, α (f_{ck}), β (f_{yk}) and γ (ρ'/ρ) are functions of the variables f_{ck}, f_{yk} and (ρ'/ρ), respectively.

In the case of high strength concrete, the coefficient A is obtained as follows

$$A = \left(\frac{132997.261}{-0.0003\, f_{ck}^{\ 2} + 0.0476\, f_{ck} - 0.367}\right) \times \left(36\rho\left(\frac{\rho'}{\rho} - \frac{1}{2}\right) - \frac{1}{3}\left(\frac{\rho'}{\rho} - \frac{7}{2}\right)\right) f_{yk}^{\ -2.268} \tag{23}$$

The proposed formula is applicable for unconfined reinforced concrete beams having a concrete strength f_{ck} from 50 up to 90 MPa, yield strength of steel f_{yk} from 400 to 600 MPa, a percentage of tension reinforcement $1 \le \rho \le 5\%$ and ratio of compression reinforcement ρ' from 0,25 ρ up to ρ.

4. Adjustment of the earlier formula

In this section, an adjustment to the exponents of the parameters (tension steel ratio (ρ) and the yield strength of steel (f_{yk})) is conducted. Also, another form of the function α (f_{ck}) which represents the effect of the concrete strength is proposed.

4.1 Effect of tension steel ratio

Table 1 shows the various values of the tension steel ratio ρ with the exponents -0.93 and -1.

From this Table, it can be seen that the average of the ratios ($\rho^{-0.93}$ / ρ^{-1}) is equal to 0.777 and the error calculated between $\rho^{-0.93}$ and $0.777\rho^{-1}$ does not exceed 7%, so we can

Table 2 Modification of the exponent of f_{yk}

f_{yk} (MPa)	$f_{yk}^{-2.268}$ (E-7)	f_{yk}^{-2} (E-6)	$f_{yk}^{-2.268}/f_{yk}^{-2}$ (E-1)	$0.190\,f_{yk}^{-2}$ (E-7)	Errors ($f_{yk}^{-2.268}$; $0.777\,f_{yk}^{-2}$) (%)
400	12.5	6.25	2.01	11.9	**5.80**
450	9.61	4.94	1.95	9.37	**2.51**
500	7.56	4.00	1.89	7.59	**0.35**
550	6.09	3.31	1.84	6.27	**2.86**
600	5.00	2.78	1.80	5.27	**5.10**
Average ($f_{yk}^{-2.268}/f_{yk}^{-2}$)			**0.190**		

Table 3 New and old form of the function $\alpha(f_{ck})$.

f_{ck} (MPa)	$-0.0003f_{ck}^2 + 0.0424\,f_{ck} - 0.367$	$-0.0003(f_{ck} - 9.25)(f_{ck} - 132)$	Errors (%)
51	1.015	1.015	**0.06**
55	1.058	1.057	**0.06**
60	1.097	1.096	**0.07**
65	1.122	1.121	**0.08**
70	1.131	1.130	**0.09**
75	1.126	1.124	**0.10**
80	1.105	1.104	**0.12**
85	1.070	1.068	**0.13**
90	1.019	1.017	**0.15**

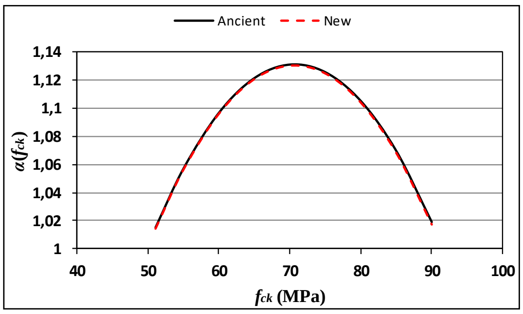

Fig. 5 New and old form of the function $\alpha(f_{ck})$

replace $\rho^{-0,93}$ by 0.777 ρ^{-1}.

4.2 Effect of yield strength of steel f_{yk}

Concerning the yield strength of steel, the earlier formula uses an exponent equal to -2.268, in the Table 2 we tried to adjust this exponent to -2.

From this Table, it can be noticed that the average of the ratios ($f_{yk}^{-2.268}/f_{yk}^{-2}$) is equal to 0.190 and the error calculated between $f_{yk}^{-2.268}$ and 0.190 f_{yk}^{-2} does not exceed 5.8%. Consequently, we can replace $f_{yk}^{-2.268}$ by 0.190 f_{yk}^{-2}.

4.3 Effect of concrete strength f_{ck}

The function which represents the effect of the concrete strength f_{ck} on the curvature ductility $\alpha(f_{ck})$ given by the following relation

$$\alpha(f_{ck}) = -0.0003 f_{ck}^{\,2} + 0.0424 f_{ck} - 0.367 \qquad (24)$$

The function $\alpha(f_{ck})$ can also be written as follows

$$\alpha(f_{ck}) = -0.0003\left(f_{ck}^{\,2} - 141.33 f_{ck} + 1223.33\right) \qquad (25)$$

Or

$$\alpha(f_{ck}) \approx -0.0003\left(f_{ck} - 9.25\right)\left(f_{ck} - 132\right) \qquad (26)$$

Table 3 summarizes the errors calculated between the earlier and the new form of the function $\alpha(f_{ck})$ given in Eq. (24) and Eq. (26). It can be seen that these errors do not exceed 0.15%, where it exists a coincidence between the two forms as shown in Fig. 5.

4.4 Final formula

The earlier formula is given in Eq. (5) as follows

$$\mu_\varphi = \left(\frac{132997.261}{-0.0003\,f_{ck}^{\,2} + 0.0476\,f_{ck} - 0.367}\right) \times \left(36\rho\left(\frac{\rho'}{\rho} - \frac{1}{2}\right) - \frac{1}{3}\left(\frac{\rho'}{\rho} - \frac{7}{2}\right)\right) f_{yk}^{\,-2.268} \rho^{-0.93} \qquad (27)$$

Replacing: the function $\alpha(f_{ck})$ by its new form, the factor of the yield strength of steel $f_{yk}^{-2.268}$ by 0,190 f_{yk}^{-2} and the factor of tension steel ratio $\rho^{-0.93}$ by $0.777\rho^{-1}$ in Eq. (10), we obtain the following formula

$$\mu_\varphi = \left(\frac{132997.261\left(36\rho\left(\frac{\rho'}{\rho} - \frac{1}{2}\right) - \frac{1}{3}\left(\frac{\rho'}{\rho} - \frac{7}{2}\right)\right)}{-0.0003\left(f_{ck} - 9.25\right)\left(f_{ck} - 132\right)}\right) \times \left(0.190 f_{yk}^{\,-2}\right)\left(0.777\,\rho^{-1}\right) \qquad (28)$$

The final form of the curvature ductility factor is obtained as follows

$$\mu_\varphi \approx \frac{-6.53\times 10^7 \left(36\rho\left(\frac{\rho'}{\rho} - \frac{1}{2}\right) - \frac{1}{3}\left(\frac{\rho'}{\rho} - \frac{7}{2}\right)\right)}{\left(f_{ck} - 9.25\right)\left(f_{ck} - 132\right) f_{yk}^{\,2}}\,\rho^{-1} \qquad (29)$$

5. Comparison between the new formula and the numerical results

The new formula given in Eq. (29) must be compared firstly with the earlier formula Eq. (5). The mean values (MV) and standard deviations (SD) of 540 ratios calculated between these two formulas are presented in Table 4. From this table, we can see that:

• The mean values (MV) of the ratios Eq. (5)/Eq. (29) are between 0.95 and 1.06;

• The standard deviations (SD) of the ratios Eq. (5) /Eq. (29) are always equal to 0.04.

As a conclusion, we can say that the new formula is in good agreement with the earlier formula.

In the second station, the new formula Eq. (29) has been compared with the numerical results of the method

Table 4 Comparison between the new and the earlier formula

f_{yk} (MPa)	$\rho'/\rho = 0.25$		$\rho'/\rho = 0.5$		$\rho'/\rho = 0.75$		$\rho'/\rho = 1$	
	MV	SD	MV	SD	MV	SD	MV	SD
400	1.06	0.04	1.06	0.04	1.06	0.04	1.06	0.04
500	0.99	0.04	0.99	0.04	0.99	0.04	0.99	0.04
600	0.95	0.04	0.95	0.04	0.95	0.04	0.95	0.04

Table 5 Comparison between the new formula and the EC2 method

f_{yk} (MPa)	$\rho'/\rho = 0.25$		$\rho'/\rho = 0.5$		$\rho'/\rho = 0.75$		$\rho'/\rho = 1$	
	MV	SD	MV	SD	MV	SD	MV	SD
400	0.95	0.05	0.94	0.04	0.95	0.05	0.98	0.05
500	1.04	0.08	0.96	0.02	0.93	0.03	0.93	0.03
600	1.13	0.11	1.01	0.04	0.94	0.02	0.91	0.03

Table 6 Comparison between the new formula and the formula of Lee (2013a) Eq. (4)

f_{yk} (MPa)	$\rho'/\rho = 0.25$		$\rho'/\rho = 0.5$		$\rho'/\rho = 0.75$		$\rho'/\rho = 1$	
	MV	SD	MV	SD	MV	SD	MV	SD
400	1.06	0.11	1.07	0.10	1.10	0.10	1.14	0.09
500	1.06	0.08	1.02	0.07	1.01	0.07	1.01	0.07
600	1.09	0.08	1.03	0.05	0.98	0.05	0.96	0.05

Table 7 Comparison between the new formula Eq. (29) and the experimental results of Maghsoudi and Bengar (2006), Maghsoudi and Sharifi (2009)

N° Beam	f_{ck} (MPa)	d (mm)	d' (mm)	ρ (%)	ρ' (%)	ρ'/ρ	f_{yk} (MPa)	Curvature ductility factor μ_{φ}			
								Experimental results	ACI	CSA	New formula
1	73.65	256	40	4.103	2.0515	0.5	400	4.33	2.75	3.51	**2.65**
2	66.81	266	40	4.773	2.3865	0.5	400	-	2.07	2.65	**2.28**
3	77.72	258	42	5.851	2.9255	0.5	400	3.38	1.76	2.18	**1.88**
4	56.31	254	42	0.61	0.61	1	400	11.84	9.89	11.91	**17.72**
5	72.98	256	40	4.81	0.61	0.13	400	3.2	1.84	2.29	**1.08**
6	63.48	250	47	1.25	0.61	0.488	400	6.84	6.68	8.13	**8.77**
7	73.42	256	40	4.81	1.23	0.26	400	3.29	2.15	2.72	**1.49**
8	63.21	251	42	2.03	1.01	0.4975	400	5.75	5.53	6.87	**5.41**
9	72.98	256	40	4.81	2.41	0.5	400	4.33	2.77	3.52	**2.26**
10	71.45	250	47	2.51	1.24	0.494	400	5.6	4.75	5.87	**4.30**

presented in section 2. The mean values (MV) and standard deviations (SD) of 600 ratios Eq. (29)/$\mu_{\varphi,numérique}$ are presented in Table 5. The results of this table showed that:

• The mean values of the ratios Eq. (29)/$\mu_{\varphi,numérique}$ are between 0.91 and 1.13;

• The standard deviations of the ratios Eq. (29)/$\mu_{\varphi,numérique}$ are between 0.02 and 0.11.

This conclusion shows that there is a good agreement between the new formula and the numerical results of the Eurocode 2.

In the last station, the new formula Eq. (29) has been compared with the formula of Lee (2013a) Eq. (4). The

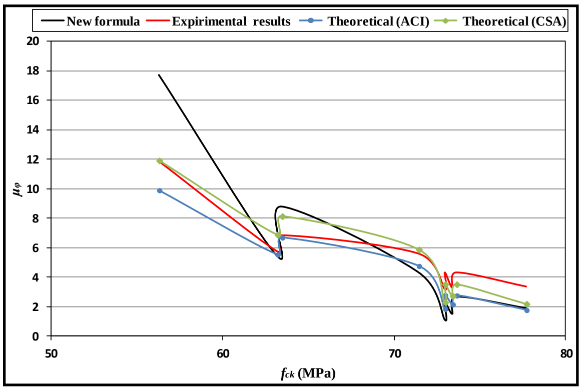

Fig. 6 Comparison between of the new formula Eq. (29) and experimental results

mean values and standard deviations of 600 ratios calculated between these two formulas are presented in Table 6. From this table, we can be noticed that:

• The mean values of the ratios Eq. (29)/Eq. (4) are between 0.96 and 1.14;

• The standard deviations of the ratios Eq. (29)/Eq. (4) are between 0.05 and 0.11.

These remarks show that there is reliability between the new formula and the prediction of Lee (2013a).

6. Comparison between the proposed formula and experimental results

In the same context, Table 7 that found above presents a comparison between the results obtained experimentally by Maghsoudi and Bengar (2006) and Maghsoudi and Sharifi (2009) and the results obtained by the new formula. From this table, it can be noticed that the results of the new formula are close to the experimental results, as well as with theoretical results of the ACI and CSA codes. Fig. 6 confirms this convergence, where there is a harmonization between the results of the new formula and the experimental results when concrete strength is above 55 MPa.

7. Conclusions

This paper presents an adjustment of the formula proposed by Bouzid and Kassoul (2016). The new formula given in Eq. (29) is simpler than the previous formula shown in Eq. (5) with less length and adjusted exponents. The comparison between the new and the earlier formula has shown a good agreement between these two expressions. Furthermore, the new formula is validated by the Eurocode 2 numerical results and other numerical and experimental results.

As the earlier expression given in Eq. (5), the new formula regroups all parameters that can affect the curvature ductility of unconfined HSC beams. This formula is applicable for beams having a concrete strength f_{ck} from 50 up to 90 MPa, yield strength of steel f_{yk} from 400 to 600 MPa, a percentage of tension reinforcement $1 \leq \rho \leq 5\%$ and a ratio of compression reinforcement ρ' from 0.25ρ up to ρ.

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