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Dynamic analysis of steel frames with semi-rigid connections

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Abstract. In the steel structures design, beam-to-column connections are usually considered either rigid or pinned, while their actual behavior lies between these two ideal cases. This consideration has a major influence on the results of the local and the global behavior of steel structures. This influence is noticed in the case of a static analysis, and has an important effect in the case of a dynamic analysis. In fact, pinned and rigid nodes can be considered as two specific cases of a semi-rigid behavior. To study the efficiency of the classification adopted in Eurocode 3, a numerical simulation of semi-rigid nodes has been carried out using the software ANSYS. In the aim to validate this simulation, the numerical results are compared to those of an analytical approach. After that, the validated numerical simulation has been used, to evaluate the efficiency of the classification adopted by the Eurocode 3, regarding semi-rigid connections. Finally, a new method is proposed to define a more accurate evaluation about semi-rigid connections.

Keywords: steel frames; semi rigid connection; fixity factor; dynamic analysis

1. Introduction

In steel construction, beam-to-column connections, Fig. 1, can be considered as rigid or pinned, according to the fact that the joint allows transmission of bending moment in the first case while this is not permitted in the second. In fact, a node is considered as perfectly rigid when it presents a sufficient rotational stiffness and can equilibrate all types of internal forces inside the connected elements. On the other hand, a node is perfectly pinned when it does not allow any transmission of bending moment from the beam to the column: the connection in this case has no rotational stiffness, but it should be able to transmit the axial and shear forces.

This field of research is increasingly treated, but few researchers are interested in the dynamic behavior of structures with semi-rigid nodes.

Ihaddoudène *et al.* (2017) developed a mechanical model taken into account the effect of beam-column joint flexibility on the elastic buckling load of plane steel frames. The model consists in the development of comprehensive approach taking into account, simultaneously, the effects of the joint rigidity, the elastic buckling load, and this for both sway and non-sway frames. Numerical results are obtained for frames with various characteristics and support conditions. Akbar and Min (2017) proposed a new method, for the seismic design of steel frame structures, based on a simplified analytical semi-rigid frame model, assuming that the structural plastic deformation is concentrated within the semi-rigid connections while beams and columns remain elastic. A finite element model is presented by Halil *et al.* (2017), that takes into account the presence of semi-rigid connections, and by implementing an accurate shear correction coefficient for I-shaped steel sections to represent shear deformation and rotary inertia in order to calculate consistent stiffness and mass matrices.

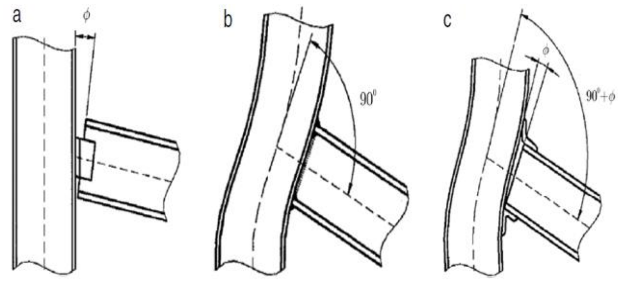

Fig. 1 Beam-column connections: (a) pinned connection; (b) rigid connection; (c) semi-rigid connection

Bouafia *et al.* (2017) carried out a detailed numerical study to examine the effects of nonlocal parameter, aspect ratio and various material compositions on the static and dynamic responses of the functionally graded nanobeam. The nonlocal elastic behavior is described by a differential constitutive model, which enables the model to become effective in the analysis and design of nanostructures. Furthermore, Zemri *et al.* (2015) presented a nonlocal shear deformation beam theory for bending, buckling and vibration of functionally graded (FG) nanobeams using the nonlocal differential constitutive relations. Analytical solutions are presented for a simply supported FG nanobeam, and the obtained results compare well with those predicted by the nonlocal Timoshenko beam theory.

In the Eurocode 3 Standard (2005), the classification of

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nodes of steel structures is established with the aim to detail the limits of the initial rotational stiffness, from which a node can be considered as pinned, rigid or semi-rigid. According to this code, a node is considered as rigid, if its initial rotational stiffness is equal or higher than $25\,E_sI_b/L_b$ in the case of unbraced frames, and $8\,E_sI_b/L_b$ in the case of braced frames. In the same way, a node is supposed to be nominally pinned if its initial rotational stiffness does not exceed $0{,}5\,E_sI_b/L_b$ for both braced and unbraced frames. Finally, nodes with an initial rotational stiffness between these two limits are classified as semi-rigid nodes and their behavior must be considered in the global analysis of structures.

Where E_s, L_b, and I_b are, respectively, the Young's modulus of the beam, its length and its moment of inertia.

In this paper, a numerical approach is developed to evaluate the natural angular frequencies of steel portal frames, with semi-rigid nodes, using the finite element software ANSYS, where the semi-rigid nodes are assimilated to linear elastic rotational springs between beams and columns, fig. 3. In a second step, an analytical study is carried out to calculate the same parameter. Then, the comparison between the results of the numerical approach and the analytical study allows to the first one to be validated.

Thereafter, a study of a multi-storey steel portal frame is numerically carried out with the aim to analyze the classification of nodes as defined by the EC3 (2005). The study focuses on a steel portal frame with semi-rigid connections, and consists in varying the stiffness of the beams, for the same type of columns, to determine its natural angular frequency as a function of the fixity factor of nodes. The results are discussed respecting the classification of nodes established by the EC3 (2005). In the same way, similar study is carried out by varying the stiffness of the columns in order to analyze the same parameters as previously, for the same types of beams.

According to the obtained results, new limits for the classification of nodes are proposed. The novelty of this paper is showing the fact that these limits consider not only the rotational stiffness of beams, but also the stiffness of columns. A formula taking into account the stiffness of columns and the rotational stiffness of beams is proposed. At the end, a practical example is presented to lead in the application of this method.

2. Numerical approach of steel portal frames with semi-rigid nodes

For the numerical approach, a simple portal frame is modeled, Fig. 2, with fixed end base columns and semi-rigid beam-column connections, in order to study a modal analysis of this portal frame and calculate its angular frequencies. The numerical simulation is performed by the finite element software ANSYS Structural 11.0, where the semi-rigid nodes are modeled by linear elastic rotational springs which have different values of the fixity factor. The characteristics of those springs can be calculated using the component method proposed by the EC3 (2005).

By means of the finite element software ANSYS, the semi-rigid parameter consists in modeling nodes as springs elements, without length, that connect the beam to each adjacent column. Those springs are represented by a fixity factor r_j, that defines the rotational rigidity of nodes relating to that of the attached element. For a pinned connection, the fixity factor is equal to zero ($r_j = 0$), and for a fully rigid connection, it is equal to the unity ($r_j = 1$). Then, a semi-rigid connection is represented by fixity factors between zero and the unity ($0 < r_j < 1$).

The sample, Fig. 2, is a steel portal frame with 4 m of span and 4 m of height. The two columns are made of HEA 260 hot rolled profiles and the beam is an IPE 360. The geometrical characteristics of these rolled sections are provided in the Euro-norms (1963).

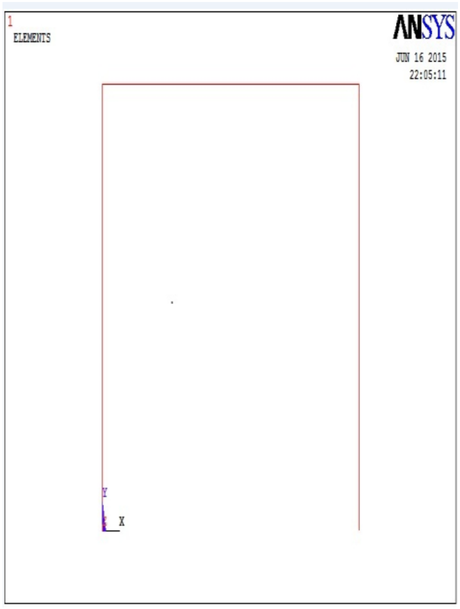

Fig. 2 Semi-rigid portal frame modeled by ANSYS

For all configurations of the studied portal frame, the same type of node at both ends of the beam is considered ($r_i = r_j$). The results are given in Table 1:

Table 1 Angular frequencies variation of the portal frame, numerically estimated

$r_i = r_j$	0,6	0,7	0,8	0,9	1
ω_i (rad/s) ANSYS	118,38	120,42	122,54	123,76	124,74

In the aim to validate this numerical study, an analytical one is developed and allows calculating the frequencies of a portal frame with semi-rigid nodes. The analytical results will confirm those of the numerical one.

3. Analytical study of plane structures with semi-rigid nodes

According to Díaz (2011), two approaches exist to introduce semi-rigid nodes in the analysis of structures. The first approach is to introduce additional elements that represent directly the beam-column connection. These are considered as elements with two nodes connecting the beams to the columns. In the second approach, the member with semi-rigid nodes is connected in each end to a spring with zero length, the beam with the two springs are considered as a single element called "hybrid", Fig. 3.

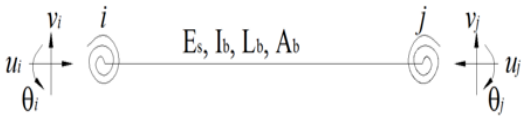

Fig. 3 Modeling of a hybrid finite element with semi-rigid nodes

Several researchers have adopted the first approach, Urbonas and Daniunas (2006), Temesgen (2011), Del Savio (2009), Piluso (2012), Bayo (2012), by developing mechanical models which represent the beam-column connection. While others, Chan and Chui (2000), Sokor (2002), Ihaddoudène (2009), Chin-Long (2009), Kartal (2010), have adopted the second approach by changing the boundary conditions of the assembled element.

This work is compared with the second approach, which consists in representing the structural elements with semi-rigid nodes as finite elements, as given by Monforton and Wu (2003). A "fixity factor" called r_j is used, taking into account the effect of semi-rigidity of the beam, which is given by the following formula (2003)

$$r_j = \frac{1}{1 + \frac{3E_s I_b}{R_j L_b}} \qquad (j = 1, 2) \tag{1}$$

Where R_j represents the stiffness of the spring, and $E_s I_b / L_b$ is the flexural rigidity of the beam. The fixity factor r_j varies between 0 and 1 ($0 < r_j < 1$), where 0 represents a pinned node and 1 a fully rigid one.

The introduction of the fixity factor in the analysis of a structure is made by considering the element with semi-rigid nodes, as a rigid finite element having an elementary stiffness matrix $[K_e]$, Clough and Penzien (1976). Thereafter, the correction stiffness matrix [C] given by Xi (2003), must be applied to have the rigidity matrix of the semi-rigid beam.

Therefore, the corrected stiffness matrix of a beam with semi-rigid nodes $[K_{e(SR)}]$ is given by

$$[K_{e(SR)}] = \begin{bmatrix} \frac{E_s A_b}{L_b} & 0 & 0 & -\frac{E_s A_b}{L_b} & 0 & 0 \\ 0 & -\frac{12E_s I_b(r_j + r_i(1+r_j))}{L_b^3(-4+r_i r_j)} & -\frac{6E_s I_b r_i(2+r_j)}{L_b^2(-4+r_i r_j)} & 0 & \frac{12E_s I_b(r_j + r_i(1+r_j))}{L_b^3(-4+r_i r_j)} & -\frac{6E_s I_b(2+r_i)r_j}{L_b^2(-4+r_i r_j)} \\ 0 & -\frac{6E_s I_b r_i(2+r_j)}{L_b^2(-4+r_i r_j)} & -\frac{12E_s I_b r_i}{L_b(-4+r_i r_j)} & 0 & \frac{6E_s I_b r_i(2+r_j)}{L_b^2(-4+r_i r_j)} & -\frac{6i_b e_s r_i r_j}{L_b(-4+r_i r_j)} \\ -\frac{E_s A_b}{L_b} & 0 & 0 & \frac{E_s A_b}{L_b} & 0 & 0 \\ 0 & \frac{12E_s I_b(r_j + r_i(1+r_j))}{L_b^3(-4+r_i r_j)} & \frac{6E_s I_b r_i(2+r_j)}{L_b^2(-4+r_i r_j)} & 0 & -\frac{12E_s I_b(r_j + r_i(1+r_j))}{L_b^3(-4+r_i r_j)} & \frac{6E_s I_b(2+r_i)r_j}{L_b^2(-4+r_i r_j)} \\ 0 & -\frac{6E_s I_b(2+r_i)r_j}{L_b^2(-4+r_i r_j)} & -\frac{6E_s I_b r_i r_j}{L_b(-4+r_i r_j)} & 0 & \frac{6E_s I_b(2+r_i)r_j}{L_b^2(-4+r_i r_j)} & -\frac{12E_s I_b r_j}{L_b(-4+r_i r_j)} \end{bmatrix} \tag{2}$$

Where A_b is the section of the beam.

3.1 Dynamic analysis of semi-rigid steel frames

In this paper, the angular frequencies of a free system are calculated, i.e., the system is not subjected to an external excitation force. The dynamic equilibrium equation of a system in free vibration is written as follows

$$[K_G] - \omega^2 \times [M_G] = 0 \tag{3}$$

where ω is the angular frequency of the system, $[K_G]$ its global stiffness matrix and $[M_G]$ its global mass matrix. Those global matrices are obtained after assembling the elementary matrices $[K_e]$ and $[M_e]$ given by Clough and Penzien (1976).

We consider a portal frame with two columns having a cross section A_c, a length L_c, a moment of inertia I_c, and a beam with a cross section A_b, a length L_b and a moment of inertia I_b. The Young's modulus of the steel is E_s and its density ρ. The solution of the Eq. (3) is given by the resolution of its determinant, which is done after applying the boundary conditions.

For the case of a fixed support, the boundary conditions are: $u = v = \theta = 0$, the solution is

$$\mathrm{Det}\begin{pmatrix} D11 & D12 & D13 & D14 & D15 & D16 \\ D21 & D22 & D23 & D24 & D25 & D26 \\ D31 & D32 & D33 & D34 & D35 & D36 \\ D41 & D42 & D43 & D44 & D45 & D46 \\ D51 & D52 & D53 & D54 & D55 & D56 \\ D61 & D62 & D63 & D64 & D65 & D66 \end{pmatrix} = 0 \tag{4}$$

The components of the determinant (4) are given as follows

$$\begin{aligned}
&D11 = \frac{A_b E_s}{L_b} + \frac{12E_s I_c}{L_c^3} - w^2 \cdot \left(\frac{1}{3}\rho A_b L_b + \frac{13}{35}\rho A_c L_c\right) \\
&D12 = 0 \\
&D13 = \frac{6E_s I_c}{L_c^2} - \frac{11}{210} w^2 \cdot \rho A_c L_c^2 \\
&D14 = -\frac{A_b E_s}{L_b} - \frac{1}{6} w^2 \cdot \rho A_b L_b \\
&D15 = 0 \\
&D16 = 0 \\
&D21 = 0 \\
&D22 = \frac{A_c E_s}{L_c} - w^2 \cdot \left(\frac{13}{35}\rho A_b L_b + \frac{1}{3}\rho A_c L_c\right) + \frac{12E_s I_b(2r_i + r_i^2)}{L_b^3(4 - r_i^2)} \\
&D23 = -\frac{11}{210} w^2 \cdot \rho A_b L_b^2 - \frac{24E_s I_b(1 - r_i)r_i}{L_b^2(4 - r_i^2)} + \frac{18E_s I_b(2 - r_i)r_i}{L_b^2(4 - r_i^2)} \\
&D24 = 0 \\
&D25 = -\frac{9}{70} w^2 \cdot \rho A_b L_b - \frac{12E_s I_b(2r_i + r_i^2)}{L_b^3(4 - r_i^2)} \\
&D26 = \frac{13}{420} w^2 \cdot \rho A_b L_b^2 - \frac{24E_s I_b(1 - r_i)r_i}{L_b^2(4 - r_i^2)} + \frac{18E_s I_b(2 - r_i)r_i}{L_b^2(4 - r_i^2)} \\
&D31 = \frac{6E_s I_c}{L_c^2} - \frac{11}{210} w^2 \cdot \rho A_c L_c^2 \\
&D32 = -\frac{11}{210} w^2 \cdot \rho A_b L_b^2 + \frac{6E_s I_b(2r_i + r_i^2)}{L_b^2(4 - r_i^2)} \\
&D33 \\
&= \frac{4E_s I_c}{L_c} - w^2 \cdot \left(\frac{1}{105}\rho A_b L_b^3 + \frac{1}{105}\rho A_c L_c^2\right) - \frac{12E_s I_b(1 - r_i)r_i}{L_b(4 - r_i^2)} + \frac{12E_s I_b(2 - r_i)r_i}{L_b(4 - r_i^2)} \\
&D34 = 0 \\
&D35 = -\frac{13}{420} w^2 \cdot \rho A_b L_b^2 - \frac{6E_s I_b(2r_i + r_i^2)}{L_b^2(4 - r_i^2)} \\
&D36 = \frac{1}{140} w^2 \cdot \rho A_b L_b^3 - \frac{12E_s I_b(1 - r_i)r_i}{L_b(4 - r_i^2)} + \frac{6E_s I_b(2 - r_i)r_i}{L_b(4 - r_i^2)} \\
&D41 = -\frac{A_b E_s}{L_b} - \frac{1}{6} w^2 \cdot \rho A_b L_b \\
&D42 = 0 \\
&D43 = 0 \\
&D44 = \frac{A_b E_s}{L_b} + \frac{12E_s I_c}{L_c^3} - w^2 \cdot \left(\frac{1}{3}\rho A_b L_b + \frac{13}{35}\rho A_c L_c\right) \\
&D45 = 0 \\
&D46 = \frac{6E_s I_c}{L_c^2} - \frac{11}{210} w^2 \cdot \rho A_c L_c^2 \\
&D51 = 0 \\
&D52 = -\frac{9}{70} w^2 \cdot \rho A_b L_b - \frac{12E_s I_b(2r_i + r_i^2)}{L_b^3(4 - r_i^2)} \\
&D53 = -\frac{13}{420} w^2 \cdot \rho A_b L_b^2 + \frac{24E_s I_b(1 - r_i)r_i}{L_b^2(4 - r_i^2)} - \frac{18E_s I_b(2 - r_i)r_i}{L_b^2(4 - r_i^2)} \\
&D54 = 0 \\
&D55 = \frac{A_c E_s}{L_c} - w^2 \cdot \left(\frac{13}{35}\rho A_b L_b + \frac{1}{3}\rho A_c L_c\right) + \frac{12E_s I_b(2r_i + r_i^2)}{L_b^3(4 - r_i^2)}
\end{aligned} \tag{5}$$

$$D56 = \frac{11}{210} w^{2\cdot} \rho A_b L_b^2 + \frac{24 E_s I_b (1 - r_i) r_i}{L_b^2 (4 - r_i^2)} - \frac{18 E_s I_b (2 - r_i) r_i}{L_b^2 (4 - r_i^2)}$$

$$D61 = 0$$

$$D62 = \frac{13}{420} w^{2\cdot} \rho A_b L_b^2 + \frac{6 E_s I_b (2 r_i + r_i^2)}{L_b^2 (4 - r_i^2)}$$

$$D63 = \frac{1}{140} w^{2\cdot} \rho A_b L_b^2 - \frac{12 E_s I_b (1 - r_i) r_i}{L_b (4 - r_i^2)} + \frac{6 E_s I_b (2 - r_i) r_i}{L_b (4 - r_i^2)}$$

$$D64 = \frac{6 E_s I_c}{L_c^2} - \frac{11}{210} w^{2\cdot} \rho A_c L_c^2$$

$$D65 = \frac{11}{210} w^{2\cdot} \rho A_b L_b^2 - \frac{6 E_s I_b (2 r_i + r_i^2)}{L_b^2 (4 - r_i^2)}$$

$$D66 = \frac{4 E_s I_c}{L_c} - w^{2\cdot} \left(\frac{1}{105} \rho A_b L_b^2 + \frac{1}{105} \rho A_c L_c^3 \right) - \frac{12 E_s I_b (1 - r_i) r_i}{L_b (4 - r_i^2)} + \frac{12 E_s I_b (2 - r_i) r_i}{L_b (4 - r_i^2)}$$

Substituting A_c, L_c, I_c, A_b, L_b, I_b, ρ and E_s in the equation of the determinant, an equation in terms of r_i, r_j and ω is obtained. By varying r_i and r_j from 0 to 1, the angular frequencies ω are then obtained for each value of fixity factors of nodes.

4. Validation of the numerical approach

In order to validate the numerical approach, the same example modeled previously in paragraph 2 is considered, Fig. 4.

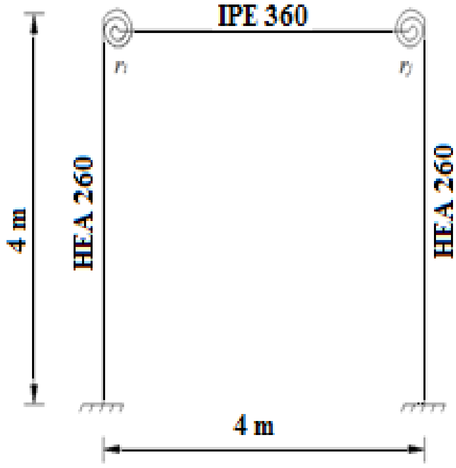

Fig. 4 Example of semi-rigid portal frame

The comparison between the angular frequencies of the portal frame analytically calculated with those numerically estimated, are given in Table 2.

Table 2 Numerical and analytical results of the portal frame angular frequencies

$r_i = r_j$	0,6	0,7	0,8	0,9	1
ω_i (rad/s) ANSYS	118,38	120,42	122,54	123,76	124,74
ω_i (rad/s) analytical	114,42	117,58	120,33	122,74	124,88

The numerical approach results tend towards those analytically calculated the fact that the difference between values is about 2%. The numerical model can be used for other configurations of portal frames.

5. Numerical study of a frame with semi-rigid nodes

After validating the obtained results with the software ANSYS, the same model is used in order to discuss the validity of the classification of joints defined by the EC3 (2005) on a multi-storey steel frame. In its part 1-8, Eurocode 3 (2005), presents a joint classification according to their rotational stiffness and allows a joint to be classified as pinned, semi-rigid or rigid.

A node is defined as pinned if its initial rotational stiffness does not exceed $0{,}5\, E_s I_b / L_b$. By replacing this limit in Eq. (1) a value of fixity factor $r_i \leq 0{,}15$ is obtained. By the same way, for an unbraced frame, a rigid joint is defined if its initial rotational stiffness is higher than $25\, E_s I_b / L_b$. By replacing this limit in Eq. (1) a value of fixity factor $r_i \geq 0{,}9$ is obtained. Joints having initial rotational stiffness between these two limits are classified as semi-rigid.

The case studied in this paragraph is a steel portal frame with one span of 4 m and three storeys of 4 m height, Fig. 5. The columns are made of HEA300 and beams are designed using IPE cross-sections.

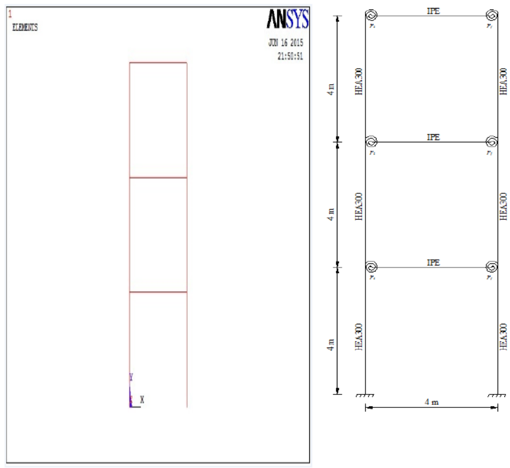

Fig. 5 Investigated semi-rigid portal frame

The same type of node in both ends of beams is considered (r_i=r_j), for all configurations of the studied portal frame.

Table 3 angular frequencies variation numerically estimated of the portal steel frame in Fig. 5

	Pinned joints according to EC3				Semi-rigid joints according to EC3							rigid joints according to EC3	
$r_i = r_j$	0	0,05	0,1	0,15	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
ω_i (rad/s) IPE300	13,5	32,3	33,9	34,6	34,9	35,2	35,4	35,5	35,6	35,6	35,7	35,7	35,7
ω_i (rad/s) IPE330	13,2	33,7	35,8	36,7	37,1	37,6	37,9	38,0	38,1	38,2	38,3	38,3	38,3
ω_i (rad/s) IPE360	12,8	34,7	37,3	38,4	39,0	39,6	40,0	40,2	40,3	40,4	40,5	40,6	40,6
ω_i (rad/s) IPE400	12,5	35,4	38,6	40,0	40,8	41,6	42,0	42,3	42,5	42,6	42,7	42,8	42,8
ω_i (rad/s) IPE450	12,1	35,8	39,6	41,2	42,2	43,2	43,7	44,1	44,3	44,5	44,6	44,7	44,8
ω_i (rad/s) IPE500	11,6	35,7	39,9	41,8	42,8	44,0	44,7	45,1	45,4	45,6	45,7	45,8	45,9

In order to develop numerical results, it has been decided to vary the rigidity of beams and calculate the corresponding angular frequencies. The results are given in Table 3.

In graphical format, the natural angular frequencies of steel portal frames with semi-rigid nodes according to the fixity factor of nodes r_i, for each value of the rigidity of the beam, are given in Fig. 6.

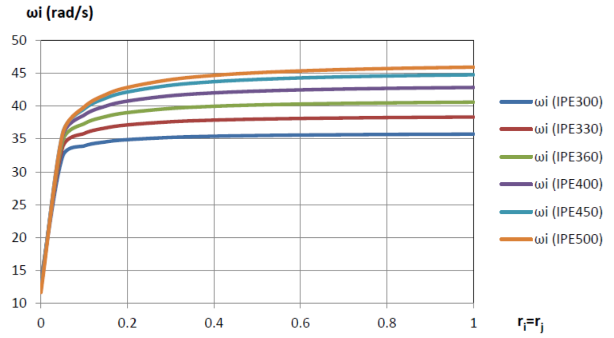

Fig. 6 Angular frequencies variation versus the fixity factor (column HEA 300)

5.1 Discussion of the results according to the classification of EC3 (2005)

According to the classification of joints adopted by the EC3 (2005), the results obtained in this study are discussed:
If $0 < r_i \leq 0{,}15$: According to the EC3 (2005), the nodes of which the fixity factors are in this interval are considered nominally pinned. According to the results of this study, a significant increase is observed in natural angular frequencies values between the case of pinned nodes $r_i = 0$ and the case of nodes that have a fixity factor $r_i = 0{,}15$. By considering the example of the steel frame with IPE 360 beams, the natural angular frequencies show a significant increase between the case of pinned nodes $r_i=0$ ($\omega_i=12{,}8$rad/s) and the case of nodes with $r_i=0{,}15$ ($\omega_i=38{,}4$rad/s).

If $0{,}15 < r_i \leq 0{,}9$: According to the EC3 (2005), the nodes of which the fixity factor are in this interval are considered as semi-rigid. Depending on the results, a slight increase can be noted in natural angular frequencies values in this case. By considering the example of the steel frame with IPE360 beams, the values of natural angular frequencies experienced a small variation between the case of nodes with $r_i=0{,}15$ ($\omega_i=38{,}4$rad/s) and the case of nodes with $r_i=0{,}9$ ($\omega_i=40{,}6$rad/s).

If $0{,}9 < r_i \leq 1$: According to the EC3 (2005), the nodes of which the fixity factors are in this interval are considered as rigid. According to the results, no variation of natural angular frequencies is underlined in this case. It has been found that the values of natural angular frequencies of the portal frame remain constant. By considering the example of the steel frame with IPE 360 beams, the natural angular frequencies have the same value between the case of nodes with $r_i=0{,}9$ ($\omega_i=40{,}6$rad/s) and the case of nodes with $r_i=1$ ($\omega_i=40{,}6$rad/s).

5.2 Calculation of the natural angular frequencies depending on the stiffness of columns

In order to confirm the previous results, it has been managed to vary the stiffness of columns and track changes in natural angular frequencies depending on the fixity factor of nodes r_i, for each type of beam. The columns considered are made of HEA400 and HEA500 cross-sections. In graphical form, the results are given as follows:

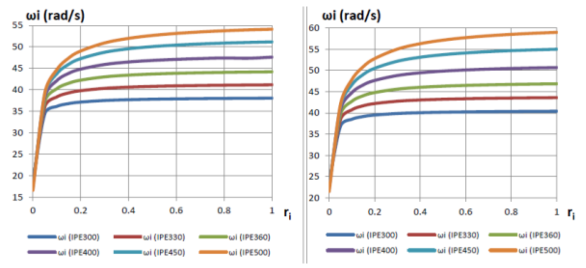

Fig. 7 Angular frequencies variation according to the fixity factor (column HEA 400 left, HEA 500 right)

By changing the stiffness of the columns, it is shown that the graphs $\omega_i = f(r_i)$ keep the same shape for different beams stiffness.

5.3 Classification of nodes and proposition of new limits

According to this study, the classification of joints in function of their rotational stiffness, as adopted by the EC3 (2005), has an influence on the dynamic response of steel portal frames. In addition, it has been shown that for all the studied cases, this rigidity does not affect in the same way the values of angular frequencies of the portal frames. For this reason, new limits for the classification of nodes are proposed:

When $0 < r_i < 0{,}05$: the values of angular frequencies of portal frames increase two times, or even three times. In this interval, the values of angular frequencies are very sensitive to the value of the nodes fixity factors.

When $0{,}05 \leq r_i \leq 0{,}6$ to $0{,}9$: The upper limit of this interval is variable because the values of the angular frequencies are comparable with those of the rigid node in some cases, and very different in other cases (see Table 3). By considering the example of the portal frame with

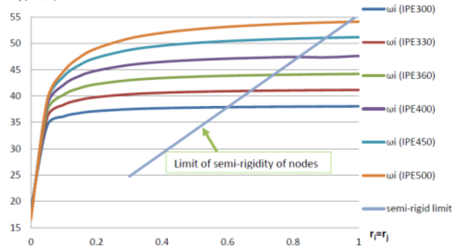

Fig. 8 Limits of the nodes semi-rigidity interval under consideration of the beams flexural stiffness (column HEA 300)

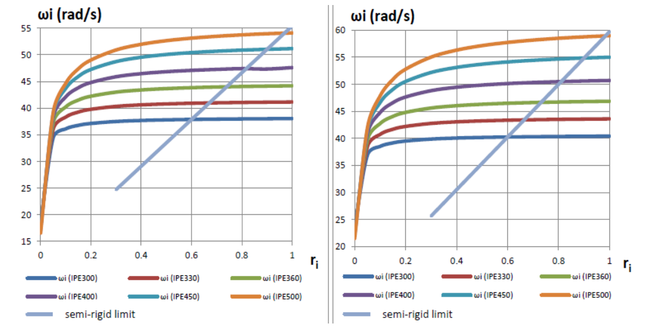

Fig. 9 Limits of the nodes semi-rigidity interval under consideration of the beams flexural stiffness (column HEA 400 left, HEA 500 right)

columns and beams made respectively of HEA300 and IPE300 cross-sections, the value of the angular frequency changes by 10% compared to the value of rigid nodes (between r_i = 0,05 and r_i = 1), while for the case of beams made of IPE 360, it changes by more than 20% compared to the value of the rigid nodes (between r_i = 0,05 and r_i = 1). This variation is due to the fact that the node semi-rigidity depends on the beam flexural stiffness, but it is linear.

When 0,6 to 0,9 < $r_i \leq 1$: depending on the value of the obtained natural angular frequency, portal frames with nodes having a coefficient r_i in this interval can be considered as rigid. The value of the natural frequency is the same as that given by r_i = 1.

The limit between the interval 0,05 ≤ r_i ≤ 0,6 to 0,9 and the interval 0,6 to 0,9 < r_i < 1 is given by a straight line between the two types of frames, and clearly not by a discrete value (r_i=0,9) as defined in the EC3 (2005).

5.4 Straight line of the semi-rigid interval

The limits of the semi-rigid interval are defined by a straight line, which varies with the flexural stiffness of beams, differentiating the rigid behavior of nodes and the semi-rigid interval. In the case of columns made of HEA300, the line is expressed by the following relationship

$$\omega_i = 35{,}5\, r_i + 14{,}3 \tag{6}$$

This equation defines the limit between the case of rigid nodes and the case of semi-rigid ones.

In the case of other types of columns which are the subject of this study, straight line giving the limit of the semi-rigidity of nodes according to the flexural stiffness of columns is given as follows.

The straight line defining the limit of the interval of semi-rigid nodes, is a function of the flexural stiffness of columns, its equation is given as follows

$$\omega_i = A\, r_i + B \tag{7}$$

In the graphs of Figs. 6-7, it may be remarked that in addition to the stiffness of beams, the stiffness of columns has an influence on the values of angular frequencies of portal frames with semi-rigid nodes. In order to take into account this stiffness, the variation of the coefficients A and B of the Eq. (8) is plotted as a function of the columns stiffness.

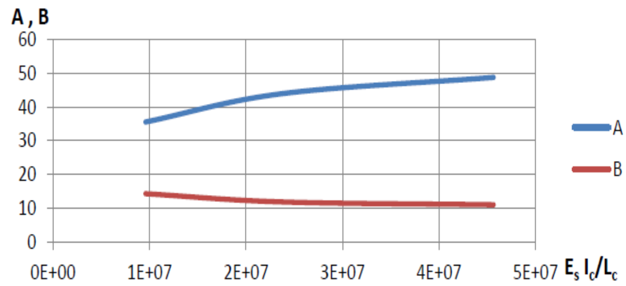

Fig. 10 Variation of coefficients A and B as a function of columns stiffness

This variation of the coefficients A and B can be approximated by

$$A = 9\times10^{-7}\times\frac{E_s\times I_c}{L_c}+27{,}46 \quad ; \quad B = -3\times10^{-7}\times\frac{E_s\times I_c}{L_c}+16{,}8 \tag{8}$$

Then, the straight line defining the relationship between the angular frequency ω_i of the portal frame and the fixity factor r_i of its nodes may be defined by

$$\omega_i = \left(9\times10^{-7}\times\frac{E_s\times I_c}{L_c}+27{,}46\right) r_i + \left(-3\times10^{-7}\times\frac{E_s\times I_c}{L_c}+16{,}8\right) \tag{9}$$

By replacing Young modulus E_s=2,1x10^{11} N/m², the column inertia by I_c=18263,5 cm^4, and the column length by L_c=4m, the same equation is obtained that defines the limit of nodes semi-rigidity in the case of a portal frame having columns with an inertia comparable to that of HEA300 and beams with inertias between that of the IPE300 and IPE500. For intermediate values, linear interpolation may be used.

5.5 Definition of the interval that takes into account flexional stiffness of columns

To obtain the value of the coefficient that takes into account the flexural stiffness of the columns and calculate the corresponding angular frequency, the following steps have been followed:

1- Calculate the equation of the straight line of the semi-rigid interval,

2- Draw this line in the corresponding graph (fig. 6 or 7).

Thereafter the value of ω_i can be calculated from the considered graph, by projection.

5.6 Practical example

By considering the example of a portal frame with beams made of IPE360 for a length of 4 m, and columns made of HEA340 cross-section, with I_c = 27690 cm^4, and height to 4 m. By using the Eq. (9), the limit of the semi-rigid interval can be given as follows:

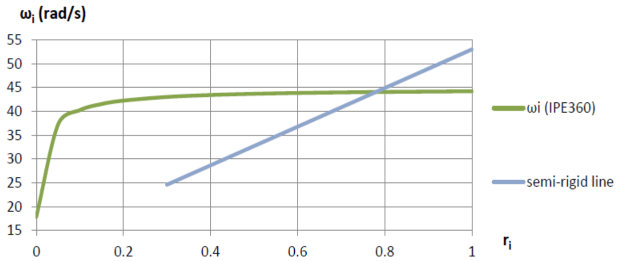

Fig. 11 The limit line of the semi-rigid interval (HEA340 column)

The equation of the straight line of the semi-rigid interval is given by

$$\omega_i = 40{,}54\ r_i + 12{,}44 \tag{10}$$

The corresponding graph is that of the Fig. 9.

It has been found that the r_i values below 0,78 define the semi-rigid interval, i.e. all nodes with $r_i \geq 0{,}78$ can be considered as rigid nodes.

• If a fixity factor r_i=0,9 is expected for this portal frame:

Its nodes are considered as rigid according to the EC3 (2005), while according to this study it is considered as semi-rigid. The correction of the natural angular frequency is about 9%.

• In a second step, if the fixity factor r_i = 0,15 is considered:

The nodes of the portal frame are considered as pinned according to the EC3 (2005), while according to this study they are considered as semi-rigid. The correction of the natural angular frequency is about 49%.

6. Conclusions

A large number of tests allow to the numerical developed program to give very close results with the analytical approach. This approach shows the efficiency of the numerical simulation used in this study.

Based on the analytical and numerical results, the introduction of semi-rigidity through the fixity factor has a significant influence on the dynamic response of structures. The proposed method can estimate the angular frequency more accurately, as well as the dynamic response.

The limit between semi-rigid nodes and rigid ones can be defined rather by a straight line, and not by one fixed value EC3 (2005).

The exploitation of the results allows verifying that for steel portal frames, not only the flexural stiffness of beams has an important role in the dynamic behavior of the structure, but also the rigidity of columns influences it. This influence is taken into account through an equation in which the rigidity of columns appears clearly.

The method described can be used for profiles whose rigidity is in the interval of profiles considered in the present study. For other types of profiles, linear interpolation or extrapolation can be used.

An important development can be made in the future, for three dimensional structures which will permit to extend similar formulations for several structural configurations. Furthermore, experimental tests should be done to confirm the results already found and consolidate the efficiency of the proposed method.

Other studies can be made to discuss the possibility to use a non-homogenous material instead of isotropic material in this type of structures. The possibility to use functionally graded materials (Ait Amar Meziane *et al.* 2014, Mahi *et al.* 2015, and Bennoun *et al.* 2016) and may also be considered.

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