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A new quasi-3D HSDT for buckling and vibration of FG plate

Mohamed Sekkal[1,2], Bouazza Fahsi[*2], Abdelouahed Tounsi[1,2,3] and S.R. Mahmoud[4]

[1]*Faculty of Technology, Civil Engineering Department, Material and Hydrology Laboratory, University of Sidi Bel Abbes, Algeria*
[2]*Laboratoire de Modélisation et Simulation Multi-échelle, Département de Physique, Faculté des Sciences Exactes, Département de Physique, Université de Sidi Bel Abbés, Algeria*
[3]*Laboratoire des Structures et Matériaux Avancés dans le Génie Civil et Travaux Publics, Université de Sidi Bel Abbes, Faculté de Technologie, Département de Génie Civil, Algeria*
[4]*Department of Mathematics, Faculty of Science, King Abdulaziz University, Saudi Arabia*

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Abstract. A new quasi-3D higher shear deformation theory (quasi-3D HSDT) for functionally graded plates is proposed in this article. The theory considers both shear deformation and thickness-stretching influences by a hyperbolic distribution of all displacements within the thickness, and respects the stress-free boundary conditions on the upper and lower surfaces of the plate without using any shear correction factor. The highlight of the proposed theory is that it uses undetermined integral terms in displacement field and involves a smaller number of variables and governing equations than the conventional quasi-3D theories, but its solutions compare well with 3D and quasi-3D solutions. Equations of motion are obtained from the Hamilton principle. Analytical solutions for buckling and dynamic problems are deduced for simply supported plates. Numerical results are presented to prove the accuracy of the proposed theory.

Keywords: buckling; vibration; sandwich plate; functionally graded materials; quasi-3D HSDT

1. Introduction

Functionally graded materials (FGMs) are considered as a new class of heterogeneous composite material in which the properties vary gradually over one or more directions. This material is fabricated from mixing two or more materials by considering certain volume ratio. Material characteristics of FGM vary within the material size depending on a function. Such material has been proposed, developed and successfully employed in industrial applications since 1980s (Koizumi 1993). Conventional composites structures such as fiber reinforced plastic (FRP) suffer from discontinuity of material characteristics at the interface of the plies and constituents. Hence the stress fields in these regions induce interface problems and thermal stress concentrations under high temperature environments. Furthermore, large plastic deformation of the interface may trigger the initiation and propagation of cracks in the material (Vel and Batra 2004). These problems can be reduced by gradually varying the volume fraction of constituent materials and tailoring the material for the desired application. Since its developments in the 1980s, FGMs are alternative materials widely utilized in aerospace, nuclear reactor, energy sources, biomechanical, optical, civil, automotive, electronic, chemical, mechanical, and shipbuilding industries

(Koizumi 1993, Larbi Chaht *et al.* 2015, Akbaş 2015, Arefi 2015a, b, Arefi and Allam 2015, Mahmoud *et al.* 2015, Zemri *et al.* 2015, Bousahla *et al.* 2016, Boukhari *et al.* 2016, Turan *et al.* 2016, El-Hassar *et al.* 2016, Ahouel *et al.* 2016, Benferhat *et al.* 2016, Celebi *et al.* 2016, Darabi and Vosoughi 2016, Ebrahimi and Shafiei 2016, Mohammadimehr *et al.* 2016, Mouaici *et al.* 2016, Trinh *et al.* 2016, Turan *et al.* 2016, Mouffoki *et al.* 2017).

Large applications of functionally graded (FG) structures have encouraged the development of various plate theories to study accurately their bending, stability and vibration behaviors. They are generally followed: classical plate theory (CPT) neglecting the transverse shear deformation influences (Feldman and Aboudi 1997, Javaheri and Eslami 2002, Mahdavian 2009, Mohammadi *et al.* 2010, Chen *et al.* 2006, Baferani *et al.* 2011, Arani and Kolahchi 2016, Bilouei *et al.* 2016, Zamanian *et al.* 2017), first-shear deformation theory (FSDT) with linear distribution of displacements (Mohammadi *et al.* 2010, Chen *et al.* 2006, Baferani *et al.* 2011, Praveen and Reddy 1998, Croce and Venini 2004, Efraim and Eisenberger 2007, Zhao *et al.* 2009a, b, Hosseini-Hashemi *et al.* 2011a, Naderi and Saidi 2010, Meksi *et al.* 2015, Adda Bedia *et al.* 2015, Bellifa *et al.* 2016, Bouderba *et al.* 2016, Kolahchi *et al.* 2016a, b, Hadji *et al.* 2016, Ebrahimi and Jafari 2016, Madani *et al.* 2016), higher-order shear deformation theory (HSDT) with nonlinear variations of displacements within the plate thickness such as third-order shear deformation plate theory (TSDT), sinusoidal shear deformation plate theory (SSDT), hyperbolic shear deformable plate theory (HDT), zigzag theories (Reddy 2000, Jha *et al.* 2013, Reddy 2011, Talha and Singh 2010, Tounsi *et al.* 2013, Bouderba *et al.* 2013, Ait Amar Meziane *et al.* 2014, Attia *et al.* 2015, Meradjah *et al.* 2015, Merazi *et al.* 2015, Bakora and Tounsi 2015, Mahi *et al.* 2015, Bouguenina *et al.* 2015, Ait

∗Corresponding author, Ph.D.
E-mail: hb_fahsi@yahoo.fr

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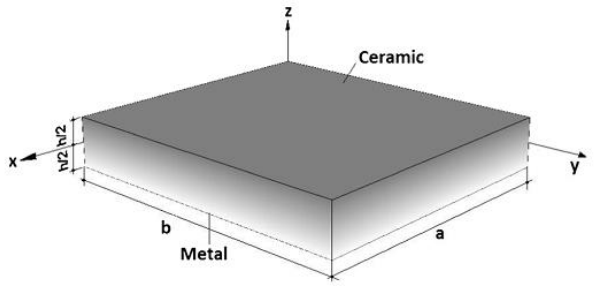

Fig. 1 Geometry of a functionally graded plate

Yahia *et al*. 2015, Bennai *et al*. 2015, Ait Atmane *et al*. 2015, Nguyen *et al*. 2015, Taibi *et al*. 2015, Zenkour 2006, Matsunaga 2008, Thai and Choi 2012, Kolahchi and Moniri Bidgoli 2016, Baseri *et al*. 2016, Chikh *et al*. 2016, Barati and Shahverdi 2016, Bourada *et al*. 2016, Merdaci *et al*. 2016, Raminnea *et al*. 2016, Saidi *et al*. 2016, Hebali *et al.* 2016, Javed *et al.* 2016, Barka *et al.* 2016, Becheri *et al.* 2016, Laoufi *et al.* 2016, Ebrahimi and Habibi, 2016, Beldjelili *et al.* 2016, El-Haina *et al.* 2017, Khetir *et al.* 2017, Klouche *et al.* 2017, Kolahchi *et al.* 2017a, b, Kolahchi, 2017, Besseghier *et al.* 2017, Chikh *et al.* 2017, Bellifa *et al.* 2017, Meksi *et al.* 2017, Zidi *et al.* 2017, Sekkal *et al.* 2017) and quasi-3D theories taking into consideration normal stretching influence (Carrera *et al.* 2008, Wu and Chiu 2011, Reddy 2011, Talha and Singh 2010, Neves *et al.* 2012a, b, Mantari and Soares 2012, 2013, Chen *et al.* 2009, Jha *et al.* 2013, Hebali *et al.* 2014, Hamidi *et al.* 2015, Akavci and Tanrikulu 2015, Pradyumna and Bandyopadhyay 2008, Bessaim *et al.* 2013, Fekrar *et al.* 2014, Kar *et al.* 2015, Swaminathan and Naveenkumar, 2014, Belabed *et al.* 2014, Bousahla *et al.* 2014, Bourada *et al.* 2015, Bounouara *et al.* 2016, Houari *et al.* 2016, Ghorbanpour Arani *et al.* 2016, Bennoun *et al.* 2016, Draiche *et al.* 2016, Benbakhti *et al.* 2016, Benchohra *et al.* 2017, Bouafia *et al.* 2017, Benahmed *et al.* 2017, Rahmani *et al.* 2017, Ait Atmane *et al.* 2017). However practically, some of these theories are computational costs because of number of additional variables included to the model (Pradyumna and Bandyopadhyay 2008, Jha *et al*. 2013, Neves *et al*. 2012a, b, Reddy 2011, Talha and Singh 2010). As a consequence, a simple quasi-3D HSDT proposed in this work is necessary.

This work aims to develop a simple quasi-3D HSDT for free vibration and buckling analysis of FG plates. By making a further assumption to the existing quasi-3D HSDT, the proposed theory contains only four unknowns and its governing equations is therefore reduced. Hamilton's principle is employed to determine equations of motion and Navier-type analytical solutions for simply-supported plates are compared with the existing solutions to check the validity of the proposed simple theory. The material properties are continuously changed within the plate thickness by the power-law form. Numerical results are found to examine the influences of the gradient index and side-to-thickness ratio on the critical buckling load and natural frequencies.

2. Theoretical formulation

Consider a FG plate as presented in Fig. 1 with a thickness h, length a and width b.

The material properties change across the thickness with a power law distribution, which is presented below (Zidi *et al.* 2014, 2017, Belkorissat *et al.* 2015, Fahsi *et al.* 2017)

$$P(z) = (P_c - P_m)V_c + P_m \tag{1}$$

Where P_c and P_m are the Young's moduli (E), Poisson's ratio (v) and mass density (ρ) of ceramic and metal materials located at the upper and lower surfaces, respectively. The volume fraction of ceramic material V_c is expressed as follows

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^p \tag{2}$$

Where p is the material index, which is positive.

2.1 Kinematics and strains

The displacement field satisfying the conditions of transverse shear stresses (and hence strains) vanishing at a point (x, y, $\pm h/2$) on the top and bottom surfaces of the plate, is expressed as follows

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,y,t)\,dx \tag{3a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2 f(z)\int \theta(x,y,t)\,dy \tag{3b}$$

$$w(x,y,z,t) = w_0(x,y,t) + g(z)\theta(x,y,t) \tag{3c}$$

The coefficients k_1 and k_2 depends on the geometry. It can be seen that the kinematic in Eq. (3) introduces only four unknowns (u_0, v_0, w_0 and θ) with considering the thickness stretching effect.

In this work, the present quasi-3D HSDT is obtained by setting

$$f(z) = -\left[\frac{3\pi\, z}{2}\sec h^2\left(\frac{1}{2}\right)\right] + \frac{3\pi}{2}h\tanh\left(\frac{z}{h}\right),$$
$$g(z) = \frac{2}{15}\frac{df}{dz} \tag{4}$$

The strain-displacement expressions, based on this formulation, are given as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} + f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}$$
$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = f'(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} + g(z)\begin{Bmatrix}\gamma_{yz}^1\\ \gamma_{xz}^1\end{Bmatrix},$$
$$\varepsilon_z = g'(z)\,\varepsilon_z^0 \tag{5}$$

Where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} \tag{6a}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1 \theta \\ k_2 \theta \\ k_1 \dfrac{\partial}{\partial y}\int \theta \, dx + k_2 \dfrac{\partial}{\partial x}\int \theta \, dy \end{Bmatrix} \tag{6b}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2 \int \theta \, dy \\ k_1 \int \theta \, dx \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^1 \\ \gamma_{xz}^1 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \theta}{\partial y} \\ \dfrac{\partial \theta}{\partial x} \end{Bmatrix} \tag{6c}$$

And

$$g'(z) = \frac{dg(z)}{dz} \tag{6d}$$

The integrals presented in the above equations shall be resolved by a Navier type method and can be expressed as follows:

$$\begin{gathered} \frac{\partial}{\partial y}\int \theta \, dx = A' \frac{\partial^2 \theta}{\partial x \partial y}, \qquad \frac{\partial}{\partial x}\int \theta \, dy = B' \frac{\partial^2 \theta}{\partial x \partial y}, \\ \int \theta \, dx = A' \frac{\partial \theta}{\partial x}, \quad \int \theta \, dy = B' \frac{\partial \theta}{\partial y} \end{gathered} \tag{7}$$

where the coefficients A' and B' are considered according to the type of solution employed, in this case via Navier method. Therefore, A', V', k_1 and k_2 are expressed as follows

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, k_1 = -\alpha^2, k_2 = -\beta^2 \tag{8}$$

Where α and β are defined in expression (25).

The linear constitutive relations are given below

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{9}$$

Where C_{ij} are the three-dimensional elastic constants defined by

$$C_{11} = C_{22} = C_{33} = \frac{(1-\nu)E(z)}{(1-2\nu)(1+\nu)}, \tag{10a}$$

$$C_{12} = C_{13} = C_{23} = \frac{\nu \, E(z)}{(1-2\nu)(1+\nu)}, \tag{10b}$$

$$C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)}, \tag{10c}$$

In consideration of different 10m height wind speed $v10$ and the power law exponent index α results shown in Table 2, the representative upstream typhoon wind fields at different directions used as the input data for training ANN model are determined, which is shown in Tables 1-2.

2.1 Equations of motion

Hamilton's principle is herein employed to determine the equations of motion

$$0 = \int_0^t (\delta U + \delta V - \delta K) \, dt \tag{11}$$

Where δU is the variation of strain energy; δV is the variation of the external work done by external load applied to the plate; and δK is the variation of kinetic energy.

The variation of strain energy of the plate is expressed by

$$\begin{aligned} \delta U &= \int_V \begin{bmatrix} \sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \sigma_z \delta \varepsilon_z + \tau_{xy} \delta \gamma_{xy} \\ + \tau_{yz} \delta \gamma_{yz} + \tau_{xz} \delta \gamma_{xz} \end{bmatrix} dV \\ &= \int_\Omega \begin{bmatrix} N_x \delta \varepsilon_x^0 + N_y \delta \varepsilon_y^0 + N_z \delta \varepsilon_z^0 \\ + N_{xy} \delta \gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b \\ + M_{xy}^b \delta k_{xy}^b + M_x^s \delta k_x^s + M_y^s \delta k_y^s \\ + M_{xy}^s \delta k_{xy}^s + Q_{yz}^s \delta \gamma_{yz}^0 + S_{yz}^s \delta \gamma_{yz}^1 \\ + Q_{xz}^s \delta \gamma_{xz}^0 + S_{xz}^s \delta \gamma_{xz}^1 \end{bmatrix} dA \end{aligned} \tag{12}$$

Where A is the top surface and the stress resultants N, M, S and Q are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} \left(1, z, f\right) \sigma_i dz, \left(i = x, y, xy\right), \tag{13a}$$

$$N_z = \int_{-h/2}^{h/2} g'(z) \sigma_z dz \tag{13b}$$

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz, \tag{13c}$$

$$\left(Q_{xz}^s, Q_{yz}^s\right) = \int_{-h/2}^{h/2} f'\left(\tau_{xz}, \tau_{yz}\right) dz \tag{13d}$$

The variation of work done by in-plane loads is given by:

$$\delta V = -\int_A \overline{N} \delta w dA \tag{14}$$

With

$$\overline{N} = \left[N_x^0 \frac{\partial^2 w}{\partial x^2} + 2N_{xy}^0 \frac{\partial^2 w}{\partial x \partial y} + N_y^0 \frac{\partial^2 w}{\partial y^2} \right] \tag{15}$$

Where (N_x^0, N_y^0, N_{xy}^0) are in-plane applied loads.

The variation of kinetic energy of the plate can be written as

$$\begin{aligned}
&\delta K = \int_V \left[\dot{u}\,\delta\,\dot{u} + \dot{v}\,\delta\,\dot{v} + \dot{w}\delta\,\dot{w}\right]\ \rho(z)\,dV \\
&= \int_A \Big\{ I_0\left[\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \dot{w}_0\delta\dot{w}_0\right] \\
&-I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial x}\delta\,\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{w}_0}{\partial y} + \frac{\partial\dot{w}_0}{\partial y}\delta\,\dot{v}_0\right) \\
&+J_1\left[\left(k_1 A'\right)\left(\dot{u}_0\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\delta\,\dot{u}_0\right)\right. \\
&\left.+\left(k_2 B'\right)\left(\dot{v}_0\frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\delta\,\dot{v}_0\right)\right] \\
&+J_1^{st}\left(\dot{w}_0\delta\,\dot{\theta} + \dot{\theta}\delta\dot{w}_0\right) \\
&+I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\,\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\,\dot{w}_0}{\partial y}\right) \\
&+K_2\left[\left(k_1 A'\right)^2\left(\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\,\dot{\theta}}{\partial x}\right) + \left(k_2 B'\right)^2\left(\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\,\dot{\theta}}{\partial y}\right)\right] \\
&-J_2\left[\left(k_1 A'\right)\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\,\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\,\dot{w}_0}{\partial x}\right)\right. \\
&\left.+\left(k_2 B'\right)\left(\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\,\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\,\dot{w}_0}{\partial y}\right)\right] \\
&+K_2^{st}\dot{\theta}\delta\,\dot{\theta}\Big\}\,dA
\end{aligned} \tag{16}$$

Where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density given by Eq. (1); and (I_j, J_i, K_i,) are mass inertias expressed by

$$\left(I_0, I_1, I_2\right) = \int_{-h/2}^{h/2}\left(1, z, z^2\right)\rho(z)dz \tag{17a}$$

$$\left(J_1, J_1^{st}, J_2, K_2, K_2^{st}\right) = \int_{-h/2}^{h/2}\left(f, g, z\,f, f^2, g^2\right)\rho(z)dz \tag{17b}$$

By substituting Eqs. (12), (14) and (16) into Eq. (11), the following equations of motion can be obtained

$$\begin{aligned}
&\delta\,u_0: \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot{u}_0 - I_1\frac{\partial\ddot{w}_0}{\partial x} + k_1 A'\,J_1\frac{\partial\ddot{\theta}}{\partial x} \\
&\delta\,v_0: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0\ddot{v}_0 - I_1\frac{\partial\ddot{w}_0}{\partial y} + k_2 B'\,J_1\frac{\partial\ddot{\theta}}{\partial y} \\
&\delta\,w_0: \quad \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x^0\frac{\partial^2 w}{\partial x^2} \\
&+2\,N_{xy}^0\frac{\partial^2 w}{\partial x\partial y} + N_y^0\frac{\partial^2 w}{\partial y^2} = I_0\ddot{w}_0 + I_1\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial y}\right) \\
&-I_2\nabla^2\ddot{w}_0 + J_2\left(k_1 A'\frac{\partial^2\ddot{\theta}}{\partial x^2} + k_2 B'\frac{\partial^2\ddot{\theta}}{\partial y^2}\right) - J_1^{st}\ddot{\theta}
\end{aligned}$$

$$\begin{aligned}
&\delta\,\theta: \quad -\mathrm{k}_1 A'\frac{\partial^2 S_x}{\partial x^2} - -\mathrm{k}_2 B'\frac{\partial^2 S_y}{\partial y^2} \\
&-\left(k_1 A' + k_2 B'\right)\frac{\partial^2 S_{xy}}{\partial x\partial y} + k_1 A'\frac{\partial R_{xz}}{\partial x} + k_2 B'\frac{\partial R_{yz}}{\partial y} \\
&+g(0)\left(N_x^0\frac{\partial^2 w}{\partial x^2} + 2\,N_{xy}^0\frac{\partial^2 w}{\partial x\partial y} + N_y^0\frac{\partial^2 w}{\partial y^2}\right) \\
&= -J_1\left(k_1 A'\frac{\partial\ddot{u}_0}{\partial x} + k_2 B'\frac{\partial\ddot{v}_0}{\partial y}\right) \\
&-K_2\left(\left(k_1 A'\right)^2\frac{\partial^2\ddot{\theta}}{\partial x^2} + \left(k_2 B'\right)^2\frac{\partial^2\ddot{\theta}}{\partial y^2}\right) \\
&+J_2\left(k_1 A'\frac{\partial^2\ddot{w}_0}{\partial x^2} + k_2 B'\frac{\partial^2\ddot{w}_0}{\partial y^2}\right) - J_1^{st}\ddot{w}_0 - K_2^{st}\ddot{\theta}
\end{aligned} \tag{18}$$

Substituting Eq. (5) into Eq. (9) and the subsequent results into Eq. (13), the stress resultants are obtained in terms of strains as following compact form

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix}\begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} + \varepsilon^0\begin{Bmatrix} L \\ L^a \\ R \end{Bmatrix}, \tag{19a}$$

$$\begin{Bmatrix} Q \\ S \end{Bmatrix} = \begin{bmatrix} F^s & X^s \\ X^s & A^s \end{bmatrix}\begin{Bmatrix} \gamma^0 \\ \gamma^1 \end{Bmatrix} \tag{19b}$$

$$\begin{aligned}
N_z &= L(\varepsilon_x^0 + \varepsilon_y^0) + L^a(k_x^b + k_y^b) \\
&+R(k_x^s + k_y^s) + R^a\varepsilon_z^0
\end{aligned} \tag{19c}$$

In which

$$N = \left\{N_x, N_y, N_{xy}\right\}^t, \quad M^b = \left\{M_x^b, M_y^b, M_{xy}^b\right\}^t, \quad M^s = \left\{M_x^s, M_y^s, M_{xy}^s\right\}^t, \tag{20a}$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}^t, \quad k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}^t, \quad k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}^t, \tag{20b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{20c}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix} \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}, \tag{20d}$$

$$Q=\left\{Q_{xz}^{s},Q_{yz}^{s}\right\}^{t},\ S=\left\{S_{xz}^{s},S_{yz}^{s}\right\}^{t},\quad \gamma^{0}=\left\{\gamma_{xz}^{0},\gamma_{yz}^{0}\right\}^{t},\quad \gamma^{1}=\left\{\gamma_{xz}^{1},\gamma_{yz}^{1}\right\}^{t} \tag{20e}$$

$$F^{s}=\begin{bmatrix}F_{55}^{s} & 0\\ 0 & F_{44}^{s}\end{bmatrix},\ X^{s}=\begin{bmatrix}X_{55}^{s} & 0\\ 0 & X_{44}^{s}\end{bmatrix},\ A^{s}=\begin{bmatrix}A_{55}^{s} & 0\\ 0 & A_{44}^{s}\end{bmatrix} \tag{20f}$$

$$\begin{Bmatrix}L\\ L^{a}\\ R\\ R^{a}\end{Bmatrix}=\int_{z}\lambda(z)\begin{Bmatrix}1\\ z\\ f(z)\\ g'(z)\dfrac{1-\nu}{\nu}\end{Bmatrix}g'(z)dz \tag{20g}$$

and stiffness components are given as

$$\begin{Bmatrix}A_{11} & B_{11} & D_{11} & B_{11}^{s} & D_{11}^{s} & H_{11}^{s}\\ A_{12} & B_{12} & D_{12} & B_{12}^{s} & D_{12}^{s} & H_{12}^{s}\\ A_{66} & B_{66} & D_{66} & B_{66}^{s} & D_{66}^{s} & H_{66}^{s}\end{Bmatrix}$$

$$=\int_{z}\lambda(z)\left(1,z,z^{2},f(z),z\,f(z),f^{2}(z)\right)\begin{Bmatrix}\dfrac{1-\nu}{\nu}\\ \nu\\ \dfrac{1-2\nu}{2\nu}\end{Bmatrix}dz, \tag{21a}$$

$$\left(A_{22},B_{22},D_{22},B_{22}^{s},D_{22}^{s},H_{22}^{s}\right)=\left(A_{11},B_{11},D_{11},B_{11}^{s},D_{11}^{s},H_{11}^{s}\right) \tag{21b}$$

$$\left(F_{44}^{s},X_{44}^{s},A_{44}^{s}\right)=\int_{-h/2}^{h/2}\frac{E(z)}{2(1+\nu)}\left([f'(z)]^{2},f'(z)g(z),g^{2}(z)\right)dz \tag{21c}$$

$$\left(F_{55}^{s},X_{55}^{s},A_{55}^{s}\right)=\left(F_{44}^{s},X_{44}^{s},A_{44}^{s}\right) \tag{21d}$$

By substituting Eq. (19) into Eq. (18), the equations of motion can be expressed in terms of displacements (u_0, v_0, w_0, θ) and the appropriate equations take the form

$$\begin{aligned}&A_{11}d_{11}u_0+A_{66}\,d_{22}u_0+\left(A_{12}+A_{66}\right)d_{12}v_0\\&-B_{11}d_{111}w_0-\left(B_{12}+2B_{66}\right)d_{122}w_0\\&+\left(B_{66}^{s}\left(k_1A'+k_2\,B'\right)+B_{12}^{s}k_2\,B'\right)d_{122}\theta\\&+B_{11}^{s}k_1A'\,d_{111}\theta+Ld_1\theta=I_0\ddot{u}_0-I_1\,d_1\ddot{w}_0\\&+J_1\,A'k_1d_1\ddot{\theta},\end{aligned} \tag{22a}$$

$$\begin{aligned}&A_{22}\,d_{22}v_0+A_{66}\,d_{11}v_0+\left(A_{12}+A_{66}\right)d_{12}u_0\\&-B_{22}\,d_{222}w_0-\left(B_{12}+2B_{66}\right)d_{112}w_0\\&+\left(B_{66}^{s}\left(k_1A'+k_2\,B'\right)+B_{12}^{s}k_1\,A'\right)d_{112}\theta\\&+B_{22}^{s}k_2\,B'd_{222}\theta+Ld_2\theta=I_0\ddot{v}_0-I_1\,d_2\ddot{w}_0\\&+J_1\,B'k_2\,d_2\ddot{\theta},\end{aligned} \tag{22b}$$

$$\begin{aligned}&B_{11}\,d_{111}u_0+\left(B_{12}+2B_{66}\right)d_{122}u_0\\&+\left(B_{12}+2B_{66}\right)d_{112}v_0+B_{22}\,d_{222}v_0\\&-D_{11}d_{1111}w_0-2\left(D_{12}+2D_{66}\right)d_{1122}w_0\\&-D_{22}\,d_{2222}w_0+D_{11}^{s}k_1\,A'd_{1111}\theta\\&+\left((D_{12}^{s}+2D_{66}^{s})\left(k_1A'+k_2B'\right)\right)d_{1122}\theta\\&+D_{22}^{s}k_2B'd_{2222}\theta+L^{a}(d_{11}\theta+d_{22}\theta)+N_{x}^{0}d_{11}w\\&+2N_{xy}^{0}d_{12}w+N_{y}^{0}d_{22}w\\&=I_0\ddot{w}_0+I_1\left(d_1\ddot{u}_0+d_2\,\ddot{v}_0\right)-I_2\left(d_{11}\ddot{w}_0+d_{22}\ddot{w}_0\right)\\&+J_2\left(k_1A'\,d_{11}\ddot{\theta}+k_2\,B'\,d_{22}\ddot{\theta}\right)+J_1^{st}\ddot{\theta},\end{aligned}$$

$$\begin{aligned}&-k_1A'B_{11}^{s}d_{111}u_0-\left(B_{12}^{s}k_2B'+B_{66}^{s}\left(k_1A'+k_2B'\right)\right)d_{122}u_0\\&-\left(B_{22}^{s}k_1A'+B_{66}^{s}\left(k_1A'+k_2B'\right)\right)d_{112}v_0-B_{22}^{s}k_2B'd_{222}v_0\\&+D_{11}^{s}k_1A'd_{1111}w_0+\left((D_{12}^{s}+2D_{66}^{s})\left(k_1A'+k_2B'\right)\right)d_{1122}w_0\\&+D_{22}^{s}k_2\,B'd_{2222}w_0-H_{11}^{s}(k_1A')^{2}\,d_{1111}\theta\\&-H_{22}^{s}(k_2B')^{2}\,d_{2222}\theta\\&-\left(2H_{12}^{s}\,k_1\,k_2A'B'+\left(k_1A'+k_2B'\right)^{2}H_{66}^{s}\right)d_{1122}\theta\\&+\left((k_1A')^{2}F_{55}^{s}+2k_1A'X_{55}^{s}+A_{55}^{s}\right)d_{11}\theta\\&+\left((k_2B')^{2}F_{44}^{s}+2k_2B'X_{44}^{s}+A_{44}^{s}\right)d_{22}\theta\\&-2R(k_1A'd_{11}\theta+k_2B'd_{11}\theta)-L(d_1u_0+d_2v_0)\\&+L^{a}(d_{11}w_0+d_{22}w_0)-R^{a}\theta\\&+g(0)\left(N_{x}^{0}d_{11}w+2N_{xy}^{0}d_{12}w+N_{y}^{0}d_{22}w\right)\\&=-J_1\left(k_1\,A'd_1\ddot{u}_0+k_2\,B'd_2\ddot{v}_0\right)\\&+J_2\left(k_1\,A'd_{11}\ddot{w}_0+k_2\,B'd_{22}\ddot{w}_0\right)\\&-K_2\left(\left(k_1A'\right)^{2}d_{11}\ddot{\theta}+\left(k_2\,B'\right)^{2}d_{22}\ddot{\theta}\right)+J_1^{st}\ddot{w}_0+K_2^{st}\ddot{\theta},\end{aligned} \tag{22c}$$

Where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij}=\frac{\partial^{2}}{\partial x_i\partial x_j},\ d_{ijl}=\frac{\partial^{3}}{\partial x_i\partial x_j\partial x_l},\ d_{ijlm}=\frac{\partial^{4}}{\partial x_i\partial x_j\partial x_l\partial x_m},\ d_i=\frac{\partial}{\partial x_i},\ (i,j,l,m=1,2). \tag{23}$$

3. Close-form solutions

The Navier solution method is utilized to deduce the analytical solutions for which the displacement variables are written as product of arbitrary parameters and known trigonometric functions to respect the equations of motion and boundary conditions.

$$\begin{Bmatrix}u_0\\ v_0\\ w_0\\ \theta\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix}U_{mn}e^{i\omega t}\cos(\alpha\,x)\sin(\beta\,y)\\ V_{mn}e^{i\omega t}\sin(\alpha\,x)\cos(\beta\,y)\\ W_{mn}e^{i\omega t}\sin(\alpha\,x)\sin(\beta\,y)\\ X_{mn}e^{i\omega t}\sin(\alpha\,x)\sin(\beta\,y)\end{Bmatrix} \tag{24}$$

Where ω is the frequency of free vibration of the plate, $\sqrt{i}=-1$ the imaginary unit.

With

$$\alpha = m\pi/a \quad \text{and} \quad \beta = n\pi/b \tag{25}$$

Considering that the plate is subjected to in-plane compressive forces of form: $N_x^0 = -N_0$, $N_y^0 = -\gamma N_0$, $N_{xy}^0 = 0$, $\gamma = N_y^0 / N_x^0$ (here γ are non-dimensional load parameter). Substituting Eq. (24) into Eq. (22), the following equation is obtained

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33}+\eta & S_{34}+g(0)\eta \\ S_{14} & S_{24} & S_{34}+g(0)\eta & S_{44}+\left[g(0)\right]^2\eta \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix} \right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{26}$$

Where

$$\begin{aligned} S_{11} &= A_{11}\alpha^2 + A_{66}\beta^2,\ S_{12} = \alpha\beta\left(A_{12}+A_{66}\right) \\ S_{13} &= -\alpha\left[B_{11}\alpha^2 + (B_{12}+2B_{66})\beta^2\right] \\ S_{14} &= \alpha\left[(k_2B'B_{12}^s + \left(k_1A'+k_2B'\right)B_{66}^s)\beta^2 \right. \\ &\left. + k_1A'B_{11}^s\alpha^2 - L\right] \\ S_{22} &= A_{66}\alpha^2 + A_{22}\beta^2 \quad S_{23} = -\beta\left[B_{22}\beta^2 + (B_{12}+2B_{66})\alpha^2\right] \\ S_{24} &= \beta\left[(k_1A'B_{12}^s + \left(k_1A'+k_2B'\right)B_{66}^s)\alpha^2 \right. \\ &\left. + k_2B'B_{22}^s\beta^2 - L\right] \\ S_{33} &= D_{11}\alpha^4 + 2(D_{12}+2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 \\ S_{34} &= -k_1A'D_{11}^s\alpha^4 - 2D_{12}^sk_2B'\alpha^2\beta^2 \\ &-2D_{66}^s(k_1A'+k_2B')\alpha^2\beta^2 - k_2B'D_{22}^s\beta^4 \\ &+L^a(\alpha^2+\beta^2) \\ \eta &= -N_0\left(\alpha^2+\gamma\beta^2\right) \end{aligned} \tag{27}$$

$$\begin{aligned} m_{11} &= I_0,\quad m_{13} = -\alpha I_1,\quad m_{14} = J_1k_1A'\alpha,\quad m_{22} = I_0, \\ m_{23} &= -\beta\ I_1,\quad m_{24} = k_2B'\ \beta J_1 \\ m_{33} &= I_0 + I_2(\alpha^2+\beta^2) \\ m_{34} &= -J_2\left(k_1A'\alpha^2 + k_2B'\beta^2\right) + J_1^{st} \\ m_{44} &= K_2\left[\left(k_1A'\right)^2\alpha^2 + \left(k_2B'\right)^2\beta^2\right] + K_2^{st} \end{aligned}$$

Eq. (26) is a general form for buckling and dynamic analysis of FG plates under in-plane loads. The critical buckling loads (N_{cr}) can be determined from the stability problem $|S_{ij}|=0$ while the free vibration problem is achieved by omitting in-plane loads.

Table 1 Material properties of metal and ceramic

Matériel	Young's modulus (GPa)	Mass density kg/m^3)	Poisson's ratio
Aluminum (Al)	70	2.702	0.3
Zirconia (ZrO_2)	151	3.000	0.3
Alumina (Al_2O_3)	380	3.800	0.3

Table 2 Comparison of the non-dimensional fundamental frequency ($\bar{\beta}$) of Al/ZrO_2 square plates

a/h	Theory	Material index							
		0	0.1	0.2	0.5	1	2	5	10
2	3D (Uymaz and Aydogdu 2007)	1.2589	1.2296	1.2049	1.1484	1.0913	1.0344	0.9777	0.9507
	Proposed	1.2671	1.2383	1.2135	1.1571	1.0999	1.0425	0.9841	0.9581
5	3D (Uymaz and Aydogdu 2007)	1.7748	1.7262	1.6881	1.6031	1.4764	1.4628	1.4106	1.3711
	Proposed	1.7830	1.7369	1.6983	1.6149	1.5392	1.4763	1.4176	1.3787
10	3D (Uymaz and Aydogdu 2007)	1.9339	1.8788	1.8357	1.7406	1.6583	1.5958	1.5491	1.5066
	Proposed	1.9388	1.8861	1.8424	1.7504	1.6708	1.6109	1.5575	1.5140
20	3D (Uymaz and Aydogdu 2007)	1.9570	1.9261	1.8788	1.7832	1.6999	1.6401	1.5937	1.5491
	Proposed	1.9854	1.9303	1.8849	1.7902	1.7101	1.6520	1.6012	1.5561
50	3D (Uymaz and Aydogdu 2007)	1.9974	1.9390	1.8920	1.7944	1.7117	1.6522	1.6062	1.5620
	Proposed	1.9990	1.9432	1.8973	1.8017	1.7215	1.6642	1.6143	1.5687
100	3D (Uymaz and Aydogdu 2007)	1.9974	1.9416	1.8920	1.7972	1.7117	1.6552	1.6062	1.5652
	Proposed	2.0009	1.9450	1.8990	1.8034	1.7232	1.6660	1.6162	1.5706

4. Numerical results and discussion

In this section, natural frequencies and critical buckling loads of simply supported FG plates are provided and compared with existing solutions to check the accuracy of the proposed new quasi-3D HSDT. FG plates made of two material combinations of metal and ceramic: Al/ZrO_2 and Al/Al_2O_3 are considered. Their material properties are presented in Table 1.

For convenience, the following non-dimensional parameters are employed

$$\begin{aligned} \bar{\omega} &= \frac{\omega a^2}{h}\sqrt{\frac{\rho_c}{E_c}},\quad \hat{\omega} = \omega h\sqrt{\frac{\rho_c}{E_c}}, \\ \bar{\beta} &= \frac{\omega ab}{\pi^2 h}\sqrt{\frac{12\left(1-\nu_c^2\right)\rho_c}{E_c}},\quad \bar{N}_{cr} = \frac{N_{cr}\,a^2}{E_m h^3}, \\ \hat{N}_{cr} &= \frac{N_{cr}\,a^2}{D_{11} - B_{11}^2/A_{11}} \end{aligned} \tag{28}$$

4.1 Results of free vibration analysis

Table 2 shows the comparison of the fundamental frequency of Al/ZrO_2 square plates obtained from the proposed quasi-3D HSDT and 3D plate model Uymaz and Aydogdu (2007).

It can be seen that the computed results agree very well with 3D solution.

Table 3 Comparison of the first three non-dimensional frequencies ($\hat{\omega}$) of Al/Al_2O_3 square plates

a/h	Mode (*m*,*n*)	Theory	Material index				
			0.1	0.2	0.5	1	2
5	1(1,1)	Quasi-3D (Matsunaga 2008)	0.2121	0.1819	0.1640	0.1383	0.1306
		TSDT (Hosseini-Hashemi *et al.* 2011)	0.2113	0.1807	0.1631	0.1378	0.1301
		FSDT (Hosseini-Hashemi *et al.* 2011)	0.2112	0.1805	0.1631	0.1397	0.1324
		Present	0.2130	0.1834	0.1665	0.1411	0.1321
	2(1,2)	Quasi-3D (Matsunaga 2008)	0.4658	0.4040	0.3644	0.3000	0.2790
		TSDT (Hosseini-Hashemi *et al.* 2011)	0.4623	0.3989	0.3607	0.2980	0.2771
		FSDT (Hosseini-Hashemi *et al.* 2011)	0.4618	0.3978	0.3604	0.3049	0.2856
		Present	0.4682	0.4064	0.3692	0.3052	0.2818
	3(2,2)	Quasi-3D (Matsunaga 2008)	0.6688	0.5803	0.5254	0.4284	0.3948
		TSDT (Hosseini-Hashemi *et al.* 2011)	0.6676	0.5779	0.5245	0.4405	0.4097
		FSDT (Hosseini-Hashemi *et al.* 2011)	0.6791	0.5924	0.5387	0.4388	0.4019
		Present	0.0578	0.0492	0.0443	0.0381	0.0364
10	1(1,1)	Quasi-3D (Matsunaga 2008)	0.0577	0.0490	0.0442	0.0381	0.0364
		TSDT (Hosseini-Hashemi *et al.* 2011)	0.0577	0.0490	0.0442	0.0382	0.0366
		FSDT (Hosseini-Hashemi *et al.* 2011)	0.0579	0.0495	0.0450	0.0390	0.0369
		Present	0.1381	0.1180	0.1063	0.0905	0.0859
	2(1,2)	Quasi-3D (Matsunaga 2008)	0.1377	0.1174	0.1059	0.0903	0.0856
		TSDT (Hosseini-Hashemi *et al.* 2011)	0.1376	0.1173	0.1059	0.0911	0.0867
		FSDT (Hosseini-Hashemi *et al.* 2011)	0.1385	0.1189	0.1080	0.0924	0.0869
		Present	0.2113	0.1807	0.1631	0.1378	0.1301
	3(2,2)	Quasi-3D (Matsunaga 2008)	0.2112	0.1805	0.1631	0.1397	0.1324
		TSDT (Hosseini-Hashemi *et al.* 2011)	0.2130	0.1834	0.1665	0.1411	0.1321
		FSDT (Hosseini-Hashemi *et al.* 2011)	0.0148	0.0125	0.0113	0.0098	0.0094
		Present	0.0148	0.0125	0.0113	0.0098	0.0094
20	1(1,1)	Quasi-3D (Matsunaga 2008)	0.0148	0.0126	0.0115	0.0100	0.0095
		TSDT (Hosseini-Hashemi *et al.* 2011)	0.2121	0.1819	0.1640	0.1383	0.1306
		FSDT (Hosseini-Hashemi *et al.* 2011)	0.2113	0.1807	0.1631	0.1378	0.1301
		Present	0.2112	0.1805	0.1631	0.1397	0.1324

Table 4 Non-dimensional natural frequencies ($\Omega = \omega h\sqrt{\rho_m / E_m}$) of Al/Al_2O_3 functionally graded plates

b/a	*a/h*	*p*	Theory		
			Jin *et al*. (2014)	Mantari (2015)	Present
1	10	0	0.1135	0.1137	0.1138
		1	0.0870	0.0883	0.0884
		2	0.0789	0.0806	0.0807
		5	0.0741	0.0756	0.0756
	5	0	0.4169	0.4183	0.4185
		1	0.3222	0.3271	0.3272
		2	0.2905	0.2965	0.2966
		5	0.2676	0.2726	0.2727
	2	0	1.8470	1.8543	1.8589
		1	1.4687	1.4803	1.4836
		2	1.3095	1.3224	1.3255
		5	1.1450	1.1565	1.1577
2	10	0	0.0719	0.0719	0.0719
		1	0.0550	0.0558	0.0558
		2	0.0499	0.0510	0.0510
		5	0.0471	0.0480	0.0480
	5	0	0.2713	0.2721	0.2722
		1	0.2088	0.2121	0.2121
		2	0.1888	0.1928	0.1929
		5	0.1754	0.1789	0.1789
	2	0	0.9570	1.3075	1.3101
		1	0.7937	1.0371	1.0390
		2	0.7149	0.9297	0.9315
		5	0.6168	0.8248	0.8253

Table 3 presents effects of the material index and thickness ratio on non-dimensional frequency of FG plate.

The results are compared with solutions of FSDT Hosseini-Hashemi *et al*. (2011b), TSDT Hosseini-Hashemi *et al*. (2011b), and quasi-3D Matsunaga (2008).

It is shown that the computed results are again found more close in many cases to 3D-quasi plate model than TSDT and FSDT.

In Table 4, non-dimensional fundamental frequencies of simply supported plate are calculated for four different material indexes and compared with the novel 3D exact solution proposed by Jin *et al*. (2014) and quasi-3D Mantari (2015).

The results obtained present good accuracy

Fig. 2 presents the variation of natural frequencies in terms of the material index and thickness ratio.

It can be observed from these results that the natural frequencies diminish with the increase of the material index. It is due to the fact that a higher value of *p* corresponds to lower value of volume fraction of the ceramic phase, and thus makes the plates become the softer ones (Nguyen 2015). Fig. 2(b) demonstrates that with an increase of the thickness ratio, the shear deformation influence becomes very effective in a relatively large region (*b/h*=30).

4.2 Results of buckling analysis

The buckling responses of Al/Al_2O_3 plates are investigated by considering three types of in-plane loads: uniaxial compression (γ=0), biaxial compressions (γ=1) and axial compression and tension (γ=−1). The computed results are given in Table 5. It can be seen that the results of present work again agree well with previous solutions HSDT Nguyen (2015) and HSDT Thai and Choi (2012).

Fig. 3 presents the critical buckling loads of rectangular plates with respect to the material index.

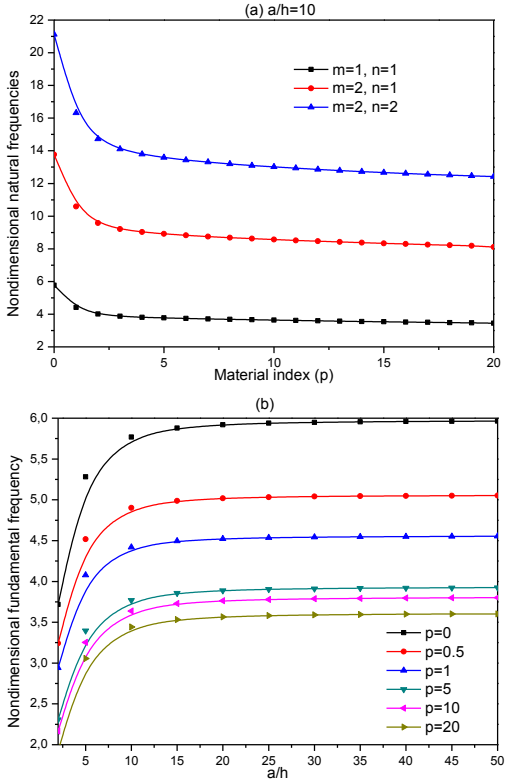

Fig. 2 Effect of the material index p and thickness ratio a/h on the natural frequency ($\bar{\omega}$) of Al/Al_2O_3 square plates

Table 5 Comparison of the critical buckling load ($\bar{N}_{cr}$) of Al/Al_2O_3 plates

γ	a/b	a/h	Theory	Material index 0	0.5	1	2	5	10
0	0.5	5	HSDT (Thai and Choi 2012)	6.7203	4.4235	3.4164	2.6451	2.1484	1.9213
			HSDT (Nguyen 2015)	6.7417	4.4343	3.4257	2.6503	2.1459	1.9260
			Present	6.7005	4.4728	3.4983	2.7347	2.2076	1.9459
		10	HSDT (Thai and Choi 2012)	7.4053	4.8206	3.7111	2.8897	2.4165	2.1896
			HSDT (Nguyen 2015)	7.4115	4.8225	3.7137	2.8911	2.4155	2.1911
			Present	7.4126	4.8904	3.8221	3.0168	2.5090	2.2374
		20	HSDT (Thai and Choi 2012)	7.5993	4.9315	3.7930	2.9582	2.4944	2.2690
			HSDT (Nguyen 2015)	7.6009	4.9307	3.7937	2.9585	2.4942	2.2695
			Present	7.6109	5.0028	3.9108	3.0968	2.5963	2.3230
	1	5	HSDT (Thai and Choi 2012)	16.0211	10.6254	8.2245	6.3432	5.0531	4.4807
			HSDT (Nguyen, 2015)	16.1003	10.6670	8.2597	6.3631	5.0459	4.4981
			Present	15.9193	10.7065	8.3828	6.5148	5.1526	4.5077
		10	HSDT (Thai and Choi 2012)	18.5785	12.1229	9.3391	7.2631	6.0353	5.4528
			HSDT (Nguyen 2015)	18.6030	12.1317	9.3496	7.2687	6.0316	5.4587
			Present	18.5846	12.2937	9.6083	7.5667	6.2535	5.5625
		20	HSDT (Thai and Choi 2012)	19.3528	12.5668	9.6675	7.5371	6.3448	5.7668
			HSDT (Nguyen 2015)	19.3593	12.5652	9.6702	7.5386	6.3437	5.7689
			Present	19.3809	12.7494	9.9658	7.8860	6.6008	5.9020
0.5	0.5	5	HSDT (Thai and Choi 2012)	5.3762	3.5388	2.7331	2.1161	1.7187	1.5370
			HSDT (Nguyen 2015)	5.3934	3.5475	2.7406	2.1202	1.7167	1.5408
			Present	5.3604	3.5783	2.7987	2.1878	1.7661	1.5568
		10	HSDT (Thai and Choi 2012)	5.9243	3.8565	2.9689	2.3117	1.9332	1.7517
			HSDT (Nguyen 2015)	5.9292	3.8580	2.9710	2.3129	1.9324	1.7529
			Present	5.9301	3.9123	3.0577	2.4134	2.0072	1.7899
		20	HSDT (Thai and Choi 2012)	6.0794	3.9452	3.0344	2.3665	1.9955	1.8152
			HSDT (Nguyen 2015)	6.0807	3.9445	3.0350	2.3668	1.9953	1.8156
			Present	6.0887	4.0022	3.1287	2.4774	2.0770	1.8584
	1	5	HSDT (Thai and Choi 2012)	8.0105	5.3127	4.1122	3.1716	2.5265	2.2403
			HSDT (Nguyen 2015)	8.0501	5.3335	4.1299	3.1815	2.5230	2.2491
			Present	7.9597	5.3533	4.1914	3.2574	2.5763	2.2539
		10	HSDT (Thai and Choi 2012)	9.2893	6.0615	4.6696	3.6315	3.0177	2.7264
			HSDT (Nguyen 2015)	9.3015	6.0659	4.6748	3.6344	3.0158	2.7293
			Present	9.2923	6.14687	4.8042	3.7834	3.1268	2.78123
		20	HSDT (Thai and Choi 2012)	9.6764	6.2834	4.8337	3.7686	3.1724	2.8834
			HSDT (Nguyen 2015)	9.6796	6.2826	4.8351	3.7693	3.1718	2.8844
			Present	9.6904	6.3747	4.9829	3.9430	3.3004	2.9510
	10	5	HSDT (Thai and Choi 2012)	8.9604	5.8980	4.5551	3.5268	2.8646	2.5617
			HSDT (Nguyen 2015)	8.9890	5.9124	4.5676	3.5337	2.8612	2.5679
			Present	8.9339	5.9637	4.6645	3.6463	2.9435	2.5946
		10	HSDT (Thai and Choi 2012)	9.8738	6.4275	4.9481	3.8529	3.2219	2.9195
			HSDT (Nguyen 2015)	9.8820	6.4299	4.9516	3.8548	3.2206	2.9214
			Present	9.8835	6.5206	5.0962	4.0224	3.3453	3.3453
		20	HSDT (Thai and Choi 2012)	10.1324	6.5753	5.0574	3.9442	3.3259	3.0253
			HSDT (Nguyen 2015)	10.1345	6.5742	5.0583	3.9447	3.3255	3.0260
			Present	10.1478	6.6704	5.2145	4.1291	3.4617	3.0974
-1	1	5	HSDT (Thai and Choi 2012)	26.2058	17.7704	13.8486	10.5589	7.9590	6.8970
			HSDT (Nguyen 2015)	26.4999	17.9424	13.9872	10.6421	7.9571	6.9626
			Present[a]	25.7567	17.6913	13.9068	10.6372	7.9346	6.8033
		10	HSDT (Thai and Choi 2012)	35.8416	23.5920	18.2206	14.1073	11.4583	10.2468
			HSDT (Nguyen 2015)	35.9559	23.6497	18.2704	14.1349	11.4447	10.2717
			Present[a]	35.7357	23.8550	18.6579	14.5850	11.7741	10.3784
		20	HSDT (Thai and Choi 2012)	39.4951	25.7100	19.7925	15.4115	12.8878	11.6779
			HSDT (Nguyen 2015)	39.5280	25.7197	19.8065	15.4190	12.8824	11.6857
			Present[a]	39.5339	26.0822	20.3846	16.0896	13.3813	11.9329

[a] Critical buckling occurs at (m; n)=(2,1)

It is seen from this figure that they diminish with the increase of the material index, and increase with the thickness ratio up to the point b/h=30 from which the curves become flatter.

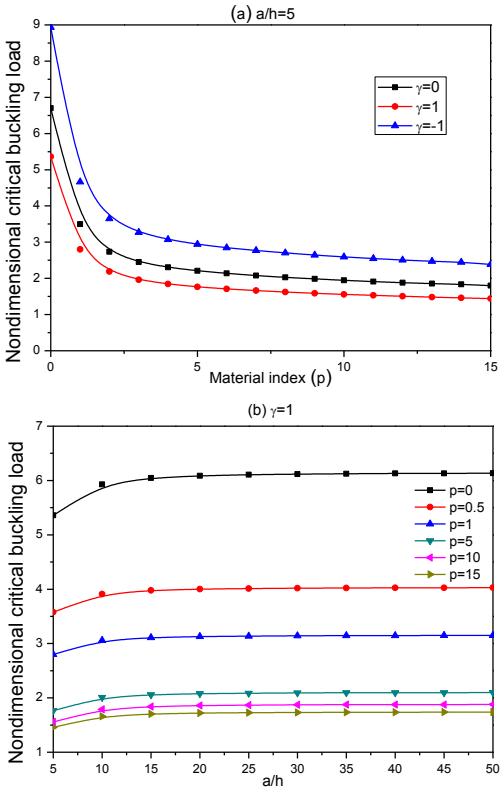

Fig. 3 Effect of the material index p and thickness ratio a/h on the critical buckling load ($\overline{N}_{cr}$) of Al/Al_2O_3 rectangular plates (a/b = 0.5)

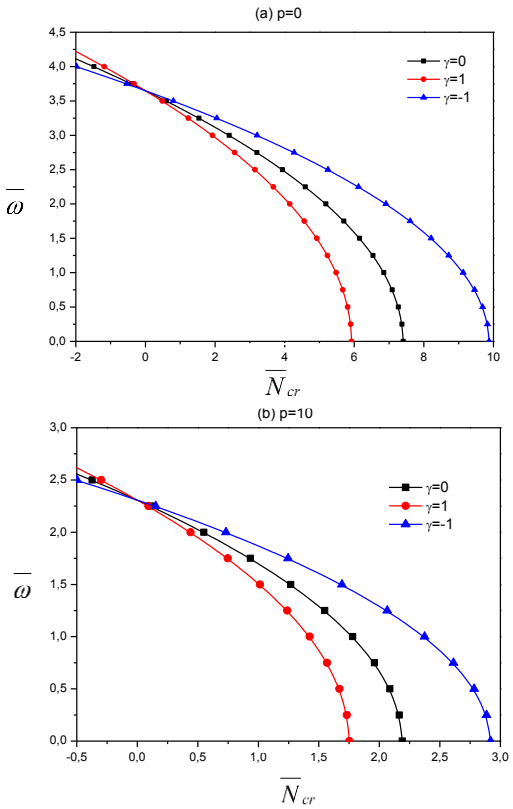

Fig. 4 Influence of in-plane loads on the non-dimensional fundamental frequency of Al/Al_2O_3 rectangular plates (a/b=0.5, a/h=10)

Fig. 4 shows the lowest load-frequency curves for both homogeneous and FG rectangular plates (a/b=0:5).

It can be observed that all fundamental frequencies decrease as in-plane loads vary from tension to compression. In compression region ($\overline{N}_{cr} > 0$), the fundamental frequencies are the largest for the plates under uniaxial compression and tension (γ=−1) and the smallest for ones under biaxial compressive force (γ=1). However, this order is varied in tension region. It is from load-frequency curves that the critical buckling forces can be obtained indirectly by vibration investigation through load-frequency curves, which corresponds to zero natural frequencies.

5. Conclusions

A new quasi-3D HSDT is proposed for free vibration and buckling analysis of FG plates in this work. The theory considers hyperbolic variation of transverse shear stress, and respects the traction-free boundary conditions on the upper and lower surfaces of the plate without employing shear correction factor. The proposed plate model contains only four unknowns and equations of motion are obtained from Hamilton's principle. Navier-type solutions are determined for simply-supported plates and compared with the existing solutions to check the validity of the proposed theory. The computed results are in well agreement with different HSDTs and quasi-3D plate models. The proposed model is found to be appropriate, simple and efficient in investigating vibration and stability problem of FG plates. In addition, it is found that the proposed quasi-3D HSDT, provides results with good accuracy compared with the CPT, FSDT and other HSDTs with higher number of unknowns Consequently, the authors suggest to consider this theory because of its simplicity.

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