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Experiments on influence of foundation mass on dynamic characteristic of structures

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Abstract. Recently, a new foundation model called "Dynamic foundation model" was proposed for the dynamic analysis of structures on the foundation. This model includes a linear elastic spring, shear layer, viscous damping and the special effects of mass density parameter of foundation during vibration. However, the relationship of foundation property parameters with the experimental parameter of the influence of foundation mass also has not been established in previous research. Hence, the purpose of the paper presents a simple experimental model in order to establish relationships between foundation properties such as stiffness, depth of foundation and experimental parameter of the influence of foundation mass. The simple experimental model is described by a steel plate connected with solid rubber layer as a single degree of freedom system including an elastic spring connected with lumped mass. Based on natural circular frequencies of the experimental models determined from FFT analysis plots of the time history of acceleration data, the experimental parameter of the influence of foundation mass is obtained and the above relationships are also discussed.

Keywords: one-parameter foundation; two-parameter foundation; dynamic foundation; foundation mass; experimental model of foundation mass

1. Introduction

The problem models of structures on foundation subjected to moving vehicles as transportation, runway, railway, have very important meaning both in learning and practical application. In almost the studies, the foundation model applied to analyze dynamic responses of structures was described by various foundation models.

The first, the one-parameter foundation model Winkler is developed on the assumption that the reaction forces per unit length at each point of the foundation are proportional to the deflection of the foundation itself which is defined by means of identical, independent, closely spaced, discrete and linear or nonlinear elastic springs as known as the modulus of subgrade reaction (Kargarnovin *et al*. 2005, Çalım 2009, Arani *et al*. 2012, Wang *et al*. 2013, Jang 2013, Coşkun *et al*. 2014, Park and Jang 2014, Jorge *et al*. 2015, Froio *et al*. 2016, Jang 2016). However, one of the most important deficiencies of the Winkler model appears a displacement discontinuity between the loaded and the unloaded part of the foundation surface while the soil surface does not show any discontinuity in reality (Teodoru 2010). Hence, it does not accurately represent the characteristic response as the true behavior of soil.

To overcome the deficiency of the above model, several other foundation models had proposed to describe more real response of soil by introducing some kind of interaction between the independent springs by visualising various types of interconnections such as an additional a thin elastic membrane stretched by a constant tension (Filonenko 1940); plate with flexural rigidity (Hetenyi 1946); an incompressible layer that resists only transverse shear deformation (Pasternak 1954, Reissener 1958, Kerr 1964), or accounts for the effect of the neglected shear strain energy in the soil and shear forces that come from surrounding soil (Vlasov and Leont'ev 1966). The common character of the above foundation models used elastic spring without density to describe the behavior of the foundation.

In reality, the foundation has mass density and it is meaningless in the problem of static analysis but it can have the influence in the problems of dynamic analysis. When the above structure is vibrating under dynamic loads, the foundation mass also causes vertical inertia force as external load subjected on the structures. The force depends on the value of mass and motion acceleration of the foundation and it completely participates in the response of the above structures (Nguyen and Pham 2016a). Hence, a new foundation model called "Dynamic foundation model" was proposed for the dynamic analysis of structures on foundation (Pham *et. al* 2015, Nguyen *et. al* 2016b and 2016c). This model includes a linear elastic spring, shear layer, viscous damping and the special effects of mass density parameters of foundation in during vibration. This model also applied to analyze dynamic response of beam

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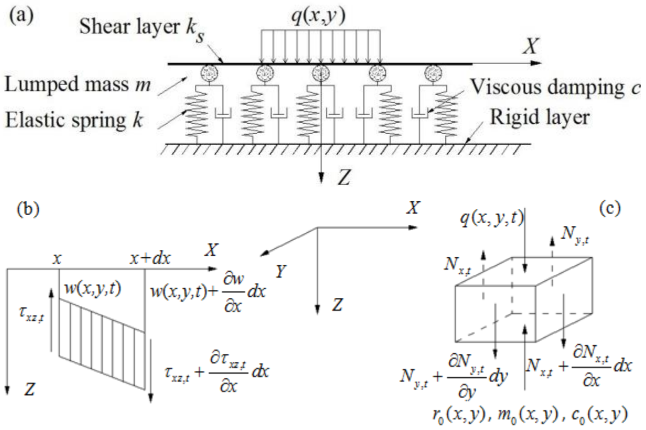


Fig. 1 The dynamic foundation model: (a) The basic model, (b) stress in the shear layer, (c) forces acting on the shear layer

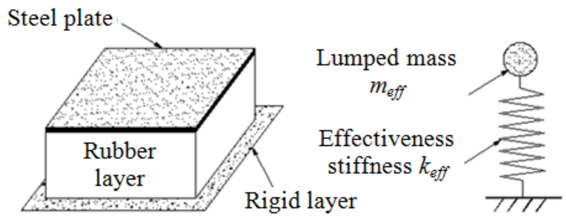


Fig. 2 The experimental model: (a) The true model, (b) The single degree of freedom

and plate structures subjected to moving the vehicle and the analyzed results showed that the foundation mass has significant effects on the dynamic responses of structures and these effects increase dynamic deflection of beam and plate structures.

It can be seen that the almost above studies rated to foundation mass have been only proposed the experimental parameter of the influence of foundation mass on the dynamic response of structures. It has been assumed as theory parameter to describe measure effects of foundation mass and the relationship between soil property parameters and the characteristic parameter also has not been established in previous research. Hence, a simple experimental model is used to confirm existence influence of the above parameter on the dynamic characteristic of structures in this study. Besides, the experimental parameter of the influence of foundation mass is quantitative, and the relationships between soil property parameters such as stiffness and depth of foundation and the above experimental parameter are established and discussed detail. It is also advanced in this study than previous works.

2. The dynamic foundation model

The dynamic foundation model (Pham *et al.* 2015), which fully describes dynamic characteristic parameters for behavior of foundation including the elastic stiffness and shear layer parameter of foundation, the viscous damping c and the mass density of foundation ρ_f are respectively replaced by lumped mass m at the top of the elastic spring connected between elastic layer and shear layer, is shown in Fig. 1.

At the time t, the pressure-deflection relationship due to a pressure $q(x,y,t)$ is determined based on the principle of dynamic balance, given by

$$\frac{\partial N_{x,t}}{\partial x}+\frac{\partial N_{y,t}}{\partial y}+q(x,y,t)-r_0(x,y,t) -m_0(x,y,t)-c_0(x,y,t)=0 \tag{1}$$

where $N_{x,t}$ and $N_{y,t}$ are the vertical forces, $r_0(x,y,t)$ is the reaction of the Winkler foundation, $m_0(x,y,t)$ is the inertia force of the density of foundation and $c_0(x,y,t)$ is the viscous damping resistance at each time t, can be expressed as follows, respectively

$$\begin{aligned} N_{x,t} &= \int_o^1 \tau_{xz,t}dz = k_s\frac{\partial w(x,y,t)}{\partial x} \\ N_{y,t} &= \int_o^1 \tau_{yz,t}dz = k_s\frac{\partial w(x,y,t)}{\partial y} \end{aligned} \tag{2}$$

$$\begin{aligned} r_0(x,y,t) &= k.w(x,y,t) \\ m_0(x,y,t) &= m\frac{\partial^2 w(x,y,t)}{\partial t^2} \\ c_0(x,y,t) &= c\frac{\partial w(x,y,t)}{\partial t} \end{aligned} \tag{3}$$

The lumped mass m of the foundation mass is given by

$$m=\alpha_f H_f \rho_f \tag{4}$$

where α_f is an experimental parameter of the influence of foundation mass and H_f is the depth of foundation.

3. The experimental model

The simple experimental model of structures on the dynamic foundation is described by a steel plate connected with solid rubber layer as a single degree of freedom including an elastic spring connected with lumped mass, shown in Fig. 2.

In this model, the foundation material is described by solid rubber layer which is specially made for the experimental models and the steel material is also chosen for the above structure because the properties of above materials such as elastic stiffness, mass density, and geometric parameters are determined very easier and more reliable than others. At the same time, the characteristic behavior of them quite agrees with assuming as homogeneous, continuous, isotropic and linear elastic. Hence, the materials are also considered as the best material for describing the dynamic characteristics of structures on the foundation.

To analyze free vibration, the acceleration sensors are put on the surface of the steel plate and connected to a vibration recorder which is connected to a computer by a wireless network, the detailed setup is shown in Fig. 3.

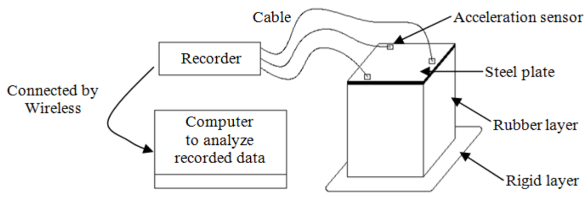


Fig. 3 The detailed setup of experimental model

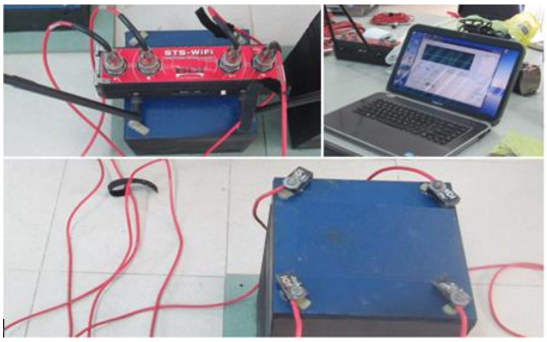


Fig. 4 The real experimental model

By acting a suddenly external force on the surface of steel plate, the time history of acceleration data is recorded. Based on FFT algorithm (Cooley *et al.* 1969) which has been used quite popular to determine the fundamental frequency of vibration systems in many last decades and it is also set into analysis software corresponding with the recorder, the fundamental natural frequency can be obtained directly by observing the FFT plot. The location of highest peak values relates to the first natural frequency of the system f_{eff} and then the first natural circular frequency $\omega_{eff}=2\pi f_{eff}$ is also determined. So, the general vibration mass of the system including the mass of plate and foundation participated in during vibration is given by

$$m_{eff} = \frac{k_{eff}}{\omega_{eff}^2} \tag{5}$$

where k_{eff} is effective stiffness of rubber layer per unit length of depth.

From Eq. (4) and Eq. (5), the experimental parameter α_f of influence of foundation mass can be given as

$$\alpha_f = \frac{m_{eff} - m_S}{\rho_{eff} H_f} \tag{6}$$

which m_S is total mass of the steel plate and acceleration sensors.

4. The experimental results

4.1 Verified experiment

This verified experiment considers a cantilever beam made up of steel and the physical and geometrical properties of the beam are given in Table 1.

By acting a suddenly external force at the free end of the beam, the natural circular frequencies of the cantilever beam based on the FFT analysis of the time history of acceleration at the free end of the cantilever beam are compared with these results obtained by finite element method and analytical solutions, shown in Table 2.

It can be seen that the above experimental results of natural circular frequencies are quite an agreement withthese results obtained by FEM and analytical solutions, the relative error is not over 4% for FEM solution and 3% for an analytical solution. At the same time, with the analysis results of the experimental model which have error percent under 5% are perfectly accepted because the experimental model can not have the results as expected and absolutely accurate. Hence, the procedure set up for analyzing experiment is reliable and it can also completely use for analyzing the influence of foundation properties on the experimental parameter α_f.

Table 1 The physical and geometrical properties of the cantilever beam

Property of the cantilever beam	Unit	Value
Young's modulus	N/m^2	1.808×10^{11}
Mass density	kg/m^3	7691.267
Length of the beam	mm	180.80
Width of the beam	mm	40.00
Depth of the beam	mm	2.80

Table 2 The natural circular frequencies of the cantilever beam

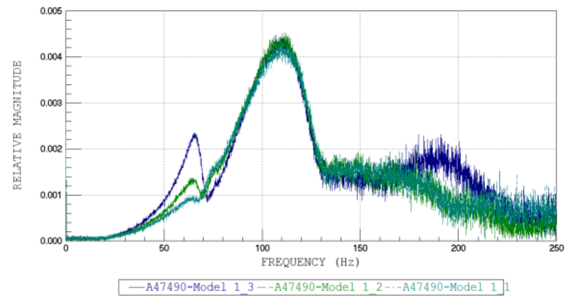
Times	f (Hz)	ω (rad/s)	FEM solution		Analytical solution	
			ω (rad/s)	Error (%)	ω (rad/s)	Error (%)
1	65.08	408.702		3.73		2.98
2	65.10	408.828	424.534	3.70	421.263	2.95
3	65.09	408.765		3.71		2.97

Table 3 The physical and geometrical properties of the experimental models

Property of experimental models	Symbol	Unit	Model 1	Model 2	Model 3	Model 4
Effective stiffness	k_{eff}	kN/mm	2.558	1.140	0.758	0.586
Depth of the beam	H_f	mm	102.675	203.500	303.475	404.775
Mass density	ρ_{eff}	kg/m	48.503	48.872	48.923	48.714
Mass of the structure	m_S	kg	1.939	1.968	1.989	1.938

4.2 Experimental analysis

The goal of the experiment performed in this study is to identify effects of the foundation properties on the experimental parameter of the influence of foundation mass α_f based on the real experimental model, shown in Fig. 4. The relationship between foundation properties and the above experimental parameter is also established such as the relationship between depth and stiffness of foundation and the experimental parameter α_f, respectively. In the experimental investigation, the foundation is described by the various depth and elastic stiffness, the physical and



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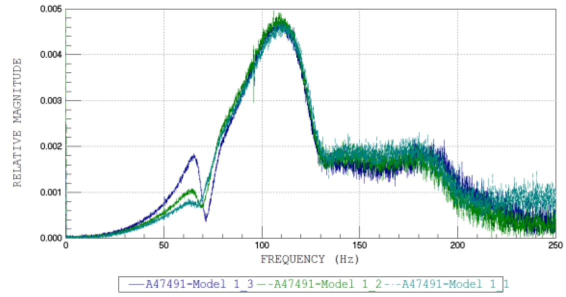

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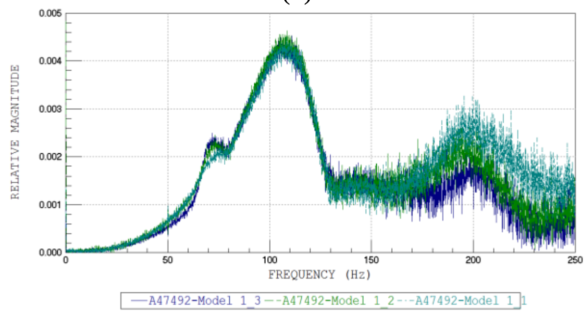

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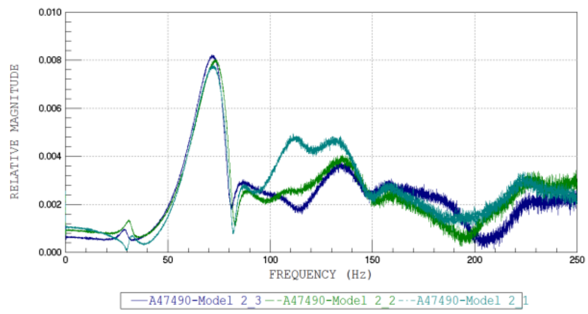
Fig. 5 The FFT analysis of the Model 1: (a) A47490, (b) A47491, (c) A47492



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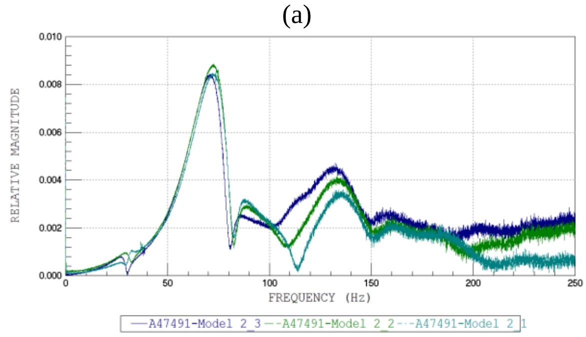

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Fig. 6 The FFT analysis of the Model 2: (a) A47490, (b) A47491, (c) A47492

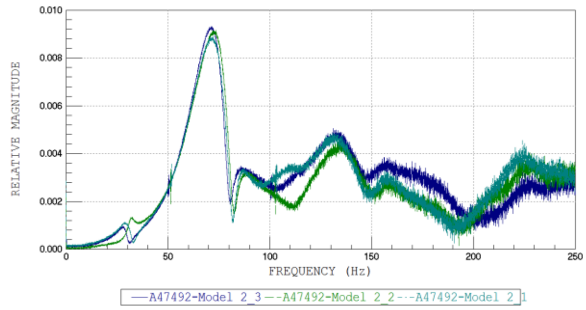

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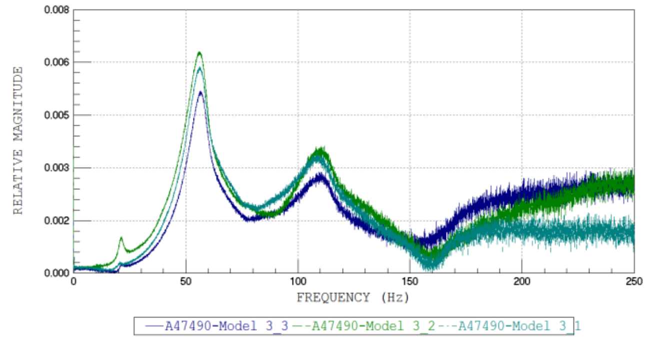
Fig. 6 Continued



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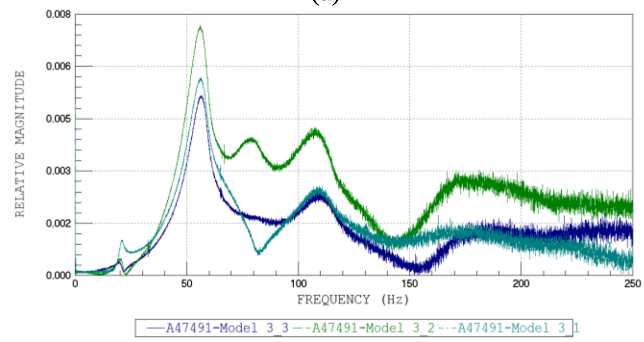

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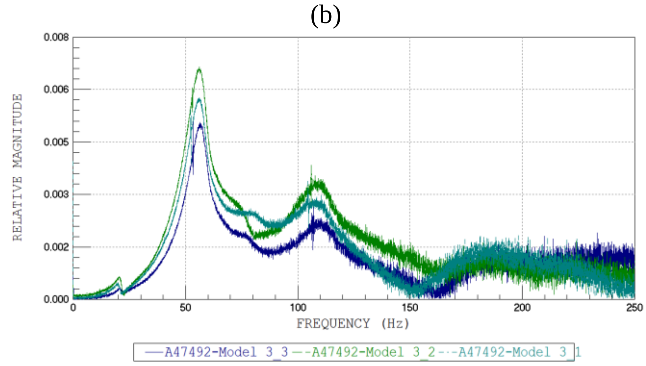
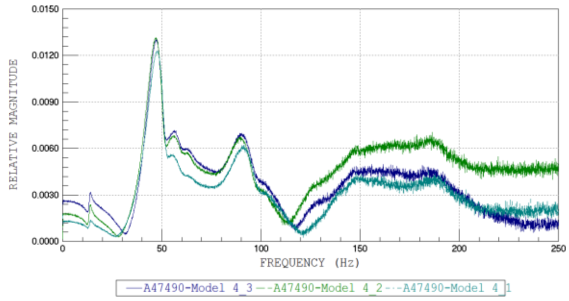


(c)

Fig. 7 The FFT analysis of the Model 3: (a) A47490, (b) A47491, (c) A47492

geometrical properties of four experimental models are given in Table 3.

By acting a suddenly external force on the surface of steel plate, the time history of acceleration at each acceleration sensor A47490, A47491 and A47492 is recorded, respectively. Each experimental mode is done three times and the value of natural circular frequency of



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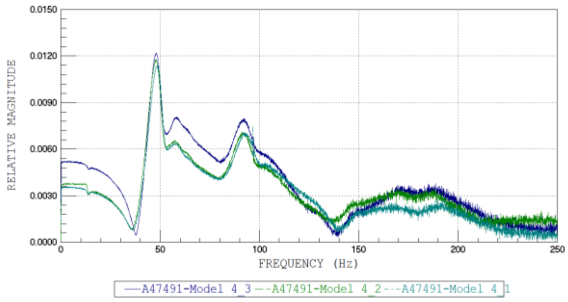

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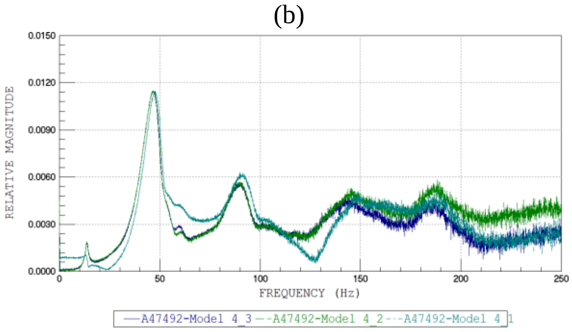

(c)

Fig. 8 The FFT analysis of the Model 4: (a) A47490, (b) A47491, (c) A47492

Table 4 The natural circular frequencies of the experimental models

Model	The frequencies at each acceleration sensor f_{eff} (Hz)			The average natural circular frequencies
	A47490	A47491	A47492	ω_{eff} (rad/s)
Model 1	110.567	109.767	108.567	688.847
Model 2	72.600	71.833	72.200	453.716
Model 3	56.400	56.267	56.300	353.883
Model 4	47.233	48.167	47.067	298.381

each time is an average of values at acceleration sensor in the times. Based on the time history of acceleration of each time of each model, the respectively FFT plot is plotted from Fig. 5 to Fig. 8.

Based on the FFT analysis plot from the time history of acceleration data at each acceleration sensor of each time (from Fig. 5 to Fig. 8), the natural circular frequencies of the experimental models are given in Table 4.

From the natural circular frequencies of the experimental models in Table 4 and based on the basic theory of experimental model, the experimental parameter of the influence of foundation mass αf is obtained and given in Table 5.

Table 5 The experimental parameter of the influence of foundation mass α_f

Model	k_{eff} (kN/mm)	ω_{eff} (rad/s)	m_{eff} (kg)	m_S (kg)	m_f (kg)	H_f (m)	α_f
Model 1	2.558	688.847	5.391	1.939	3.450	102.675	0.693
Model 2	1.140	453.716	5.538	1.968	3.560	203.500	0.359
Model 3	0.758	353.883	6.053	1.989	4.064	303.475	0.274
Model 4	0.586	298.381	6.582	1.938	4.644	404.775	0.260

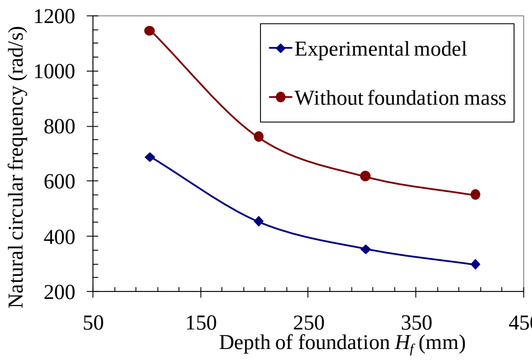

Fig. 9 The natural circular frequency of the structure

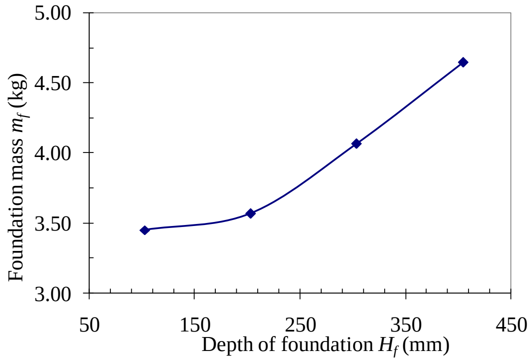

Fig. 10 The relationship between depth of foundation H_f and foundation mass m_f

The above analyzing results show that the foundation mass has significant effects on the dynamic characteristic of the structure; it causes foundation mass participating in during vibration m_f and is more increasing general vibration mass of the structures m_{eff} than without effect of foundation mass. In each experimental model, the effective stiffness is constant but the general vibration mass increases caused foundation mass in during vibration. Hence, the system is softer than other without effects of the foundation mass, and then it is decreasing natural circular frequencies of the structure, plotted in Fig. 9.

At the same time, the relationship between foundation depth H_f and foundation mass m_f is plotted in Fig. 10. It can be also seen that an increase of foundation depth H_f increases foundation mass m_f because the foundation mass m_f is proportional to the depth of foundation H_f, shown in the Eq. (4). It means that an increase of the depth of foundation H_f increases effective depth of foundation joined in during vibration, and then the total effective foundation mass per unit length also increases proportionally with increasing depth of foundation. Hence, it increases the

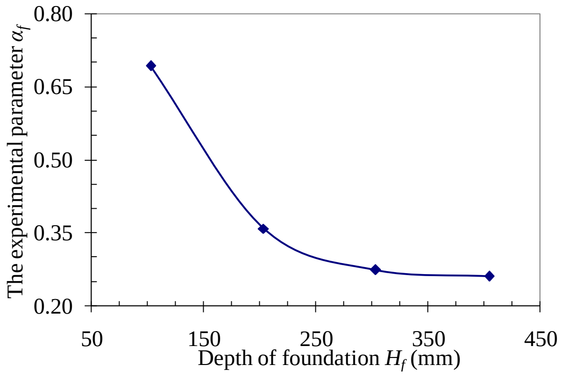

Fig. 11 The relationship between depth of foundation H_f and experimental parameter α_f

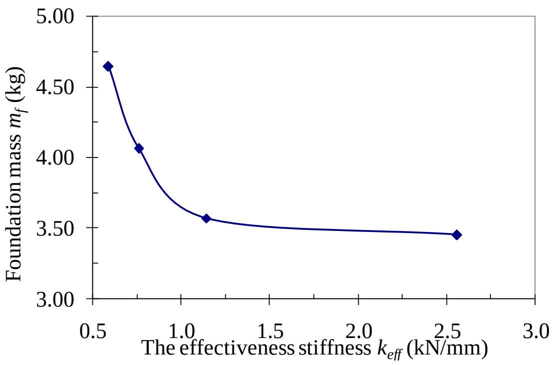

Fig. 12 The relationship between effective stiffness k_{eff} and foundation mass m_f

foundation mass in during vibration, shown in Fig. 10.

Fig. 11 presents the effect of depth of foundation H_f on the experimental parameter α_f. It can be seen that an increase of depth of foundation H_f decreases the values of experimental parameter α_f. For analyzing the above comment, the Eq. (6) shows that the experimental parameter α_f depends on both the effective mass density of foundation per unit length of depth ρ_{eff} and depth of foundation layer H_f, so the relationship between the experimental parameter α_f and depth of foundation H_f is a nonlinear inverse function. It can be also seen that the experimental parameter α_f is also a characteristic parameter for the sensitivity of influence of foundation mass corresponding with foundation depth. Hence, an increase of foundation depth decreases the sensitivity of foundation, and then the value of experimental parameter α_f decreases with an increase of foundation depth H_f, shown in Fig. 11.

The relationship between effective stiffness of foundation k_{eff} and the foundation mass m_f is also plotted in Fig. 12. The decrease of effective stiffness of foundation, k_{eff} increases foundation mass m_{eff} in during vibration. It can be clearly seen that a decrease of the effective stiffness k_{eff} causes the foundation to become softer. It means that a decrease of the effective stiffness k_{eff} increases strongly vibration of foundation, and then the influence of foundation mass m_{eff} in during vibration is also more increasing significantly than others, shown in Fig. 12.

From the above Eq. (5) and Eq. (6) can be seen that the relationship between the experimental parameter α_f and

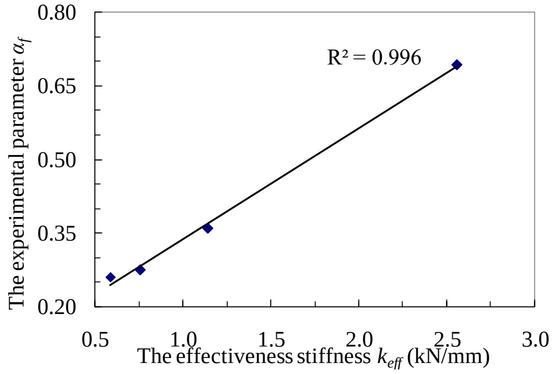

Fig. 13 The relationship between effective stiffness k_{eff} and experimental parameter α_f

effective stiffness k_{eff} is directly proportional and the experimental results also confirmed the above comment, shown in Fig. 13. It can be also known that the experimental parameter α_f is also a characteristic parameter for the sensitivity of influence of foundation mass. Hence, the more increasing the effective stiffness k_{eff} is, the clearer the sensitivity of vibration system gets. Another way, the experimental parameter α_f increases with the increase of the effective stiffness k_{eff}, shown in Fig. 13.

From the above comments, it can be seen that the contents of experimental research quite agree with the dynamic foundation model which considers to effects of foundation mass on the dynamic characteristic of the structures. The relationships between the experimental parameter α_f and soil property parameters such as effective stiffness k_{eff} and foundation depth H_f are established and discussed detail. Besides, the experimental results are also confirmed the basic theory and the estimates rated to effects of the foundation properties on the experimental parameter α_f are presented. At the same time, the experimental results also show that the foundation mass has affected significantly on dynamic characteristic and it is also more increasing general vibration mass of the structures than without foundation mass.

5. Conclusions

Based on the foundation mass considering in during vibration in the dynamic foundation model, the influence of the foundation mass on the dynamic characteristic of the systems has been studied by the experimental model, the following conclusions of this paper can be drawn as follows:

- The simple experimental model is established detail to determine the parameter of the influence of foundation mass α_f in the dynamic foundation model.
- The experimental results are also confirmed that the foundation mass has affected significantly on the dynamic characteristic of the structures and it causes softer the systems, decreasing of the natural frequency, than without effects of foundation mass.
- The relationships between the foundation properties such as depth and stiffness of foundation and the

parameter of the influence of foundation mass α_f are also derived; an increasing of foundation depth H_f decreases the value of the parameter α_f; At the same time, the relationship between the parameter α_f and effective stiffness k_{eff} is directly proportional.

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