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Wave dispersion characteristics of nonlocal strain gradient double-layered graphene sheets in hygro-thermal environments

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Abstract. Importance of procuring adequate knowledge about the mechanical behavior of double-layered graphene sheets (DLGSs) incensed the authors to investigate wave propagation responses of mentioned element while rested on a visco-Pasternak medium under hygro-thermal loading. A nonlocal strain gradient theory (NSGT) is exploited to present a more reliable size-dependent mechanical analysis by capturing both softening and hardening effects of small scale. Furthermore, in the framework of a classical plate theory the kinematic relations are developed. Incorporating kinematic relations with the definition of Hamilton's principle, the Euler-Lagrange equations of each of the layers are derived separately. Afterwards, combining Euler-Lagrange equations with those of the NSGT the nonlocal governing equations are written in terms of displacement fields. Interaction of the each of the graphene sheets with another one is regarded by the means of vdW model. Then, a widespread analytical solution is employed to solve the derived equations and obtain wave frequency values. Subsequently, influence of each participant variable containing nonlocal parameter, length scale parameter, foundation parameters, temperature gradient and moisture concentration is studied by plotting various figures.

Keywords: wave propagation; nonlocal strain gradient theory (NSGT); double-layered graphene sheet (DLGS); visco-Pasternak medium; hygro-thermal environment

1. Introduction

These days, small size structures have possessed a huge application in many micro/nano electromechanical systems (MEMSs/NEMSs) as sensors, actuators, transistors and resonators (Ebrahimi and Salari 2015a). Owing to this fact, lots of researchers utilize such tiny elements in their probes analyzing the mechanical responses of those called structures. Clearly, small size effects shall be regarded once studying the mechanical responses of size-dependent beams, plates or shells than ones of macro scales. Due to this fact, the nonlocal continuum theories are developed to capture the small-scale effects while investigating mechanical characteristics of nanodevices. Eringen (1972, 1983) presented the first nonlocal theory, called nonlocal elasticity theory (NET), which expresses the stress state in a desired point to be a function of the strains of all other adjacent points besides the strain of that specific point. This theory has been employed by a widespread range of authors, thus, it is worth mentioning to remonstrate some of the previous works gaining NE during their study on the mechanical responses of nanostructures. Wang and Varadan (2007) studied the wave dispersion properties of nanosize shells in the framework of a nonlocal shell theory. Reddy and Pang (2008) tried to magnify small scale influences investigating the bending, buckling and free vibration responses of carbon nanotubes (CNTs). Bending, buckling and free vibration answers of nanobeams are studied by Aydogdu (2009). Wang *et al.* (2010) analyzed the wave dispersion characteristics of nanoplates employing NE. Malekzadeh *et al.* (2011) showed the free vibration responses of orthotropic arbitrary straight-sided quadrilateral nanoplates. The study of temperature effects on wave dispersion properties of size-dependent plates is performed by Narendar and Gopalakrishnan (2012a). Also, Narendar and Gopalakrishnan (2012b) could explain surface effects on wave propagation behaviors of a nanoplate. Eltaher *et al.* (2013) described the vibrational properties of nanobeams in the framework of a finite element method (FEM) and Euler-Bernoulli beam theory. In another attempt, Ebrahimi and Salari (2015b) surveyed the vibration properties of CNTs by the means of NET. Moreover, the coupled influences of thermal environments and surface elasticity are regarded by Ebrahimi *et al.* (2016g) while studying the vibration and buckling answers of nanotubes. The bending vibration analysis of nanobeams is performed by Ghadiri and Shafiei (2016) utilizing differential quadrature method (DQM). In many researches, authors utilized the NET to investigate the mechanical responses of size-dependent beams and plates subjected to various external loads (Ebrahimi and Barati 2016a, b, c, d, e, f, 2017a, b, c, d, Ebrahimi and Dabbagh 2017a, Ebrahimi and Hosseini 2016, Ebrahimi *et al.* 2016a, c, e, f, Ebrahimi and Hosseini 2016). Even though Eringen's theory has been employed by a large number of authors, this theory is not powerful enough to depict the size-dependent behaviors of nanostructures entirely. Remarking this fact, some researches are allocated to investigate the deficiencies of

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NE (Fleck and Hutchinson 1993, Lam *et al.* 2003). Lam *et al.* (2003) proved the crucial role of elastic strain gradient in the size-dependent responses of small structures. Coupling the NET and strain gradient theory, Lim *et al.* (2015) organized a new nonlocal strain gradient theory (NSGT) in which considers both of the former effects. In this new theory, the stiffness-hardening influence is considered in addition to the stiffness-softening effect. Furthermore, Lim *et al.* (2015) could show the ingenuity of this novel theory in predicting wave dispersion responses of CNTs. Thereafter, many researchers tried to use this theory studying vibration, bending, buckling or wave propagation answers of nanostructures. The examination of the thermo-mechanical buckling properties of orthotropic nanoplates has been performed by Farajpour *et al.* (2016) in the framework of the NSGT. The wave dispersion properties of nano-beams and-plates are investigated in detail in the framework of the NSGT by Ebrahimi *et al.* (2016b, d) and Ebrahimi and Dabbagh (2017b, c). Lately, some authors have analyzed the vibration, stability, and wave dispersion responses of composite nanosize beams, rods, and tubes using NSGT (Li and Hu 2015, 2017, Li *et al.* 2015). Also, Ebrahimi and Barati (2017e, f) employed the NSGT in order to highlight effect of various parameters in their researches on vibrational responses of compositionally graded nanobeams. Most recently, Ebrahimi and Barati (2017g) presented a NSG based theory for vibration analysis of viscoelastic nanoplates whenever rested on visco-Pasternak substrate. For better understanding of the characteristics of the size-dependent elements, the readers are advised to see other attempts performed by researchers dealing with the statical and dynamical behaviors of small scale structures (Ahouel *et al.* 2016, Al-Basyouni *et al.* 2015).

On the other hand, a large variety of carbon-based structures including CNTs, carbon nanocones and nanorings can be achieved by generating some controlled distortions in single-layered graphene sheets (SLGSs) (Arani and Jalaei 2016). In addition, graphene sheets encompass some superiorities compared with other small size structures made of many various types of materials like higher elastic potential (Lee *et al.* 2008) and larger thermal conductivity (Seol *et al.* 2010). According to above information, it is necessary to obtain detailed results about the mechanical responses of these types of nanostructures. Thus, Liew *et al.* (2006) studied the vibration characteristics of multi-layered graphene sheets (MLGSs) embedded in an elastic medium. Murmu and Pradhan (2009) tried to show dynamic answers of embedded SLGSs employing Eringen's nonlocal theory. Moreover, Pradhan and Phadikar (2009) investigated the vibrational responses of MLGSs rested in polymer matrix. Also, the study of size-dependent buckling properties of SLGSs is performed by Pradhan and Murmu (2010). Ansari *et al.* (2010) tried to introduce a new FE based method for vibration analysis of embedded MLGSs. In addition, Ansari *et al.* (2011) utilized a generalized DQM in order to solve the vibration problem of a MLGS considering diverse boundary conditions. Pradhan and Kumar (2011) used DQM to show the reliability of this solution approach in solving vibration problems of orthotropic SLGSs. Rouhi and Ansari (2012) presented an atomistic FE based model for vibration and axial buckling analysis of SLGSs. Natsuki *et al.* (2012) employed the NET to show the buckling responses of double-layered circular graphene sheets. In another attempt, Arani *et al.* (2013) could exactly highlight the influences of substrate parameters on the nonlinear thermo-mechanical vibration answers of orthotropic DLGSs in the framework of DQM. Murmu *et al.* (2013) investigated the transverse vibrational responses of a magnetically affected SLGS using NET. Farajpour *et al.* (2013) analyzed the Postbuckling characteristics of MLGSs once a non-uniform biaxial compression is applied. Also, Anjomshoa *et al.* (2014) used NET incorporated with FEM to study the buckling responses of embedded MLGSs. Wang *et al.* (2015) employed the nonlocal relations of Eringen in order to investigate the nonlinear vibration problem of a double-layered viscoelastic graphene sheets (DLVGSs). Also, Hashemi *et al.* (2015) presented an exact solution for vibration analysis of orthotropic DLGSs. Furthermore, magneto-mechanical vibration and stability analysis of SLGSs rested on viscoelastic foundation is the issue of another research performed by Arani *et al.* (2016). Zenkour (2016) surveyed the transient vibration problem of a SLGS rested on a viscoelastic foundation. Influence of initial shear stress is regarded by Ebrahimi and Shafiei (2017) analyzing vibrational characteristics of SLGSs rested on Winkler-Pasternak foundation. Most recently, Ebrahimi and Barati (2018) could show the viscoelastic dynamic characteristics of SLGSs embedded on three-parameter viscoPasternak medium utilizing NSGT.

In addition, thermo-mechanical analysis of various elements under different thermal loadings are studied by several researchers (Ebrahimi and Habibi 2017a, b, Ebrahimi and Shaghaghi 2016).

Obviously, the vibration and buckling responses of single- or multi-layered graphene sheets has been performed by a large number of authors, whereas, the wave propagation analysis of the same structure is not studied a lot. Even though this academic lack, nobody doubts in the crucial role of wave propagation-based methods in various fields. For instance, wave propagation analysis can be useful for defect detection in the applications in which performing non-destructive tests is not practical. However, some of the authors tried to investigate the wave propagation behaviors of desirable structures in the recent years (Karami *et al.* 2017, Yahia *et al.* 2015). Lately, a few endeavors on the wave dispersion properties of graphene sheets has been performed by some of the authors. For example, size-dependent mechanical properties of propagating waves in graphene sheets are exactly studied by Arash *et al.* (2012) by the means of the nonlocal elasticity. Also, Liu and Yang (2012) employed a nonlocal model to investigate the wave propagation problem of an embedded isotropic graphene sheet. Most recently, Xiao *et al.* (2017) presented a nonlocal strain gradient-based theory to examine wave propagation behaviors of viscoelastic monolayer graphene sheets. Thereafter, Ebrahimi and Dabbagh (2017d) surveyed the thermo-mechanical wave propagation behavior of DLGSs, rested on a viscoelastic substrate, based on the NE.

A brief literature review reveals a lack in the scientific researches about the wave propagation responses of embedded DLGSs in hygro-thermal environments. Actually, it can be found that in the previous researches the authors

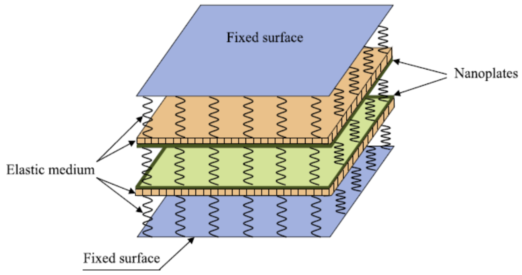

Fig. 1 Geometry of a double-layered graphene sheet rested on Winkler-Pasternak foundation

focused on the some of the issues concerned in this attempt individually. In other words, although a wide range of endeavors are performed to analyze the mechanical behaviors of GSs, there is no article available dealing with the hygro-thermo-elastic wave dispersion behaviors of DLGSs using NSGT. Herein, by coupling the principle of virtual work with the kinematic relations of Kirchhoff plate theory the governing equations are derived. Furthermore, an analytical solution is utilized to solve the final nonlocal differential equations. Then, a separate section is allocated to investigate effect of each parameter on the wave frequency and phase velocity of propagated waves.

2. Theory and formulation

2.1 Kinematic relations

Present part is devoted to describe the kinematic behaviors of graphene sheets. The schematic of an embedded DLGS can be seen in Fig. 1.

The basic assumptions of the classical theory of plates can be summarized as:

• straight lines normal to the mid-surface of the plate remain straight after deformation.

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It is of significance to point that in some of the newly developed theories, shear deformation effects are included based on a three-variable plate model (Houari *et al.* 2016). Ones who are interested in studying different kinematic theories are referred to study more references (Ebrahimi and Heidari 2017, Ebrahimi and Jafari 2017, 2018).

Here, the displacement fields can be written as

$$\begin{cases} U_i(x,y,z) = -z\dfrac{\partial w_i}{\partial x} \\ V_i(x,y,z) = -z\dfrac{\partial w_i}{\partial y} \\ W_i(x,y,z) = w_i(x,y) \end{cases}, \quad i=(1,2) \tag{1}$$

where w_i is bending deflection of the i-th plate in the thickness direction. Now, the nonzero strains for each of the graphene sheets can be stated as follows

$$\begin{Bmatrix} \varepsilon_{xx,i} \\ \varepsilon_{yy,i} \\ \gamma_{xy,i} \end{Bmatrix} = z\begin{Bmatrix} -\dfrac{\partial^2 w_i}{\partial x^2} \\ -\dfrac{\partial^2 w_i}{\partial y^2} \\ -2\dfrac{\partial^2 w_i}{\partial x\,\partial y} \end{Bmatrix}, \quad i=(1,2) \tag{2}$$

Besides, the Hamilton's principle can be defined as

$$\int_0^t \delta(U - T + V)\,dt = 0 \tag{3}$$

in which U is strain energy, T is kinetic energy and V is work done by external loads. The variation of strain energy for each plate can be calculated as

$$\delta U = \int_V \sigma_{mn,i}\,\delta\varepsilon_{mn,i}\,dV = \int_V \begin{pmatrix} \sigma_{xx,i}\,\delta\varepsilon_{xx,i} + \\ \sigma_{yy,i}\,\delta\varepsilon_{yy,i} + \\ \sigma_{xy,i}\,\delta\gamma_{xy,i} \end{pmatrix} dV, \quad i=(1,2) \tag{4}$$

Substituting Eq. (2) in Eq. (4) reveals

$$\delta U_i = \int_0^a\!\!\int_0^b \begin{pmatrix} -M_{xx,i}\dfrac{\partial^2 \delta w_i}{\partial x^2} \\ -2M_{xy,i}\dfrac{\partial^2 \delta w_i}{\partial x\,\partial y} \\ -M_{yy,i}\dfrac{\partial^2 \delta w_i}{\partial y^2} \end{pmatrix} dy\,dx, \quad i=(1,2) \tag{5}$$

in Eq. (5) the unknown parameters can be defined in the following form

$$M_{j,i} = \int_{-h/2}^{h/2} z\,\sigma_{j,i}\,dz, \; j=(xx,yy,xy), i=(1,2) \tag{6}$$

Furthermore, variation of work done by external forces can be shown as follows

$$\delta V_i = \int_0^a\!\!\int_0^b \begin{pmatrix} N_x^0\dfrac{\partial w_i}{\partial x}\dfrac{\partial \delta w_i}{\partial x} + N_y^0\dfrac{\partial w_i}{\partial y}\dfrac{\partial \delta w_i}{\partial y} \\ -k_w\,\delta w_i + \\ k_p\left(\dfrac{\partial w_i}{\partial x}\dfrac{\partial \delta w_i}{\partial x} + \dfrac{\partial w_i}{\partial y}\dfrac{\partial \delta w_i}{\partial y}\right) \\ -C_d\,\delta\dfrac{\partial w_i}{\partial t} \end{pmatrix} dy\,dx, i=(1,2) \tag{7}$$

where N_x^0, N_y^0 are in-plane applied loads; k_w, k_p and C_d

are Winkler, Pasternak and damping coefficients, respectively. The variation of the kinetic energy will be written as

$$\delta K = \int_0^a\int_0^b \begin{pmatrix} I_0\left(\frac{\partial w_i}{\partial t}\frac{\partial \delta w_i}{\partial t}\right)+ \\ I_2\begin{pmatrix}\frac{\partial w_i}{\partial x\partial t}\frac{\partial \delta w_i}{\partial x\partial t}+ \\ \frac{\partial w_i}{\partial y\partial t}\frac{\partial \delta w_i}{\partial y\partial t}\end{pmatrix}\end{pmatrix} dydx \tag{8}$$

in which

$$\left(I_0, I_2\right) = \int_{-h/2}^{h/2}\left(1, z^2\right)\rho dz \tag{9}$$

Inserting Eqs. (5), (7) and (8) in Eq. (3) and setting the coefficients of δw_i to zero, the Euler-Lagrange equations of each of the graphene sheets can be rewritten as

$$\begin{aligned} &\frac{\partial^2 M_{xx,i}}{\partial x^2}+2\frac{\partial^2 M_{xy,i}}{\partial x\partial y}+\frac{\partial^2 M_{yy,i}}{\partial y^2}+N_x^0\frac{\partial^2 w_i}{\partial x^2} \\ &+N_y^0\frac{\partial^2 w_i}{\partial y^2}-k_w w_i - C_d\frac{\partial w_i}{\partial t}+k_p\nabla^2 w_i = \\ &I_0\frac{\partial^2 w_i}{\partial t^2}-I_2\nabla^2\left(\frac{\partial^2 w_i}{\partial t^2}\right),\ i=(1,2) \end{aligned} \tag{10}$$

where $N_x^0 = N_y^0 = N^T + N^H$, in which N^T and N^H stand for applied loads made of temperature and moisture change, respectively.

2.2 The nonlocal strain gradient elasticity

According to the nonlocal strain gradient theory, the stress field takes into consider the effects of nonlocal elastic stress field besides strain gradient stress field. So, the theory can be expressed as follows for elastic solids

$$\sigma_{ij} = \sigma_{ij}^{(0)} - \frac{d\sigma_{ij}^{(1)}}{dx} \tag{11}$$

in above equation, the stresses $\sigma_{xx}^{(0)}$ (classical stress) and $\sigma_{xx}^{(1)}$ (higher-order stress) are corresponding to strain ε_{xx} and strain gradient $\varepsilon_{xx,x}$, respectively as follows

$$\begin{cases} \sigma_{ij}^{(0)} = \int_0^L C_{ijkl}\alpha_0(x,x',e_0a)\varepsilon'_{kl}(x')dx' \\ \sigma_{ij}^{(1)} = l^2\int_0^L C_{ijkl}\alpha_1(x,x',e_1a)\varepsilon'_{kl,x}(x')dx' \end{cases} \tag{12}$$

in which C_{ijkl} is the elastic modulus tensor; e_0a and e_1a are introduced to account for the nonlocality effects which is a decreasing (softening) effect physically. Also, l captures the strain gradient effects to take into consider the hardening impact in nanoscale. In this research the influences of structure's thickness are not included. However, Li *et al.* (2018) have recently presented a new NSG based theory to investigate the mechanical responses of nanobeams which can be employed in other future researches. Once the nonlocal kernel functions α_0 and α_1 satisfy the developed conditions, the constitutive relation of nonlocal strain gradient theory can be expressed as below

$$\begin{aligned} &\left(1-(e_1a)^2\nabla^2\right)\left(1-(e_0a)^2\nabla^2\right)\sigma_{ij} = \\ &C_{ijkl}\left(1-(e_1a)^2\nabla^2\right)\varepsilon_{kl} - C_{ijkl}\, l^2\left(1-(e_0a)^2\nabla^2\right)\nabla^2\varepsilon_{kl} \end{aligned} \tag{13}$$

in which ∇^2 denotes the Laplacian operator. Herein, the assumption of $e_0=e_1=e$ is employed for simplicity. Afterwards, the general constitutive relation in Eq. (15) becomes

$$\left(1-(ea)^2\nabla^2\right)\sigma_{ij} = C_{ijkl}\left(1-l^2\nabla^2\right)\varepsilon_{kl} \tag{14}$$

Finally, the simplified constitutive relation can be written as follows

$$\begin{aligned} &\left(1-\mu^2\nabla^2\right)\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix} = \\ &\left(1-\eta^2\nabla^2\right)\begin{pmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{pmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{Bmatrix} \end{aligned} \tag{15}$$

in above equation

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2},\ Q_{12} = \nu Q_{11},\ Q_{66} = \frac{E}{2(1+\nu)} \tag{16}$$

where $\mu=e_0a$ and $\eta=l$. Now, inserting Eq. (6) in Eq. (15) gives

$$\begin{aligned} &\left(1-\mu^2\nabla^2\right)\begin{Bmatrix}M_{xx}\\ M_{yy}\\ M_{xy}\end{Bmatrix} = \left(1-\eta^2\nabla^2\right) \\ &\left(\begin{pmatrix}D_{11} & D_{12} & 0\\ D_{12} & D_{22} & 0\\ 0 & 0 & D_{66}\end{pmatrix}\begin{Bmatrix}-\frac{\partial^2 w_i}{\partial x^2}\\ -\frac{\partial^2 w_i}{\partial y^2}\\ -2\frac{\partial^2 w_i}{\partial x\partial y}\end{Bmatrix}\right) \end{aligned} \tag{17}$$

in Eq. (17), the cross-sectional rigidities can be formulated as follows

$$\begin{Bmatrix}D_{11}\\ D_{12}\\ D_{66}\end{Bmatrix} = \int_{-h/2}^{h/2} Q_{11}z^2\begin{Bmatrix}1\\ \nu\\ \frac{1-\nu}{2}\end{Bmatrix}dz \tag{18}$$

By substituting Eq. (17) in Eq. (10), the nonlocal governing equation of each layer of DLGSs can be directly derived in terms of displacements as follows

$$\left(1-\eta^2\nabla^2\right)\begin{pmatrix} D_{11}\dfrac{\partial^4 w_i}{\partial x^4}+2\left(D_{12}+2D_{66}\right)\dfrac{\partial^4 w_i}{\partial x^2\partial y^2} \\ +D_{22}\dfrac{\partial^4 w_i}{\partial y^4}\end{pmatrix}+\left(1-\mu^2\nabla^2\right)\begin{pmatrix} I_0\dfrac{\partial^2 w_i}{\partial t^2}-I_2\begin{pmatrix}\dfrac{\partial^4 w_i}{\partial x^2\partial t^2}+ \\ \dfrac{\partial^4 w_i}{\partial y^2\partial t^2}\end{pmatrix} \\ +k_w w_i+C_d\dfrac{\partial w_i}{\partial t}- \\ \left(k_p-N^T-N^H\right)\begin{pmatrix}\dfrac{\partial^2 w_i}{\partial x^2}+ \\ \dfrac{\partial^2 w_i}{\partial y^2}\end{pmatrix}\end{pmatrix}=0,\ i=(1,2) \tag{19}$$

The above equation is the nonlocal governing equations of each of the layers without any attention to the interactions between the layers. Herein, van der Waals (vdW) model is employed to account for this phenomenon as follows

$$\left(1-\eta^2\nabla^2\right)\begin{pmatrix} D_{11}\dfrac{\partial^4 w_1}{\partial x^4}+2\left(D_{12}+2D_{66}\right)\dfrac{\partial^4 w_1}{\partial x^2\partial y^2} \\ +D_{22}\dfrac{\partial^4 w_1}{\partial y^4}\end{pmatrix}+\left(1-\mu^2\nabla^2\right)\begin{pmatrix} I_0\dfrac{\partial^2 w_1}{\partial t^2}-I_2\left(\dfrac{\partial^4 w_1}{\partial x^2\partial t^2}+\dfrac{\partial^4 w_1}{\partial y^2\partial t^2}\right) \\ +k_w w_1+C_d\dfrac{\partial w_1}{\partial t}- \\ \left(k_p-N^T-N^H\right)\left(\dfrac{\partial^2 w_1}{\partial x^2}+\dfrac{\partial^2 w_1}{\partial y^2}\right) \\ +C\left(w_1-w_2\right)\end{pmatrix}=0 \tag{20}$$

$$\left(1-\eta^2\nabla^2\right)\begin{pmatrix} D_{11}\dfrac{\partial^4 w_2}{\partial x^4}+2\left(D_{12}+2D_{66}\right)\dfrac{\partial^4 w_2}{\partial x^2\partial y^2} \\ +D_{22}\dfrac{\partial^4 w_2}{\partial y^4}\end{pmatrix}+\left(1-\mu^2\nabla^2\right)\begin{pmatrix} I_0\dfrac{\partial^2 w_2}{\partial t^2}-I_2\left(\dfrac{\partial^4 w_2}{\partial x^2\partial t^2}+\dfrac{\partial^4 w_2}{\partial y^2\partial t^2}\right) \\ +k_w w_2+C_d\dfrac{\partial w_2}{\partial t}- \\ \left(k_p-N^T-N^H\right)\left(\dfrac{\partial^2 w_2}{\partial x^2}+\dfrac{\partial^2 w_2}{\partial y^2}\right) \\ +C\left(w_2-w_1\right)\end{pmatrix}=0 \tag{21}$$

in above equations, C stands for vdW interaction coefficient. In this research, the dimensions of the nanoplates in both longitudinal and transverse directions are too long and actually there in no need to introduce boundary conditions. However, different types of boundary conditions are discussed in some attempts conducted by authors (Ebrahimi and Jafari 2016, Ebrahimi and Salari 2017).

3. Analytical solution

In this part, the nonlocal governing equations derived in previous section are going to be solved analytically. The displacement fields are assumed to be exponential and can be defined as follows

$$\begin{Bmatrix} w_1(x,y,t) \\ w_2(x,y,t)\end{Bmatrix}=\begin{Bmatrix} W_1\exp\left[i\left(\beta_1 x+\beta_2 y-\omega t\right)\right] \\ W_2\exp\left[i\left(\beta_1 x+\beta_2 y-\omega t\right)\right]\end{Bmatrix} \tag{22}$$

where W_1 and W_2 are the unknown coefficients; β_1 and β_2 are the wave numbers of wave propagation along x and y directions respectively, and finally ω is wave's angular frequency. Now, substituting Eq. (22) into Eqs. (20) and (21) results

$$\left(\left[K\right]_{2\times2}-\omega^2\left[M\right]_{2\times2}\right)\{\Delta\}=\{0\} \tag{23}$$

where the corresponding k_{ij}, m_{ij} are as written in appendix. The unknown parameters of Eq. (23) can be noted as follows

$$\{\Delta\}=\{W_1,W_2\}^T \tag{24}$$

In order to attaining wave's angular frequency, the determinant of the left-hand side of Eq. (23) should be set to zero

$$\left|\left[K\right]_{2\times2}-\omega^2\left[M\right]_{2\times2}\right|=0 \tag{25}$$

In above equation by setting $\beta_1=\beta_2=\beta$ and solving the obtained equation for ω, the wave's angular frequency of embedded DLGSs can be calculated. If the angular frequency is divided by wave number, the phase velocity can be obtained as below

$$c_p=\frac{\omega}{\beta} \tag{26}$$

Also, the escape frequency of DLGSs can be derived by tending wave number to infinity

$$f_{esc}=\lim_{\beta\to\infty}\frac{\omega}{2\pi} \tag{27}$$

4. External forces

Here, the external applied forces can be expressed as follows

Table 1 Comparison of frequency of FG nanoplates for various nonlocal parameters (p=5)

μ	a/h=10		a/h=20	
	Present	Natarajan *et al.* (2012)	Present	Natarajan *et al.* (2012)
0	0.043803	0.0441	0.011255	0.0113
1	0.040051	0.0403	0.010288	0.0103
2	0.037123	0.0374	0.009534	0.0096
4	0.032791	0.033	0.008418	0.0085

$$\begin{cases} N^T = \int_{-h/2}^{h/2} \frac{E}{1-\nu}.\alpha.\Delta T dz \\ N^H = \int_{-h/2}^{h/2} \frac{E}{1-\nu}.\beta.\Delta C dz \end{cases} \tag{28}$$

in above relation, E, ν, α, β, ΔC and ΔT are Young's modulus, poison's ratio, thermal expansion coefficient, moisture expansion coefficient, moisture concentration and temperature gradient, respectively.

5. Results and discussion

Herein, the wave propagation responses of DLGSs are compared once various parameters are supposed to be changed. The material properties of graphene sheets are defined as: E=1 TPa, ν=0.19, ρ=2300 kg/m^3, α=1.6×10^{-6} 1/K, β=0.0026 (Ebrahimi and Barati 2018). Also, the thickness is presumed to be h=0.34 nm. In addition, the vdW interaction coefficient can be supposed to be C=-108 GPa/nm (Liew *et al.* 2006). In the following diagrams wave frequencies are calculated by dividing wave's angular frequency to 2π ($f=\omega/2\pi$). Moreover, the validity of reported results is proven setting a comparison between results of present research with those of antecedent works.

Also, it is of importance to point that in some of the recent researches, the nonlocal and length scale parameters are determined on the basis of data obtained from experiments or molecular dynamic simulations. Volunteers are offered to read Karami *et al.* (2017) and Zhu and Li (2017a).

Fig. 2 is devoted to show the influence of nonlocal and length scale parameters on wave frequency of DLGSs with respect to variations of wave number. It is clear that in the case of the NE (η=0), a rise in the amount of nonlocal parameter reveals a decrease in the value of wave frequency. In this condition, by adding wave number wave frequency increases gradually until obtaining its peak amount. Once the strain gradient elasticity is considered ($\eta\neq0$), wave frequency tends to infinity as wave number becomes greater. Also, in this situation length scale parameter acts in the way of increasing wave frequency. In other words, if nonlocal parameter is supposed to be constant, an increase in the value of length scale parameter can be resulted in a raise in the amount of wave frequency.

In Fig. 3, variation of phase velocity versus wave number is plotted for various nonlocal and length scale parameters. Obviously, it is clear that phase velocity rises to its maximum amount and then starts to diminish continuously as wave number increases when the NET is utilized (η=0). Also, it is worth mentioning that nonlocal parameter has a softening effect on the phase velocity of DLGSs as same as their wave frequency. Indeed, phase velocity can be easily detracted by choosing a bigger nonlocal parameter. In addition, if NSGT is applied ($\eta\neq0$), by increasing wave number phase velocity becomes bigger and whenever reached to its maximum amount remains constant. It shall be mentioned that a larger phase velocity value can be obtained once a greater length scale parameter is utilized.

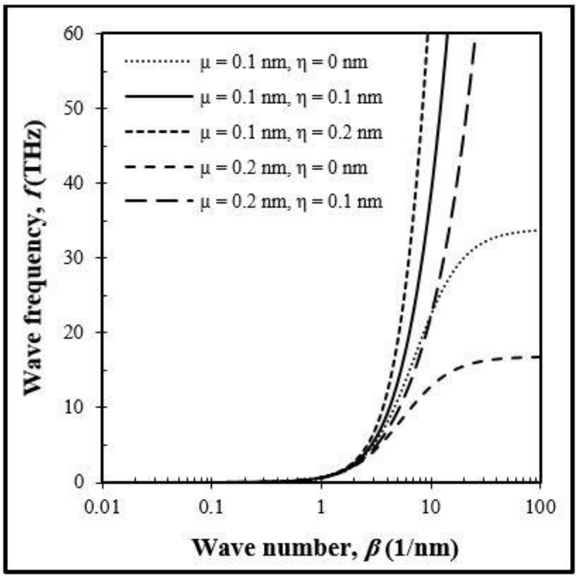

Fig. 2 Coupled effect of nonlocal and length scale parameters on wave frequency of DLGSs (k_w=k_p=0, C_d=0, ΔT= ΔC=0)

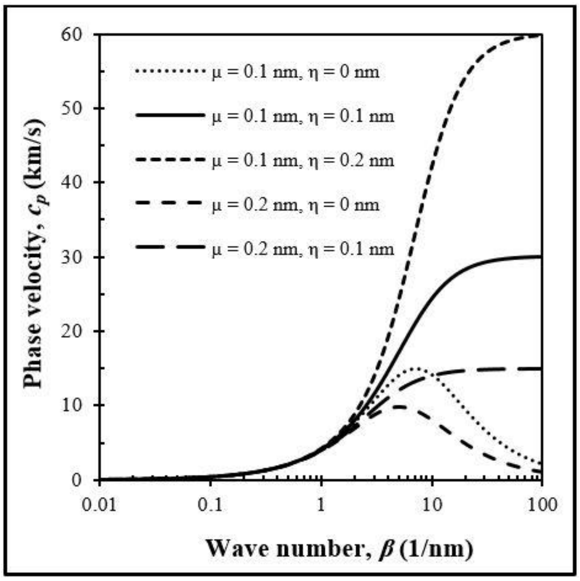

Fig. 3 Coupled effect of nonlocal and length scale parameters on phase velocity of DLGSs (k_w=k_p=0, C_d=0, ΔT= ΔC=0)

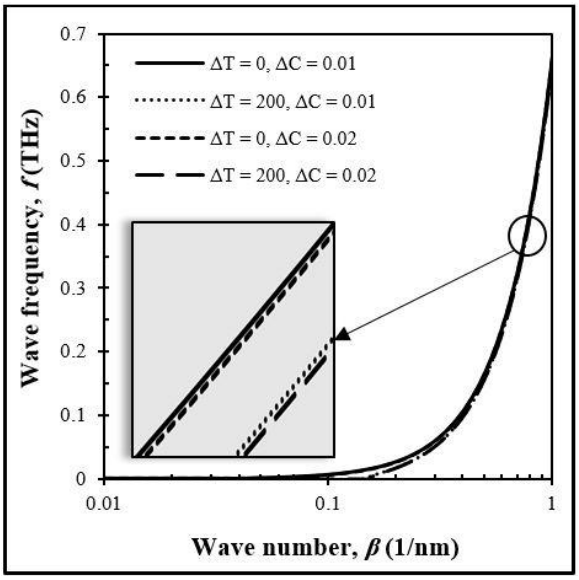

Fig. 4 Coupled effect of temperature gradient and moisture concentration on wave frequency of DLGSs (μ=η=0.1 nm, k_w=k_p=0, C_d=0)

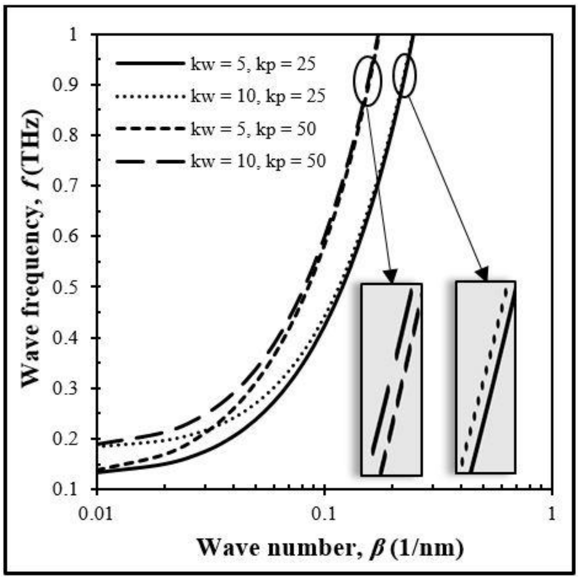

Fig. 5 Coupled effect of Winkler and Pasternak coefficients on wave frequency of DLGSs (μ=η=0.1 nm, C_d=0, ΔT= ΔC=0)

Fig. 4 is presented in order to characterize the variation of wave frequency versus wave number for different values of temperature gradient and moisture concentrations. It is clear that wave frequency can be affected by making a change in the temperature gradient or moisture concentration in small wave numbers. As predicted before, increasing temperature gradient leads to a decrease in the amount of wave frequency. Moreover, a similar behavior can be observed by changing moisture concentration. In other words, smaller wave frequencies are achieved once a raise is produced in the moisture concentration. Once wave number is tended to infinity, variations of wave frequency become insensible. Therefore, one of the ways of obtaining smaller wave frequencies is to utilize hygro-thermal environment than thermal one. Effect of various Winkler and Pasternak coefficients on the wave frequency of DLGSs is shown in Fig. 5. It is clear that in a constant amount of each of these parameters bigger wave frequencies can be obtained by choosing a higher value for another coefficient. It shall be considered that in small wave numbers Winkler coefficient can influence wave frequency more than Pasternak coefficient. However, in wave numbers bigger than β=0.2 (1/nm) effect of Pasternak coefficient is more observable. Thus, it can be concluded that a higher wave frequency value can be reached by employing bigger amounts for linear or nonlinear medium parameters.

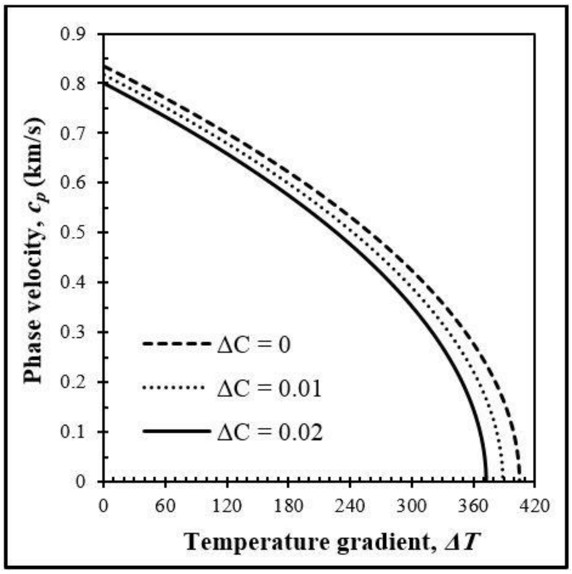

Fig. 6 Variation of phase velocity of a DLGS versus temperature gradient for different moisture concentrations (μ=η=0.1 nm, k_w=k_p=0, C_d=0, β=0.2)

Furthermore, in Fig. 6 variation of phase velocity versus temperature gradient is plotted for different moisture concentrations. Depending on this figure, both of the temperature gradient and moisture concentration are able to decrease phase velocity values whenever they are raised. In each desired moisture concentration, phase velocity starts from its maximum amount and diminishes to zero in a continuous manner. This phenomenon happens in a smaller temperature gradient if moisture concentration is intensified.

Besides, Fig. 7 is devoted to study the variation of phase velocity versus damping coefficient for both thermal and hygro-thermal conditions. It can be understood that once graphene sheets are rested on a viscoelastic substrate their wave dispersion response can be damped in comparison with a Winkler-Pasternak foundation. Also, it shall be mentioned that wave propagation responses of DLGSs are not too different in thermal and hygro-thermal situations. However, whenever hygro-thermal condition is chosen the outcome can be a very tiny decrease in the phase velocity value compared with thermal condition. So, despite phase velocity is not sensitive to thermal or hygro-thermal being of the environment, selecting a hygro-thermal environment can be considered as one of the alternatives for obtaining smaller phase velocities.

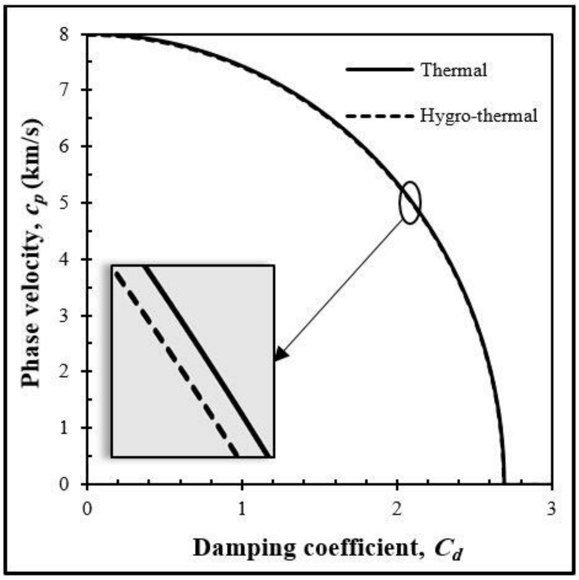

Fig. 7 Variation of phase velocity of a DLGS versus damping coefficient for both thermal and hygro-thermal conditions (μ=η=0.1 nm, k_w=k_p=0, β=2

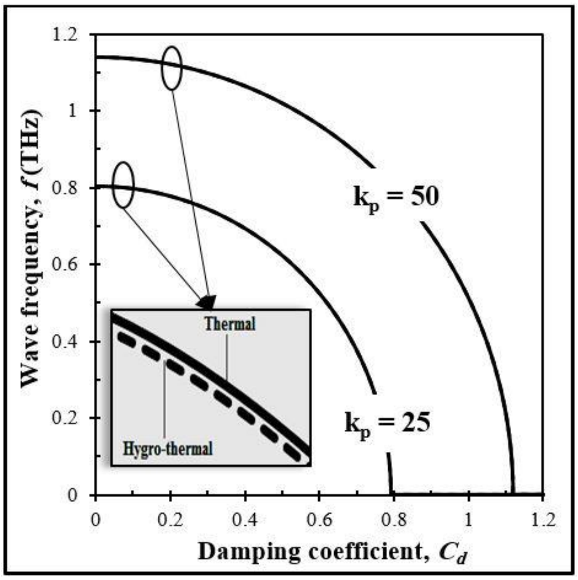

Fig. 9 Variation of wave frequency of a DLGS versus damping coefficient for various Pasternak coefficients and both thermal and hygro-thermal conditions (μ=η=0.1 nm, k_w=0, β=0.2)

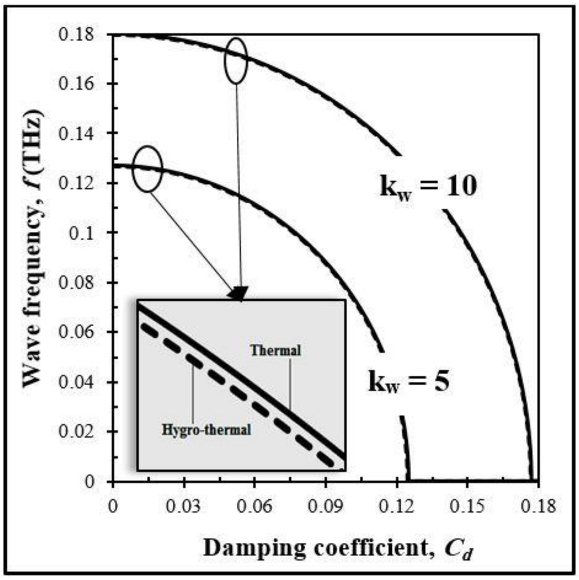

Fig. 8 Variation of wave frequency of a DLGS versus damping coefficient for various Winkler coefficients and both thermal and hygro-thermal conditions (μ=η=0.1 nm, k_p=0, β=0.2)

At last, Figs. 8 and 9 are plotted to magnify the influences of viscoelastic medium on wave frequency of DLGSs under thermal and hygro-thermal conditions. It can be well observed that wave frequency shows a damping influence whenever its variations are plotted with respect to damping coefficient of visco-Pasternak foundation. As estimated before, wave frequency will be of a smaller magnitude once a hygro-thermal environment is considered.

As a matter of fact, both of the linear (Winkler) and nonlinear (Pasternak) coefficients are powerful enough to enlarge the amount of wave frequency. It is worth mentioning that effect of Winkler coefficient can be damped by using a smaller damping coefficient. Furthermore, it is of significance to point that Pasternak coefficient requires bigger values to generate an increase in the amount of wave frequency once compared with Winkler coefficient. Obviously, whenever higher wave frequency is the main purpose, it is better to pay more attention to Pasternak coefficient; because, in a same amount of linear and nonlinear foundation parameters this coefficient can enormously amplify wave frequency compared with Winkler coefficient.

6. Conclusions

Presented article aims to survey wave propagation responses of DLGSs under hygro-thermal environments once it is rested on a visco-Pasternak foundation. Also, a more accurate size-dependent analysis is performed on the basis of the NSGT. Moreover, mixing the principle of virtual work with the kinematic relations, the final equation of motion is derived for each graphene sheet. Then, these equations are coupled to each other by the means of vdW interaction model. Finally, wave frequencies are achieved in the framework of an analytical solution method. Now, it is time to recall some of the most significant effects as follows:

• Wave frequency and phase velocity can be easily strengthened if length scale parameter is added or nonlocal parameter is decreased.

• Linear and nonlinear medium parameters are able to increase wave frequency or phase velocity once they are amplified.

• Whenever damping coefficient is captured, wave frequency or phase velocity can be finally damped in each desired wave number.

• Wave dispersion responses of DLGSs are smaller once hygro-thermal condition is applied compared with thermal condition.

• Increasing temperature gradient or moisture concentration is a practical way of decreasing wave frequency and phase velocity amounts of DLGSs.

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Appendix

The k_{ij} and m_{ij} arrays in Eq. (23) are defined in the following form

$$
\begin{aligned}
k_{11} = k_{22} &= \left(1+\eta^2\left(\beta_1^2+\beta_2^2\right)\right)\begin{pmatrix} D_{11}\beta_1^4 + \\ 2\left(D_{12}+2D_{66}\right)\beta_1^2\beta_2^2 \\ +D_{22}\beta_2^4 \end{pmatrix} \\
&+\left(1+\mu^2\left(\beta_1^2+\beta_2^2\right)\right)\begin{pmatrix} k_w + \\ \left(k_p - N^T - N^H\right)\left(\beta_1^2+\beta_2^2\right) \\ -i\,\omega C_d + C \end{pmatrix}, \\
k_{12} = k_{21} &= -\left(1+\mu^2\left(\beta_1^2+\beta_2^2\right)\right)C
\end{aligned} \tag{A.1}
$$

$$
\begin{aligned}
m_{11} = m_{22} &= \left(1+\mu^2\left(\beta_1^2+\beta_2^2\right)\right)\left(I_0 + I_2\left(\beta_1^2+\beta_2^2\right)\right), \\
m_{12} = m_{21} &= 0
\end{aligned} \tag{A.2}
$$