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A refined four variable plate theory for thermoelastic analysis of FGM plates resting on variable elastic foundations

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Abstract. In this paper, an efficient higher-order shear deformation theoryis presented to analyze thermomechanical bending of temperature-dependentfunctionally graded (FG) plates resting on an elastic foundation. Further simplifying supposition are made to the conventional HSDT so that the number of unknowns is reduced, significantly facilitating engineering analysis. These theory account for hyperbolic distributions of the transverse shear strains and satisfy the zero traction boundary conditions on the surfaces of the plate without using shear correction factors. Power law material properties and linear steady-state thermal loads are assumed to be graded along the thickness. Nonlinearthermal conditions are imposed at the upper and lower surface for simply supported FG plates. Equations of motion are derived fromthe principle of virtual displacements. Analytical solutions for the thermomechanical bending analysis are obtained based on Fourier series that satisfy the boundary conditions (Navier's method). Non-dimensional results are compared for temperature-dependent FG plates and validated with those of other shear deformation theories. Numerical investigation is conducted to show the effect of material composition, plate geometry, and temperature field on the thermomechanical bending characteristics. It can be concluded that the present theory is not only accurate but also simple in predicting the thermomechanical bending responses of temperature-dependentFG plates.

Keywords: functionally graded plate; higher-order plate theory; thermomechanical bending; temperature-dependent properties

1. Introduction

Functionally graded materials (FGMs) are a class of composites that have continuous variation of material properties from one surface to another and thus eliminate the stress concentration found in laminated composites. The concept of FGM has been widely explored in various engineering applications including mechanical, aerospace, nuclear, and civil engineering (Shahrjerdi *et al.* 2011, Kar and Panda 2014, Swaminathanand Naveenkumar 2014, Kolahchi *et al.* 2015, Belkorissat *et al.* 2015, Ghorbanpour Arani *et al.* 2016, Bellifa *et al.* 2016, Boukhari *et al.* 2016, Bounouara *et al.* 2016, Bousahla *et al.* 2016, Barati and Shahverdi 2016, Bellifa *et al.* 2017a). The increase in FGM applications requires accurate models to predict their responses. Since the shear deformation has significant effects on the responses of functionally graded (FG) plates, shear deformation theories are used to capture such shear deformation effects. The first-order shear deformation theory FPT (Mindlin 1951, Reissner 1945) accounts for shear deformation effects, but violates the equilibrium conditions at the top and bottom surfaces of the plate. A shear correction factor is therefore required (Yaghoobi and Yaghoobi 2013, Araniand Kolahchi 2016, Madani *et al.* 2016, Zamanian *et al.* 2017). The higher-order shear deformation theories (HSDT) (Reddy 1984, Reddy 2000, Ren 1986, Touratier 1991, Soldatos 1992, Xiang *et al.* 2009, Akavci 2010, Grover *et al.* 2013, Karama *et al.* 2003, Pradyumna and Bandyopadhyay 2008, Ait Atmane *et al.* 2010, Mantari *et al.* 2012, Zidi *et al.* 2014, Larbi Chaht *et al.* 2015, Mahi *et al.* 2015, Taibi *et al.* 2015, Zemri *et al.* 2015, Kolahchi and Moniri Bidgoli 2016, Saidi *et al.* 2016, Houari *et al.* 2016, Baseri *et al.* 2016, Kolahchi *et al.* 2016a, b, Kolahchi 2017, Kolahchi *et al.* 2017, Zidi *et al.* 2017, Abdelaziz *et al.* 2017, Youcef *et al.* 2018) account for shear deformation effects and satisfy the equilibrium conditions at the top and bottom surfaces of the plate without requiring any shear correction factors. In a number of recent articles, a new refined and robust plate theory for bending response and vibration of simply supported FGM plate with only four unknown functions has been developed (Bourada *et al.* 2012, Bachir Bouiadjra *et al.* 2012, Tounsi *et al.* 2013, Kettaf *et al.* 2013). In addition, many of the

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above-mentioned papers deal with temperature-independent materials with shear deformation theories. Temperature-dependent materials in temperature variations with surface-to-surface heat flow through the thickness direction were considered in other research by applying first, third and higher order shear deformation theories. However, the bending analyses of FGMs resting on an elastic foundation are quite limited. Many models related to soil-structure interaction were proposed by scientists (Kerr 1964), the simplest one is the Winkler model, which regards the foundation as a series of closed-spaced springs without coupling effects between them, the disadvantage of this model is that it assumes no interaction between the springs. To overcome this problem, several two-parameter models have been proposed. Zhemochkin and Sinitsyn (1947) introduced a combined elastic foundation, which is the classical foundation covered by a layer of Winkler foundation. Filonenko-Borodich (1940) developed an improved Winkler model by connecting the top ends of the springs with an elastic membrane stretched to a constant tensile stress. Hetényi (Hetényi 1946, 1950) created an interaction between the springs by incorporating an additional plate into the Winkler foundation. Vlasov (1949) also suggested a more refined two-parameter model. Gorbunov-Posadov (1949) considered problems on the flexure of plates and beams lying on a linearly deform able foundation. Pasternak (1954) improved the Winkler model by connecting the ends of the springs to a plate, or "shear layer," consisting of in compressible vertical elements able to de form only in lateral shear. Since then, the Pasternak model has been widely used to describe the mechanical behavior of structure-foundation interactions (Shen 1995, Omurtag *et al.* 1997, Matsunaga 2000, Filipich and Rosals 2002, Zhou *et al.* 2004, Huang *et al.* 2008, Bilouei *et al.* 2016). Kolahchi *et al.* (2016) studied the dynamic stability response of temperature-dependent functionally graded CNT-reinforced visco-plates resting on orthotropic elastomeric medium. All of the previous work has been analyzed plates or beams resting on elastic foundations with constant moduli. However, studies on structures resting on variable elastic foundation are limited in literature. The vibration and buckling of beams on variable two-parameter elastic foundations were discussed by Eisenberger and Clastornik (1987). Zhou (1993) studied vibration of a uniform single span beam resting on variable Winkler elastic foundation. Pradhan and Murmu (2009) illustrated the thermo-mechanical vibration of FG sandwich beam resting on variable elastic foundations.

The aim of this work is to develop a simple higher-order shear deformation theory for thermo mechanical bending of temperature-dependentfunctionally graded (FG) plates resting on variable two-parameter elastic foundations. The proposed theory contains fewer unknowns and equations of motion than the first-order shear deformation theory, but satisfies the equilibrium conditions at the top and bottom surfaces of the plate without using any shear correction factors. Further simplifying supposition are made to the conventional HSDT so that the number of unknowns is reduced, and consequently, makes the present theory much more amenable to mathematical implementation. The temperature is assumed to be constant in the plane of the plate. The variation of temperature is assumed to occur in the thickness direction only. The FG plates are assumed to be simply supported with temperature-dependent material properties with a power law distribution in terms of the volume fractions of the constituents and subjected to nonlinear temperature rise. Equations of motion are derived from the principle of virtual displacements. The accuracy of obtained solutions is verified by comparing the present results with those predicted by solutions available in the literature.

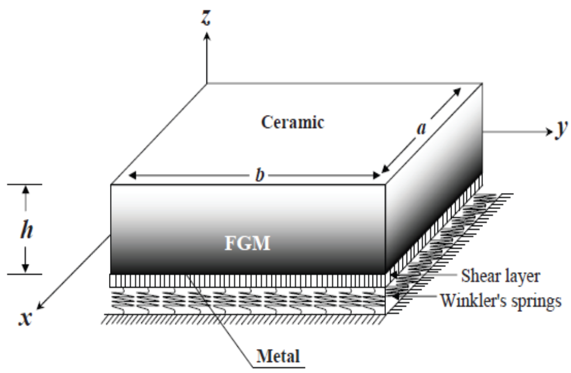

Fig. 1 Schematic representation of FGM plate resting on elastic foundation

2. Theoretical developments

Consider a simply supported rectangular FG plate with the length a width b, and thickness h. The x-, y-, and z-coordinates are taken along the length, width, and height of the plate, respectively (Fig. 1). The plate lies on two-parameter elastic foundation model which consists of closely spaced springs interconnected through a shear layer made of incompressible vertical elements, which deform only by transverse shear. The formulation is limited to linear elastic material behavior. The FG plate isisotropic with its material properties vary smoothly through the thickness of the plate.

2.1 Kinematics and strains

In this article, further simplifying supposition are made to the conventional HSDT so that the number of unknowns is reduced. The displacement field of the conventional HSDT is given by

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + f(z)\varphi_x(x,y,t) \tag{1a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + f(z)\varphi_y(x,y,t) \tag{1b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{1c}$$

Where u_0; v_0; w_0, φ_x, φ_y are five unknown displacements of the mid-plane of the plate, $f(z)$ denotes shape function representing the variation of the transverse shear strains and stresses within the thickness. By considering that

$\varphi_x = \int \theta(x,y)dx$, and $\varphi_y = \int \theta(x,y)dy$ (Besseghier *et al.* 2017, Sekkal *et al.* 2017a, Khetir *et al.* 2017, Menasria *et al.* 2017, Yazid *et al.* 2018), the displacement field of the present model can be expressed in a simpler form as

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y,t)\,dx \tag{2a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y,t)\,dy \tag{2b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{2c}$$

In this work, the present higher-order shear deformation plate theory is obtained by setting

$$f(z) = z\left(\frac{5}{4} - \frac{5z^2}{3h^2}\right) \tag{3}$$

It can be seen that the displacement field in Eq. (2) introduces only four unknowns (u_0, v_0, w_0 and θ). The nonzero strains associated with the displacement field in Eq. (2) are

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} + f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \tag{4}$$

Where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix},$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix} = \begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\int\theta\,dx + k_2\dfrac{\partial}{\partial x}\int\theta\,dy\end{Bmatrix}, \tag{5a}$$

$$\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} = \begin{Bmatrix}k_2\int\theta\,dy\\ k_1\int\theta\,dx\end{Bmatrix}$$

and

$$g(z) = \frac{df(z)}{dz} \tag{5b}$$

The integrals defined in the above equations shall be resolved by a Navier type method and can be written as follows

$$\frac{\partial}{\partial y}\int\theta\,dx = A'\frac{\partial^2\theta}{\partial x\partial y},\quad \frac{\partial}{\partial x}\int\theta\,dy = B'\frac{\partial^2\theta}{\partial x\partial y},$$
$$\int\theta\,dx = A'\frac{\partial\theta}{\partial x},\quad \int\theta\,dy = B'\frac{\partial\theta}{\partial y} \tag{6}$$

Where the coefficients A' and B' are expressed according to the type of solution used, in this case via Navier. Therefore, A', B', k_1 and k_2 are expressed as follows

$$A' = -\frac{1}{\alpha^2},\quad B' = -\frac{1}{\beta^2},\ k_1 = \alpha^2,\ k_2 = \beta^2 \tag{7}$$

Where α and β are defined in expression (27).

2.2 Constitutive relations

FGMs are composite materials made of ceramic and metal. There are some models in the literature that express the variation of material properties in FGMs (Chi and Chung 2006a, b). The most commonly used is the power law distribution of the volume fraction. According to this model, the material properties of FG plates are assumed to be position and temperature- dependent and can be expressed as the following (Kim 2005, Bouderba *et al.* 2013, Attia *et al.* 2015, El-Haina *et al.* 2017, Mouffoki *et al.* 2017)

$$\Gamma(z,T) = \left(\Gamma_c(T) - \Gamma_m(T)\right)V_c + \Gamma_m(T) \quad \text{and}$$
$$V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^p \tag{8}$$

Where Γ denotes a generic material property such as elastic modulus E, the Poisson's ratio ν, mass density ρ and thermal expansion coefficient α of FG plates; furthermore, subscripts m and c refer to the pure metal and ceramic plates, respectively. V_c denotes the ceramic volume fraction, where $p\geq 0$ is a namely grading index that is the volume fraction exponent. The non-linear FG plate's material can be expressed as the following

$$P(T) = P_0\left(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3\right) \tag{9}$$

Where P denotes material property; P_{-1}, P_0, P_1, P_2 and P_3 are the coefficients of temperature-dependent material properties unique to the constituent materials, and ΔT is the temperature rise only through the thickness direction. Temperature-dependent typical values for some functionally graded materials components are in Table 1.

Taking into account the thermal effects and using Eqs. (4) and (2), the stress-strain relationships of the FGM plate can be written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix} = \begin{bmatrix}C_{11} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{22} & 0 & 0 & 0\\ 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}\left\{\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix} - \alpha\Delta T\begin{Bmatrix}1\\1\\0\\0\\0\end{Bmatrix}\right\} \tag{10}$$

where (σ_x, σ_y, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively. Using the material properties defined in Eq. (8), stiffness coefficients, C_{ij}, can be expressed as

$$C_{11} = C_{22} = \frac{E(z,T)}{1-\nu^2(z,T)}, \tag{11a}$$

$$C_{12} = \nu(z,T)C_{11}, \tag{11b}$$

$$C_{44} = C_{55} = C_{66} = \frac{E(z,T)}{2(1+\nu(z,T))}, \tag{11c}$$

The plate is assumed to rest on two parameter elastic foundation model which consists of closely spaced springs interconnected through a shear layer made of incompressible vertical elements, which deform only by transverse shear. The response equation of this foundation is given by

$$R(x,y) = \overline{K}(x)\, w(x,y) - \overline{G}\, \nabla^2 w(x,y) \tag{12}$$

Where R is the density of the reaction force of elastic foundation. $\overline{K}$ is Winkler parameter depended on x only. It is assumed to be linear, parabolic or sinusoidal (Pradhan and Murmu 2009, Sobhy 2015, Beldjelili *et al.* 2016)

$$\overline{K}(x) = \frac{J_1 h^3}{a^4}\begin{cases} 1+\varsigma \dfrac{x}{a} & \text{linear} \\ 1+\varsigma\left(\dfrac{x}{a}\right)^2 & \text{parabolic} \\ 1+\varsigma \sin\left(\pi \dfrac{x}{a}\right) & \text{sinusoidal} \end{cases} \tag{13}$$

In witch J_1 is a constant and ς is a varied parameter. $\overline{G}$ is the shear layer foundation stiffness

∇^2 is the Laplace operator in x and y, and w is the deflection of the plate.

Note that, if ς=0, the elastic foundation becomes Pasternak foundation and if the shear layer foundation stiffness is neglected, the Pasternak foundation becomes the Winkler foundation.

2.3 Equations of motion

The principle of virtual work is herein utilized to determine the equations of motion (Bellifa *et al.* 2017b, Benadouda *et al.* 2017, Chikh *et al.* 2017, Ahouel *et al.* 2016, Bouderba *et al.* 2016, Al-Basyouni *et al.* 2015, Ait Yahia *et al.* 2015, Ait Amar Meziane *et al.* 2014)

$$\int_{-h/2}^{h/2}\int_{\Omega}\left[\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right] d\Omega dz - \int_{\Omega} q\delta w d\Omega + \int_{\Omega} R\delta w d\Omega = 0 \tag{14}$$

Where Ω is the top surface, and q is the applied transverse load.

Subtitling Eqs. (4) and (2) into equations. (14) and integrating through the thickness of the plate, Eq. (14) can be rewritten as

$$\int_{\Omega}\left[N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b\delta k_x^b + M_y^b\delta k_y^b + M_{xy}^b\delta k_{xy}^b + M_x^s\delta k_x^s + M_y^s\delta k_y^s + M_{xy}^s\delta k_{xy}^s + S_{yz}^s\delta\gamma_{yz}^s + S_{xz}^s\delta\gamma_{xz}^0\right] d\Omega - \int_{\Omega} q\delta w d\Omega + \int_{\Omega} R\delta w d\Omega = 0 \tag{15}$$

Where the stress resultants N, M, and S are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2}(1, z, f)\sigma_i dz, \quad (i = x, y, xy) \text{ and}$$
$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz \tag{16}$$

By substituting Eqs. (2) and (5) into Eq. (15), the following can be derived

$$\delta u_0: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$
$$\delta v_0: \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0$$
$$\delta w_0: \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} - R + q = 0 \tag{17}$$
$$\delta\theta: -k_1 M_x^s - k_2 M_y^s - (k_1 A' + k_2 B')\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1 A'\frac{\partial S_{xz}^s}{\partial x} + k_2 B'\frac{\partial S_{yz}^s}{\partial y} = 0$$

Substituting Eq. (10) into Eq. (16) and the subsequent results into Eq. (15), the stress resultants are obtained in terms of strains as following compact form

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix}\begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix}, \quad S = A^s\gamma, \tag{18}$$

In which

$$N = \{N_x, N_y, N_{xy}\}^t, \quad M^b = \{M_x^b, M_y^b, M_{xy}^b\}^t, \quad M^s = \{M_x^s, M_y^s, M_{xy}^s\}^t, \tag{19a}$$

$$\varepsilon = \{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\}^t, \quad k^b = \{k_x^b, k_y^b, k_{xy}^b\}^t, \quad k^s = \{k_x^s, k_y^s, k_{xy}^s\}^t, \tag{19b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{19c}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}, \tag{19d}$$

$$S=\left\{S^s_{xz},S^s_{yz}\right\}^t,\quad \gamma=\left\{\gamma^0_{xz},\gamma^0_{yz}\right\}^t,\quad A^s=\begin{bmatrix}A^s_{44} & 0\\ 0 & A^s_{55}\end{bmatrix},\tag{19e}$$

And stiffness components are given as

$$\begin{Bmatrix}A_{11} & B_{11} & D_{11} & B^s_{11} & D^s_{11} & H^s_{11}\\ A_{12} & B_{12} & D_{12} & B^s_{12} & D^s_{12} & H^s_{12}\\ A_{66} & B_{66} & D_{66} & B^s_{66} & D^s_{66} & H^s_{66}\end{Bmatrix}= \int_{-h/2}^{h/2} C_{11}\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)\begin{Bmatrix}1\\ \nu\\ \dfrac{1-\nu}{2}\end{Bmatrix}dz,\tag{20a}$$

$$\left(A_{22},B_{22},D_{22},B^s_{22},D^s_{22},H^s_{22}\right)=\left(A_{11},B_{11},D_{11},B^s_{11},D^s_{11},H^s_{11}\right),\tag{20b}$$

$$A^s_{44}=A^s_{55}=\int_{-h/2}^{h/2} C_{44}\left[g(z)\right]^2 dz,\tag{20c}$$

Introducing Eq. (18) into Eq. (17), the equations of motion can be expressed in terms of displacements (u_0, v_0, w_0, θ) and the appropriate equations take the form

$$\begin{aligned}&A_{11}d_{11}u_0+A_{66}\,d_{22}u_0+\left(A_{12}+A_{66}\right)d_{12}v_0-\\ &B_{11}d_{111}w_0-\left(B_{12}+2B_{66}\right)d_{122}w_0\\ &+\left(B^s_{66}\left(k_1A'+k_2\,B'\right)\right)d_{122}\theta+\left(B^s_{11}k_1+B^s_{12}\,k_2\right)d_1\theta=0,\end{aligned}\tag{21a}$$

$$\begin{aligned}&A_{22}\,d_{22}v_0+A_{66}\,d_{11}v_0+\left(A_{12}+A_{66}\right)d_{12}u_0-B_{22}\,d_{222}w_0\\ &-\left(B_{12}+2B_{66}\right)d_{112}w_0\\ &+\left(B^s_{66}\left(k_1A'+k_2\,B'\right)\right)d_{112}\theta+\left(B^s_{22}k_2+B^s_{12}k_1\right)d_2\theta=0\end{aligned}\tag{21b}$$

$$\begin{aligned}&B_{11}\,d_{111}u_0+\left(B_{12}+2B_{66}\right)d_{122}u_0+\left(B_{12}+2B_{66}\right)d_{112}v_0\\ &+B_{22}\,d_{222}v_0-D_{11}d_{1111}w_0\\ &-2\left(D_{12}+2D_{66}\right)d_{1122}w_0-D_{22}\,d_{2222}w_0+\\ &\left(D^s_{11}\,k_1+D^s_{12}\,k_2\right)d_{11}\theta+2\left(D^s_{66}\left(k_1A'+k_2B'\right)\right)d_{1122}\theta\\ &+\left(D^s_{12}k_1+D^s_{22}k_2\right)d_{22}\theta+q-R=0\end{aligned}\tag{21c}$$

$$\begin{aligned}&-\left(B^s_{11}k_1+B^s_{12}k_2\right)d_1u_0-\left(B^s_{66}\left(k_1A'+k_2B'\right)\right)d_{122}u_0\\ &-\left(B^s_{66}\left(k_1A'+k_2B'\right)\right)d_{112}v_0-\left(B^s_{12}k_1+B^s_{22}k_2\right)d_2v_0\\ &+\left(D^s_{11}k_1+D^s_{12}k_2\right)d_{11}w_0+2\left(D^s_{66}\left(k_1A'+k_2B'\right)\right)d_{1122}w_0\\ &+\left(D^s_{12}k_1+D^s_{22}k_2\right)d_{22}w_0\\ &-H^s_{11}\,k_1^2\,\theta-H^s_{22}\,k_2^2\,\theta-2H^s_{12}\,k_1\,k_2\theta\\ &-\left(\left(k_1A'+k_2B'\right)^2H^s_{66}\right)d_{1122}\theta\\ &+A^s_{44}\left(k_2B'\right)^2d_{22}\theta+A^s_{55}\left(k_1\,A'\right)^2d_{11}\theta=0\end{aligned}\tag{21d}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij}=\frac{\partial^2}{\partial x_i\partial x_j},\quad d_{ijl}=\frac{\partial^3}{\partial x_i\partial x_j\partial x_l},\quad d_{ijlm}=\frac{\partial^4}{\partial x_i\partial x_j\partial x_l\partial x_m},\quad d_i=\frac{\partial}{\partial x_i},\quad (i,j,l,m=1,2).\tag{22}$$

2.4 Temperature field

The nonlinear temperature rise across the thickness of the plate is determined by solving the one-dimensional heat conduction equation. The one dimensional steady-state heat conduction equation in the z-direction is given by

$$-\frac{d}{dz}\left(k(z)\frac{dT}{dz}\right)=0\tag{23}$$

With the boundary condition $T(h/2)T_t$ and $T(-h/2)=T_b=T_0$. Here a stress-free state is assumed to exist at T_0=300 K. The analytical solution of Eq. (23) is

$$T(z)=T_b-\left(T_t-T_b\right)\frac{\int_{-h/2}^{z}\frac{1}{k(z)}dz}{\int_{-h/2}^{h/2}\frac{1}{k(z)}dz}\tag{24}$$

In the case of power-law FG plate, the solution of Eq. (18) also can be expressed by means of a polynomial series

$$\begin{aligned}T(z)=T_b+\frac{(T_t-T_b)}{C_{tb}}\Bigg[&\left(\frac{2z+h}{2h}\right)-\frac{k_{tb}}{(p+1)k_b}\left(\frac{2z+h}{2h}\right)^{p+1}\\ &+\frac{k_{tb}^2}{(2p+1)k_b^2}\left(\frac{2z+h}{2h}\right)^{2p+1}\\ &-\frac{k_{tb}^3}{(3p+1)k_b^3}\left(\frac{2z+h}{2h}\right)^{3p+1}+\frac{k_{tb}^4}{(4p+1)k_b^4}\left(\frac{2z+h}{2h}\right)^{4p+1}\\ &-\frac{k_{tb}^5}{(5p+1)k_b^5}\left(\frac{2z+h}{2h}\right)^{5p+1}\Bigg]\end{aligned}\tag{25}$$

With

$$C_{tb}=1-\frac{k_{tb}}{(p+1)k_b}+\frac{k_{tb}^2}{(2p+1)k_b^2}-\frac{k_{tb}^3}{(3p+1)k_b^3}+\frac{k_{tb}^4}{(4p+1)k_b^4}-\frac{k_{tb}^5}{(5p+1)k_b^5}\tag{26}$$

Where $k_{tb}=k_t-kb$, with k_t and k_b are the thermal conductivity of the top and bottom faces of the plate, respectively.

3. Analytical solutions

Based on the Navier approach with simply supported boundary conditions, the displacement fields are expressed as

$$\begin{Bmatrix}u_0\\ v_0\\ w_0\\ \theta\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix}U_{mn}\cos(\alpha\,x)\sin(\beta\,y)\\ V_{mn}\sin(\alpha\,x)\cos(\beta\,y)\\ W_{0mn}\sin(\alpha\,x)\sin(\beta\,y)\\ \theta_{mn}\sin(\alpha\,x)\sin(\beta\,y)\end{Bmatrix}\tag{27}$$

where U_{mn}, V_{mn}, W_{bmn} and θ_{mn} are arbitrary parameters to be determined, and $\alpha=m\pi/a$ and $\beta=n\pi/b$.

4. Numerical results

Table 1 Temperature-dependent coefficients for ZrO_2/Ti-6Al-4V (Reddy and Chin 1998)

	Material	P_{-1}	P_0	P_1	P_2	P_3
E	Ti-6Al-4V	0	$122.56e^{+9}$	$-4.586e^{-4}$	0	0
	ZrO_2	0	$244.27e^{+9}$	$-1.371e^{-3}$	$1.214e^{-6}$	$-3.681e^{-10}$
α	Ti-6Al-4V	0	$7.5788e^{-6}$	$6.638e^{-4}$	$-3.147e^{-6}$	0
	ZrO_2	0	$12.766e^{-6}$	$-1.491e^{-3}$	$1.006e^{-5}$	$-6.778e^{-11}$
k	Ti-6Al-4V	0	7.82	0	0	0
	ZrO_2	0	1.80	0	0	0

We present the bending results of a FG plate simply supported and resting on variable elastic foundations. This plate is subjected to thermal and mechanical loads. We choose the constituent materials of the FGM plate to be composed of a titanium alloy (Ti-6Al-4V) and zicronia (ZrO_2). Temperature-dependent coefficients of Young's modulus E, thermal expansion α and thermal conductivity k are given in Table 1. While Poisson's ratio is assumed to be a constant ν=0.3.

The used non-dimensional parameters are

$$\bar{w} = \frac{100h}{q_0 a^2} w\left(\frac{a}{2}, \frac{b}{2}\right)$$

$$\bar{\sigma}_x = -\frac{10h^2}{q_0 a^2} \sigma_x \left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2}\right)$$

$$\bar{\tau}_{xz} = -\frac{10hb}{q_0 a^2} \tau_{xz} \left(0, \frac{b}{2}, 0\right) \tag{28}$$

$$\bar{\tau}_{xy} = \frac{100h^2 b}{q_0 a^3} \tau_{xy} \left(0, 0, -\frac{h}{2}\right)$$

$$\bar{\tau}_{yz} = \frac{10h}{q_0 a} \tau_{yz} \left(\frac{a}{2}, 0, z\right) \quad J_2 = \frac{\bar{G} a^2}{h^3}$$

As is described in references (Sobhy 2015). The top surface is ceramic-rich and the bottom surface is metal-rich. Verification is carried out by assuming the the following values (unless otherwise stated) a/h=10; J_1=J_2=100; b/a=1; m=n=1; ζ=10; q_0=10^5.

4.1 Validation of the results

In this section, various numerical results for temperature-dependent FG plates computed using the present theory having four unknowns are compared to those of other higher-order shear deformation theories with more unknowns (Sobhy 2015).

Example 1:

In the first example, a FG ZrO_2/Ti–6Al–4V square and rectangular plates resting on elastic foundations with a parabolic Winkler modulus is considered for various values of mod numbers (m, n) and the dimensionless deflections are tabulated in Table 2. It can be seen from Table 2 that the results computed using the present efficient higher-order shear deformation theory is in a good agreement with other results from Refs (Sobhy 2015). It is clear that the deflection of the rectangular plate is higher than that of the square plate. It should be noted that deflection decreases with the increase of m and n.

Table 2 the deflection $\bar{w}$ of FGM square and rectangular plate simply supported and resting on elastic foundations (ΔT=300; p=1)

m	n	Theory	Square plate	Rectangular plate (b/a = 2)
1	1	SPT[a]	1.57312	3.11472
		present	1.58118	3.14664
	3	SPT[a]	-0.10770	-0.72382
		present	-0.10770	-0.72551
2	1	SPT[a]	0.00000	0.00000
		present	0.00000	0.00000
	3	SPT[a]	0.00000	0.00000
		present	0.00000	0.00000
3	1	SPT[a]	-0.10765	-0.12701
		present	-0.10773	-0.12296
	3	SPT[a]	0.04059	0.08825
		present	0.04060	0.08831
4	1	SPT[a]	0.00000	0.00000
		present	0.00000	0.00000
	3	SPT[a]	0.00000	0.00000
		present	0.00000	0.00000

[a]Sobhy (2015)

Table 3 the deflection $\bar{w}$ of FGM square plates without or resting on elastic foundations (ΔT=300; p=1, ζ=0)

J_1	J_2	Theory	a/h 5	10	15	20	25	30	50
0	0	FPT[a]	0.72464	2.50034	5.45984	9.60314	14.93025	21.44115	59.32280
		HPT[a]	0.72413	2.50010	5.45965	9.60296	14.93007	21.44098	59.32257
		SPT[a]	0.72385	2.49988	5.45944	9.60276	14.92988	21.44079	59.32241
		present	0.72382	2.49972	5.45916	9.60237	14.92936	21.44014	59.32124
10^3	0	FPT[a]	0.56180	2.00022	4.39367	7.74398	12.05137	17.31589	47.94570
		HPT[a]	0.56149	2.00006	4.39355	7.74386	12.05126	17.31578	47.94558
		SPT[a]	0.56132	1.99992	4.39367	7.74373	12.05113	17.31566	47.94544
		present	0.56130	1.99982	4.39323	7.74347	12.0508	17.31526	47.94468
10^3	10^3	FPT[a]	0.10335	0.40422	0.90506	1.60613	2.50748	3.60913	10.01867
		HPT[a]	0.10334	0.40421	0.90505	1.60612	2.50748	3.60912	10.01865
		SPT[a]	0.10333	0.40421	0.90505	1.60612	2.50747	3.60912	10.01865
		present	0.10333	0.40420	0.90504	1.60612	2.50746	3.60910	10.01862

[a]Sobhy (2015)

Example 2:

In the next example, aZrO_2/Ti–6Al–4V square plate without or resting on one-parameter ortwo-parameter elastic foundations is considered for different values of the side-to-thickness ratio a/h. the obtained results are compared to those of Sobhy (2015), as shown in Tables 3. It can be seen that the computed results are in good agreement with the previously published results (Sobhy 2015).

Table 4 The transverse shear stress $\bar{\tau}_{xz}$ in FGM square plates without or resting on elastic foundations (ΔT=300; p=1, ζ = 0)

J_1	J_2	Theory	a/h 5	10	15	20	25	30	50
0	0	FPT[a]	1.91547	1.91547	1.91547	1.91547	1.91547	1.91547	1.91547
		HPT[a]	2.38438	2.38915	2.39004	2.39035	2.39049	2.39057	2.39069
		SPT[a]	2.45911	2.46518	2.46632	2.46671	2.46687	2.46700	2.46714
		present	2.45911	2.46517	2.46630	2.46669	2.46683	2.46698	2.46713
10^3	0	FPT[a]	1.48501	1.53232	1.54142	1.54462	1.54612	1.54694	1.54811
		HPT[a]	1.84885	1.91131	1.92334	1.92758	1.92956	1.93062	1.93220
		SPT[a]	1.90696	1.97216	1.98473	1.98918	1.99122	1.99235	1.99399
		present	1.90698	1.97219	1.98474	1.98918	1.99124	1.99236	1.99398
10^3	10^3	FPT[a]	0.27321	0.30969	0.31754	0.32037	0.32171	0.32243	0.32349
		HPT[a]	0.34029	0.38629	0.39624	0.39981	0.40149	0.40245	0.40379
		SPT[a]	0.35107	0.39860	0.40886	0.41257	0.41433	0.41525	0.41667
		present	0.35107	0.39862	0.40887	0.41258	0.41432	0.41527	0.41666

[a]Sobhy (2015)

Table 5 the in-plane shear stress $\bar{\tau}_{xy}$ in FGM square plates without or resting on elastic foundations (ΔT=300; p=1, ζ=0)

J_1	J_2	Theory	a/h 5	10	15	20	25	30	50
0	0	FPT[a]	9.81157	9.81156	9.81156	9.81153	9.81156	9.81159	9.81157
		HPT[a]	10.16623	9.90037	9.85112	9.83369	9.82579	9.82149	9.81513
		SPT[a]	10.18859	9.90608	9.85362	9.83516	9.82662	9.82216	9.81546
		present	10.18068	9.89877	9.84800	9.83074	9.82299	9.81893	9.81338
10^3	0	FPT[a]	7.60670	7.84902	7.89561	7.91202	7.91967	7.92383	7.92990
		HPT[a]	7.88289	7.92022	7.92750	7.92991	7.93117	7.93182	7.93276
		SPT[a]	7.90093	7.92493	7.92954	7.93116	7.93116	7.93239	7.93300
		present	7.89490	7.91920	7.92512	7.92763	7.92899	7.92986	7.93138
10^3	10^3	FPT[a]	1.39937	1.58622	1.62642	1.64099	1.64783	1.65155	1.65701
		HPT[a]	1.45083	1.60071	1.63304	1.64472	1.65023	1.65023	1.65763
		SPT[a]	1.45449	1.60174	1.63349	1.64499	1.65040	1.65335	1.65769
		present	1.45344	1.60065	1.63264	1.64431	1.64982	1.65286	1.65735

[a]Sobhy (2015)

Example 3: Tables 4 and 5 show the comparison between the results of the present theory and those of Sobhy (2015) for transverse shear stress $\bar{\tau}_{xz}$and in-plane shear stress $\bar{\tau}_{xy}$in bending of FGM square plates without or resting on one-parameter ortwo-parameter elastic foundations for different values of the side-to-thickness ratio a/h. Again, a good agreement between the present results and those of Sobhy (2015) is observed. The difference observed in the results between the present theory and HPT, EPT and SPT of Sobhy (2015) are due to the displacement fields assumed by these theories. It should be noted that the present theories give the accurate

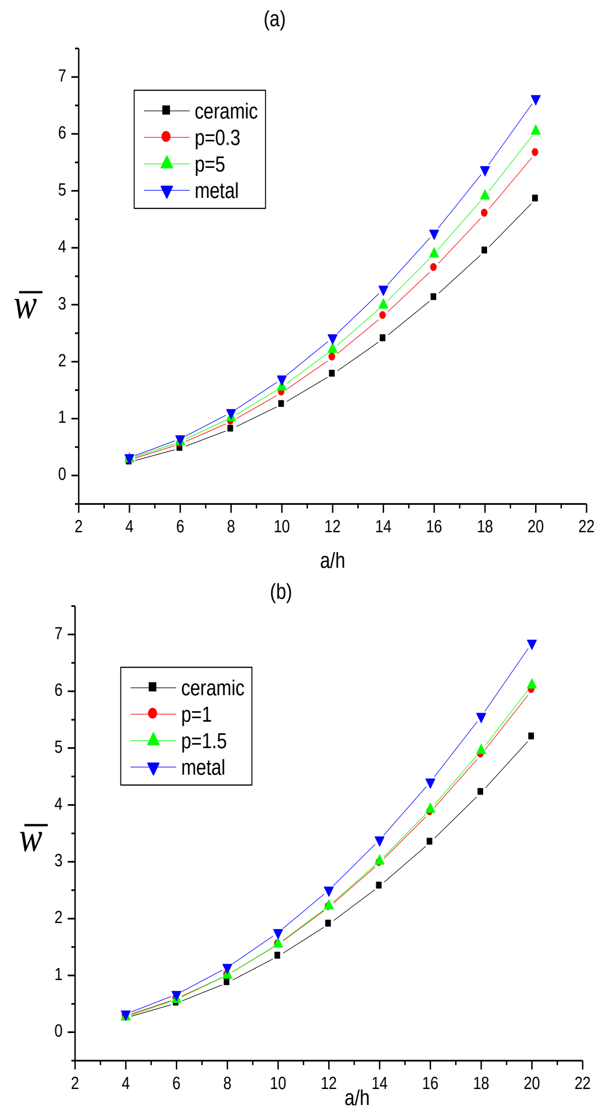

Fig. 2 Effect of the power-law index p on the deflection $\bar{W}$ of FGM plates under (a) mechanical load and (b) thermomechanicalload ΔT=200

representation of the transverse shear strain, the transverse shear correction factor is not needed as against in the case of the FPT. It can be concluded that the present theory is not only accurate but response of FG plates. In addition, all displacements and stresses are decreasing with increasing J_1 and J_2. It is indicated that large moduli of elastic foundation can enhance bending rigidity of the plate.

4.3 Results of present study

The effects different parameters such as the power law index, elastic foundation, plate geometry, and temperature field on the bending of FG plates are investigated here.
The central deflection versus the side-to-thickness ratio a/h of the simply supported FGM square plates for different values of the power-law index $p$is plotted in Fig. 2 the plate is resting on parabolic elastic foundations and subjected to mechanical or thermomechanical loads. As expected, the deflection $\bar{W}$ increases with the increasing of the ratio a/h.

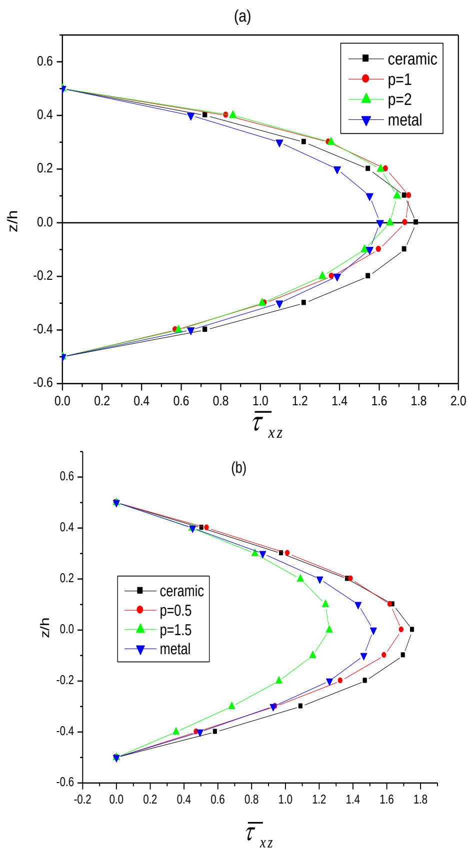

Fig. 3 Variation of the transverse shear stress $\overline{\tau}_{xz}$ through the thickness of FGM plates under (a) mechanical load and (b) thermomechanicalload ΔT=200

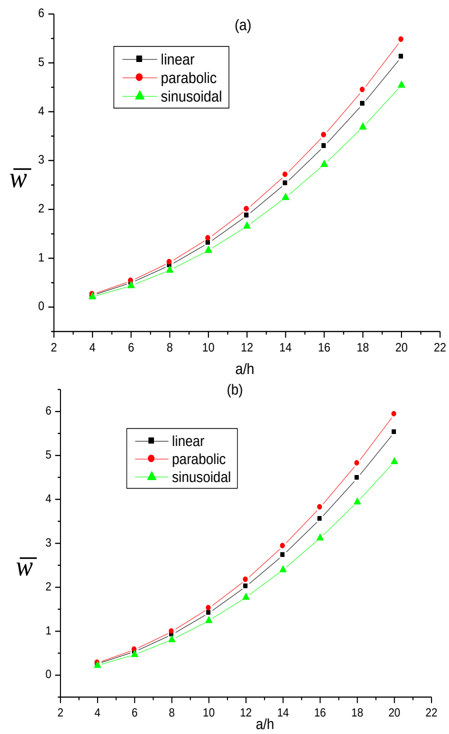

Fig. 4 the deflection $\overline{w}$ of FGM plate (p=1, ζ=20) versus to the side to thickness ratio a/h under (a) mechanical load and (b) thermomechanicalload ΔT=300 for various types of Winkler parameter

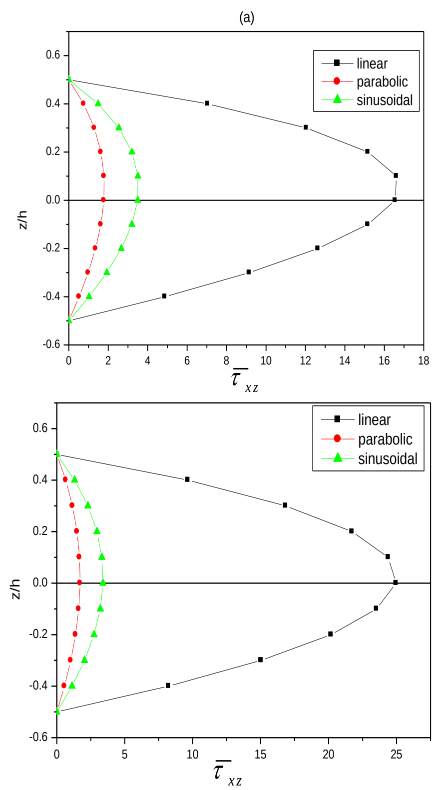

Fig. 5 Variation of the transverse shear stress $\overline{\sigma}_{xz}$ of FGM plate (p=1, ζ=20) versus to the side to thickness ratio z/h under (a) mechanical load and (b) thermomechanicalload ΔT=300 for various types of Winkler parameter

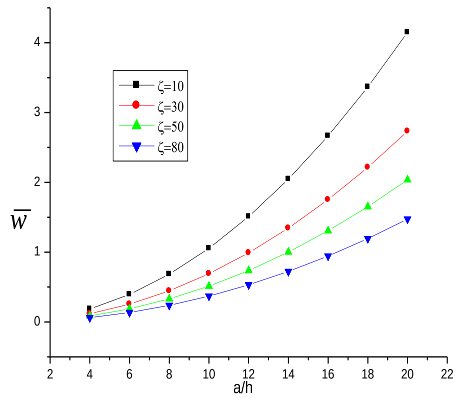

Fig. 6 The deflection $\overline{w}$ of FGM plate (p=1) against the side to thickness ratio a/h under thermomechanicalload (ΔT=300) for different values of the parabolic parameter ζ (J_1 =1000)

The FG platedeflection is between those of plate made of ceramic (ZrO_2) and metal (Ti–6Al–4V). It can be observed that the deflection of metal rich plates is larger when compared to ceramic rich FGMplates.

Fig. 3 shows the through-the-thickness distributions of the stress $\overline{\tau}_{xz}$ axial stress of FG plate resting on parabolic elastic foundations for different values of the power-law index p. the maximum value $\overline{\tau}_{xz}$ occurs at a point which is not at the plate center as in the homogeneous case. It is evident that the maximum stresses decrease with the increase of the power-law index p.

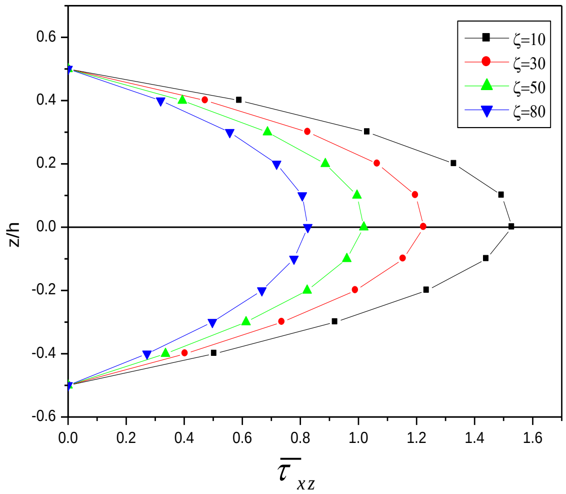

Fig. 7 Variation of the transverse shear stress $\bar{\tau}_{xz}$ through the thickness of FGM plate (p=1) under thermomechanicalload (ΔT=300) for different values of the parabolic parameter ζ (J_1 =1000)

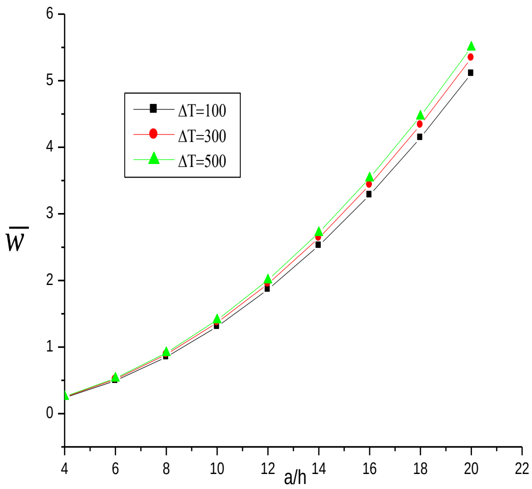

Fig. 8 Effect of the temperature difference ΔT on the deflection $\bar{w}$ of the FGM plate (p=1) against the ratio a=h

The deflection $\bar{w}$ and the stress $\bar{\tau}_{xz}$ of simply supported FGM (p=1) square plate subjected to mechanical and thermomechanical loads are compared in Figs. 4-5 for different type of elastic foundations (parabolic, linear, sinusoidal). It can be noted that the results depend on the type of elastic foundations. The thermomechanical load always over predicts the deflection $\bar{w}$ whereas the mechanical load always over predicts the $\bar{\tau}_{xz}$ magnitude.

Figs. 6-7 shows the variation of the deflection and stress of simply supported FGM plate (p=1) restingon parabolic elastic foundation and under thermomechanical load for ζ=10, 30, 50, 80. Also, as is evident, as the parabolic parameter ζ decrease, the deflection $\bar{w}$ *and* the transverse shear stress $\bar{\tau}_{xz}$ decrease.

The effect of the temperature difference ΔT on the deflection $\bar{w}$ of simply supported FGM plate (p=1) resting on parabolic elasticfoundations (ζ=50) is explained in Fig. 8. As expected, the deflectionsincrease as the temperature difference increases.

5. Conclusions

In this research study, we have analyzed thermoelastic bending of FG plates subjected to thermomechanical loads and resting on two-parameterelastic foundations. One of these parameters is varying in the direction of x-axis aslinear, parabolic or sinusoidal functions of x. This parameter represents Winklersprings modulus. The second parameter represents the shear layer modulus that takes constant values. The main advantage of the proposed theory over the existing higher-order shear deformation theories is that the present ones involve fewer variables as well as equations of motion. The computational cost can therefore be reduced. Material properties of FG plates are assumed to be temperature-dependent and graded through the thickness according to a power-law distribution in terms of volume fractions of constituents. Numerical results show that the proposed theory give results close to those of existing higher-order shear deformation theories. Applications of this work for the thicker FG structures can be extended in future with considering new formulations developed by other works (see, e.g., Bessaim *et al.* 2013, Belabed *et al.* 2014, Hebali *et al.* 2014, Bousahla *et al.* 2014, Fekrar *et al.* 2014, Hamidi *et al.* 2015, Bourada *et al.* 2015, Bennoun *et al.* 2016, Draiche *et al.* 2016, Bouafia *et al.* 2017, Sekkal *et al.* 2017b, Benahmed *et al.* 2017, Abualnour *et al.* 2018, Benchohra *et al.* 2018).

References

Abdelaziz, H.H., Ait Amar Meziane, M., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Alwabli, A.S. (2017), "An efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FGM sandwich plates with various boundary conditions", *Steel Compos. Struct.*, **25**(6), 693-704.

Abualnour, M., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), "A novel quasi-3D trigonometric plate theory for free vibration analysis of advanced composite plates", *Compos. Struct.*, **184**, 688-697.

Ahouel, M., Houari, M.S.A., Adda Bedia, E.A. and Tounsi, A. (2016), "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept", *Steel Compos. Struct.*, **20**(5), 963-981.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

Ait Atmane, H., Tounsi, A. and Adda Bedia, E.A. (2010), "Free vibration analysis of functionally graded plates resting on Winkler-Pasternak elastic foundations using a new shear deformation theory", *J. Mech. Mater. Des.*, **6**(2), 113-121.

Ait Yahia, S., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech.*, **53**(6), 1143-1165.

Akavci, S. (2010), "Two new hyperbolic shear displacement models for orthotropic laminated composite plates", *Mech. Compos. Mater.*, **46**(2), 215-226.

Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), "Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position", *Compos. Struct.*, **125**, 621-630.

Arani, A.J. and Kolahchi, R. (2016), "Buckling analysis of embedded concrete columns armed with carbon nanotubes", *Comput. Concrete*, **17**(5), 567-578.

Attia, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct.*, **18**(1), 187-212.

Bachir Bouiadjra, M., Houari, M.S.A. and Tounsi, A. (2012), "Thermal buckling of functionally graded plates according to a four-variable refined plate theory", *J. Therm. Stress.*, **35**, 677-694.

Barati, M.R. and Shahverdi, H. (2016), "A four-variable plate theory for thermal vibration of embedded FG nanoplates under non-uniform temperature distributions with different boundary conditions", *Struct. Eng. Mech.*, **60**(4), 707-727.

Baseri, V., Jafari, G.S. and Kolahchi, R. (2016), "Analytical solution for buckling of embedded laminated plates based on higher order shear deformation plate theory", *Steel Compos. Struct.*, **21**(4), 883-919.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos. Part B*, **60**, 274-283.

Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), "Hygrothermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory", *Smart Struct. Syst.*, **18**(4), 755-786.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct.*, **18**(4), 1063-1081.

Bellifa, H., Bakora, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017a), "An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates", *Steel Compos. Struct.*, **25**(3), 257-270.

Bellifa, H., Benrahou, K.H., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017b), "A nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams", *Struct. Eng. Mech.*, **62**(6), 695-702.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J. Braz. Soc. Mech. Sci. Eng.*, **38**(1), 265-275.

Benadouda, M., Ait Atmane, H., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2017), "An efficient shear deformation theory for wave propagation in functionally graded material beams with porosities", *Earthq. Struct.*, **13**(3), 255-265.

Benahmed, A., Houari, M.S.A., Benyoucef, S., Belakhdar, K. and Tounsi, A. (2017), "A novel quasi-3D hyperbolic shear deformation theory for functionally graded thick rectangular plates on elastic foundation", *Geomech. Eng.*, **12**(1), 9-34.

Benchohra, M., Driz, H., Bakora, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), "A new quasi-3D sinusoidal shear deformation theory for functionally graded plates", *Struct. Eng. Mech.*, In Press.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**(4), 423-431.

Bessaim, A., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Adda Bedia, E.A. (2013), "A new higher order shear and normal deformation theory for the static and free vibration analysis of sandwich plates", *J. Sandw. Struct. Mater.*, **15**(6), 671-703.

Besseghier, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Free vibration analysis of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory", *Smart Struct. Syst.*, **19**(6), 601-614.

Bilouei, B.S., Kolahchi, R. and Bidgoli, M.R. (2016), "Buckling of concrete columns retrofitted with nano-fiber reinforced polymer (NFRP)", *Comput. Concrete*, **18**(5), 1053-1063.

Bouafia, K., Kaci, A., Houari, M.S.A., Benzair, A. and Tounsi, A. (2017), "A nonlocal quasi-3D theory for bending and free flexural vibration behaviors of functionally graded nanobeams", *Smart Struct. Syst.*, **19**(2), 115-126.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Compos. Struct.*, **14**(1), 85-104.

Bouderba, B., Houari, M.S.A. and Tounsi, A. and Mahmoud, S.R. (2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech.*, **58**(3), 397-422.

Boukhari, A., Ait Atmane, H., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech.*, **57**(5), 837-859.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct.*, **20**(2), 227-249.

Bourada, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2012), "A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates", *J. Sandw. Struct. Mater.*, **14**, 5-33.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct.*, **18**(2), 409-423.

Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech.*, **60**(2), 313-335.

Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A., (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *J. Comput. Meth.*, **11**(6), 1350082.

Cheng, Z.Q. and Batra, B.C. (2000), "Exact correspondence between eigenvalues of membranes and function ally graded simply supported polygonal plate", *J. Sound Vibr.*, **229**(4), 879-895.

Cheng, Z.Q. and Kitipornchai, S. (1999), "Membrane analogy of buckling and vibration of inhomogeneous plates", *J. Eng. Mech.*, **125**(11), 1293-1297.

Chi, S. and Chung, Y. (2006a), "Mechanical behavior of functionally graded material plates under transverse load. Part I: Analysis", *J. Sol. Struct.*, **43**, 3657-3674.

Chi, S. and Chung, Y. (2006b), "Mechanical behavior of functionally graded material plates under transverse load. Part II: Numerical results", *J. Sol. Struct.*, **43**, 3675-3691.

Chikh, A., Tounsi, A., Hebali, H. and Mahmoud, S.R. (2017), "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Struct. Syst.*, **19**(3), 289-297.

Draiche, K., Tounsi, A. and Mahmoud, S.R. (2016), "A refined theory with stretching effect for the flexure analysis of laminated composite plates", *Geomech. Eng.*, **11**(5), 671-690.

Eisenberger, M. and Clastornik, J. (1987), “Vibrations and buckling of a beam on a variable Winkler elastic foundation”, *J. Sound Vibr.*, **115**, 233-241.

El-Haina, F., Bakora, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017), “A simple analytical approach for thermal buckling of thick functionally graded sandwich plates", *Struct. Eng. Mech.*, **63**(5), 585-595.

Fekrar, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2014), “A new five-unknown refined theory based on neutral surface position for bending analysis of exponential graded plates”, *Meccan.*, **49**, 795-810.

Filipich, C.P. and Rosales, M.B. (2002), “A further study about the behavior of foundation piles and beams in a Winkler-Pasternak soil”, *J. Mech. Sci.*, **44**(1), 21-36.

Filonenko-Borodich, M.M. (1940), “Some approximate theories of die elastic foundation”, *Uch. ZapiskiMosk. Gos. Univ. Mekhanika*, **46**, 3-18.

Filonenko-Borodich, M.M. (1940) “Some approximate theories of die elastic foundation”, *Uch. ZapiskiMosk. Gos. Univ. Mekhanika*, **46**, 3-18.

GhorbanpourArani, A., Cheraghbak, A. and Kolahchi, R. (2016), “Dynamic buckling of FGM viscoelastic nano-plates resting on orthotropic elastic medium based on sinusoidal shear deformation theory”, *Struct. Eng. Mech.*, **60**(3), 489-505.

Gorbunov-Posadov, M.I. (1949), *Beams and Plates on Elastic Foundation*, Gosstroiizdat.

Grover, N., Maiti, D.K. and Singh, B.N. (2013), “A new inverse hyperbolic shear deformation theory for static and buckling analysis of laminated composite and sandwich plates”, *Compos. Struct.*, **95**, 667-675.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), “A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates”, *Steel Compos. Struct.*, **18**(1), 235-253.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and AddaBedia, E.A. (2014), “New quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates”, *J. Eng. Mech.*, **140**, 374-383.

Hetényi, M. (1950), “A general solution for the bending of beams on an elastic foundation of arbitrary continuity”, *J. Appl. Phys.*, **21**, 55-58.

Hetényi, M. (1946), *Beams on Elastic Foundations*, The Univ. Michigan Press, Ann Arbor, Michigan, U.S.A.

Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2016), “A new simple three-unknown sinusoidal shear deformation theory for functionally graded plates”, *Steel Compos. Struct.*, **22**(2), 257-276.

Huang, Z.Y., Lu, C.F. and Chen, W.Q. (2008) “Bench mark solutions for functionally graded thick plates resting on Winkler-Pasternak elastic foundations”, *Compos. Struct.*, **85**, 95-104.

Huang, Z.Y., Lu, C.F. and Chen, W.Q. (2008), “Bench mark solutions for functionally graded thick plates resting on Winkler-Pasternak elastic foundations”, *Compos. Struct.*, **85**, 95-104.

Kar, V.R. and Panda, S.K. (2014), “Large deformation bending analysis of functionally graded spherical shell using FEM”, *Struct. Eng. Mech.*, **53**(4), 661-679.

Karama, M., Afaq, K.S. and Mistou, S. (2003), “Mechanical behaviour of laminated composite beam by the new multi-layered laminated composite structures model with transverse shear stress continuity”, *J. Sol. Struct.*, **40**(6), 1525-1546.

Kerr, A.D. (1964), “Elastic and viscoelastic foundation models”, *ASME J. Appl. Mech.*, **31**(3), 491-498.

Kettaf, F.Z., Houari, M.S.A., Benguediab, M. and Tounsi, A. (2013), “Thermal buckling of functionally graded sandwich plates using a new hyperbolic shear displacement model”, *Steel Compos. Struct.*, **15**(4), 399-423.

Khetir, H., Bachir Bouiadjra, M., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), “A new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates”, *Struct. Eng. Mech.*, **64**(4), 391-402.

Kim, Y. (2005), “Temperature dependent vibration analysis of functionally graded rectangular plates”, *J. Sound Vibr.*, **284**(3-5), 531-549.

Kolahchi, R. (2017), “A comparative study on the bending, vibration and buckling of viscoelastic sandwich nano-plates based on different nonlocal theories using DC, HDQ and DQ methods”, *Aerosp. Sci. Technol.*, **66**, 235-248.

Kolahchi, R., Bidgoli, A.M.M. and Heydari, M.M. (2015), “Size-dependent bending analysis of FGM nano-sinusoidal plates resting on orthotropic elastic medium”, *Struct. Eng. Mech.*, **55**(5), 1001-1014.

Kolahchi, R., Hosseini, H. and Esmailpour, M. (2016a), “Differential cubature and quadrature-Bolotin methods for dynamic stability of embedded piezoelectric nanoplates based on visco-nonlocal-piezoelasticity theories”, *Compos. Struct.*, **157**, 174-186.

Kolahchi, R. and MoniriBidgoli, A.M. (2016), “Size-dependent sinusoidal beam model for dynamic instability of single-walled carbon nanotubes”, *Appl. Math. Mech.*, **37**(2), 265-274.

Kolahchi, R., Safari, M. and Esmailpour, M. (2016), “Dynamic stability analysis of temperature-dependent functionally graded CNT-reinforced visco-plates resting on orthotropic elastomeric medium”, *Compos. Struct.*, **150**, 255-265.

Kolahchi, R., Safari, M. and Esmailpour, M. (2016b), “Dynamic stability analysis of temperature-dependent functionally graded CNT-reinforced visco-plates resting on orthotropic elastomeric medium”, *Compos. Struct.*, **150**, 255-265.

Kolahchi, R., Zarei, M.S., Hajmohammad, M.H. and Oskouei, A.N. (2017), “Visco-nonlocal-refined Zigzag theories for dynamic buckling of laminated nanoplates using differential cubature-Bolotin methods”, *Thin-Wall. Struct.*, **113**, 162-169.

Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég, O. and Mahmoud, S.R. (2015), “Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect”, *Steel Compos. Struct.*, **18**(2), 425-442.

Madani, H., Hosseini, H. and Shokravi, M. (2016), “Differential cubature method for vibration analysis of embedded FG-CNT-reinforced piezoelectric cylindrical shells subjected to uniform and non-uniform temperature distributions”, *Steel Compos. Struct.*, **22**(4), 889-913.

Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), “A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates”, *Appl. Math. Modell.*, **39**(9), 2489-2508.

Mantari, J.L., Oktem, A.S. and GuedesSoares, C. (2012), “A new trigonometric shear deformation theory for isotropic, laminated composite and sandwich plates”, *J. Sol. Struct.*, **49**(1), 43-53.

Matsunaga, H. (2000), “Vibration and stability of thick plates on elastic foundations”, *J. Eng. Mech.*, **126**(1), 27-34.

Menasria, A., Bouhadra, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017), “A new and simple HSDT for thermal stability analysis of FG sandwich plates”, *Steel Compos. Struct.*, **25**(2), 157-175.

Mindlin, R.D. (1951), “Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates”, *J. Appl. Mech.*, **18**(1), 31-38.

Mouffoki, A., Adda Bedia, E.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), “Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory", *Smart Struct. Syst.*, **20**(3), 369-383.

Omurtag, M.H., Ozutok, A. and Akoz, A.Y. (1997), "Free vibration analysis of Kirchhoff plates resting on elastic foundation by mixed finite element formulation based on Gateaux differential", *J. Numer. Meth. Eng.*, **40**(2), 295-317.

Pasternak, P.L. (1954), *On a new Method of Analysis of an Elastic Foundation by Means of Two Foundation Constant*, Gosstroiizdat, Moscow, Russia.

Pradhan, S.C. and Murmu, T. (2009), "Thermo-mechanical vibration of FGM sandwich beam under variable elastic foundations using differential quadrature method", *J. Sound Vibr.*, **321**, 342-362.

Pradhan, S.C. and Murmu, T. (2009), "Thermo-mechanical vibration of FGM sandwich beam under variable elastic foundations using differential quadrature method", *J. Sound Vibr.*, **321**, 342-362.

Pradyumna, S. and Bandyopadhyay, J.N. (2008), "Free vibration analysis of functionally graded curved panels using a higher-order finite element formulation", *J. Sound Vibr.*, **318**(1-2), 176-192.

Reddy, J.N. and Chin, C.D. (1998), "Thermoelastical analysis of functionally graded cylinders and plates", *J. Therm. Stress.*, **21**, 593-626.

Reddy, J.N. (1984), "A simple higher-order theory for laminated composite plates", *J. Appl. Mech.*, **51**(4), 745-752.

Reddy, J.N. (2000), "Analysis of functionally graded plates", *J. Numer. Meth. Eng.*, **47**(1-3), 663-684.

Reddy, J.N. (2004), *Mechanics of Laminated Composite Plates and Shells*, CRC Press.

Reddy, J.N. and Cheng, Z.Q. (2002), "Frequency correspondence between membranes and functionally graded spherical shallow shells of polygonal plan form", *J. Mech. Sci.*, **44**(5), 967-985.

Reissner, E. (1945), "The effect of transverse shear deformation on the bending of elastic plates", *J. Appl. Mech.*, **12**(2), 69-72.

Ren, J.G. (1986), "A new theory of laminated plate", *Compos. Sci. Technol.*, **26**(3), 225-239.

Saidi, H., Tounsi, A. and Bousahla, A.A. (2016), "A simple hyperbolic shear deformation theory for vibration analysis of thick functionally graded rectangular plates resting on elastic foundations", *Geomech. Eng.*, **11**(2), 289-307.

Sekkal, M., Fahsi, B., Tounsi, A. and Mahmoud, S.R. (2017a), "A novel and simple higher order shear deformation theory for stability and vibration of functionally graded sandwich plate", *Steel Compos. Struct.*, **25**(4), 389-401.

Sekkal, M., Fahsi, B., Tounsi, A. and Mahmoud, S.R. (2017b), "A new quasi-3D HSDT for buckling and vibration of FG plate", *Struct. Eng. Mech.*, **64**(6), 737-749.

Shahrjerdi, A., Mustapha, F., Bayat, M. and Majid, D.L.A. (2011), "Free vibration analysis of solar functionally graded plates with temperature-dependent material properties using second order shear deformation theory", *J. Mech. Sci. Technol.*, **25**(9), 2195-2209.

Shen, H.S. (1995), "Postbuckling analysis of composite laminated plates on two parameter elastic foundations", *J. Mech. Sci.*, **37**(12), 1307-1316.

Sobhy, M. (2015), "Thermoelastic response of FGM plates with temperature-dependent properties resting on variable elastic foundations", *J. Appl. Mech.*, **7**(6), 1550082.

Soldatos, K.P. (1992), "A transverse shear deformation theory for homogeneous monoclinic plates", *Acta Mech.*, **94**(3), 195-220.

Swaminathan, K. and Naveenkumar, D.T. (2014), "Higher order refined computational models for the stability analysis of FGM plates: Analytical solutions", *Eur. J. Mech. A/Sol.*, **47**, 349-361.

Taibi, F.Z., Benyoucef, S., Tounsi, A., Bachir Bouiadjra, R., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "A simple shear deformation theory for thermo-mechanical behaviour of functionally graded sandwich plates on elastic foundations", *J. Sandw. Struct. Mater.*, **17**(2), 99-129.

Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol.*, **24**(1), 209-220.

Touratier, M. (1991), "An efficient standard plate theory", *J. Eng. Sci.*, **29**(8), 901-916.

Vlasov, V.Z. (1949), *Structural Mechanics of Thin Walled Three-Dimensional Systems*, Stroiizdat.

Xiang, S., Wang, K.M., Ai, Y.T., Sha, Y.D. and Shi, H. (2009), "Analysis of isotropic, sandwich and laminated plates by a meshless method and various shear deformation theories", *Compos. Struct.*, **91**(1), 31-37.

Yaghoobi, H. and Yaghoobi, P. (2013), "Buckling analysis of sandwich plates with FGM face sheets resting on elastic foundation with various boundary conditions: An analytical approach", *Meccan.*, **48**, 2019-2035.

Yazid, M., Heireche, H., Tounsi, A., Bousahla, A.A. and Houari, M.S.A. (2018), "A novel nonlocal refined plate theory for stability response of orthotropic single-layer graphene sheet resting on elastic medium", *Smart Struct. Syst.*, **21**(1).

Youcef, D.O., Kaci, A., Benzair, A., Bousahla, A.A., Tounsi, A. (2018), "Dynamic analysis of nanoscale beams including surface stress effects", *Smart Struct. Syst.*, Accepted.

Zamanian, M., Kolahchi, R. and Bidgoli, M.R. (2017), "Agglomeration effects on the buckling behaviour of embedded concrete columns reinforced with SiO_2 nano-particles", *Wind Struct.*, **24**(1), 43-57.

Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), "A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory", *Struct. Eng. Mech.*, **54**(4), 693-710.

Zhemochkin, B.N. and Sinitsyn, A.P. (1947), *Practical Methods of Calculation of Foundation Beams and Slabs on Elastic Foundation*, Stroiizdat, Moscow, Russia.

Zhemochkin, B.N. and Sinitsyn, A.P. (1947), *Practical Methods of Calculation of Foundation Beams and Slabs on Elastic Foundation*, Stroiizdat, Moscow, Russia.

Zhou, D., Cheung, Y.K., Lo, S.H. and Au, F.T.K. (2004), "Three-dimensional vibration analysis of rectangular thick plates on Pasternak foundations", *J. Numer. Meth. Eng.*, **59**(10), 1313-1334.

Zhou, D., Cheung, Y.K., Lo, S.H. and Au, F.T.K. (2004), "Three-dimensional vibration analysis of rectangular thick plates on Pasternak foundations", *J. Numer. Meth. Eng.*, **59**(10), 1313-1334.

Zhou, D. (1993), "A general solution to vibrations of beams on variable Winkler elastic foundation", *Comput. Struct.*, **47**(1), 83-90.

Zidi, M., Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2017), "A novel simple two-unknown hyperbolic shear deformation theory for functionally graded beams", *Struct. Eng. Mech.*, **64**(2), 145-153.

Zidi, M., Tounsi, A., Houari M.S.A., Adda Bedia, E.A. and Anwar Bég, O. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four-variable refined plate theory", *Aerosp. Sci. Technol.*, **34**, 24-34.

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