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Stability analysis of porous multi-phase nanocrystalline nonlocal beams based on a general higher-order couple-stress beam model

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Abstract. This article investigates buckling behavior of a multi-phase nanocrystalline nanobeam resting on Winkler-Pasternak foundation in the framework of nonlocal couple stress elasticity and a higher order refined beam model. In this model, the essential measures to describe the real material structure of nanocrystalline nanobeams and the size effects were incorporated. This non-classical nanobeam model contains couple stress effect to capture grains micro-rotations. Moreover, the nonlocal elasticity theory is employed to study the nonlocal and long-range interactions between the particles. The present model can degenerate into the classical model if the nonlocal parameter, and couple stress effects are omitted. Hamilton's principle is employed to derive the governing equations and the related boundary conditions which are solved applying an analytical approach. The buckling loads are compared with those of nonlocal couple stress-based beams. It is showed that buckling loads of a nanocrystalline nanobeam depend on the grain size, grain rotations, porosities, interface, elastic foundation, shear deformation, surface effect, nonlocality and boundary conditions.

Keywords: buckling; higher-order theory; nanocrystalline nanobeam; nonlocal couple stress theory

1. Introduction

Small scale beams are the basic structures used in several applications such as nano-electro-mechanical systems, nano-probes, atomic force microscope (AFM), nanoactuators and nanosensors. At nano scale, the physical and mechanical properties of small scale structures render evident size effects, which are quite different from their bulk counterparts. Such micro/nano structural components are constructed from nanostructured materials due to possessing small size. It is known that nanostructured materials such as nanocrystalline materials (NcMs) and nanoparticle composites (NpCs) have an inhomogeneous structure and their properties are significantly influenced by the essence of their material structure (Shaat and Abdelkefi 2016, Shaat 2015). In fact, nanocrystalline materials are multi-phase composites consist of grain phase, porosities and interface phase. In NcMs, several atoms are separated from the grains and create a new phase which is called as the interface. The interface phase shows a softening impact on the structure by reducing the elastic moduli (Wang *et al.* 2003). Also, properties of nanocrystalline materials depend on the grain and porosity size which can change from 0.5 to 100 nm (Kim and Bush 1999).

Recently, some physical phenomena have been reported in micro/nano scale structures such as the translation and rotation of grains or crystals within the material structure. Translational motion of grains is the observable degree of freedom in macro-size structures. However, rotational motion of grains inside the material shows an important influence on the behavior of a micro/nano structure. Several higher-order continuum theories are suggested accounting for the size influences by considering additional degrees of freedom. The modified couple stress theory (Yang *et al.* 2002) has been implemented to examine the influences of the grains rigid rotations on the behavior of nanobeams (Shaat and Abdelkefi 2015). Analysis of mechanical response of nanocrystalline nanobeams are very limited in the literature. Shaat *et al.* (2016) examined vibration behavior of a cracked nanocrystalline nanobeam based on modified couple stress theory. Also, Shaat and Abdelkefi (2015) researched pull-in instability of multi-phase nanocrystalline nanobeams exposed to an electrostatic force. Nanostructures are also significantly influenced by their surface tension and surface energy (Gurtin and Murdoch 1975, Ebrahimi *et al.* 2016). The free surfaces (external surfaces creating the outer boundary of materials) and interfaces (interfacial surfaces between the non-homogeneities) have a major role on the behavior of nanostructures made of nano-structured materials (Shaat and Abdelkefi 2015). Moreover, surface effects have a major role in dynamic behavior of nanostructures (Ebrahimi *et al.* 2016, Wang and Feng 2009). The free surfaces (external surfaces creating the outer boundary of materials) and interfaces (interfacial surfaces between the non-homogeneities) have a major role on the behavior of nanostructures made of nano-structured materials (Shaat and Abdelkefi 2015b). Gurtin and Murdoch (1975) suggested a surface elasticity theory for modeling of the continuum surface as a two-dimensional membrane having zero thickness lying on the material bulk. In the literature, there is no relevant paper to surface elasticity effects on vibration and buckling behavior of multi-phase

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nanocrystalline nanobeams. However, there are many papers on vibration and buckling of homogenous nanobeams incorporating surface effects. Gheshlaghi and Hasheminejad (2011) investigated surface stress effects on nonlinear vibrational behavior of homogenous nanobeams. Also, Ansari *et al.* (2014) presented vibration analysis of Timoshenko nanobeams based on surface stress elasticity theory. Ansari *et al.* (2014) performed post-buckling analysis of homogenous nanoscale beams considering surface stress effects. Also, Sahmani *et al.* (2014) explored surface energy effects on vibrational behavior of post-buckled higher order nanobeams. Ebrahimi and Boreiry (2016) examined various surface effects on vibrational behavior of nanobeams based on classical beam theory.

In fact, literature survey indicates that all of previous papers on nanocrystalline nanobeams have not considered nonlocal effects in their analysis. It is reported that the mechanical behavior of nanobeams is significantly influenced by the presence of nonlocality. Therefore, there is a strong scientific need to investigate the behavior of nanocrystalline nanobeams incorporating both surface elasticity and nonlocal effects. Up to now, several investigations are performed to incorporate the nonlocal effects in vibration and buckling analysis of nanobeams based on nonlocal elasticity theory of Eringen (Eringen 1972, 1983, Ebrahimi and Barati 2016a, b, Ebrahimi *et al.* 2016b, Li *et al.* 2016, Li and Hu 2017). Reddy (2007) presented nonlocal modeling of nanobeams for static, buckling and vibration analysis of small scale beams based on Euler-Bernoulli, Timoshenko and third-order theories. Eltaher *et al.* (2013) explored both nonlocal and surface energy effects on vibrational response of nanobeams. Şimşek (2014) examined nonlocal effects on nonlinear free vibration of nanobeams under different boundary conditions. Also, Eltaher, *et al.* (2013) performed vibration analysis of nonlocal nanobeams employing finite element approach. Berrabah *et al.* (2013) proposed different higher order nonlocal beam models for static, vibration and buckling analysis of nanoscale beams. Also, Tounsi *et al.* (2013) investigated thermal buckling response of nanobeams by developing a nonlocal higher order beam model. Investigation of vibration behavior of preloaded nonlocal coupled nanobeams is performed by Murmu and Adhikari (2012). In another research, Zenkour *et al.* (2015) studied vibration of nanobeams via a nonlocal thermoelasticity model without energy dissipation implementing a state space method. Ebrahimi *et al.* (2015) examined the application of differential transformation approach in vibration analysis of nonlocal inhomogeneous nanobeams. Also, Ansari *et al.* (2016) carried out nonlocal nonlinear free vibration analysis of fractional viscoelastic nanobeams. Ebrahimi and Barati (2016c, d, e, f) proposed a nonlocal third-order shear deformable beam model for vibration and buckling analysis of nanobeams under various physical fields. Most recently Ebrahimi and Barati (2016g, h, i) explored thermal and hygro-thermal effects on nonlocal vibration behavior of small scale of temperature-dependent nonhomogeneous nanoscale beams. All these papers based on nonlocal elasticity theory of Eringen only introduced a stiffness-softening effect. In the recent area of nanoscience and nanotechnology, some factors are responsible for the deficiency of nonlocal elasticity theory. Among of them are the effects of microstructure degrees of freedoms and the surface energy. In only one paper, Attia *et al.* (2016) investigated combined effects of nonlocal stress field, couple stress and surface energy on mechanical behavior of nanobeams.

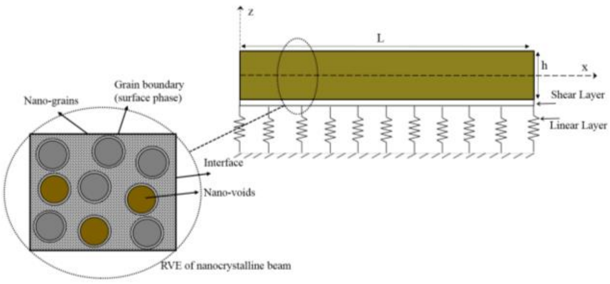


Fig. 1 Configuration of an embedded nanocrystalline nanobea

Also, it is evident that all of afore-mentioned papers related to vibration and buckling of nonlocal nanobeams have not used size-dependent material properties and they are independent of grains and voids size. So, it is crucial to consider size-dependent material properties in analysis of nonlocal nanobeams by using a micromechanical model in which the effects of nano-grains, nano-voids and interface size are considered. Based on above discussions, analysis of buckling behavior of nanocrystalline nanobeams considering combined effects of nonlocal stress field, couple stress and grains surface energy is very important for accurate analysis nanoscale beams by taking into account both size-dependency of structure and material properties.

This research deals with the buckling analysis of nanocrystalline nanobeams resting on Winkler-Pasternak foundation based on nonlocal couple stress theory incorporating grains surface energy effects. The model contains the modified couple-stress theory to explore the influence of rotational degree of freedom of particles. Moreover, nonlocal elasticity theory is employed to study the nonlocal and long-range interactions between the particles. A micromechanical model is employed to describe a multi-phase nanobeam with grain and void size dependent material properties. The governing differential equations of motion are derived by using Hamilton's principle and an analytical approach is employed to solve the equations for various boundary conditions. The results of present study are compared with those of previously published papers. The influences of couple stress, nonlocality, nanograin/nanopore size, nanopore percentage, elastic foundation and shear deformation on buckling loads of nanocrystalline nanobeam are discussed in detail.

2. Theory and formulation

2.1 Effective elastic constants of nanocrystalline nanobeam

Consider a nanocrystalline silicon nanobeam which is a three-phase composite having nano-grains and nano-voids randomly distributed in the interface region, as indicated in Fig. 1. In this figure, a Representative Volume Element

Table 1 Material properties of nanocrystalline nanobeam

Phase-1 (Interface)	E_{in}=45.56 GPa, v_{in}=0.064, ρ_{in}=2004.3 kg/m^3
Phase-2 (Si-nanograins)	E_g=169 GPa, v_g=0.064, ρ_g=2300 kg/m^3
Phase-3 (nanovoids)	E_v=0
Surface coefficients of grains and voids	λ_s=-4.488 N/m, μ_s=-2.774 N/m

Table 2 Effective material properties of nanocrystalline nanobeam for various grain size

R_g=R_v	T_{in}	f_v	f_g	K_{NcM}	γ_{NcM}	E_{NcM}	ρ_{NcM}
100 nm	1.02		0.97	61.72	75.33	160.66	2291.1
20 nm	3.7126	0%	0.6	37.34	43.54	94.07	2181.7
0.5 nm	0.13		0.5	28.09	35.85	75.46	2152.2
100 nm	1.02		0.873	45.55	52.06	113.1	2062
20 nm	3.7126	10%	0.54	29.12	32.51	71.10	1963.5
0.5 nm	0.13		0.45	6.570	58.10	44.16	1936.9

(RVE) is proposed in which distinct surface phases of inhomogeneities are indicated. A size-dependent micromechanical model is used to describe the effective material constants (Shaat and Abdelkefi 2015).

In this model, influences of the size of grains and voids and their surface energies are included in the Mori-Tanaka micromechanical model. Elastic properties of interface or grain boundary, nano-grains and nano-voids are presented in Table 1.

According to the suggested model, the elastic properties of a two-phase RVE considering nano-grains can be expressed by

$$k_{eff} = k_{in} + f_g(k_g - k_{in})A^{(k)} + f_g k_g^s B^{(k)} \tag{1}$$

$$\mu_{eff} = \mu_{in} + f_g(\mu_g - \mu_{in})A^{(\mu)} + f_g \mu_g^s B^{(\mu)} \tag{2}$$

in which $A^{(k)}$, $B^{(k)}$, $A^{(\mu)}$ and $B^{(\mu)}$ are four scalars which are determined in the context of the used micromechanical model. Based on Mori-Tanaka model for two-phase composites having spherical inclusion, the effective bulk modulus k_{eff} and shear modulus μ_{eff} can be defined as (Shaat 2015)

$$k_{eff} = \frac{3k_g\left(4f_g\mu_{in} + 3k_{in}\right) + 2\mu_{in}\left(4f_g\mu_{in}k_s^* + 3k_{in}\left(2 - 2f_g + k_s^*\right)\right)}{3\left(3\left(1 - f_g\right)k_g + 3f_g k_{in} + 2\mu_{in}\left(2 + k_s^* - f_g k_s^*\right)\right)} \tag{3}$$

$$\mu_{eff} = \frac{\mu_{in}\left(5 - 8f_g\xi_3(7 - 5v_{in})\right)}{5 - f_g(5 - 84\xi_1 - 20\xi_2)} \tag{4}$$

in which

$$\xi_1 = \frac{15(1 - v_{in})(k_s^* + 2\mu_s^*)}{4H} \tag{5}$$

$$\xi_2 = \frac{-15(1 - v_{in})((\frac{\mu_g}{\mu_{in}})(7 + 5v_g) - 8v_g(5 + 3k_s^* + \mu_s^*) + 7(4 + 3k_s^* + 2\mu_s^*))}{4H} \tag{6}$$

$$\xi_3 = \frac{5}{16H}[2(\frac{\mu_g}{\mu_{in}})^2(7 + 5v_g) - 4(7 - 10v_g)(2 + k_s^*)(1 - \mu_s^*) + (\frac{\mu_g}{\mu_{in}})(7(6 + 5k_s^* + 4\mu_s^*) \\ - v_g(90 + 47k_s^* + 4\mu_s^*))] \tag{7}$$

where

$$H = -2(\frac{\mu_g}{\mu_{in}})^2(7 + 5v_g)(4 - 5v_{in}) + 7(\frac{\mu_g}{\mu_{in}})(-39 - 20k_s^* - 16\mu_s^* + 5v_g(9 + 5k_s^* + 4\mu_s^*)) \\ + (\frac{\mu_g}{\mu_{in}})v_g(285 + 188k_s^* + 16\mu_s^* - 5v_{in}(75 + 47k_s^* + 4\mu_s^*)) \\ + 4(7 - 10v_g)(-7 - 11\mu_s^* - k_s^*(5 + 4\mu_s^*) + v_{in}(5 + 13\mu_s^* + k_s^*(4 + 5\mu_s^*))) \tag{8}$$

in which $k_s^* = k_g^s / R_g\mu_{in}$ and $\mu_s^* = \mu_g^s / R_g\mu_{in}$ are surface bulk modulus and shear modulus, respectively and $k_g^s = 2(\mu_g^s + \lambda_g^s)$. Also, R_g is average radius of nano-grains.

The present form of Eqs. (3) and (4) cannot be used for multi-phase composites, since it is unable to capture the effect of nano-voids or other inclusions. To overcome this problem, the decoupling method introduced by Huang *et al.* (1994) is implemented to decompose a multi-phase composite into a set of two-phase composites. Based on decoupling method, a two-phase composite for every kind of inclusion is considered with a matrix material as the matrix of multi-phase composite. Hence, the effective bulk K_{MP} and shear μ_{MP} moduli of a multi-phase composite can be represented by

$$\frac{K_{MP}}{k_{in}} \cong \prod_{n=1}^{N-1} \frac{k_{H_n}}{k_{in}} \tag{9}$$

$$\frac{\mu_{MP}}{\mu_{in}} \cong \prod_{n=1}^{N-1} \frac{\mu_{H_n}}{\mu_{in}} \tag{10}$$

in which k_{H_n} and μ_{H_n} are effective elastic constants of each two-phase composite. Also, N is number of phases in multi-phase composite. It should be mentioned that multi-phase composite is decoupled into N-1 two-phase composite. For a nanocrystalline material, the atoms inside the crystallites vibrate at their equilibrium positions r_0 with a elastic modulus of E(r_0) which is identical to that of a perfect crystal, E_g. Also, the average atomic spacing r in the grain boundary regions is larger than r_0, with a elastic modulus of E_{in}=E(r) which has the following relation with elastic modulus of the perfect crystal as

$$\frac{E_{in}}{E_g} = \frac{E(r)}{E(r_0)} = \frac{1}{n - m}\left((n+1)(\frac{r_0}{r})^{n+3} - (m+1)(\frac{r_0}{r})^{m+3}\right) \tag{11}$$

in which interatomic spacing r is related to the mass density of interface $\rho(r)$ by

$$\frac{r_0}{r} = (\frac{\rho(r)}{\rho(r_0)})^{1/3} \tag{12}$$

where $\rho(r_0)$ is mass density of perfect crystal. Also, for a nanocrystalline Si, the values of m and n are identical to 8 and 12, respectively.

By defining $\eta = E_{in}/E_g$, the shear and bulk moduli of interface should be related to those of the grain as

$$k_{in} = \eta k_g, \quad \mu_{in} = \eta \mu_g \tag{13}$$

Hence, implementing decoupling method leads to the following relations for the elastic properties of nanocrystalline material (NcM) as

$$K_{NcM} \cong k_{H_1} \times k_{H_2} \times \frac{1}{\eta k_g} \tag{14}$$

$$\gamma_{NcM} \cong \gamma_{H_1} \times \gamma_{H_2} \times \frac{1}{\eta \gamma_g} \tag{15}$$

in which using Eqs. (3) and (4) gives

$$k_{H_1} = k_{eff}(k_{in} = \eta k_g, \mu_{in} = \eta \mu_g, k_g, \mu_g, f_g, k_g^s, \mu_g^s, v_{in} = v_g, R_g) \tag{16}$$

$$\mu_{H_1} = \mu_{eff}(k_{in} = \eta k_g, \mu_{in} = \eta \mu_g, k_g, \mu_g, f_g, k_g^s, \mu_g^s, v_{in} = v_g, R_g) \tag{17}$$

$$k_{H_2} = k_{eff}(k_{in} = \eta k_g, \mu_{in} = \eta \mu_g, k_g = 0, \mu_g = 0, f_v, k_v^s, \mu_v^s, v_v, R_v) \tag{18}$$

$$\mu_{H_2} = \mu_{eff}(k_{in} = \eta k_g, \mu_{in} = \eta \mu_g, k_g = 0, \mu_g = 0, f_v, k_v^s, \mu_v^s, v_v, R_v) \tag{19}$$

Hence, the effect of nano-voids is included in the present size-dependent micromechanical model. Also, the grain volume fraction f_g as a function of porosity percent f_v can be determined as follows

$$f_g = r(1 - f_v), \quad r = \frac{R_g^3}{(R_g + T_{in})^3} \tag{20}$$

in which R_g and T_{in} are average radius of grain and interface thickness, respectively. Finally, Young's modulus and Poisson's ratio of NcM can be obtained as

$$E_{NcM} = \frac{9K_{NcM}\mu_{NcM}}{3K_{NcM} + \mu_{NcM}} \tag{21}$$

$$v_{NcM} = \frac{3K_{NcM} - 2\mu_{NcM}}{2(3K_{NcM} + \mu_{NcM})} \tag{22}$$

Also, the effective mass density of nanocrystalline is determined by the following relation

$$\rho_{NcM} = (1 - f_g - f_v)\rho_{in} + f_g \rho_g \tag{23}$$

So, using Eqs. (21)-(23) the size-dependent material properties of multi-phase composites could be obtained incorporating the surface energy effects of inclusions.

2.2 The modified couple stress theory

According to the modified couple stress model, the strain energy, U of an elastic material occupying region Ω is related to the strain and curvature tensors as

$$U = \frac{1}{2}\int_{\Omega} (\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij})dV, \quad (i.j = 1.2.3) \tag{24}$$

where σ, ε, m and χ are Cauchy stress tensor, classical strain tensor, deviatoric part of the couple stress tensor and symmetric curvature tensor, respectively. The strain and curvature tensors can be defined by

$$\varepsilon_{ij} = \frac{1}{2}(u_{i.j} + u_{j.i}) \tag{25a}$$

$$\chi_{ij} = \frac{1}{2}(\theta_{i.j} + \theta_{j.i}) \tag{25b}$$

where u and θ are the components of the displacement and rotation vectors written by

$$\theta_i = \frac{1}{2}e_{ijk}u_{k.j} \tag{26}$$

in which e_{ijk} is the permutation symbol. The constitutive relations can be expressed as

$$\sigma_{ij} = \lambda\epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij} \tag{27a}$$

$$m_{ij} = 2\mu l^2 X_{ij} \tag{27b}$$

where δ_{ij} is the Kroenke delta, l is the material length scale parameter which reflects the effect of couple stress. Also, the Láme's constants can be defined by

$$\lambda = \frac{vE}{(1+v)(1-2v)} \tag{28}$$

$$\gamma = \frac{E}{2(1+v)} \tag{29}$$

2.3 The refined nanobeam model

The displacement field of FG nanobeam according to the refined shear-deformable beam model can be expressed by

$$u_x(x,z,t) = -z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{29a}$$

$$u_y(x,z,t) = 0 \tag{29b}$$

$$u_z(x,z,t) = w_b(x,t) + w_s(x,t) \tag{29c}$$

in which u is displacement component of the mid-axis and w_b and w_s denote the bending and shear transverse displacement, respectively.

Also:

- For the classical beam theory (CBT)

$$w_s(x.t) = 0 \tag{30a}$$

- For the first order beam theory (FBT)

$$f(z) = 0 \tag{30b}$$

• For the new parabolic beam theory (PBT)

$$f(z) = z - \frac{1}{2} z[\frac{h^2}{4} - \frac{1}{3} z^2] \tag{30c}$$

Finally, the non-zero strains of the present refined beam model are achieved as

$$\epsilon_x = -z \frac{\partial^2 w_b}{\partial x^2} - f(z) \frac{\partial^2 w_s}{\partial x^2} \tag{31a}$$

$$\epsilon_y = \epsilon_z = \gamma_{xy} = \gamma_{yz} = 0 \tag{31b}$$

$$\gamma_{xz} = 2\epsilon_{xz} = g(z) \frac{\partial w_s}{\partial x} \tag{31c}$$

where $g(z) = 1 - f'(z)$. In addition, Eqs. (26) and (31) give

$$\begin{gathered} \theta_y = -\frac{\partial w_b}{\partial x} - \frac{1}{2} \psi(z) \frac{\partial w_s}{\partial x}, \\ \theta_x = \theta_z = 0 \end{gathered} \tag{32}$$

with, $\psi(z) = 1 + f'(z)$.

Substitution of Eq. (32) into (25b) leads to the following expression for the non-zero components of the symmetric curvature tensor

$$\begin{gathered} \chi_{xy} = -\frac{1}{2} \frac{\partial^2 w_b}{\partial x^2} - \frac{1}{4} \psi(z) \frac{\partial^2 w_s}{\partial x^2} \\ \chi_{yz} = -\frac{1}{4} f''(z) \frac{\partial w_s}{\partial x} \\ \chi_{xx} = \chi_{yy} = \chi_{zz} = \chi_{xz} = 0 \end{gathered} \tag{33}$$

2.4 The nonlocal constitutive relations

Through the nonlocal elasticity model (Eringen 1983), the stress state at a point inside a body is regarded to be function of strains of all points in the neighbor regions. The equivalent differential form of nonlocal constitutive equation can be expressed by

$$(1-(e_0 a)\nabla^2)\sigma_{kl} = t_{kl} \tag{34}$$

where ∇^2 is the Laplacian operator. Thus, the scale length $e0a$ considers the influences of small scale on the response of nano-structures. Finally, the constitutive relations of NL-CS nanobeams can be expressed as

$$\sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial x^2} = [\lambda + 2\mu]\varepsilon_{xx} \tag{35}$$

$$\sigma_{xz} - \mu \frac{\partial^2 \sigma_{xz}}{\partial x^2} = 2\mu\gamma_{xz} \tag{36}$$

$$m_{xy} - \mu \frac{\partial^2 m_{xy}}{\partial x^2} = 2\mu l^2 \chi_{xy} \tag{37}$$

$$m_{yz} - \mu \frac{\partial^2 m_{yz}}{\partial x^2} = 2\mu l^2 \chi_{yz} \tag{38}$$

where $\mu=(e_0 a)^2$.

2.5 The governing equations

The governing equations and boundary conditions of a nanocrystalline nanobeam could be derived via Hamilton's principle as

$$\int_0^t \delta(U+V)\, dt = 0 \tag{39}$$

In which U is strain energy and V is the work done by applied loads. The variation of the strain energy can be stated as

$$\begin{aligned} \delta U &= \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\sigma_{ij}\delta\epsilon_{ij} + m_{ij}\delta X_{ij}\right) dz_{ns} dx \\ &= \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\sigma_x \delta\epsilon_x + \tau_{xz}\delta\gamma_{xz} \right. \\ &\quad \left. + 2m\delta X_{xy} + 2m_{yz}\delta X_{yz}\right) dz dx \\ &= \int_0^L \left(-(M_b + Y_1) \frac{d^2\delta w_b}{dx^2} \right. \\ &\quad - \left(M_s + \frac{1}{2}Y_1 + \frac{1}{2}Y_2\right) \frac{d^2\delta w_s}{dx^2} \\ &\quad \left. + \left(Q - \frac{1}{2}Y_3\right) \frac{d\delta w_s}{dx} \right) dx \end{aligned} \tag{40}$$

where the stress resultants are presented as

$$\begin{gathered} (M_b, M_s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (z, f)\sigma_x dz, \\ Q = \int_{-\frac{h}{2}}^{\frac{h}{2}} g\tau_{xz} dz_{ns} \end{gathered} \tag{41}$$

$$\begin{gathered} (Y_1, Y_2) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, f') m_{xy} dz_{ns}, \\ Y_3 = \int_{-\frac{h}{2}}^{\frac{h}{2}} f'' m_{yz} dz_{ns} \end{gathered} \tag{42}$$

Also, the work done variation by external loads can be written as

$$\delta V = \int_0^L [+N_0 \frac{\partial^2 \delta w}{\partial x^2} - k_w \delta w + k_p \frac{\partial^2 \delta w}{\partial x^2}] dx \tag{43}$$

where kw, k_p are Winkler and Pasternak coefficients and N_0 is buckling load. Inserting the expressions from Eqs. (40)-(43) and integrating by parts versus both space and time variables, and collecting the coefficients of δw_b and δw_s, the following equations of motion of the nanocrystalline nanobeam are obtained

$$\delta w_b : \frac{d^2 M_b}{dx^2} + \frac{d^2 Y_1}{dx^2} - N_0 \frac{\partial^2 w}{\partial x^2} - k_w w + k_p \frac{\partial^2 w}{\partial x^2} = 0 \quad (44)$$

$$\delta w_s : \frac{d^2 M_s}{dx^2} + \frac{1}{2}\frac{d^2 Y_1}{dx^2} + \frac{1}{2}\frac{d^2 Y_2}{dx^2} - \frac{1}{2}\frac{dY_3}{dx} + \frac{dQ}{dx} - N_0 \frac{\partial^2 w}{\partial x^2} - k_w w + k_p \frac{\partial^2 w}{\partial x^2} = 0 \quad (45)$$

Integrating Eqs. (35)-(38) over the beam's cross-section area, we obtain the force-strain and the moment-strain of the nonlocal couple stress beam theory can be obtained as follows

$$M_b - \mu \frac{d^2 M_b}{dx^2} = -D_{11} \frac{d^2 w_b}{dx^2} - D_{11}^s \frac{d^2 w_s}{dx^2} \quad (46)$$

$$M_s - \mu \frac{d^2 M_s}{dx^2} = -D_{11}^s \frac{d^2 w_b}{dx^2} - H_{11}^s \frac{d^2 w_s}{dx^2} \quad (47)$$

$$Q - \mu \frac{d^2 Q}{dx^2} = A_{55}^s \frac{dw_s}{dx} \quad (48)$$

$$Y_1 - \mu \frac{d^2 Y_1}{dx^2} = -A_{13} \frac{d^2 w_b}{dx^2} - \frac{1}{2}(A_{13} + B_{13}) \frac{d^2 w_s}{dx^2} \quad (49)$$

$$Y_2 - \mu \frac{d^2 Y_2}{dx^2} = -B_{13} \frac{d^2 w_b}{dx^2} - \frac{1}{2}(B_{13} + D_{13}) \frac{d^2 w_s}{dx^2} \quad (50)$$

$$Y_3 - \mu \frac{d^2 Y_3}{dx^2} = -\frac{1}{2} E_{13} \frac{dw_s}{dx} \quad (51)$$

and the following boundary conditions are obtained at $x = 0$ and $x = L$.

Specify

$$w_b \ or \ V_b = \frac{dM_b}{dx} + \frac{dY_1}{dx} \quad (52)$$

Specify

$$w_s \ or \ V_s = \frac{dM_s}{dx} + \frac{1}{2}\frac{dY_1}{dx} + \frac{1}{2}\frac{dY_2}{dx} - \frac{1}{2}Y_3 + Q \quad (53)$$

$$\text{Specify } \frac{dw_b}{dx} \ or \ M_b + Y_1 \quad (54)$$

$$\text{Specify } \frac{dw_s}{dx} \ or \ M_s + \frac{1}{2}Y_1 + \frac{1}{2}Y_2 \quad (55)$$

where A_{11} , B_{11}^s , etc., are the beam stiffness, defined by

$$(A_{11}, D_{11}, D_{11}^s, H_{11}^s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \lambda \frac{1-v}{v}(1, z^2, zf, f^2) dz \quad (56)$$

$$(A_{13}, B_{13}, D_{13}, E_{13}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \mu l^2 [1, f', (f')^2, (f'')^2] dz \quad (57)$$

$$A_{55}^s = \int_{-\frac{h}{2}}^{\frac{h}{2}} \mu g^2 dz \quad (58)$$

By employing Eqs. (46)-(51), the equations of motion of nonlocal couple stress nanobeam in terms of the displacements are calculated as

$$-(D_{11} + A_{13})\frac{\partial^4 w_b}{\partial x^4} - (D_{11}^s + \frac{1}{2}(A_{13} + B_{13}))\frac{\partial^4 w_s}{\partial x^4} + \mu(+N_0 \frac{\partial^4 w}{\partial x^4} + k_w \frac{\partial^2 w}{\partial x^2} - (k_p)\frac{\partial^4 w}{\partial x^4}) - N_0 \frac{\partial^2 w}{\partial x^2} - k_w w + k_p \frac{\partial^2 w}{\partial x^2} = 0 \quad (59)$$

$$-(D_{11}^s + \frac{1}{2}[A_{13} + B_{13}])\frac{\partial^4 w_b}{\partial x^4} - (H_{11}^s + \frac{1}{4}[A_{13} + 2B_{13} + D_{13}])\frac{\partial^4 w_s}{\partial x^4} + (A_{55}^s + \frac{1}{4}E_{13})\frac{\partial^2 w_s}{\partial x^2} + \mu(+N_0 \frac{\partial^4 w}{\partial x^4} + k_w \frac{\partial^2 w}{\partial x^2} - (k_p)\frac{\partial^4 w}{\partial x^4}) - N_0 \frac{\partial^2 w}{\partial x^2} - k_w w + k_p \frac{\partial^2 w}{\partial x^2} = 0 \quad (60)$$

3. Solution method

Through this section, solution procedure of the governing equations of anonlocal couple stress based nanocrystalline nanobeam under different types of boundary conditions (S-S, C-S and C-C) is presented.

Simply-supported (S):

$w_b = w_s = M = 0$ at $x = 0, L$

Clamped (C):

$u = w_b = w_s = 0$ at $x = 0, L$

Thus, the following expansions of displacements are supposed as

$$\begin{Bmatrix} w_b \\ w_s \end{Bmatrix} = \sum_{n=1}^{\infty} \begin{Bmatrix} W_{bn} X_m(x) \\ W_{sn} X_m(x) \end{Bmatrix} \quad (61)$$

where W_{bn} and W_{sn} are Fourier coefficients. Substituting Eq. (61) into Eqs. (59) and (60) leads to

$$\left\{ \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} \right\} \begin{Bmatrix} W_{bn} \\ W_{sn} \end{Bmatrix} = 0$$

where

$$k_{1,1} = -(D_{11} + A_{13})\kappa_9 - k_w \kappa_5 + \mu k_w \kappa_7 + (k_p - N_0)\kappa_7 - \mu(k_p - N_0)\kappa_9,$$
$$k_{1,2} = -(D_{11}^s + 0.5[A_{13} + B_{13}])\kappa_9 - k_w \kappa_5 + \mu k_w \kappa_7 + (k_p - N_0)\kappa_7 - \mu(k_p - N_0)\kappa_9,$$
$$k_{2,2} = -(H_{11}^s + 0.25[A_{13} + 2B_{13} + D_{13}])\kappa_9 + (A_{55}^s + 0.25E_{13})\kappa_7 - k_w \kappa_5 + \mu k_w \kappa_7 + (k_p - N_0)\kappa_7 - \mu(k_p - N_0)\kappa_9,$$

in which

$$(\kappa_1, \kappa_3, \kappa_5, \kappa_7, \kappa_9) = \int_0^L \left(X_m' X_m', X_m X_m''', X_m X_m, X_m'' X_m, X_m'''' X_m \right) dx \quad (62)$$

The function X_m for different boundary conditions is defined by

S-S:
$$X_m(x) = \sin(\lambda_n x) \qquad \lambda_n = \frac{n\pi}{L} \tag{63}$$

C-C:
$$X_m(x) = \sin(\lambda_n x) - \sinh(\lambda_n x) - \xi_m(\cos(\lambda_n x) - \cosh(\lambda_n x))$$
$$\xi_m = \frac{\sin(\lambda_n x) - \sinh(\lambda_n x)}{\cos(\lambda_n x) - \cosh(\lambda_n x)} \tag{64}$$
$$\lambda_1 = 4.730, \lambda_2 = 7.853, \lambda_3 = 10.996, \lambda_4 = 14.137, \quad \lambda_{n\geq 5} = \frac{(n+0.5)\pi}{L}$$

C-S:
$$X_m(x) = \sin(\lambda_n x) - \sinh(\lambda_n x) - \xi_m(\cos(\lambda_n x) - \cosh(\lambda_n x))$$
$$\xi_m = \frac{\sin(\lambda_n x) + \sinh(\lambda_n x)}{\cos(\lambda_n x) + \cosh(\lambda_n x)} \tag{65}$$
$$\lambda_1 = 3.927, \lambda_2 = 7.069, \lambda_3 = 10.210, \lambda_4 = 13.352, \quad \lambda_{n\geq 5} = \frac{(n+0.25)\pi}{L}$$

The following relation is accomplished in order to compute the non-dimensional buckling loads and foundation parameters

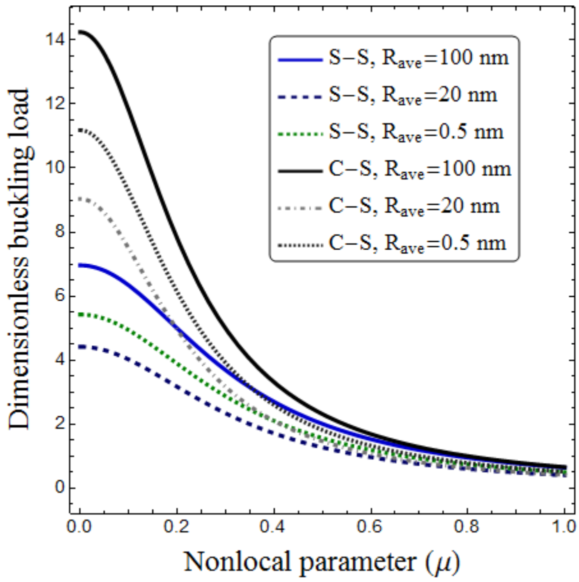
$$K_p = k_p \frac{L^2}{E_g I}, \quad K_w = k_w \frac{L^4}{E_g I}, \quad \bar{N} = N^0 \frac{L^2}{E_g I}, \quad \mu = \frac{(e_0 a)}{L} \tag{66}$$

4. Numerical results and discussions

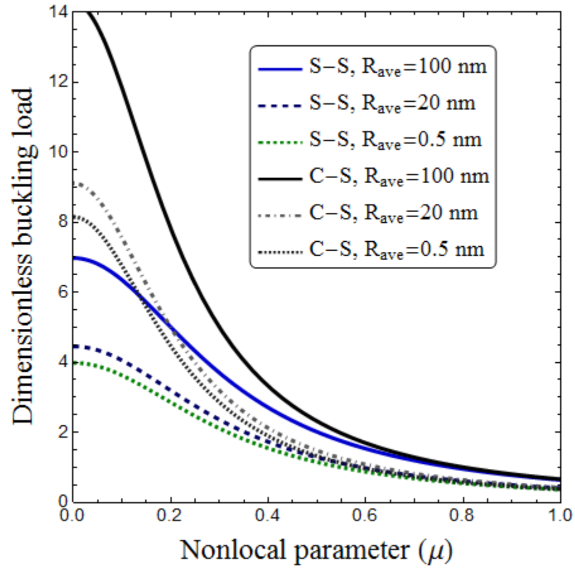
In this section, influences of elastic foundation, shear deformation, nonlocality parameter, surface elasticity, grain size, grain rotation, porosities and interface on the buckling loads of a nanocrystalline nanobeam will be investigated. The nanocrystalline silicon (Nc-Si) is a three-phase composite with two inhomogeneity types. Phase 2 is nano crystals (grains) of an average radius R_g and Young's modulus E_g. Phase 3 is nano-void (pores) of an average radius $R_v = R_g$ and Young's modulus $E_v = 0$. Here, the surface of voids could be supposed to have the same surface parameters of the grains' surface. In fact, the surface parameters of solids rely on the intermolecular bonds at the surface, the nature of the surrounding medium and the bulk material parameters. Also, the nanobeam thickness in this study is taken as $h = 100\,nm$.

Table 3 Comparison of the dimensionless buckling load for nonlocal couple stress nanobeams (l=0.25h)

μ	L/h=10		L/h=20	
	CBT (Attia and Mahmoud 2016)	present	CBT (Attia and Mahmoud 2016)	present
0	14.451	14.1174	14.451	14.3658
2	12.069	11.7901	13.771	13.6902
3	11.150	10.8923	13.455	13.3757
4	10.361	10.1216	13.153	13.0753



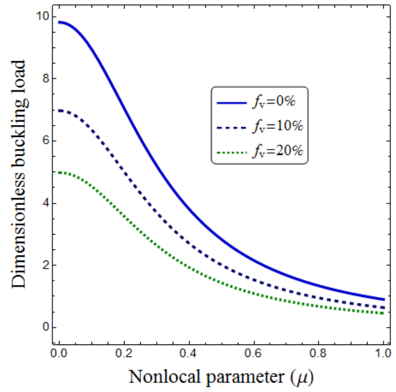
(a) WS



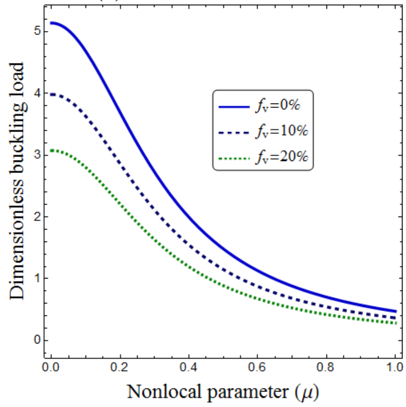
(b) NS

Fig. 2 Variation of dimensionless buckling load of S-S and C-S nanocrystalline nanobeam versus nonlocal parameter for different values of average radius (l=0.1h, Kw=Kp=0, L/h=10, fv=10%)

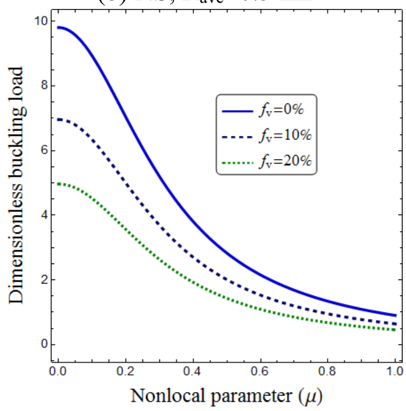
Buckling loads are compared with those of nonlocal couple stress homogenous nanobeams presented by Attia and Mahmoud (2016). They considered the material



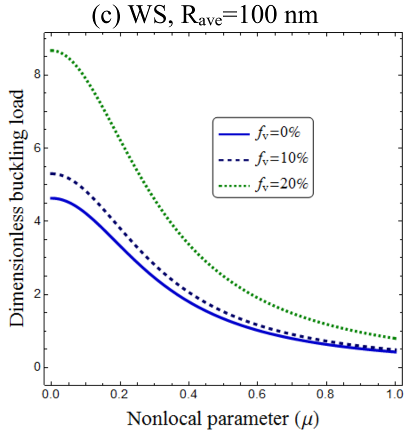
(a) NS, R_{ave}=100 nm



(b) NS, R_{ave}=0.5 nm

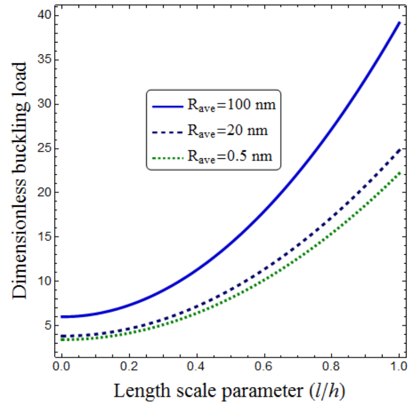


(c) WS, R_{ave}=100 nm

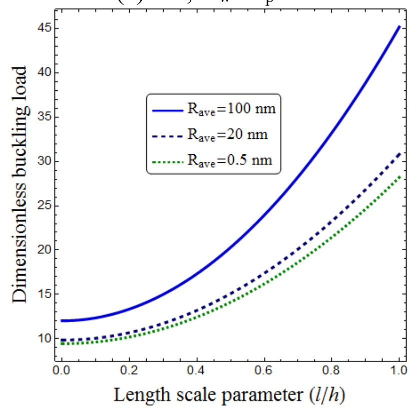


(d) WS, R_{ave}=0.5 nm

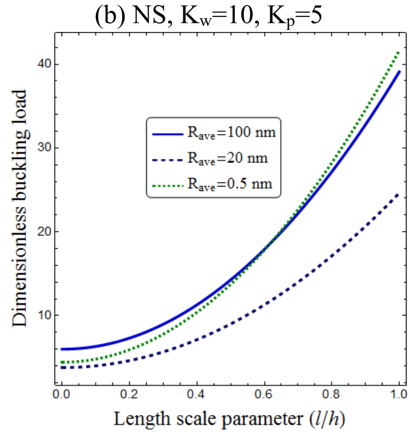
Fig. 3 Variation of dimensionless buckling load of nanocrystalline nanobeam versus nonlocal parameter for different values of porosity percentage (l=0.1h, Kw=Kp=0, L/h=10)



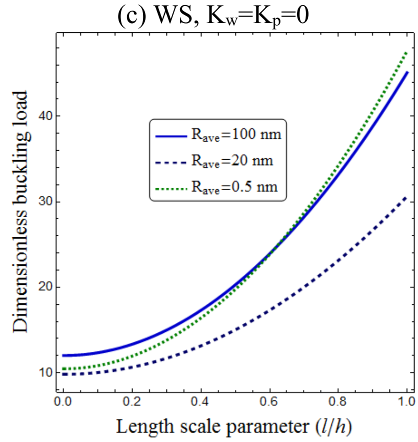
(a) NS, K_w=K_p=0



(b) NS, K_w=10, K_p=5

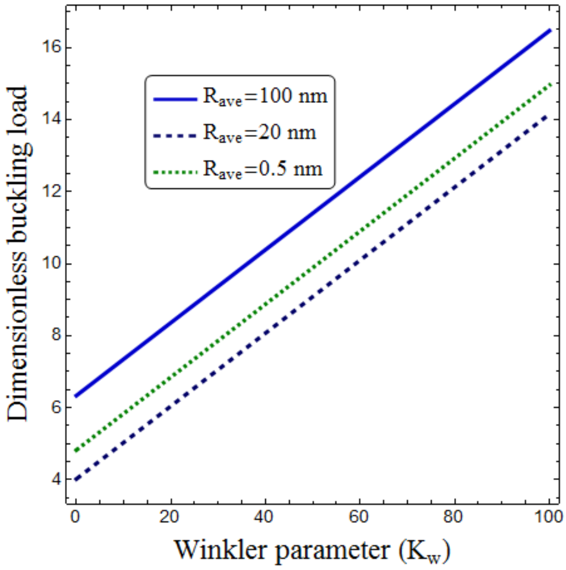


(c) WS, K_w=K_p=0

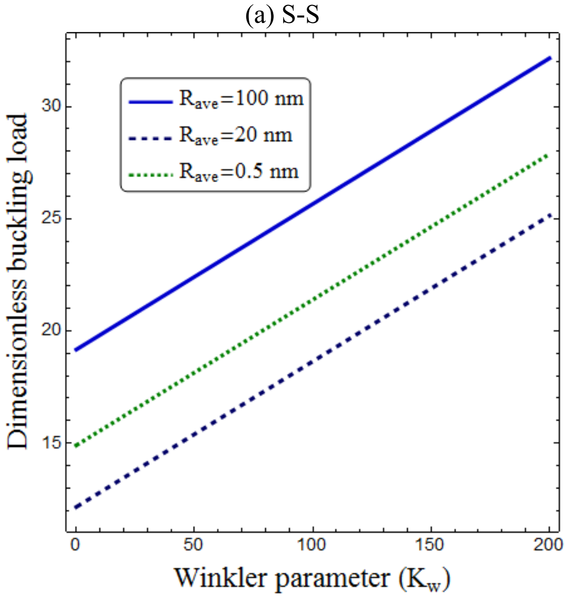


(d) WS, K_w=10, K_p=5

Fig. 4 Variation of dimensionless buckling load of S-S nanocrystalline nanobeam versus length scale parameter for different values of average radius (μ=0.1, L/h=10)

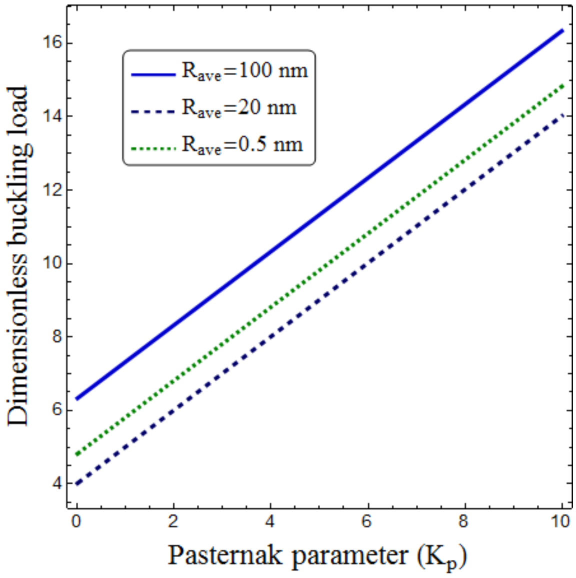


(a) S-S

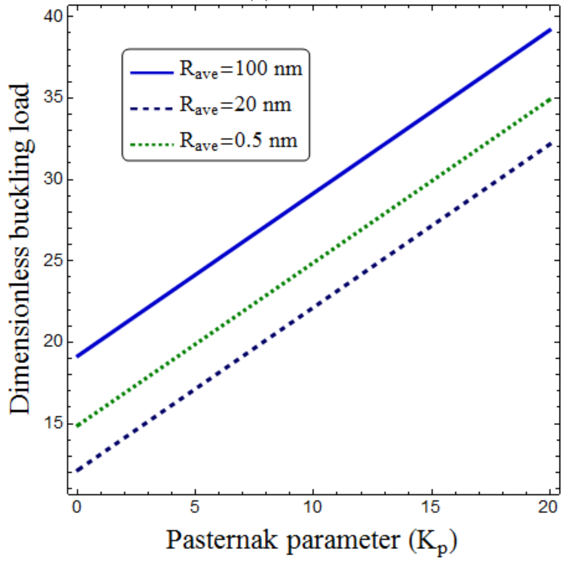


(b) C-C

Fig. 5 Variation of dimensionless buckling load of S-S and C-C nanocrystalline nanobeam versus Winkler parameter for different values of average radius (l=0.1h, Kp=0, L/h=10, fv=10%)



(a) S-S



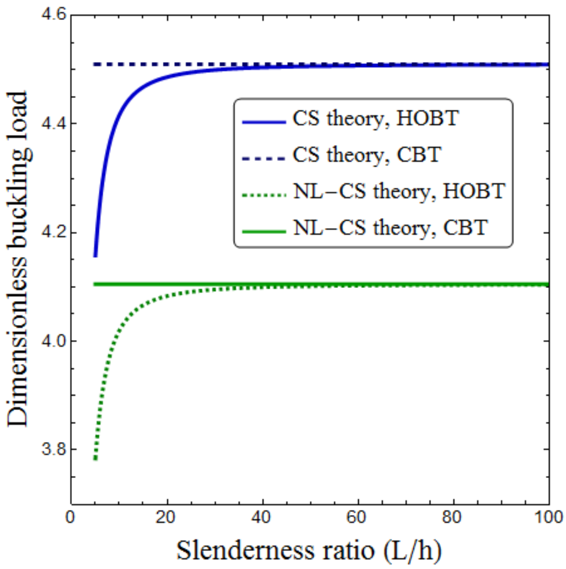
(b) C-C

Fig. 6 Variation of dimensionless buckling load of S-S and C-C nanocrystalline nanobeam versus Pasternak parameter for different values of average radius (l=0.1h, Kw=0, L/h=10, fv=10%)

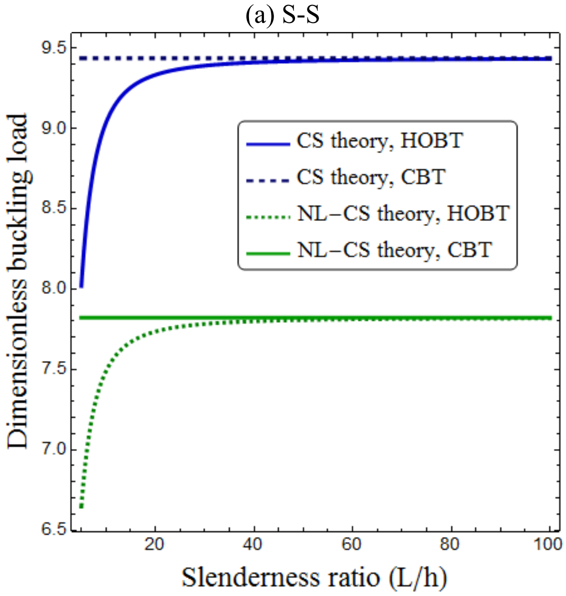
properties as: E=90 GPa, ν=0.23. Table 3 presents the comparison of the buckling load of a nonlocal beam of S-S boundary conditions with those of Euler-Bernoulli beam model. According to this table, the results are presented for different nonlocal parameters and a good agreement is observed. Examination of the effect of porosity percentage on buckling load of nanobeams versus nonlocal parameter for different average radius is showed in Fig. 3 when l=0.1h. In this figure, a softening mechanism is illustrated due to the porosities or nano-voids inside the material structure.

So, an increase in the porosity percentage leads to reduction in the nanobeam rigidity and magnitude of buckling load when the surface phase of pores is considered (WS). So, the buckling loads of a nanocrystalline nanobeams are overestimated by neglecting the porosity effect. But, an opposite trend is observed when R_g=R_v=0.5 nm and surface phase is neglected (NS). This is due to the fact that the effect of surface phase of porosities becomes more significant when R_v=0.5 nm. In this case, increase of porosity percentage leads to larger buckling loads.

Fig. 4 illustrates the effect of couple stress parameter (grains rigid rotation) on buckling load of nanocrystalline nanobeam for various inhomogeneities sizes at μ=0.1 and f_v=10%. The plotted curves of this figure reflect the prominent influences of the micro-rotations where dimensionless buckling loads are obtained by changing of couple stress parameter l. It is noticed that enlargement of couple stress parameter leads to increment in the buckling load values which highlights the stiffness enhancement of



(a) S-S



(b) C-S

Fig. 7 Variation of dimensionless buckling load of S-S and C-S nanocrystalline nanobeam versus slenderness ratio for classical and higher order beam theories (l=0.1h, μ=0.1, Rave=20, fv=10%)

nanobeam. Enlargement of dimensionless buckling load with respect to couple stress parameter depends on the value of inhomogeneities sizes. At large values of couple stress parameter, the nanobeam with R_g=0.5 has the maximum buckling load because surface energy of nano-grains when nm has a dominant role by exerting a stiffness-hardening impact. But, the nanobeam with R_g=0.5 has minimum buckling loads when the grains surface energy is ignored for all ranges of couple stress parameter.

Figs. 5 and 6 present the variation of dimensionless buckling load of nanocrystalline nanobeam with respect to Winkler and Pasternak parameters for various inhomogeneities sizes and boundary conditions when μ=0.1 and f_v=10%. As mentioned, by reducing the grains and porosities size from 100 nm to 20 nm, the magnitude of buckling load will decrease. But, by reducing the average radius from 20 nm to 0.5 nm the magnitude of buckling load will rise. It can be concluded that the largest and smallest critical loads are obtained for R_g=R_v=100 nm and R_g=R_v=20 nm, respectively. This figure indicates the prominence of accurate modeling of nanobeams by incorporating the essential measures to describe the size-dependency of material structure. It is also found that presence of elastic medium has a significant effect on the buckling behavior nanocrystalline nanobeam. In fact, elastic medium makes the nanocrystalline nanobeam more rigid and buckling loads increase at a constant value of inhomogeneities size. Another observation is that the shear layer or Pasternak foundation has more significant impact in enlargement of buckling loads of nanocrystalline nanobeam than Winkler foundation. In fact, Pasternak layer has a continuous interaction with the nanocrystalline nanobeam, while Winkler layer is modeled as parallel springs with a discontinuous interaction with nanobeam.

Variation of dimensionless buckling load of S-S and C-S nanocrystalline nanobeams versus slenderness ratio (L/h) based on classical and higher order beam models at μ=0.1, f_v=10% is plotted in Fig. 7. It is found that increase of slenderness ration leads to higher buckling loads. However, shear deformation effect on buckling loads of nanocrystalline nanobeam is important at lower slenderness ratios and it is negligible at large slenderness ratios. In fact, higher order beam model (HOBT) for nanocrystalline nanobeams gives smaller buckling loads compared with classical beam model (CBT). So, selecting an appropriate beam model is crucial for accurate design of nanocrystalline nanostructures.

5. Conclusions

In this paper, buckling behavior of porous nanocrystalline nanobeams lying on Winkler-Pasternak foundation is investigated based on nonlocal couple stress elasticity theory. Nanocrystalline nanobeam is composed from three phases which are nano-grains, nano-voids and interface. Nano-voids or porosities inside the material have a stiffness-softening impact on the nanobeam. Nonlocal elasticity theory of Eringen is applied in analysis of nanocrystalline nanobeams for the first time. Also, modified couple stress theory is employed to capture grains rigid rotations. Applying an analytical approach which satisfies various boundary conditions the governing equations obtained from Hamilton’s principle are solved. The reliability of present approach is verified by comparing obtained results with those provided in literature. It is seen that inclusion of nonlocal parameter leads to buckling loads by reducing the bending stiffness of nanocrystalline nanobeam, regardless of the size of inhomogeneities. Also, inclusion of grains surface effects gives larger buckling loads than when surface effects are ignored. It is observed that couple stress effect leads to larger buckling loads. By ignoring the couple stress effect in analysis of nanocrystalline nanobeam, the buckling loads are underestimated. The buckling loads may increase or decrease with the reduction in the inhomogeneities sizes.

Also, an increase in the porosity percentage leads to reduction in the nanobeam rigidity and magnitude of buckling load. Also, it is observed that presence of elastic foundation makes the nanocrystalline nanobeam more rigid and leads to larger buckling loads.

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