Structural Engineering and Mechanics, *Vol. 66, No. 2 (2018) 151-160*
DOI: https://doi.org/10.12989/sem.2018.66.2.151

Strain energy-based fatigue life prediction under variable amplitude loadings

Shun-Peng Zhu[1, 2a], Peng Yue[1, 3], José Correia[4b], Sergio Blasón[5c], Abílio De Jesus[4d] and Qingyuan Wang*[2, 6]

[1]Institute of Reliability Engineering, School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China
[2]Key Laboratory of Deep Earth Science and Engineering, Ministry of Education, Sichuan University, Chengdu 610065, China
[3]Bazhong Professional Technology College, Bazhong 636000, China
[4]INEGI, Faculty of Engineering, University of Porto, Porto 4200-465, Portugal
[5]Department of Construction and Manufacturing Engineering, University of Oviedo, Gijón 33203, Spain
[6]School of Architecture and Civil Engineering, Chengdu University, Chengdu 610106, China

(Received November 2, 2017, Revised January 10, 2018, Accepted February 14, 2018)

Abstract. With the aim to evaluate the fatigue damage accumulation and predict the residual life of engineering components under variable amplitude loadings, this paper proposed a new strain energy-based damage accumulation model by considering both effects of mean stress and load interaction on fatigue life in a low cycle fatigue (LCF) regime. Moreover, an integrated procedure is elaborated for facilitating its application based on S-N curve and loading conditions. Eight experimental datasets of aluminum alloys and steels are utilized for model validation and comparison. Through comparing experimental results with model predictions by the proposed, Miner's rule, damaged stress model (DSM) and damaged energy model (DEM), results show that the proposed one provides more accurate predictions than others, which can be extended for further application under multi-level stress loadings.

Keywords: fatigue; life prediction; mean stress; load interaction; S-N curve

1. Introduction

For fatigue critical parts, like aircraft engine components, various theories and models have been developed for fatigue life prediction and reliability assessment in practice, from perspective of damage accumulation, remaining usage life prediction and reliability assessment (Wang *et al.* 2016, Hu *et al.* 2012, Hu *et al.* 2016, Zhu *et al.* 2017, Zhu *et al.* 2017, Zhu *et al.* 2017, Zhu *et al.* 2017, Wu *et al.* 2016). However, fatigue damage estimation and/or life prediction is still one of the difficult and hot spots that need to be resolved for structural design and integrity assessment under variable amplitude loadings (Correia *et al.* 2017, Zhu *et al.* 2016, Zhu *et al.* 2012, Sanches *et al.* 2015, Wang *et al.* 2017). In general, fatigue damage accumulation methods can be classified roughly into two types, namely linear and nonlinear damage accumulation theories (Fatemi and Yang 1998).

Among them, linear damage accumulation theory, also named as Palmgren-Miner's rule (Miner 1945) (just the Miner rule for short in this analysis) has been commonly used in engineering application due to its simplicity (Correia *et al.* 2015). On the other hand, various nonlinear damage accumulation theories have been presented to address different fatigue damage characteristics from uniaxial to multiaxial fatigue loadings (Ince and Glinka 2014, Huffman and Beckman 2013, Kujawski 2014), including damage curve method (Manson and Halford 1981, Rege and Pavlou 2017) and methods based on ductility exhaustion (Pavlou 2002, Zhu *et al.* 2013), strain energy (Halford 1966, Meneghetti 2007, Zhu *et al.* 2012, Huffman *et al.* 2017), continuum damage mechanics (CDM) (Chaboche and Lesne 1988, Shang and Yao 1998, Besson 2010, Dattoma *et al.* 2006), physical properties degradation (Ye and Wang 2001, Łagoda *et al*. 2005, Peng *et al.* 2016) and loading interaction (Freudenthal and Heller 1959, Dai *et al.* 2013, Yu *et al.* 2017b). Recently, Dattoma *et al.* (2006) presented a nonlinear CDM model for evaluating damage evolution of metals under complex load sequences according to thermodynamic framework. Mesmacque *et al.* (2005) introduced a concept of damage stress and developed a damaged stress model (DSM) to consider the effect of loading history under variable amplitude loadings. In order to characterize the stochastic nature of fatigue damage or fatigue life, a statistical approach of consistent damage criterion is firstly proposed under variable amplitude loadings (Sun *et al.* 2014). Through exploring physical property degradation and load interaction effects, a modified fatigue damage accumulation model was presented under two stage loadings (Giancane *et*

*Corresponding author, Professor
E-mail: wangqy@scu.edu.cn
[a]Associate Professor
E-mail: zspeng2007@uestc.edu.cn
[b]Ph.D.
E-mail: jacorreia@inegi.up.pt
[c]Ph.D. Student
E-mail: blasonsergio@uniovi.es
[d]Professor
E-mail: ajesus@fe.up.pt

http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

al. 2010). In consideration of detrimental effect of pre-hardening on fatigue life in strain-controlled fatigue tests, a conservative method for modeling fatigue damage accumulation under variable amplitude loadings was put forward by Taheri *et al*. (2013).

From the damage accumulation point of view, the effects of mean stress and load interaction have shown significant influences on fatigue life under variable amplitude loadings, which should be taken into account during the development of improved analysis procedures for fatigue of engineering structures. In this regard, a new nonlinear damage accumulation model, considering both effects of load interaction and mean stress on fatigue life, is proposed by combining with strain energy-based criterion. The rest of this paper is organized as follows. Section 2 introduces several commonly-used theories for fatigue damage modeling, including linear and nonlinear damage accumulation models. Section 3 develops a new strain energy-based damage accumulation model for fatigue life prediction under variable amplitude loadings, as well as an integrated procedure for facilitating its application based on *S-N* curve and loading conditions. Then the proposed model is verified and compared with three other models using eight experimental datasets in Section 4; Finally, conclusion has been drawn in Section 5.

2. Linear and nonlinear damage accumulation theories

One of the commonly-used cumulative damage concepts is the linear damage accumulation rule proposed by Palmgren and Miner (Miner 1945), which has been practically adopted and recommended within several international standards/codes for mechanical design/assessment due to its simplicity

$$\boldsymbol{D}=\sum_{i=0}^{k}\frac{\boldsymbol{n_i}}{\boldsymbol{N_{fi}}} \tag{1}$$

where n_i is the number of loading cycles at the stress amplitude σ_{ai}; N_{fi} is the number of cycles to failure at σ_{ai}; D is the cumulative fatigue damage; k is the number of loading stress levels. Once the total cumulative damage D under multi-levels of loadings reaches the critical damage D_c, generally $D_c=1$, failure occurs.

However, it should be pointed out that the Miner rule ignored the effects of load interaction, load sequence and the damage caused by the stress below the fatigue limit (Zhu *et al*. 2011). Accordingly, several modified versions have been presented to address these issues for fatigue life prediction under complex loading conditions (Mesmacque *et al.* 2005, Zhu *et al*. 2011, Blason *et al.* 2016, Lv *et al.* 2014, Yu *et al.* 2017a, Correia *et al.* 2017, Correia 2013, Fernández-Canteli *et al.* 2014, Zhu *et al.* 2017). In general, for a specimen under multi-level stress loadings, its crack formation life relates to the maximum applied stress, and the crack length at the end of one stress level generally depends on the current applied stress level and the subsequent one. Note from (Zhu *et al*. 2011, Gathercole *et al.* 1994) that Miner expectations (namely the total cumulative damage) for increasing (low-high) loadings are larger than one due to the strengthen effect, and smaller than one for decreasing (high-low) loadings due to the acceleration effect. Morrow (1986) indicated that initial micro-cracks might be introduced by the large strain during the earlier life stage and propagate under small number of loading cycles, which may cause more damage sooner than that under constant amplitude loadings. Recently, Zhu *et al*. (2011) pointed out that the influence of load sequence on fatigue life can be explained well by the indeterminacy of fatigue limit, in particular the fatigue limit is reduced due to the high-amplitude loads under high-low loadings and strengthened by the low-amplitude loads under low-high loadings. For the load interaction effect accompanied by the load sequence effect under variable amplitude loadings, it often leads to the indeterminacy of damage accumulation process, and the larger the difference of loading stress amplitudes, the stronger damage interaction effect it results. Accordingly, several researchers (Freudenthal and Heller 1959, Corten and Dolan 1956, Xu *et al.* 2012) introduced an exponent parameter $\alpha_{i-1,i}$ to take into account the load interaction effect by using a form of a minimum value of load ratio

$$\boldsymbol{\alpha_{i-1,i}}=\left(\frac{\boldsymbol{N_{f(i-1)}}}{\boldsymbol{N_{fi}}}\right)^{0.4*min\left\{\frac{\sigma_{i-1}}{\sigma_i},\frac{\sigma_i}{\sigma_{i-1}}\right\}} \tag{2}$$

Similar to the Corten-Dolan's model (Corten and Dolan 1956), Morrow's plastic work interaction rule (Gathercole *et al.* 1994), Freudenthal-Heller's model (Freudenthal and Heller 1959) and Carpinteri's model (Carpinteri *et al.* 2003), Lv *et al.* (2014) defined the fatigue damage D_n by introducing an interaction factor (σ_i/σ_{max}) to model the physical property degradation of the material as

$$\boldsymbol{D_n}=-\frac{\boldsymbol{D_{(N_f-1)}}}{\boldsymbol{lnN_f}}\boldsymbol{ln}\left(\boldsymbol{1}-\frac{\boldsymbol{n}}{\boldsymbol{N_f}}\right)\cdot\frac{\boldsymbol{\sigma_i}}{\boldsymbol{\sigma_{max}}} \tag{3}$$

Recently, from a nonlinear damage accumulation point of view, a nonlinear damage indicator, namely, damaged stress model (DSM), was presented by Mesmacque *et al.* (2005) to define fatigue damage from the Wöhler curve (also known as *S-N* curve), which takes into account the effects of loading history under variable amplitude loadings and gives a satisfactory estimation comparing with the Miner rule

$$\boldsymbol{D_i}=\frac{\boldsymbol{\sigma_{edi}}-\boldsymbol{\sigma_i}}{\boldsymbol{\sigma_u}-\boldsymbol{\sigma_i}} \tag{4}$$

where σ_{edi} is the stress of damage; σ_i is the applied stress; σ_u is the ultimate tensile stress.

Since the usage of DSM in Eq. (4) for fatigue damage estimation requires only the fully known *S-N* curve of materials, and it includes the loading history effects, Djebli *et al.* (2013) extended this concept for strain energy density-based fatigue analysis, and presented a damaged energy model (DEM) for fatigue damage estimation and life prediction by

$$\boldsymbol{D_i}=\frac{\boldsymbol{W_{edi}}-\boldsymbol{W_i}}{\boldsymbol{W_u}-\boldsymbol{W_i}} \tag{5}$$

where W_{edi} is the strain energy due to the stress damage; W_i is the strain energy due to the applied stress; W_u is the strain energy due to the ultimate tensile stress of the material.

According to the boundary conditions of the damage indicator in Eq. (5), W_{edi} is equal to W_i due to the applied stress at the first loading cycle, and equal to W_u due to the ultimate tensile stress at the final loading cycle. Note that the damage indicator ranges from 0 to one at the rupture point.

From the concept of equivalent damage, fatigue damage introduced by the *i*th level damage stress can be transferred into the $(i+1)th$ level damage stress by the following relation

$$\boldsymbol{D_i = \frac{W_{edi} - W_i}{W_u - W_i} = \frac{W_{equiv} - W_{i+1}}{W_u - W_{i+1}}} \tag{6}$$

where W_{equiv} is the strain energy due to the equivalent damage from the *i*th level damage stress to the $(i+1)th$ one; W_{i+1} is the strain energy due to the $(i+1)th$ level of applied stress.

Comparing with the Miner rule, both DSM and DEM are derived from the *S-N* curve, then the damage corresponding to the ith damage stress level can be obtained by using this curve. Note from (Garcia *et al.* 2005, Siriwardane *et al.* 2008, Aid *et al.* 2012) that the DSM can be utilized for damage estimation and life prediction of several materials under tension-compression, bending, torsion, and multiaxial loadings. In practical engineering, the effects of mean stress and load interaction under cyclic loadings on fatigue damage accumulation and life prediction of components should not be ignored (Yuan *et al.* 2016, Zhu *et al.* 2016, Zhu *et al.* 2017, Ince and Glinka 2011, Ince 2017a, Ince 2017b). According to this, a strain energy-based damage accumulation model is proposed to simultaneously consider both effects of loading history and mean stress for fatigue life prediction in Section 3.

3. Proposed strain energy-based damage accumulation model

As is well known, the effect of mean stress under cyclic loadings is an important factor during fatigue damage estimation and life prediction. To overcome this issue, Ince and Glinka (2011) extended the Morrow and SWT models for mean stress corrections in fatigue life prediction of metals. In particular, a concept of equivalent elastic strain amplitude $\varepsilon_{ea,eq}$ is presented to consider the effect of mean stress on fatigue life as

$$\boldsymbol{\varepsilon_{ea,eq} = \frac{\sigma_{max}}{\sigma'_f}\frac{\Delta\varepsilon_e}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^{2b}} \tag{7}$$

where σ'_f is the fatigue strength coefficient; b is the fatigue strength exponent; E is the Young's modulus; $\Delta\varepsilon_e$ is the elastic strain range; σ_{max} is the maximum stress.
Through considering the strain energy density as a damage parameter for fatigue analysis, an equivalent elastic strain energy density $W_{e,eq}$ can be derived based on Eq. (7) under fully reversed loadings

$$\boldsymbol{W_{e,eq} = \frac{1}{2}\varepsilon_{ea,eq}\sigma_a = \frac{1}{2}\frac{\sigma_{max}}{\sigma'_f}\varepsilon_{ea}\sigma_a = \frac{\left(\sigma'_f\right)^2}{2E}\left(2N_f\right)^{3b}} \tag{8}$$

Then the total strain energy density W_t with mean stress corrections is given by

$$\boldsymbol{W_t = W_{e,eq} + W_p = \frac{1}{2}\frac{\sigma_{max}}{\sigma'_f}\varepsilon_{ea}\sigma_a + \frac{1-n'}{4(1+n')}\sigma_a\varepsilon_{pa}} \tag{9}$$

where W_p is the plastic strain energy density, n' is the cyclic strain hardening exponent, ε_{pa} is the plastic strain amplitude.

Combining Eq. (6) with Eq. (9) and replacing W_{edi} with $W_{t,i}$ in Eq. (6), leads to

$$\boldsymbol{D_i = \frac{W_{t,i} - W_i}{W_u - W_i}} \tag{10}$$

Under multi-level stress loadings, the strain energy due to equivalent damage stress can be calculated by

$$\boldsymbol{W_{equiv} = \frac{W_u - W_{i+1}}{W_u - W_i}\left(W_{t,i} - W_i\right) + W_{i+1}} \tag{11}$$

The equivalent strain energy density W_{equiv} in Eq. (11) is expressed by a function of the number of cycles to failure N_f

$$W_{equiv} = f\left(2N_f\right) \tag{12}$$

Combining Eq. (10) with Eq. (12), a damage parameter φ is defined as

$$\varphi = \frac{W_u - W_{i+1}}{W_u - W_i} = \frac{f\left(2N_{f(i+1)}\right)}{f\left(2N_{f(i)}\right)} \tag{13}$$

For the case of high-low loadings, $\varphi > 1$, and $\varphi < 1$ for low-high loadings. To take into account the load interaction effect, an interaction factor $\gamma_i = \sigma_{i+1}/\sigma_i$ can be introduced to modify the equivalent strain energy density as

$$\boldsymbol{W_{equiv} = \frac{W_u - W_{i+1}}{W_u - W_i}\left(W_{t,i} - W_i\right)\frac{\sigma_{i+1}}{\sigma_i} + W_{i+1}} \tag{14}$$

Note that the interaction factor $\gamma_i < 1$ for high-low loadings, and $\gamma_i > 1$ for low-high loadings. Then the calculation result would close to 1 after introducing the interaction factor. Using the damage indicator in Eq. (10) and Eq. (14) as well as the *S-N* curve, cumulative damage

Procedure 1 Strain energy-based sequential law for damage modeling and life prediction	
Input: n_i number of loading cycles at stress level σ_i, for $i = 1,2,3,\cdots,k$; *S-N* curve; failure criterion $D = 1$	
Obtainment of $W_t - N_f$ curve (W_t: strain energy density, N_f: number of cycles at failure)	
1:	Calculate W_t using Eq. (9)
2:	Identify the $W_t - N_f$ curve parameters
Strain energy-based damage accumulation and life prediction	
3:	$i = 1$
4:	N_1: number of cycles to failure at σ_1 from $W_t - N_f$ curve
5:	$N_{1R} = N_1 - n_1$: residual life
6:	$W_{t,eq(1)}$: damage energy for N_{1R} loading cycles from $W_t - N_f$ curve
7:	Damage indicator $D_1 = \frac{W_{t,eq(1)} - W_1}{W_u - W_1}$
8:	$D = D_1$
9:	While the failure criterion is not met do
10:	$i \leftarrow i + 1$
11:	Damage transformation from previous step to next step $D_1 = D_i' = \frac{W'_{t,eq(i)} - W_i}{W_u - W_i} \Rightarrow W'_{t,eq(i)}$
12:	$W'_{t,eq(i)}$ associated number of loading cycles $N'_{(i)R}$ from $W_t - N_f$ curve
13:	$N_{(i)R} = N'_{(i)R} - n_i$: residual life
14:	$W_{t,eq(i)}$: damage energy for $N_{(i)R}$ number of loading cycles from $W_t - N_f$ curve
15:	$D_i = \frac{W_{t,eq(i)} - W_i}{W_u - W_i} = D$
16:	end while
Output: Fatigue damage accumulation and life prediction	

and fatigue life can be calculated by following the algorithm outlined in Procedure 1.

4. Model verification and comparison

In this section, experimental data of eight materials under two-stress level loadings were utilized for model validation and comparison, including Al 2024 T42 (Pavlou 2002, Mesmacque *et al.* 2005, Dattoma *et al.* 2006, Garcia *et al.* 2005), 300CVM steel (Manson *et al.* 1967), 16Mn steel (Shang and Yao 1998), 30NiCrMoV12 steel (Dattoma *et al.* 2006), welded Aluminum alloy joint of ENAW6005 (Tian *et al*. 2012), short carbon fiber reinforeced poly-ether-ether-ketone (CFR PEEK) (Noguchi *et al.* 1995), type 316 stainless steel (Kamaya and Kawakubo 2015) and 45 steel (Shang and Yao 1998). More details on fatigue testing and material properties of the eight materials can be found from above-mentioned references.

Particularly, these fatigue tests were conducted under two-stress level loadings including low-high and high-low

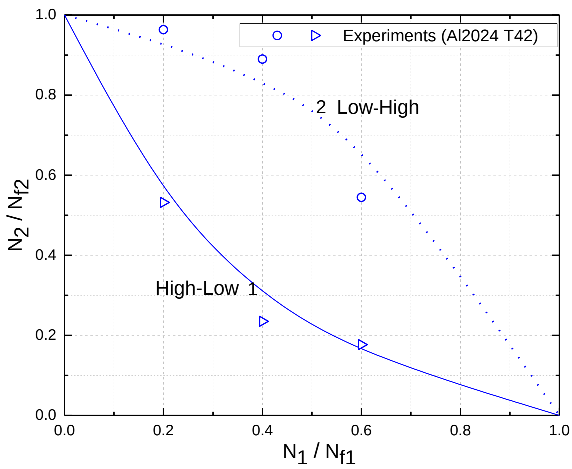


Fig. 1 Comparison between experimental results and model predictions for Al2024 T42 (Pavlou 2002, Mesmacque *et al.* 2005, Dattoma *et al.* 2006, Garcia *et al.* 2005) (1: predicted curve under high-low loading; 2: predicted curve under low-high loading)

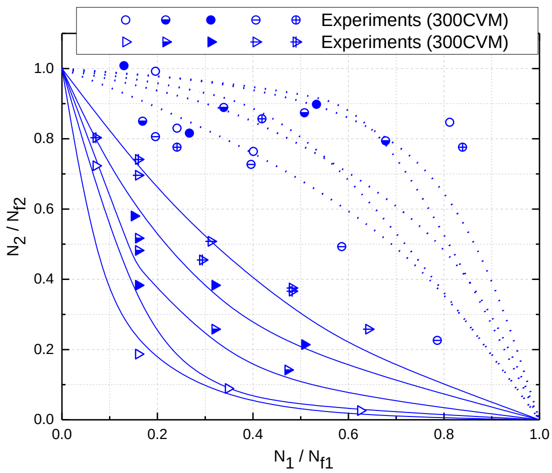


Fig. 2 Comparison between experimental results and model predictions for 300CVM steel (Manson *et al.* 1967) (1: predicted curve under high-low loading (2000-724 MPa); 2: predicted curve under high-low loading (2000-827 MPa); 3: predicted curve under high-low loading (2000-1103 MPa); 4: predicted curve under high-low loading (2000-1379 MPa); 5: predicted curve under high-low loading (2000-1655 MPa); 6: predicted curve under low-high loading (1655-2000 MPa); 7: predicted curve under low-high loading (1103-2000 MPa); 8: predicted curve under low-high loading (965-1655 MPa); 9: predicted curve under low-high loading (900-2086 MPa); 10: predicted curve under low-high loading (827-2000 MPa))

loadings in the axial strain/stress-controlled modes. All the specimens with given stress amplitude σ_{a1} for duration n_1 after fatigue loading at the first stress level is further loaded with defined stress amplitude σ_{a2} at the second stress level up to fracture. In other words, experimental results obtained are subjected to two consecutive stage loading until fracture failure occurs. Based on these tests, fatigue performance of bibliographic materials was investigated. Results for different life fractions at the first stress level and the

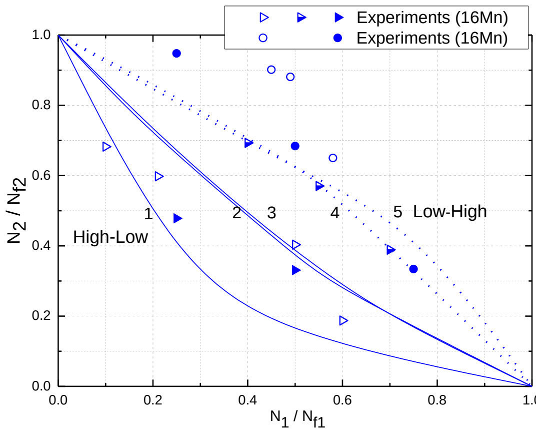


Fig. 3 Comparison between experimental results and model predictions for16Mn steel (Shang and Yao 1998) (1: predicted curve under high-low loading (294.2-166.71 MPa); 2: predicted curve under high-low loading (394-345 MPa); 3: predicted curve under high-low loading (366-324 MPa); 4: predicted curve under low-high loading (345-394 MPa); 5: predicted curve under low-high loading (166.71-294.2 MPa))

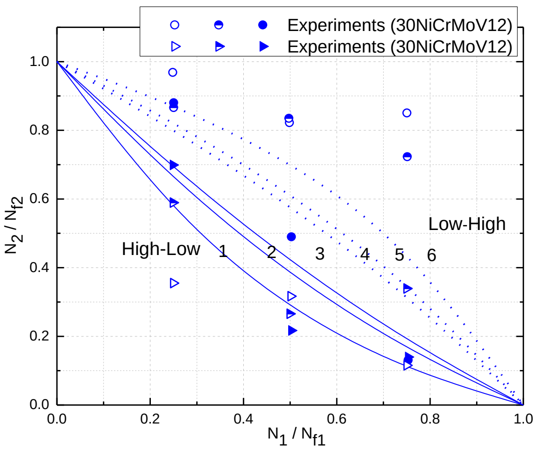


Fig. 4 Comparison between experimental results and model predictions for 30NiCrMoV12 steel (Dattoma *et al.* 2006) (1: predicted curve under high-low loading (485-400 MPa); 2: predicted curve under high-low loading (465-420 MPa); 3: predicted curve under high-low loading (450-420 MPa); 4: predicted curve under low-high loading (420-450 MPa); 5: predicted curve under low-high loading (420-465 MPa); 6: predicted curve under low-high loading for (400-485 MPa))

residual fractions of life at the second stress level, are obtained.

By following the proposed procedure 1, predicted values according to the proposed model, Miner rule, DSM and DEM are compared with experimental results under two-stress level loadings. Comparisons between experimental results and model predictions for the two-stress level tests including high-low and low-high loadings are presented in Figs. 1-8, in which the abscissa and the ordinate represent the fraction of life spent at the first stress level and second stress level, respectively. Specifically, the predicted curves with different line types by the proposed model are depicted

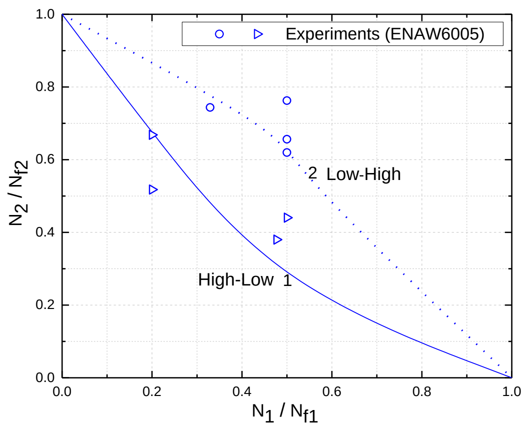


Fig. 5 Comparison between experimental results and model predictions for ENAW6005 (Tian *et al.* 2012) (1: predicted curve under high-low loading; 2: predicted curve under low-high loading)

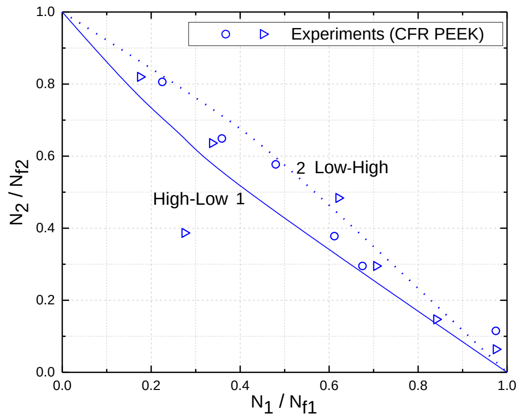


Fig. 6 Comparison between experimental results and model predictions for short carbon fiber reinforced poly-ether ether-ketone (CFR PEEK) (Noguch *et al.* 1995) (1: predicted curve under high-low loading; 2: predicted curve under low-high loading)

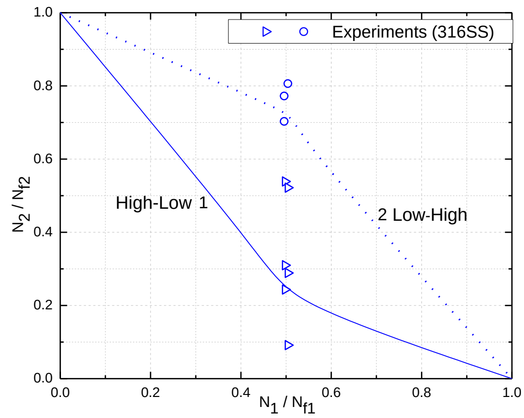


Fig. 7 Comparison between experimental results and model predictions for type 316 stainless steel (Kamaya and Kawakubo 2015) (1: predicted curve under high-low loading; 2: predicted curve under low-high loading)

in these figures, in which have shown good agreements with those experimental points.

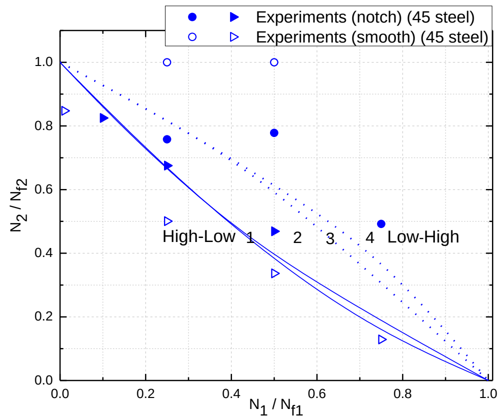
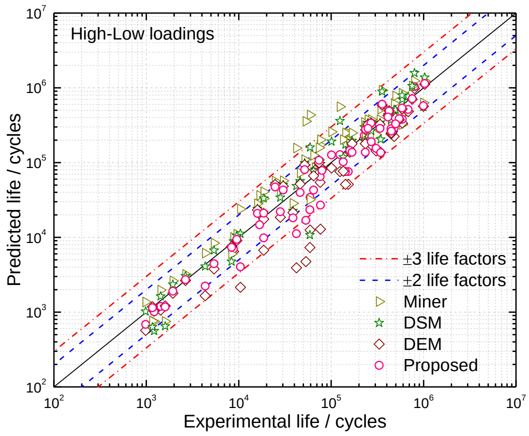
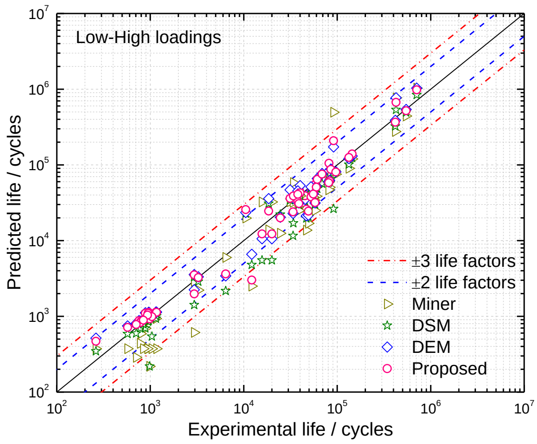


Fig. 8 Comparison between experimental results and model predictions for 45 steel (Shang and Yao 1998) (1: predicted curve of smooth normalized under high-low loading (331.46-284.4 MPa); 2: predicted curve of notch normalized under high-low loading (331.46-284.4 MPa); 3: predicted curve of notch normalized under low-high loading (284.4-331.46 MPa); 4: predicted curve of smooth normalized under low-high loading (284.4-331.46 MPa))



(a) High-low loadings



(b) Low-high loadings

Fig. 9 Comparison between experimental results and model predictions by the Miner, DSM, DEM and proposed models

Moreover, a comparison with different scatter bands between experimental results and model predictions by the Miner rule, DSM, DEM and proposed model for model comparison under high-low and low-high loadings are given in Fig. 9.

Note from Fig. 9 that model prediction errors of the Miner rule are relatively large, while the proposed model provides better predictions than others according to the dispersion of model predictions. Also, both the DSM and DEM provide more accurate predictions than the Miner rule. The proposed model provides a better prediction with a smaller dispersion compared to other three models, in which most of experimental points are predicted within the ±3 life factor for the eight materials under different loading conditions.

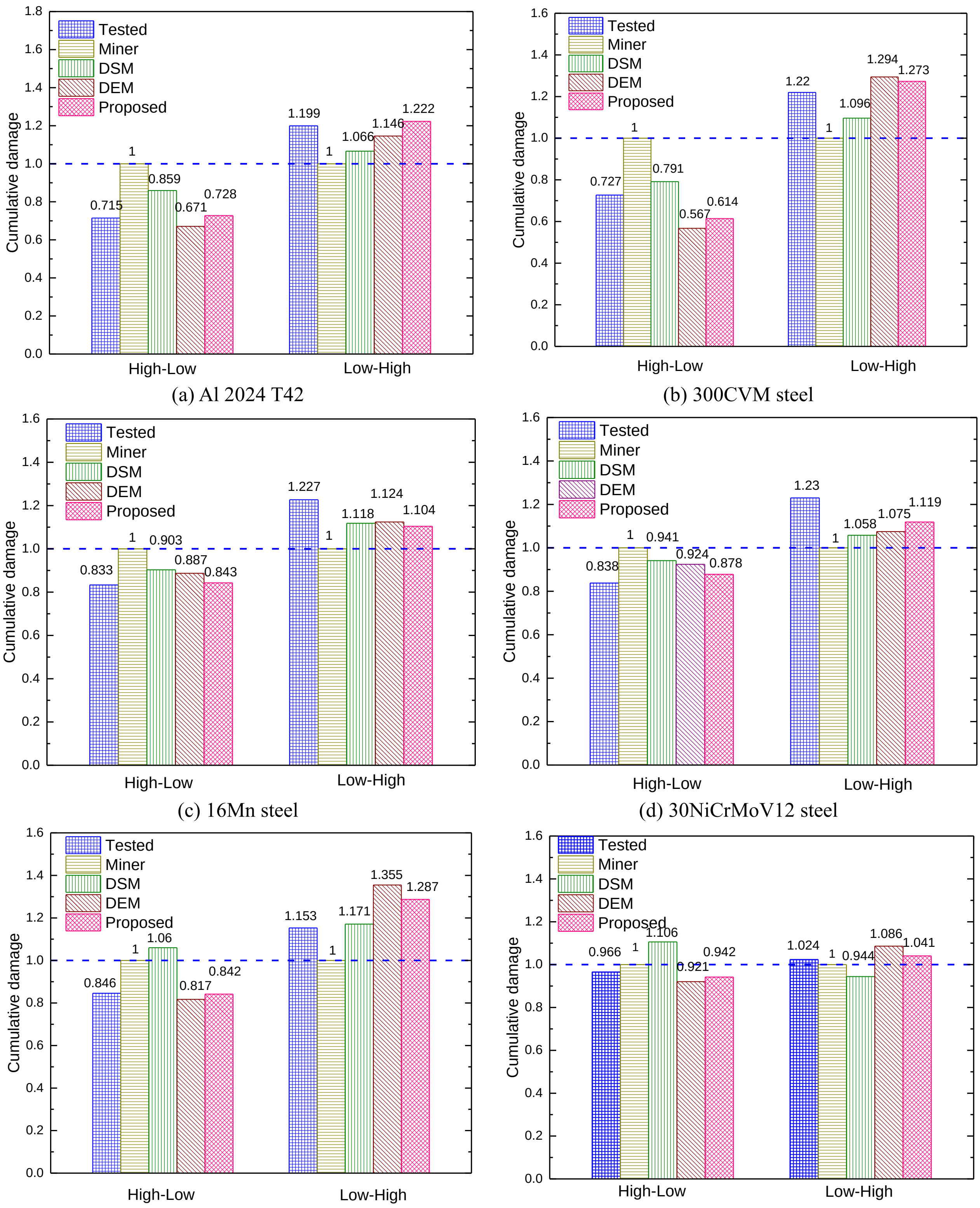
In order to study the effect of loading sequence on fatigue damage, the cumulative damage is analyzed statistically. The mean cumulative damage is compared with the tested ones for different materials under high-low and low-high loadings, respectively, as shown in Fig. 10. As it can be seen, the effect of load sequence has been addressed when referring the total strain energy W_t from the $W_t - N_f$ curve at different loading stresses, particularly, the influence of mean stress on fatigue life was incorporated within W_t by using Eq. (9). For structural components subjected to complex loading spectra, the effect of load sequence can make a significant difference to the subsequent operating mean stress of the loading cycles which in turn can also affect the fatigue life prediction, as pointed out by Knop *et al*. (2000).

Results show that predictions by the proposed model agree well with the experimental results and the relative errors are lower than others for the eight materials under different loading conditions. Particularly, it estimates the crack initiation life well by considering the effect of load interaction on fatigue damage accumulation. During the crack propagation phase, however, the load interaction mechanism is fully different due to interaction of the crack tip plastic zones and the associated crack growth retardation effects (Pavlou 2000) (Pavlou *et al.* 2004). As it can be seen, the proposed model provides better correlations with life under stepwise constant amplitude fatigue loadings. Moreover, through extracting the composition of a field load spectrum by using methods of cycle counting, like the rain flow counting method, the proposed procedure can be extended for engineering application.

5. Conclusions

In the present study, from the linear and nonlinear damage accumulation points of view, a brief summary of several criteria commonly used for fatigue damage estimation and life prediction is presented. With the aim to evaluate fatigue damage accumulation and predict residual life of engineering components under variable amplitude loadings, strain energy-based fatigue life predictions of eight materials have been carried out under stepwise constant amplitude fatigue loadings. From this investigation, the following conclusions can be drawn as:

(1) A new strain energy-based damage accumulation model is proposed for fatigue life prediction by incorporating both effects of mean stress and load interaction;



(a) Al 2024 T42 (b) 300CVM steel

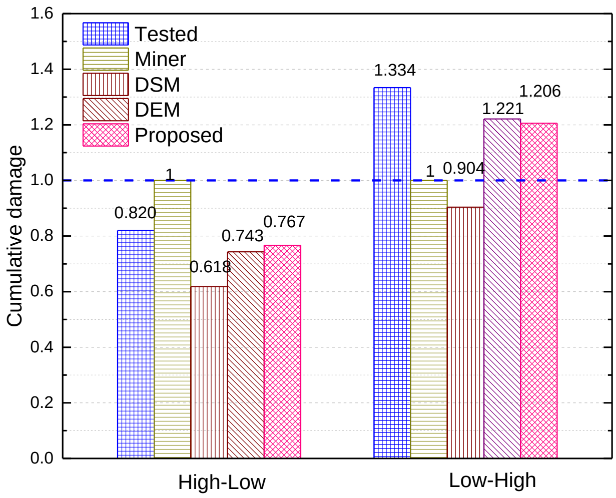
(c) 16Mn steel (d) 30NiCrMoV12 steel

(e) ENAW6005 (f) CFR PEEK

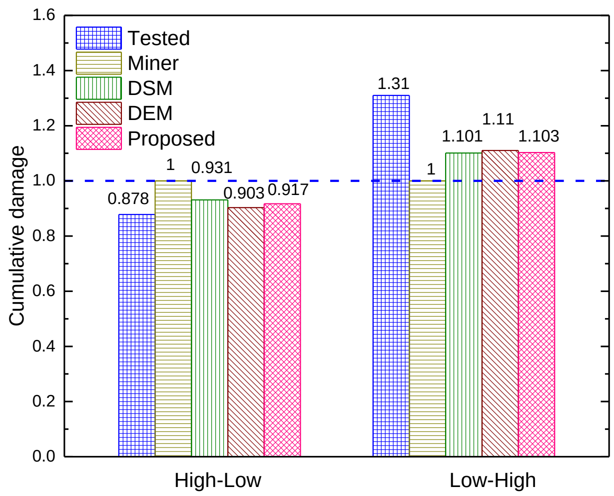
Fig. 10 Comparison between cumulative damage and experimental values of the four models under different loadings

(2) With this model, an integrated procedure is elaborated for facilitating its application based on *S-N* curve and loading conditions, which makes it easier to estimate fatigue damage accumulation and predict fatigue lifetime;

(3) Through model comparison by experimental data sets of eight materials, the proposed model provides more accurate predictions than the Miner rule, DSM and DEM models. Moreover, it can be extended for fatigue life estimation under multi-level stress loadings using the concept of equivalent damage, and the strain energy parameter can be transformed from the ith stress level to the $(i+1)th$ stress level.



(g) 316 stainless steel



(h) 45 steel

Fig. 10 Continued

Acknowledgments

The present work was partially supported by the National Natural Science Foundation of China (No. 11672070 and 11302044) and the China Postdoctoral Science Foundation Funded Project (No. 2015M582549 and No. 2017T100697) and the Fundamental Research Funds for the Central Universities (No. ZYGX2016J208) and the Open Project Program of Key Laboratory of Deep Earth Science and Engineering (Sichuan University), Ministry of Education (No. DUSE201701).

References

Aid, A., Bendouba, M., Aminallah, L., Amrouche, A., Benseddiq, N. and Benguediab, M. (2012), "An equivalent stress process for fatigue life estimation under multiaxial loadings based on a new non linear damage model", *Mater. Sci. Eng. A*, **538**, 20-27.

Besson, J. (2010), "Continuum models of ductile fracture: A review", *Int. J. Dam. Mech.*, **19**(1), 3-52.

Blason, S., Correia, J.A.F.O., De Jesus, A.M.P., Calçada, R. and Fernández-Canteli, A. (2016), "A probabilistic analysis of Miner's law for different loading conditions", *Struct. Eng. Mech.*, **60**(1), 71-90.

Carpinteri, A., Spagnoli, A. and Vantadori, S. (2003), "A multiaxial fatigue criterion for random loading", *Fatig. Fract. Eng. Mater. Struct.*, **26**(6), 515-522.

Chaboche, J.L. and Lesne, P.M. (1988), "A non-linear continuous fatigue damage model", *Fatig. Fract. Eng. Mater. Struct.*, **11**(1), 1-17.

Correia, J., Apetre, N., Arcari, A., De Jesus, A., Muñiz-Calvente, M., Calçada, R., Berto, F. and Fernández-Canteli, A. (2017), "Generalized probabilistic model allowing for various fatigue damage variables", *Int. J. Fatig.*, **100**, 187-194.

Correia, J.A.F.O., De Jesus, A.M.P., Fernández-Canteli, A. and Calçada, R.A.B. (2015), "Fatigue damage assessment of a riveted connection made of puddle iron from the Fão bridge using the modified probabilistic interpretation technique", *Proc. Eng.*, **114**, 760-767.

Correia, J.A.F.O., Raposo, P., Muniz-Calvente, M., Blasón, S., Lesiuk, G., De Jesus, A., Moreira, P., Calçada, R., Canteli, A. (2017), "A generalization of the fatigue Kohout-Věchet model for several fatigue damage parameters", *Eng. Fract. Mech.*, **185**, 284-300.

Correia, J.A.F.O., De Jesus, A.M.P. and Fernández-Canteli, A. (2013), "Local unified probabilistic model for fatigue crack initiation and propagation: Application to a notched geometry", *Eng. Struct.*, **52**, 394-407.

Dai, J., Das, D. and Ohadi, M. (2013), "Reliability risk mitigation of free air cooling through prognostics and health management", *Appl. Energy*, **111**, 104-112.

Dattoma, V., Giancane, S., Nobile, R. and Panella, F.W. (2006), "Fatigue life prediction under variable loading based on a new non-linear continuum damage mechanics model", *Int. J. Fatig.*, **28**(2), 89-95.

Djebli, A., Aid, A., Bendouba, M., Amrouche, A., Benguediab, M. and Benseddiq, N. (2013), "A non-linear energy model of fatigue damage accumulation and its verification for Al-2024 aluminum alloy", *Int. J. Non-Lin. Mech.*, **51**, 145-151.

Fatemi, A. and Yang, L. (1998), "Cumulative fatigue damage and life prediction theories: A survey of the state of the art for homogeneous materials", *Int. J. Fatig.*, **20**, 9-34.

Fernández-Canteli, A., Blasón, S., Correia, J.A.F.O. and De Jesus A.M.P. (2014), "A probabilistic interpretation of the miner number for fatigue life prediction", *Fratt. Integrita Strutturale*, **30**, 327-339.

Freudenthal, A.M. and Heller, R.A. (1959), "On stress interaction in fatigue and cumulative damage rule", *J. Aerosp. Sci.*, **26**(7), 431-442.

Garcia, S., Amrouche, A., Mesmacque, G., Decoopman, X. and Rubio, C. (2005), "Fatigue damage accumulation of cold expanded hole in aluminum alloys subjected to block loading", *Int. J. Fatig.*, **27**(10-12), 1347-1353.

Gathercole, N., Adam, T., Reiter, H. and Harris, B. (1994), "Life prediction for fatigue of T800/5245 carbon-fiber composites. 1. constant-amplitude loading", *Int. J. Fatig.*, **16**(8), 523-532.

Giancane, S., Nobile, R., Panella, F.W. and Dattoma, V. (2010), "Fatigue life prediction of notched components based on a new nonlinear continuum damage mechanics model", *Proc. Eng.*, **2**(1), 1317-1325.

Hu, D., Wang, R., Fan, J. and Shen, X. (2012), "Probabilistic damage tolerance analysis on turbine disk through experimental data", *Eng. Fract. Mech.*, **87**, 73-82.

Hu, D., Ma, Q., Shang, L., Gao, Y. and Wang, R. (2016), "Creep-fatigue behavior of turbine disc of superalloy GH720Li at 650 C and probabilistic creep-fatigue modeling", *Mater. Sci. Eng. A*,

670, 17-25.

Huffman, P.J. and Beckman, S.P. (2013), "A non-linear damage accumulation fatigue model for predicting strain life at variable amplitude loadings based on constant amplitude fatigue data", *Int. J. Fatig*., **48**, 165-169.

Huffman, P.J., Ferreira, J. and Correia, J. (2017), "Fatigue crack propagation prediction of a pressure vessel mild steel based on a strain energy density model", *Fract. Struct. Integr*., **42**, 74-84.

Ince, A. and Glinka, G. (2011), "A modification of Morrow and Smith-Watson-Topper mean stress correction models", *Fatig. Fract. Eng. Mater. Struct*., **34**(11), 854-867.

Ince, A. and Glinka, G. (2014), "A generalized fatigue damage parameter for multiaxial fatigue life prediction under proportional and non-proportional loadings", *Int. J. Fatig*., **62**, 34-41.

Ince, A. (2017a), "A generalized mean stress correction model based on distortional strain energy", *Int. J. Fatig*., **104**, 273-282.

Ince, A. (2017b), "A mean stress correction model for tensile and compressive mean stress fatigue loadings", *Fatig. Fract. Eng. Mater. Struct*., **40**(6), 939-948.

Kamaya, M. and Kawakubo, M. (2015), "Loading sequence effect on fatigue life of Type 316 stainless steel", *Int. J. Fatig*., **81**, 10-20.

Kujawski, D. (2014), "A deviatoric version of the SWT parameter", *Int. J. Fatig*., **67**, 95-102.

Knop, M., Jones, R. and Molent, L. (2000), "On the Glinka and Neuber methods for calculating notch tip strains under cyclic load spectra", *Int. J. Fatig*., **22**(9), 743-755.

Łagoda, T., Macha, E. and Niesłony, A. (2005), "Fatigue life calculation by means of the cycle counting and spectral methods under multiaxial random loading", *Fatig. Fract. Eng. Mater. Struct*., **28**(4), 409-420.

Lv, Z., Huang, H.Z., Zhu, S.P., Gao, H. and Zuo, F. (2014), "A modified nonlinear fatigue damage accumulation model", *Int. J. Dam. Mech*., **24**(2), 168-181.

Manson, S.S., Freche, J.C. and Ensign, C.R. (1967), "Application of a double linear damage rule to cumulative fatigue", *ASTM STP*, **415**, 384-412.

Manson, S.S. and Halford, G.R. (1981), "Practical implementation of the double linear damage rule and damage curve approach for treating cumulative fatigue damage", *Int. J. Fract*., **17**(2), 169-192.

Meneghetti, G. (2007), "Analysis of the fatigue strength of a stainless steel based on the energy dissipation", *Int. J. Fatig*., **29**(1), 81-94.

Mesmacque, G., Garcia, S., Amrouche, A. and Rubio-Gonzalez, C. (2005), "Sequential law in multiaxial fatigue, a new damage indicator", *Int. J. Fatig*., **27**(4), 461-467.

Miner, M.A. (1945), "Cumulative damage in fatigue", *J. Appl. Mech*., **67**, A159-A164.

Noguchi, H., Kim, Y.H. and Nisitani, H. (1995), "On the cumulative fatigue damage in short carbon fiber reinforced poly-ether-ether-ketone", *Eng. Fract. Mech*., **51**(3), 457-465.

Pavlou, D.G. (2002), "A phenomenological fatigue damage accumulation rule based on hardness increasing for the 2024-T42 aluminum", *Eng. Struct*., **24**, 1363-1368.

Pavlou, D.G. (2000), "Prediction of fatigue crack growth under real stress histories", *Eng. Struct*., **22**(12), 1707-1713.

Pavlou, D.G., Vlachakis, N.V. and Pavlou, M.G. (2004), "Estimation of fatigue crack growth retardation due to crack branching", *Comput. Mater. Sci*., **29**(4), 446-452.

Peng, W., Li, Y.F., Yang, Y.J., Zhu, S.P. and Huang, H.Z. (2016), "Bivariate analysis of incomplete degradation observations based on inverse Gaussian processes and copulas", *IEEE Trans. Reliab*., **65**(2), 624-639.

Rege, K. and Pavlou, D.G. (2017), "A one-parameter nonlinear fatigue damage accumulation model", *Int. J. Fatig*., **98**, 234-246.

Sanches, R.F., De Jesus, A.M.P. and Correia, J.A.F.O. (2015), "A probabilistic fatigue approach for riveted joints using Monte Carlo simulation", *J. Constr. Steel Res*., **110**, 149-162.

Siriwardane, S., Ohga, M., Dissanayake, R. and Taniwaki, K. (2008), "Application of new damage indicator-based sequential law for remaining fatigue life estimation of railway bridges", *J. Constr. Steel Res*., **64**(2), 228-237.

Sun, Q., Dui, H.N. and Fan, X.L. (2014), "A statistically consistent fatigue damage model based on Miner's rule", *Int. J. Fatig*., **69**, 16-21.

Taheri, S., Vincent, L. and Le-roux, J.C. (2013), "A new model for fatigue damage accumulation of austenitic stainless steel under variable amplitude loading", *Proc. Eng*., **66**, 575-586.

Wang, R., Liu, X. and Hu D. (2017), "Zone-based reliability analysis on fatigue life of GH720Li turbine disk concerning uncertainty quantification", *Aerosp. Sci. Technol*., **70**, 300-309.

Wang, R.Z., Zhang, X.C., Tu, S.T., Zhu, S.P. and Zhang, C.C. (2016), "A modified strain energy density exhaustion model for creep-fatigue life prediction", *Int. J. Fatig*., **90**, 12-22.

Wu, S.C., Zhang, S.Q., Xu, Z.W., Kang, G.Z. and Cai, L.X. (2016), "Cyclic plastic strain based damage tolerance for railway axles in China", *Int. J. Fatig*., **93**, 64-70.

Ye, D. and Wang, Z. (2001), "A new approach to low-cycle fatigue damage based on exhaustion of static toughness and dissipation of cyclic plastic strain energy during fatigue", *Int. J. Fatig*., **23**(8), 679-687.

Yu, Z.Y., Zhu, S.P., Liu, Q. and Liu, Y. (2017a), "A new energy-critical plane damage parameter for multiaxial fatigue life prediction of turbine blades", *Mater*., **10**(5), 513.

Yu, Z.Y., Zhu, S.P., Liu, Q. and Liu, Y. (2017b), "Multiaxial fatigue damage parameter and life prediction without any additional material constan ts", *Mater*., **10**(8), 923.

Yuan, X., Yu, W., Fu, S., Yu, D. and Chen, X. (2016), "Effect of mean stress and ratcheting strain on the low cycle fatigue behavior of a wrought 316LN stainless steel", *Mater. Sci. Eng. A*, **677**, 193-202.

Zhu, S.P., Huang, H.Z., Ontiveros, V., He, L.P. and Modarres, M. (2012), "Probabilistic low cycle fatigue life prediction using an energy-based damage parameter and accounting for model uncertainty", *Int. J. Dam. Mech*., **21**(8), 1128-1153.

Zhu, S.P., Huang, H.Z., Liu, Y., Yuan, R. and He, L.P. (2013), "An efficient life prediction methodology for low cycle fatigue-creep based on ductility exhaustion theory", *Int. J. Dam. Mech*., **22**(4), 556-571.

Zhu, S.P., Lei, Q., Huang, H.Z. and Yang, Y.J. (2016), "Mean stress effect correction in strain energy-based fatigue life prediction of metals", *Int. J. Dam. Mech*., **26**(8), 1219-1241.

Zhu, S.P., Yue, P., Yu, Z.Y. and Wang, Q. (2017), "A combined high and low cycle fatigue model for life prediction of turbine blades", *Mater*., **10**(7), 698.

Zhu, S.P., Yang, Y.J., Huang, H.Z., Lv, Z. and Wang, H.K. (2017), "A unified criterion for fatigue–creep life prediction of high temperature components", *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, **231**(4), 677-688.

Zhu, S.P., Huang, H.Z. and Wang Z.L. (2011), "Fatigue life estimation considering damaging and strengthening of low amplitude loads under different load sequences using fuzzy sets approach", *Int. J. Dam. Mech*., **20**(6), 876-899.

Zhu, S.P., Huang, H.Z., Peng, W., Wang H.K. and Mahadevan, S. (2016), "Probabilistic Physics of Failure-based framework for fatigue life prediction of aircraft gas turbine discs under uncertainty", *Reliab. Eng. Syst. Safety*, **146**, 1-12.

Zhu, S.P., Foletti, S. and Beretta, S. (2017), "Probabilistic framework for multiaxial LCF assessment under material variability", *Int. J. Fatig*., **103**, 371-385.

Zhu, S.P., Lei, Q. and Wang, Q.Y. (2017), "Mean stress and ratcheting corrections in fatigue life prediction of metals", *Fatig*. *Fract*. *Eng*. *Mater*. *Struct*., **40**(9), 1343-1354.
Zhu, S.P., Liu, Q., Zhou, J. and Yu, Z.Y. (2018), "Fatigue reliability assessment of turbine discs under multi-source uncertainties", *Fatig*. *Fract*. *Eng*. *Mater*. *Struct*., In Press.
Zhu, S.P., Liu, Q., Lei, Q. and Wang, Q.Y. (2018), "Probabilistic fatigue life prediction and reliability assessment of a high pressure turbine disc considering load variations", *Int*. *J*. *Dam*. *Mech*., In Press.

PL