Structural Engineering and Mechanics, *Vol. 65, No. 6 (2018) 751-760*
DOI: https://doi.org/10.12989/sem.2018.65.6.751

Reliability-based assessment of damaged concrete buildings

Zafer I. Sakka*, Ibrahim A. Assakkaf[a] and Jamal S. Qazweeni[b]

Energy and Building Research Center, KISR, P.O. Box 24885 Safat 13109, Kuwait

(Received June 17, 2017, Revised December 24, 2017, Accepted January 17, 2018)

Abstract. Damages in concrete structures due to aging and other factors could be a serious and immense matter. Making the best selection of the most viable and practical repairing and strengthening techniques are relatively difficult tasks using traditional methods of structural analyses. This is due to the fact that the traditional methods used for assessing aging structure are not fully capable when considering the randomness in strength, loads and cost.

This paper presents a reliability-based methodology for assessing reinforced concrete members. The methodology of this study is based on probabilistic analysis, using statistics of the random variables in the performance function equations. Principles of reliability updating are used in the assessment process, as new information is taken into account and combined with prior probabilistic models. The methodology can result in a reliability index β that can be used to assess the structural component by comparing its value with a standard value. In addition, these methods result in partial safety factor values that can be used for the purpose of strengthening the R/C elements of the existing structure. Calculations and computations of the reliability indices and the partial safety factors values are conducted using the First-order Reliability Method and Monte Carlo simulation.

Keywords: structural reliability; first order reliability method; Monte Carlo simulation; reinforced concrete structures; existing reinforced concrete structures; partial safety factors

1. Introduction

Due to social and economic pressures, owners all over the world in the desire to keep and use old, existing R/C structures, including aging ones. Therefore, in some cases it is extremely difficult, if not impossible, to demolish these existing structures for the reasons revealed above. However, these aging structures, especially those that are dear, need extremely and exceptionally accurate condition assessment and proper safety evaluation. More than half of the budget spent for construction activities in developed countries is related to repair and maintenance of these R/C structures (Bayerische Ingenieurkammer Bau 2004). Also, in accordance with Paul (2002) periodical evaluation and assessment of relatively old concrete structures are awfully vital and imperative. If the evaluation and assessment of structural components of a particular aging R/C structure reveals that repairs are essential for these components, these repairs should not be delayed. Delaying the repairs has the potential of losing serviceability of the whole structure and /or causing total failure and collapse of the structure. In addition, if repairs are delayed, the cost of maintenance will skyrocket as well. It can also be concluded from the above that the assessment of existing structures' components has and will receive more consideration and thought from the structural engineering societies' and literature.

Traditionally, deterministic method of structural analyses for R/C structural components that includes 'Allowable Stress Design (ASD)', or sometimes called 'Working Stress Design (WSD)', has been, and is still, used in some countries. However, the problem with these methods is that they are inefficient to provide a full structural picture of what is going on in the existing structures. While the ASD is very well-established, it does not provide, especially for the analysis part, a quantitative risk and reliability measure such as the reliability index β or the probability of failure (P_f) and risk of an aging structure. Subjective judgment may not be enough for decision-making when the penalty for a mistake is high. Therefore, more rigorous, systematic and quantitative probability-based approaches are needed. Probabilistic (reliability) methods and risk analysis are among such approaches that have been seen widely and commonly used in recent years to assess and evaluate R/C existing structures in an efficient, cost-effective and practical manner.

1.1 Background

All structural designs and analyses involve uncertainty; uncertainty is deemed to be ubiquitous in structural engineering (ASME 2005). Uncertainty is unavoidable and ever-presents in loading conditions, in building materials, in the modeling of strength equations, in environmental loads, in geometric properties, in fabrication and installation precision, in examination and inspection results, in construction, and in actual usage. Conventionally and over the years, engineering design and analysis methodology have addressed uncertainty through the common and

*Corresponding author, Ph.D.
E-mail: zsakka@kisr.edu.kw
[a]Ph.D.
E-mail: iassakkaf@kisr.edu.kw
[b]M.Sc.
E-mail: jqazweni@kisr.edu.kw

http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

popular deterministic factor of safety (FS). This methodology could lead to inconsistent reliability levels and sometimes overly conservative designs that do not provide insight into the effects of individual uncertainties and the actual margin of safety. Therefore, whether it is a new design or an existing structure, the engineer or analyst has to deal with such uncertainty. Several approaches and methods have been used over the years to deal with such uncertainty. These methods and approaches include, but not limited to, the old ASD method and the most recent and efficient approach of the Load and Resistance Factor Design (LRFD).

1.2 Allowable stress design (ASD) versus load and resistance factor design (LRFD)

The ASD has been used and is still used in many countries in the design and analysis of structural components. In the ASD method, the popular and common FS is used, for example, in R/C design strength equation to amplify the right side of the equation that includes loading effects or in the left side of the equation to reduce the strength stress. This FS treats all load variables as if they have the same uncertainty, which of course is not rational or logical.

The LRFD, which is probability based, uses a different approach. Unlike the ASD approach, which is stress-based, the LRFD approach uses ultimate strength design and analysis of structural components, and also various factors for each type of load effect. These factors are called partial safety factors (PSF's). As alluded to earlier, the ASD uses one FS to account for the entire uncertainty in loads and strength, while the LRFD utilizes different partial safety factors for different load and strength types. This allows for taking into account uncertainties in strength and load effects, and to scale their characteristic values accordingly, in the limit-state design equation.

Although both approaches are very well-established and both are used and suited for design and analysis of structures, the LRFD is gradually and progressively replacing the old ASD in numerous civil, mechanical, marine, and other structural codes because of its superiority and efficiency. It is to be noted that both methods do not provide in a direct way, especially in structural analysis, a quantitative risk measure such as the reliability index β or the probability of failure (P_f) for aging or damaged R/C structural elements. Therefore, more rigorous systematic and quantitative approaches are required such as using direct reliability approach. Direct probabilistic and reliability methods and risk analysis are among such approaches that have been developed in recent years, which include both the First-order Reliability Method (FORM) and the Second-order Reliability Method (SORM) that is more proficient and accurate.

2. Theory of structural reliability-analysis and design

In recent years, analysis and design of R/C structural elements and components have been moving toward a more rational and probability-based analysis and design procedure referred to as limit states design. Such analysis and design procedures take into account more information than the deterministic methods in the design, maintenance, and life expectancy of the structural components. This information includes uncertainties in the strength of various structural elements, in load effects, and modelling errors in the analysis procedures. Probability-based design and analysis formats are more flexible and rational than the working stress formats because they provide consistent levels of safety over various types of structural elements.

2.1 The failure probability of structural member

Reliability analysis determines the degree of reliability of a structure taking into consideration the uncertainties of the variables used in designing or analyzing a structural element or the uncertainties that have a probabilistic distribution property. When the effective external force L is greater than the resistance R, the structural element fails. The probability of failure, which corresponds to the area in Fig. 1, marked by shaded area, can be calculated using Eq. (1).

$$P_f = P(R - L < 0) = \int_{g(x) \leq 0} f(x)dx \tag{1}$$

where P_f = probability of failure, R = strength or resistance, L = load effect, $f(x)$ = strength and load random variables function, and $g(x)$ = limit state function, which can be expressed in its simplest of two random variables as shown in Eq. (2).

$$g(x) = R - L \tag{2}$$

The reliability index β is the ratio of the standard deviation σ_g and the mean μ_g from 0 to the probability variable g as shown in Fig. 1 (Seo *et al.* 2010). As the probability of failure decreases as β increases, the degree of safety level of the structural element increases. The reliability index β can be defined, herein, as the mean ratio relative to the standard deviation (Melchers 1999).

2.2 Reliability-based design and analysis

The reliability-based design and analysis of any structural component requires the consideration of the following three components: (1) loads, (2) structural strength, and (3) methods of reliability analysis. These three components are indispensable and essential for the development of reliability-based LRFD and analysis for R/C elements. There are two primary approaches for reliability-based design and analysis (Ayyub *et al.* 1998): (a) direct reliability-based design and (b) load and resistance factor design. The LRFD approach is called a Level 1 reliability method. Level 1 reliability methods utilize partial safety factors (PSF's) that are reliability based; but the methods do not require the explicit use of the probabilistic description of the variables (Assakkaf *et al.* 2013).

2.3 Direct reliability-based design and analysis

The direct reliability-based design method uses all available information about the basic variables and does not simplify the limit state in any manner (Assakkaf 2012a, Assakkaf 1998, Ayyub *et al.* 1998). It requires performing spectral analysis and extreme analysis of the loads. In addition, the linear or nonlinear structural analysis can be used to develop a stress frequency distribution. Then, stochastic load combinations can be performed. Linear or nonlinear structural analysis can then be used to obtain deformation and stress values. The appropriate loads, strength variables, and failure definitions need to be selected for each failure mode. Using reliability assessment methods such as the first-order reliability method (FORM), reliability indices β's for all modes at all levels need to be computed and compared with the target reliability indices β_T's.

Ideally, the safety measure should not depend on the way in which loads and resistance(s) are defined. An important form of invariant safety measure is the performance function, g, as given by the following equation where the applied load effect component L_i is compared with the resistance R

$$g = R - \sum_{i=1}^{n} L_i = 0 \tag{3}$$

The performance function or limit-state equation is by nature considered to be a random entity because it contains the basic random variables and parameters of strength and loads. It is usually expressed in such a way that failure of the structural system or component results in a negative sign of the function (i.e., $g < 0$), survival of the system or component results in a positive sign for the function (i.e., $g > 0$), and limit-state results in $g = 0$ (see Fig. 1).

The relationship between the reliability index β and the probability of failure is given by Ang and Tang (1984), Ayyub and McCuen (2011), and Assakkaf (2012b) as

$$P_f = 1 - \Phi(\beta) \quad , \beta = \mu_g / \sigma_g \tag{4}$$

where $\Phi(.)$ = cumulative probability distribution function of the standard normal distribution, β = reliability index and μ_g and σ_g the mean and the standard deviation of the performance function g. Eq. (4) assumes that all random variables in the linear limit state equation to have normal probability distribution. For all practical purposes, Eq. (1) can be used to estimate the failure probability P_f with sufficient accuracy.

2.4 The first-order reliability method

The First-Order Reliability Method (FORM) is a convenient and powerful computational tool used to assess the reliability of a structural element. In addition, it provides a means for calculating the partial safety factors ϕ and γ_i for a specified target reliability level (Assakkaf 2012b, Assakkaf 2012a). The simplicity of the first-order reliability method stems from the fact that this method, in addition to the requirement that the distribution types of a random variable must be known, requires only the first and second statistical moments; namely the mean values and the standard deviations of the respective random variables. Knowledge of the joint probability density function (PDF) of the design or analysis basic variables is not needed, as in the case of the direct integration method for calculating the reliability index. Even if the joint PDF of the basic random variables is known, the computation of β by the direct integration method can be a very difficult task.

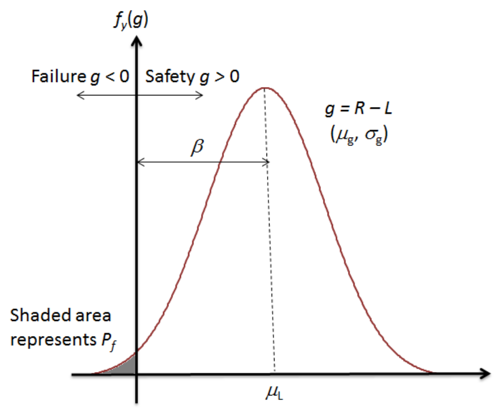

Fig. 1 Probability distribution showing reliability index and probability of failure

In design practice, there are usually two types of limit states: the ultimate limit states and the serviceability limit state. Both types can be represented by the following performance or limit-state function

$$g(X) = g(X_1, X_2, \ldots, X_n) \tag{5}$$

in which X is a vector of basic random variables (X_1, X_2, ..., X_n) for the strengths and load variables. The performance function $g(X)$ is sometimes called the limit-state function. It relates to the random variables for the limit-state of interest. The limit state is defined when $g(X) = 0$, and therefore, failure occurs when $g(X) < 0$. The reliability index β is defined as the shortest distance to the failure surface at the design point as shown in Fig. 2.

Low and Tang (2004) presented a practical procedure for reliability analysis involving correlated nonnormals and enhanced their procedure in Low and Tang (2007). Keshtegar (2016) proposed a robust approach using chaotic conjugate map to overcome the unstable results in nonlinear problems that are associated with the Hasofer and Lind (1974) and Rackwitz and Flessler (1978) algorithm.

As alluded to earlier, the basic approach to develop a reliability-based analysis for strength standard is to determine the relative reliability of designs based on current practice. In order to do that, reliability assessment of existing components of an R/C structure is needed to estimate a representative value of the reliability index β. The first-order-reliability method is very well-suited to perform such a reliability assessment.

2.5 Monte Carlo simulation

Due to the advent and the state-of-the-art fast computers and vast advancement in the computational methods, simulation methods can be used effectively for assessing the reliability of a structural system (Schueller 2007). Simulation is a process of replicating the real world based on a set of assumptions and conceived models of reality.

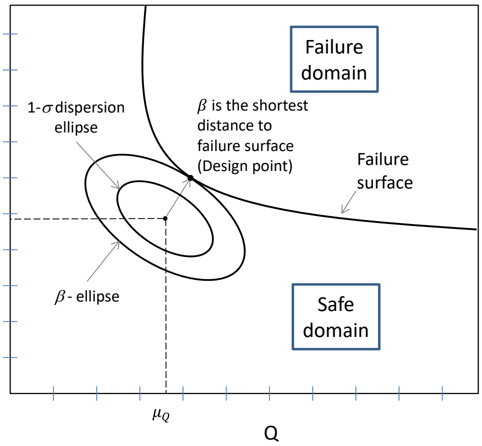

Fig. 2 Dispersion ellipses and reliability index in space of random variables (Low and Tang 2004, 2007)

Simulation can be performed either experimentally or theoretically, however in practice; theoretical simulation is preferred because it is inexpensive. Simulation may be applied in structural engineering to predict or study the performance and response of a structural element or system. Also, simulation can be used to verify the accuracy of structural reliability methods with little background in probability and statistics.

Simulation is also the process of conducting experiments on a model instead of applying experiments directly on the system or the components. A model, physical or mathematical, is a representation of the real system or the component for the purpose of studying its performance. Monte Carlo simulation (MCS) is the most popular and common technique used to replicate the real physical phenomenal situation. However, the MCS is considered to have, relatively, high computational cost and several authors suggested different techniques to improve it (Krakovski 1995, Wang *et al.* 1997, Picard *et al.* 1992).

3. Reliability-based assessment and evaluation of existing structures

In the preceding sections, the reliability of structural components involving a single failure mode, defined by a single limit state function, was discussed. This is referred to as 'reliability of components." However, in practice, a vast majority of engineering structures involve multiple failure modes; in other words, there may be a potential for several modes of failure. In these cases, the occurrence of any one of the potential failure modes will definitely constitute collapse or nonperformance of the system or component. A structure can fail in flexure, shear, or buckling, or a combination of all. If multiple structural components of a structure are to be reliability analyzed, in this situation, we are dealing with what is called "System Reliability," which requires fault tree and an event tree analyses.

The accuracy of the structural system reliability is dependent on the accuracy of the individual structural components that make up the whole system. It also depends on the correlations among the basic random variables of the components. Therefore, it is a function of the reliabilities of the individual basic structural components that compose the engineering system.

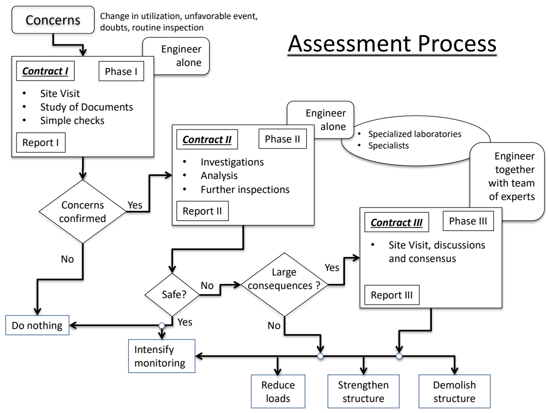

Fig. 3 Illustration of the three phases approach (RILEM 2001)

The relationship among the components in the system and the degree of redundancies are important aspects in the reliability assessment of engineering systems. Several methods are available and can be used to assess the reliability of a structural system based on the relationship among system components. The fault tree analysis can be used in general for assessing the structural reliability of systems.

Although system reliability has its own merit in producing more accurate results and fairly predicting the remaining life of a structure or a building (Assakkaf *et al.* 2013), it requires a more rigorous and detailed analysis and it can be costly and time-consuming. In addition, it requires more probabilistic information and statistics on strength, loads, materials, methods of construction, etc., that might not be available for performing such an analysis in a proper and straightforward manner. System reliability involves evaluating and assessing the whole structure as a unit rather than individual structural components and elements. This method is recommended and justified if money and time are not a problem, and if all the above mentioned needed information and data for this analysis are available. In this study, only reliability analyses of the structural components such as an R/C beam or a column were performed.

3.1 General assessment procedure

Engineering experience shows that the assessment of existing structures can be divided into three phases as shown in Fig. 3 (RILEM 2001) and (Assakkaf *et al.* 2016). Each of these phases should be completed in its specific unit and own merit with the required deterministic or reliability-based analyses. The assessment process updates the knowledge about the structure through inspections and investigations that are conducted during the assessment. Depending on the seriousness of a particular phase, either sound engineering judgment can be drawn (not very serious) or detailed and rigorous reliability-based analysis be performed (very or extremely serious) for each phase, provided sufficient statistics and full probabilistic data of

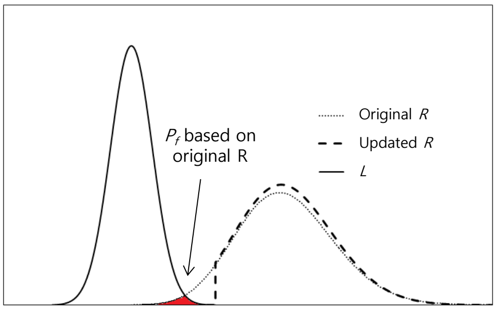

Fig. 4 An example of updated reliability analysis for a beam that shows the resistance distributions and load functions before and after the updated reliability analysis

reliability parameters are available.

3.2 FORM as a tool for structural assessment and repair

FORM can be used in the analysis of an aging or damaged R/C structural element by producing new PSF's for the investigated element based on the existing probabilistic characteristics and statistics of its condition. These PSF's can be computed for a specific value for target reliability index β_T based on international standards for R/C beams and columns. The resulting partial safety factors can be used to redesign or strengthen these aging or damaged structural elements using the proper limit-state equations.

3.3 Data updating

Updating information for an existing structure for its present and future use is an important procedure in assessing the reliability of the structure. One of the good advantages of structural reliability is that it lends itself in the process of updating the basic random variables based on prior information that was available during the design phase and on collected observations and measurements that were made available during the assessment phase. This process results in posterior information that serves for assessing the structure and can be evaluated by utilizing Bayes rule as shown in Eq. (6). In some cases, the field inspections can include proof load test. If the structural element passed a proof load test, the reliability can be updated using Eq. (7) (RILEM 2001), where the load test event I proved that the structural element has a minimum capacity of L. Fig. 4 illustrates the effect of updating the performance function on the probability of failure, P_f, for a structural element that proved to successfully pass a load test.

$$P(F|I) = \frac{P(F \cap I)}{P(I)} \tag{6}$$

$$P(g < 0|I > L) = \frac{P(g < 0 \cap I > L)}{P(I > L)} \tag{7}$$

In recent years, many researchers have been focusing on the different novel techniques and methods in reliability. Leira (2016) presented an application of updated reliability on short and long-term monitored structural response parameters. Liu *et al.* (2016) proposed a novel interval uncertainty formulation for exploring the impact of epistemic uncertainty on reliability-constrained design performance. Wang *et al.* (2016) proposed a computational method for the calculation of non-probabilistic reliability for linear structural systems. Xu *et al.* (2016) proposed a method for reliability assessment of structural dynamic systems. In the proposed method, the reliability is evaluated by the equivalent extreme value distribution of structural dynamic systems.

In this paper, a selection of two different existing buildings were reached for the purpose of reliably and in probabilistic manner analysing and evaluating them. Probabilistic and reliability methods were implemented in assessing and evaluating the safety of the structural elements of these two buildings. The first building is a 7-year-old recreational building, whereas the other is a 42-year old office building. The traditional or deterministic

Table 1 Dimensions and reinforcement of the overstressed beams

Element name	b (mm)	h (mm)	Reinforcement			
			Longitudinal		Transverse	
			Bars	No. of Layers	No. of Stirrups and Size	Spacing (mm)
B1	1000	1000	30 No. 32	3	2 No. 16	150
B2	1000	1000	33 No. 32	3	2 No. 16	125
B3	1200	1000	38 No. 32	3	3 No. 13	100
B4	1000	1000	24 No. 32	3	2 No. 16	150
B5	800	1000	21 No. 32	3	2 No. 16	150

Table 2 Dimensions and reinforcement of the overstressed columns

Column	b (mm)	h (mm)	Reinforcement				
			Longitudinal		Transverse		ρ (%)
			Bars	Distribution	Tie Size	s (mm)	
C1	600	600	16 No. 32	All Sides equal	3 No. 8	200	3.64
C2	400	400	8 No. 19	All Sides equal	2 No. 6	200	1.42

Table 3 Nominal moments acting on selected beams in the recreational building

Beam	Dead Load Moment (kN.m)	Live Load Moment (kN.m)	Moment due to Support Settlement (kN.m)
B1	2,945	1008	-
B2	3,400	1145	-
B3	3,693	1244	-
B4	2,590	884	993
B5	2,448	878	-

Table 4 Nominal loads acting on selected columns in the office building

Column	Load	Dead Load	Live Load	Wind Load
C1	Axial (kN)	4,150	641	84.7
	Moment (kN.m)	95	43	12.8
C2	Axial (kN)	450	54.8	-
	Moment (kN.m)	109	42.6	

Table 5 Statistical parameters of concrete compressive strength f'_c

Building	No. of cores	Mean (MPa)	Min. (MPa)	Max. (MPa)	*COV**	Distribution
Recreational	91	27.5	17.6	45.8	27.0%	Lognormal
Office	37	11.0	4.84	23.5	28.0%	Lognormal

*Coefficient of variation

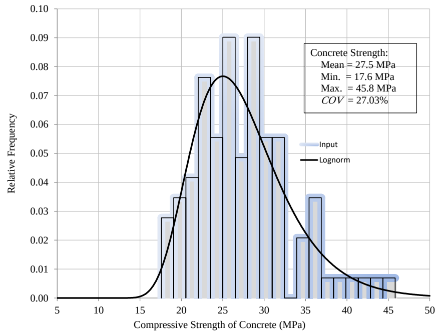

Fig. 5 Probabilistic best-fit of concrete cylinder compressive strength in the recreational building

approach for evaluation, verification of design, and analysis of the buildings' structural components was conducted and it resulted in the unsatisfactory performance of some R/C beams and columns, according to the ACI 318 (2014)) and the ACI 562 (2016) strength requirements. Table 1 and Table 2 show the geometrical properties and reinforcement details for these elements while Table 3 and Table 4 give the deterministic nominal internal forces acting on them which were obtained from the analysis of 3D models that were created using commercially available software. The dead, live, and wind loads specified in these tables are the maximum values over a referenced return period of 50 years.

4. Probabilistic parameters and random variables

This section presents the probabilistic parameters and random variables needed for the structural assessment of the two investigated buildings.

The probabilistic parameters for the concrete compressive strength were obtained from a statistical analysis of the test results for 128 concrete cores that were extracted from the investigated buildings. Table 5 shows the probabilistic parameters for the concrete compressive strength in both building while Fig. 5 shows the probabilistic best-fit for the compressive strength of the 91 cores that were extracted from the recreational building.

The statistical parameters for the acting loads were estimated based on Nowak *et al.* (2012) and Ellingwood *et al.* (1980) and listed in Table 6. Loads due to support settlements were assumed to be deterministic as they were based on field measurements.

Fabrication and professional variables are also vital to reliability analysis. The fabrication factor represents the variation in dimensions, while the professional factor represents the variation in the ratio of the actual resistance and what is analytically predicted. Table 7 and Table 8 show the implemented fabrication and professional parameters, respectively, as per Nowak *et al.* (2012).

Table 6 Statistical parameters of structural loads

Loading	Bias Factor, λ	*COV*	Distribution
Dead	1.05	0.10	Normal
Live	1.00	0.25	Type I
Wind	0.78	0.37	Type I

Table 7 Statistical information on fabrication random variables

	Bias Factor, λ	*COV*	Distribution Type
Width of Cross Section, b	1.01	0.04	Normal
Height of Cross Section, h	0.99	0.04	Normal

Table 8 Statistical information on professional factor

	Bias Factor, λ	*COV*	Distribution Type
Beams in flexure	1.02	0.06	Normal
Tied columns	1.00	0.08	Normal

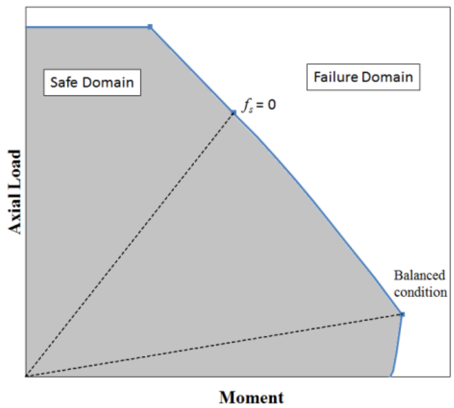

Fig. 6 Typical interaction diagram for an eccentrically loaded short column

5. Ultimate resistance capacity of R/C structural components

5.1 Strength capacity of flexural members

The ultimate capacity of R/C members in flexure is determined using the following most common equation for R/C beam under pure bending moment

$$M_n = A_s f_y \left(d - \frac{A_s f_y}{1.7 f'_c b} \right) \tag{8}$$

where M_n = ultimate moment capacity, A_s = cross sectional area of reinforcement steel, b = width of rectangular section of the beam, d = distance from the center of reinforcement to the upper edge of the rectangular section of the beam, f_y = yield strength of steel, and f'_c = compression strength of concrete.

Table 9 Statistical characteristics of load and resistance for the weakened beams

Beam	M_D (kN.m)		M_L (kN.m)		M_{ss} (kN.m)	M_n (kN.m)	
	μ	σ	μ	σ		μ	σ
B1	3,092	309	1,008	252	-	7,682	836
B2	3,570	357	1,145	286	-	8,354	909
B3	3,877	388	1,244	311	-	9,722	1,052
B4	2,720	272	884	221	993	6,349	690
B5	2,570	257	878	219	-	5,474	595

Table 10 Statistical characteristics of load and resistance for the weakened columns

Column			C1		C2	
			M (kN.m)	P (kN)	M (kN.m)	P (kN)
Internal forces	Dead	μ	99.8	4,350	115	473
		σ	9.65	417	10.9	45.3
	Live	μ	53.5	799	53.6	68.5
		σ	12.5	185	12.6	15.8
	Wind	μ	10.6	70.1	-	-
		σ	4.79	32.1	-	-
Resistance (M_n, P_n)		μ	238	7,570	176	564
		σ	39.7	763	23.7	107

Table 11 Reliability index β and corresponding P_F for selected weakened elements

Beam	β	P_f
B1	3.89	5.01×10^{-5}
B2	3.61	15.3×10^{-5}
B3	3.95	3.91×10^{-5}
B4	2.26	0.0119
B5	2.96	154×10^{-5}
C1	3.26	55.7×10^{-5}
C2	0.14	0.444

5.2 Column ultimate strength

The ultimate resistance capacity of a loaded column is dependent on the applied moment(s) and axial force(s). The limit state can be defined in the form of an interaction diagram (Szerszen *et al.* 2005). A typical interaction diagram is shown in Fig. 6. The points within the bell-shaped schematic are considered safe loading cases, while the points outside the shape are considered failure loading cases.

6. Results and outcome of reliability analysis

Reliability analysis was used to compute the reliability indices and failure probabilities for the selected weakened and deteriorating reinforced concrete beams using FORM. The performance function for the flexural limit state in beams is shown in Eq. (9). The reliability analysis results for all investigated beams and columns are summarized in Tables 9 to 11. The probability of failures (P_f) listed in Table 11 were calculated based on Eq. (3).

$$g(x) = M_n - (M_D + M_L + M_{SS}) \tag{9}$$

Table 12 Target reliability indices (β_T) related to ultimate limit states

Cost of safety measure	Consequences of failure		
	Minor	Moderate	Large
Large	3.1	3.3	3.7
Normal	3.7	4.2	4.4
Small	4.2	4.4	4.7

Table 13 Values of β and β_T for selected weakened beams

Beam	β	β_T	Action
B1	3.89	3.7	Beam deemed safe
B2	3.61	3.7	Further investigation needed
B3	3.95	3.7	Beam deemed safe
B4	2.26	3.7	Further investigation needed
B5	2.96	3.7	Further investigation needed
C1	3.26	4.4	Further investigation needed
C2	0.14	4.4	Strengthen/replace

7. Reliability requirements for existing structures

Existing structures differ from new ones in several aspects including: (1) the higher cost associated with increasing levels of safety, (2) the remaining working life is often less, and more information on the actual structural conditions may be available during the assessment process (inspections, tests, measurements). Therefore, it is considered to be uneconomical to require the same target reliabilities for existing and new structures. Table 12 lists the target reliability indices β_T as per RILEM (2001). Accordingly, for the investigated buildings, the target reliability for the beams is estimated to be 3.7 (minor consequences of failure with normal cost of safety measures) and 4.4 (large consequences of failure with normal cost of safety measures) for the columns. Accordingly, Table 13 summarizes the required reliability (β) and the target reliability (β_T) for the weakened elements.

It can be seen from Table 13 that the β for B1 and B3 exceed the β_T, therefore, these beams are deemed to be safe. However, B2, B4, B5 and C1 have β which are lower than the β_T which requires further investigation. On the other hand, C2 has a very high probability of failure P_f = 44% (Table 11). Thus, it can be concluded that this column needs either strengthening or replacement.

One option in a further investigation is to examine the feasibility and viability of carrying out a proof load test on the doubtful elements. This investigation is done using Monte Carlo simulation to estimate the value of the test

Table 14 Proof load test that if passed, the beam will give an updated β = 3.7

Beam	Original M_n (kN.m)		Proof load (kN.m)	Ratio	Updated M_n (kN.m)		Action
	μ	σ			μ	σ	
B2	8,354	909	6,500	78%	8,383	877	Do proof load test
B4	6,349	690	6,175	97%	6,809	475	Strengthen and/or decrease loads
B5	5,474	595	4,900	90%	5,643	485	Strengthen and/or decrease loads

Table 15 Design of repairs for the beams

Beam	Repair type	*COV*	β_T	μ_R (*kN.m*)	ϕ_R	ϕ_E	γ_D	γ_L
B4	A	0.1	3.7	958	0.95	0.73	1.13	1.65
	B	0.2		1,040	0.80	0.73	1.13	1.60
B5	A	0.1	3.7	406	0.98	0.75	1.13	1.81
	B	0.2		420	0.91	0.75	1.13	1.80

load in which the doubted element has to successfully pass it to achieve an updated reliability equal to the β_T. Table 14 summarizes the values of the testing load for each beam that if passed, the beam will deem to be safe. It can be concluded from the updated reliability analysis that it is feasible to carry out a load test for B2 and if the beam successfully passes the test, it will prove to be safe. On the other hand, beams B4 and B5 will require test load values that are equivalent to 97% and 90%, respectively, of the average nominal capacity. Thus, it can be concluded that these beams should be strengthened rather than load tested. In a similar manner, it was concluded that C2 needs strengthening.

8. Probabilistic design of repairs

Reliability analysis can also be used in the design of repairs by calculating the design point and, consequently, the PSFs (ϕ and γ), which are important tools to assure consistency in reliability among loads, materials, and modes of failure. In the case of design of repairs, the performance function can be rewritten as shown in Eq. (10).

$$g = R_R + R_E - \sum_{i=1}^{n} S_i = 0 \tag{10}$$

where R_R is the added resistance by repair, R_E is the existing resistance, and S_i is the acting stresses due to different loads. The exiting resistance that was used in the design of the repairs is the simplified single random variable that was obtained after applying the MCS on the multi-variable nominal resistance functions.

As an example, two repair methods, A and B, are assumed for beams B4 and B5. The coefficients of variation for these methods are 0.1 and 0.2, respectively, and both have normal probability distribution function. FORM was used to calculate the PSFs from Eq. (9). The results are summarized in Table 15. The PSFs in Table 15 provide us with the factors to be used in the LRFD design approach.

For example, the design equations of the repair works for B4 will become as shown in Eq. (11) and Eq. (12). Using these equations will assure that the design of the repairs will achieve a reliability index β of 3.7 for the repaired elements. It should be noted here that the moment due to the support settlement (M_{SS}) has a deterministic value of 993 kN.m based on field measurement.

Repair Type A

$$\phi R = 0.95 R_R + 0.73 R_E \geq 1.13 M_D + 1.65 M_L + M_{SS} \tag{11}$$

Repair Type B

$$\phi R = 0.8 R_R + 0.73 R_E \geq 1.13 M_D + 1.60 M_L + M_{SS} \tag{12}$$

9. Conclusions

The paper presents a probabilistic approach for the assessment of existing structures with reliability methods used to implement the MCS and FORM to determine the probability/reliability of strength limit state failure. FORM was proven to be a convenient computational tool that can be employed in assessing and evaluating the reliability of R/C structural components or systems as well as to develop and establish partial safety factors. Also, in this study the reliability analysis has been confirmed to be a powerful means that can be utilized as a decision-making process in the assessment of existing R/C structures or any different structure. The approached presented in the paper demonstrated that combining the MCS and the FORM can simplify the reliability analysis by converting the multi-random variable functions into a single random variable, which can be easily analysed.

Based on the development of reliability analysis in this paper, the following conclusions can be drawn:

1. The reliability-based analysis and assessment can be performed for different components and elements of R/C structures such as beams, columns, and other elements that involve shear stress, etc. It can also be performed on fatigue of reinforced concrete structures, although in this study neither shear nor fatigue were explored. But they could be investigated in future studies.
2. The probabilistic and statistical characteristics of both the strength and load variables play a fundamental role in reliability assessment and reliability-based design for R/C structural elements. The outcome of the assessment will be as good as the probabilistic characteristics that were collected and applied in the reliability procedures. Therefore, the proper quantification of the probabilistic characteristics of these variables for the weakened structural elements is an essential facet for assessment of R/C structural elements. For example, determination of reliability index in the limit-state function (equation) depends on these characteristics. Thus, reliability indices from limit state equations are as good as the probabilistic characteristics from which they were determined.
3. The probabilistic approach can establish partial safety factors that account for the variation on the different repair methods providing a proper tool to shift back to the LRFD method in the design of repairs.

For future studies on cases similar to this one, it is recommended that other structural components such as joints, one-way slabs, two-way slabs, and other structural components should be considered in the reliability analysis. In addition, other types of loading, such as wind, seismic, dynamic, fatigue, etc., should be taken into consideration Assakkaf and Shaikha (2015).

Although system reliability has its own merit in producing more accurate results and fairly predicting the remaining life of the structure or building, it requires more rigorous and detailed analysis, and it can be costly and time-consuming. Also, it requires more probabilistic information and statistics on strength, loads, materials, method of construction, etc., that might not be all available for performing such an analysis in a proper and straightforward manner. System reliability involves evaluating and assessing the whole structure or building as a unit (engineering system) rather than the individual structural components and elements. This method is recommended and justified only if money and time are not a problem, and if all of the above-mentioned needed information and data for this analysis are available.

Acknowledgments

The authors would like to thank engineers Mahmoud Fawzy, Mona Al-Saffar, and Ahmad Yousif for their contributions toward obtaining the field data and conducting the necessary laboratory testing and site investigation.

References

ACI 318 (2014), *Building Code Requirements for Structural Concrete and Commentary*, American Concrete Institute, U.S.A.

ACI 562 (2016), *Code Requirements for Evaluation, Repair, and Rehabilitation of Concrete Buildings and Commentary*, American Concrete Institute, U.S.A.

Ang, H.S. and Tang, H.T. (1984), *Probability Concepts in Engineering Planning and Design, Vol. 2: Decision, Risk, and Reliability*, John Wiley & Sons Inc. New York, U.S.A.

ASME (2005), *Development of Reliability-Based Load and Resistance Factor Design (LRFD) Methods for Piping*, ASME, 1828 L St. NW Suite 906, Washington, U.S.A.

Assakkaf, I., Sakka, Z. and Qazweeni, J. (2016), "Structural assessment and reliability of existing buildings", *Proceedings of the 6th Asian-Pacific Symposium on Structural Reliability and its Applications*, Tongji University Press, China, June.

Assakkaf, I.A. (1998), *Reliability-Based Design of Panels and Fatigue Details of Ship Structures*, University of Maryland, College Park, U.S.A.

Assakkaf, I.A. (2012a), *Reliability Design of Doubler Plates for Sea Tankers*, Advances in Civil Engineering and Building Materials, Hong Kong, November.

Assakkaf, I.A. (2012b), *Reliability-Based Design of Tanker Unstiffened Steel Panels*, Advances in Civil Engineering and Building Materials, Hong Kong, November.

Assakkaf, I.A., Kamal, H. and Al-Saffar, M. (2013), *Assessment of Reinforced Concrete Structural Components for An Aging Complex Building Using Reliability Methodologies*, Safety, Reliability, Risk and Life-Cycle Performance of Structures and Infrastructures, New York, U.S.A., June.

Ayyub, B.M. and McCuen, R.H. (2011), *Probability, Statistics, and Reliability for Engineers and Scientists*, CRC Press, Boca Raton, Florida, U.S.A.

Ayyub, B.M., Assakkaf, I., Atua, K., Engle, A., Hess, P., Karaszewski, Z., Kihl, D., Melton, W., Sielski, R.A., Sieve, M., Waldman, J. and White, G. J. (1998), *Reliability-based Design of Ship Structures: Current Practice and Emerging Technologies*, R-53, SNAME, T & R, U.S.A.

Bayerische Ingenieurkammer Bau (2004), *Report 2004*.

Ellingwood, B.R., Galambos, T.V., McGregor, J.G. and Cornell, C.A. (1980), *Development of a Probability Based Load Criterion for American National Standard A58*, NBS Special Report 577, U.S. Department of Commerce, National Bureau of Standards, U.S.A.

Hasofer, A.M. and Lind, N.C. (1974), "Exact and invariant second-moment code format", *J. Eng. Mech. Div.*, **100**(1), 111-121.

Keshtegar, B. (2016), "Chaotic conjugate stability transformation method for structural reliability analysis", *Comput. Meth. Appl. Mech. Eng.*, **310**, 866-885.

Krakovski, M. (1995), "Structural reliability estimation using Monte Carlo simulation and Pearson's curves", *Struct. Eng. Mech.*, **3**(3), 201-213.

Leira, B.J. (2016), "Reliability updating based on monitoring of structural response parameters", *Reliab. Eng. Syst. Safety*, **155**, 212-223.

Liu, Y., Jeong, H.K. and Collette, M. (2016), "Efficient optimization of reliability-constrained structural design problems including interval uncertainty", *Comput. Struct.*, **177**, 1-11.

Low, B. and Tang, W. (2007), "Efficient spreadsheet algorithm for first-order reliability method", *J. Eng. Mech.*, **133**(12), 1378-1387.

Low, B.K. and Tang, W.H. (2004), "Reliability analysis using object-oriented constrained optimization", *Struct. Safety*, **26**(1), 69-89.

Melchers, R.E. (1999), *Structural Reliability Analysis and Prediction*, John Wiley & Sons Ltd, West Sussex, U.K.

Nowak, A.S., Rakoczy, A.M. and Szeliga, E.K. (2012), "Revised statistical resistance models for R/C structural components", *Spec. Publ.*, **284**.

Paul, J.H. (2002), *Repair, Renovation and Strengthening of Concrete Structures*, Evaluation and Rehabilitation of Concrete Structures, Mexico, September.

Picard, Y., Thompson, C.J. and Marrett, S. (1992), "Improving the precision and accuracy of Monte Carlo simulation in positron emission tomography", *IEEE Trans. Nucl. Sci.*, **39**(4), 1111-1116.

Rackwitz, R. and Flessler, B. (1978), "Structural reliability under combined random load sequences", *Comput. Struct.*, **9**(5), 489-494.

RILEM (2001), *Probabilistic Assessment of Existing Structures-JCSS Report*, Rep. No. rep032, RILEM Publications SARL.

Schueller, G.I. (2007), "On procedures for reliability assessment of mechanical systems and structures", *Struct. Eng. Mech.*, **25**(3), 275-289.

Seo, D., Shin, S. and Han, B. (2010), "Reliability-based structural safety evaluation of reinforced concrete members", *J. Asian Architect. Build. Eng.*, **9**(2), 471-478.

Szerszen, M., Szwed, A. and Nowak, A. (2005), "Reliability analysis for eccentrically loaded columns", *Struct. J.*, **102**(5), 676-688.

Wang, G.S., Ang, H.S. and Lee, J. (1997), "Adaptive kernal method for evaluating structurl system reliability", *Struct. Eng. Mech.*, **5**(2), 115-126.

Wang, R., Wang, X., Wang, L. and Chen, X. (2016), "Efficient computational method for the non-probabilistic reliability of linear structural systems", *Acta Mech. Sol. Sin.*, **29**(3), 284-299.

Xu, J., Zhang, W. and Sun, R. (2016), "Efficient reliability assessment of structural dynamic systems with unequal weighted quasi-Monte Carlo simulation", *Comput. Struct.*, **175**, 37-51.

CC