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Optimum design of partially prestressed concrete beams using Genetic Algorithms

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Abstract. This paper deals with the optimum cost design of partially prestressed concrete I crosssectioned beams by using Genetic Algorithms. For this purpose, the optimum cost design of two selected example problems that have different characteristics in behavior are performed via Genetic Algorithms by determining their objective functions, design variables and constraints. The results obtained from the technical literature are compared with the ones obtained from this study. The interpretation of the results show that the design of partially prestressed concrete I crossectioned beams from cost point of view by using Genetic Algorithms is 35~50 % more economical than the traditional ones (technical literature) without conceding safety.

Keywords: optimum; partially prestressed concrete; beam; Genetic Algorithm; cost

1. Introduction

Partially prestressed concrete (PPC) is the construction technique that comprises the transition between reinforced concrete (RC) and fully prestressed concrete (FPC). In RC structures, the design and construction of the structural elements are carried out by using only traditional (*conventional*) reinforcement. Furthermore, the design and construction of FPC structures are performed only with prestressed reinforcements where the other conventional reinforcements are used for the ease of construction. However, in PPC structures, the combination of conventional and prestressed reinforcement is used for the load carrying capacity of the whole structure.

There are so many technical definitions of PPC structures given in the technical literature (Nilson 1976, Naaman 1977, Lee 1984, Nilson 1987, Naaman and Hamza 1993, Al-Gahtani *et al*. 1995, Agrawal and Bhattacharya 2010, Karayannis and Chalioris 2013). In this study, PPC is defined as the concrete that contains both conventional and prestressed reinforcement in both tension and/or compression regions that are considered in the load carrying capacity of the structure without conceding usability and safety requirements. Also, in some conditions, it is allowed for PPC structures that under dead and live loads to have the formation of tension stress and therefore, the formation of cracks (Türkeli 2016).

In chronological order, RC was first established in the technical literature. Then, FPC was described by Freyssinet (1933) which is accepted as the founder of the method. At last, a different design method other than FPC, PPC method, was proposed by von Emperger (1939) and Abeles (1940). Also, Abeles termed this design approach as "partially prestressed concrete".

In technical literature, there are so many studies dealing with the analytical and experimental structural behavior of PPC structural elements and optimization studies. Abeles discussed the design of PPC beams, covering also the two limiting cases, i.e., fully prestressed and ordinary RC beams (Abeles 1967). Nilson (1976) presented a method to calculate elastic flexural stresses in PPC beams after cracking. Naaman (1977) dealt with a non-linear analysis procedure which attempts to predict the behavior at ultimate of prestressed and partially prestressed sections by strain compatibility. Uber (1983) dealt with a design method that permits a rapid solution of partially prestressed T-beams. Saouma and Murad (1984) dealt with a comprehensive method for minimum cost design of simply supported, uniformly loaded, PPC beams. In the study, nine design variables i.e., six geometrical dimensions, area of prestressing steel and area of tensile and compressive mild reinforcement was used. Cohn and MacRae (1984a, 1984b) studied the minimum cost design of fully prestressed and PPC I-beams with fixed cross-sectional geometry using a nonlinear programming technique. Harajli and Naaman (1985) experimentally investigated the fatigue behavior of twelve different sets of PPC beams. Al-Zaid and Naaman (1986) developed a general analysis procedure that is based on equilibrium and compatibility for cracked prestressed and PPC composite sections in the elastic range of behavior. Dilger and Suri (1986) represented two methods to directly calculate the steel stresses in PPC members. A simplified flexural design of PPC members was suggested by Peterson and Tadros (1986). Abendroth and Salmon (1986) dealt with a parametric study on the sensitivity of the optimum cost of

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partially or fully endrestrained RC T-section beams in terms of various parameters such as allowable deflections, material strength, support conditions, and unit material costs. Harajli and Alameh (1989) developed a theoretical model for computing the load-deflection response of progressively cracking PPC beams. Harajli and Hijazi (1991) evaluated the ultimate steel stress in PPC members. Khaleel and Itani (1993) presented a comprehensive study on the optimization of simply supported PPC girders by using sequential quadratic programming. Naaman and Hamza (1993) dealt with the prestress losses in partially prestressed high strength concrete beams. In the study, the results of a comprehensive parametric study on prestress losses in prestressed and partially prestressed high strength concrete beams are reported. Al-Gahtani *et al.* (1995) dealt with an effective formulation for optimum design of two-span continuous PPC beams. Han *et al.* (1996) presented the minimum cost design of multispan PPC beams using discretized continuum-type optimality criteria (DCOC). Chowdhury (1999) studied about the damping characteristics of reinforced and PPC beams. Agrawal and Bhattacharya (2010) described the methodology in details for developing a set of optimal reliability-based partial safety factors for given limit state, load combination and target reliability. Lee and Park (2011) studied about a unified procedure to combine both topology and shape optimization by using genetic algorithm. Toğan *et al.* (2011) studied about the minimum weight of a space truss under the uncertainties on the load, material and cross-section areas with harmony search using reliability index and performance measure approaches. Zandi *et al.* (2012) dealt with the use of high performance concrete in partially prestressed beams and optimization of partially prestressed ratio. In the book of Naaman (2012) namely “Prestressed Concrete Analysis and Design”, the relevant structural behavior equations of PPC beams were studied in details. Toğan (2013) developed a procedure employing a Teaching-Learning Based Optimization method to design discrete pin jointed structures. Dede and Ayvaz (2013) dealt with a new efficient optimization algorithm called Teaching-Learning-Based Optimization (TLBO) for the least weight design of trusses with continuous design variables. Majumdar *et al.* (2013) presented a numerical method to detect and assess structural damages from changes in natural frequencies using Ant Colony Optimization algorithm. Karayannis and Chalioris (2013) dealt with the design of PPC beams based on the cracking control provisions. Kutylowski and Rasiak (2014) dealt with the design of bridge girder structures by applying topology optimization for obtaining optimal bridge girder designs. Niğdeli *et al.* (2015) proposed a novel optimization approach for RC biaxially loaded columns. Temur and Bekdaş (2016) proposed a methodology based on Teaching Learning-Based Optimization algorithm for optimum design of RC retaining walls. Artar (2016) presented an optimization process using Harmony Search Algorithm for minimum weight of steel space frames under earthquake effects according to Turkish Earthquake Code (2007) specifications. Türkeli (2016) dealt with the cost optimization of PPC beams by using different optimization techniques. Xu *et al.* (2016) introduced a new formula to the discovering probability process to improve the convergence rate and the Tournament Selection Strategy to enhance global search ability of the Cuckoo search algorithm. Kaveh and Shokohi (2016) applied the recently developed meta-heuristic algorithm called tug of war optimization to optimal design of castellated beams. Du *et al.* (2016) developed a method to convert the cross sectional area of unbonded prestressed tendons to the equivalent cross sectional area of non-prestressed steel in order to determine the deflection of unbonded PPC continuous beams. Esfahani *et al.* (2016) evaluated the seismic response of PPC and prefabricated PPC frames and damage plasticity theory is implemented in order to investigate the behavior of concrete in tension and compression.

The purpose of this study is to perform minimum cost design of I cross-sectioned PPC beams by using Genetic Algorithms (GA). For this purpose, two I cross-sectioned PPC beams are selected from the technical literature as the case study. Then, the objective function, design variables, design parameters, structural and behavioral constraints are determined and a MATLAB (2008) code is developed in order to perform optimization process by using GA. At the end of the process, the obtained results are compared with the ones given in the technical literature.

In technical literature, this study comprises a place that is being the first about the optimum design of PPC beams by using GA. Also, by this study, it is shown that GA can be used effectively in the optimum cost design I-crossectioned PPC beams.

2. Genetic algorithms

The Genetic Algorithms was developed by John Holland (1989). In technical literature, this optimization algorithm is known as a frequently preferred and used among the other ones. Therefore, it is not needed to give detailed procedures or information about GA. In the GA described in this study, only the relevant and important procedures were proposed. In this study, adaptable penalty function method developed by Toğan and Daloğlu (2006) was used. According to this developed method, the penalty coefficient can be determined by using Eq. (1).

$$PC = \begin{cases} g(i) \geq g_{avg} \Rightarrow (g_{\max} + g(i))/(g_{\max} - g_{avg}) \\ g(i) < g_{avg} \Rightarrow (g_{avg} + g(i))/(g_{avg} - g_{\min}) \\ g(i) = 0 \Rightarrow 0 \end{cases} \quad (1)$$

In Eq. (1), $g(i)$, $g_{\min}$, g_{avg} and $g_{\max}$ are denoting the violation amount of i^{th} constraint, the minimum, average and maximum violation value in the generation, respectively. By using penalty coefficient given in Eq. (1), penalized objective function can be found from Eq. (2). In Eq. (2), $f(x)$ is denoting the objective function.

$$\Phi(x) = f(x) \cdot (1 + PC) \quad (2)$$

Also, the fitness degree and the fitness value of an individual in the generation can be determined from Eqs. (3) and (4) in which $\Phi_{\max}(x)$, $\Phi_{\min}(x)$ and f_{avg} are denoting the maximum and minimum penalized objective function value in the generation, the average fitness degree of individuals in the generation, respectively.

$$f_i^d(x) = \left[\Phi_{\max}(x) + \Phi_{\min}(x)\right] - \Phi_i(x) \tag{3}$$

$$f_i^{'}(x) = \frac{f_i^d(x)}{f_{avg}} \tag{4}$$

The other adaptable techniques used in the GA given in this study are cross-over and mutation techniques. According to these methods, the cross-over (p_c) and mutation (p_m) probability values can be found from Eqs. (5) and (6), respectively.

$$p_c = \begin{cases} f_m \geq f_{avg} \Rightarrow (f_{\max} - f_m)/(f_{\max} - f_{avg}) \\ f_m < f_{avg} \Rightarrow 1.0 \end{cases} \tag{5}$$

$$p_m = \begin{cases} f^{'} \geq f_{avg} \Rightarrow 0.5 \cdot (f_{\max} - f^{'})/(f_{\max} - f_{avg}) \\ f^{'} < f_{avg} \Rightarrow (f_{avg} - f^{'})/(f_{avg} - f_{\min}) \end{cases} \tag{6}$$

In Eqs. (5) and (6), f', $f_{\min}$, f_{avg}, $f_{\max}$ and f_m are denoting, fitness value of any individual, minimum, average and maximum fitness value in a generation, the value of cross-overed individual that has the smallest fitness value, respectively. For the purpose of this study, it is unnecessary to give the detailed version of the GA used in the optimization process. Therefore, the adaptive Pseudo-code of GA algorithm used in this study is given below:

```
Initialize the population
Evaluate the population
cycle=1
While cycle=MCN do
    Set the adaptive penalty coefficient by Eq. (1)
    Calculate penalized objective functions by Eq. (2)
    Calculate fitness by Eq. (4)
    Select parents for crossover
    Set the crossover and mutation probability by Eqs. (5-6)
    Perform crossover and mutation
    Evaluate population
    Memorize the best solution
    cycle =cycle + 1
EndWhile
```

3. Cost optimum design of I-crossectioned PPC beams

3.1 Objective function

The objective of this study is to achieve the minimum cost design of I crossectioned PPC beams without violating the constraints. Therefore, the objective function includes the cost of prestressing steel (C_p), the cover of prestressing steel (C_{cov}), labor of placing prestressing steel (C_{lab}), prestressed concrete (C_c), placement of precast beam (C_{place}), ordinary reinforcing steel (C_{ste}). The ordinary reinforcing steel can be either in tension (C_{ste1}) or in compression (C_{ste2}) region of the beam. The objective

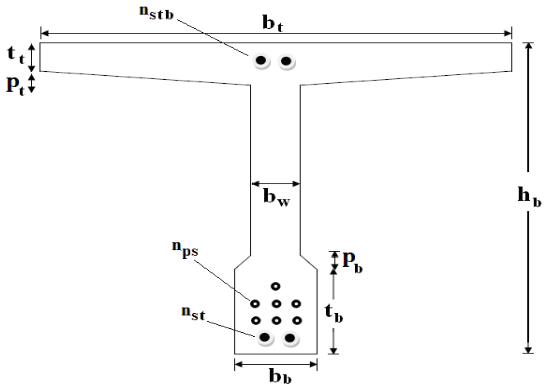

Fig. 1 A typical cross-section of a PPC beam with design variables

function of the optimized beam that includes ordinary reinforcing steel only in tension region is given in Eq. (7).

$$f(x) = C_p + C_{\text{cov}} + C_{lab} + C_c + C_{place} + C_{ste} \tag{7}$$

Moreover, the objective function of the optimized beam that includes ordinary reinforcing steel both in tension and in compression region is given in Eq. (8).

$$f(x) = C_p + C_{\text{cov}} + C_{lab} + C_c + C_{place} + C_{ste} + C_{ste2} \tag{8}$$

3.2 Design variables

In this study, design variables were classified into two categories namely variables associated with dimensions of the PPC beam and the number of utilized ordinary reinforcing and prestressed steel bars. The design variables associated with dimensions of the PPC beam are: Upper flange thickness (t_t), width of upper flange (b_t), bottom flange thickness (t_b), width of bottom flange (b_b), web width (b_w), total height of the PPC beam (h_b), top slopped thickness (p_t) and bottom slopped thickness (p_b). Moreover, the design variables associated with the number of utilized ordinary reinforcing and prestressed steel are: Number of prestressing steel bars (n_{ps}), number of ordinary reinforcing steel bar in tension region of the PPC beam (n_{st}) and number of ordinary reinforcing steel bar in compression region of the PPC beam (n_{stb}). The design variables cited above are highlighted on a typical cross-section of a PPC beam in Fig. 1.

3.3 Design parameters

In this study, the design parameters (*constant throughout the optimum design process*) are previously selected bridge dimensions, the mechanical and structural properties of materials, loads, the unit costs of labor and materials, other related design parameters. The design parameter related with the bridge dimensions is the total span length (L). Also, design parameters related with the mechanical and structural properties of materials are unit weight of ordinary reinforcing steel (γ_s), characteristic compressive strength of concrete ($f_c^{'}$), modulus of rupture of concrete (f_r), diameter of ordinary reinforcing steel (d_{ste}), diameter of prestressed steel (d_{pre}), unit weight of concrete (γ_c), the modulus of elasticity of concrete (E_c), prestressing steel (E_{ps}) and ordinary reinforcing steel (E_s), creep coefficient (C_u), ultimate strength of prestressing steel (σ_{pu}), yield strength of ordinary reinforcing steel (f_y) and material factors for concrete (m_c) and ordinary reinforcing steel (m_s). The design parameters related with the loads are: Superimposed dead load (w_{SD}), moment due to superimposed dead load (M_{SD}), live loads (w_L) and moment due to live load (M_L). Also, the unit costs of labor and materials were determined from the source published from Turkish Republic of General Directorate of Highways (GDH 2014). Other related design parameters are: Net concrete cover (t_{cov}), initial prestressing force at the time of release (P_i), capacity reduction factor used for PPC beams ($\emptyset$), factor used for normal weight concrete (λ) and prestress loss ratio occurred in prestressed steel (R).

3.4 Constraints

In this study, the constraints are considered consistent with the studies published in the technical literature (Saouma and Murad 1984, Nawy 2003, Aydın 2006) and provisions given in national and international specifications (TS 1979, AASHTO 2002, ACI 2002). These 16 constraints are evaluated under eight categories given below:

1- Flexural stress constraints.
2- Initial camber constraint.
3- Dead and live load deflection constraints.
4- Ductility constraints.
5- Shear strength constraint.
6- Flexural ultimate strength constraint.
7- Beam geometrical constraints.
8- Ordinary reinforcing steel constraints (*fitting to beam cross-section*).

These constraints cited above are explicitly given below.

3.4.1 Flexural stress constraints

In this study, flexural stresses occurred on top and bottom fibers of the PPC beams are checked in two different stages namely initial stage and service stage. At initial stage, only the dead load and initial prestressing force is acting on the beam. However, at service stage, besides dead load and initial prestressing force, the superimposed dead load, live load and prestress losses were taken into account. According to these stages explained above, the flexural stress constraints at initial and service stage in normalized form are given in Eq. (9)-Eq. (12)

$$g_1 = \frac{\sigma_i^t}{\sigma_{ti}} - 1 \tag{9}$$

$$g_2 = \frac{\sigma_i^b}{\sigma_{ci}} - 1 \tag{10}$$

$$g_3 = \frac{\sigma_s^t}{\sigma_{cs}} - 1 \tag{11}$$

$$g_4 = \frac{\sigma_s^b}{\sigma_{ts}} - 1 \tag{12}$$

where σ^t and σ^b are in order denoting the stress occurred on top and bottom fibers of the beam, σ_c and σ_t are also denoting the allowable compression and tension stresses where the initials **i** and **s** are representing the initial and service stage, respectively.

3.4.2 Initial camber constraint

At initial stage, the prestressing force applied to the beam caused an upward camber which is opposed by the deflection due to the self-weight of the beam. The sum of these downward and upward deflections (*self-weight plus initial camber due to prestressing force*) is controlled by the maximum allowable camber. The initial camber constraint is given in Eq. (13)

$$g_5 = \frac{\Delta_{wo} - \Delta_{pi}}{\Delta_{up}^{\max}} - 1 \tag{13}$$

where Δ_{wo}, Δ_{pi} and $\Delta_{\mathrm{up}}^{\max}$ are denoting the deflection due to the self-weight of the beam, the upward camber due prestressing force and maximum allowable camber, respectively. The maximum allowable camber is determined as L/180 in which L is denoting the length of the beam.

3.4.3 Dead and live load deflection constraints

The constraints including long term dead load deflection with time dependent effects (Δ_{DL}) and the superimposed live load deflections (Δ_{LL}) are given in Eq. (14) and Eq. (15), respectively.

$$g_6 = \frac{\Delta_{DL}}{\Delta_{DL}^{\max}} - 1 \tag{14}$$

$$g_7 = \frac{\Delta_{LL}}{\Delta_{LL}^{\max}} - 1 \tag{15}$$

In Eq. (14) and Eq. (15), $\Delta_{\mathrm{DL}}^{\max}$ and $\Delta_{\mathrm{LL}}^{\max}$ are denoting the maximum allowable dead and live load deflections which are determined as L/180 and L/240, respectively. In these equations, L is denoting the length of the beam.

3.4.4 Ductility constraints

The minimum and maximum reinforcement limitation of the PPC beams are determined by using the constraints given in Eq. (16) and Eq. (17), respectively.

$$g_8 = \frac{1.2 \cdot M_{cr}}{\phi M_n} - 1 \tag{16}$$

$$g_9 = \frac{c}{d_e} - 0.42 \tag{17}$$

In Eq. (16) and Eq. (17), M_{cr}, M_n, $\emptyset$, c and d_e are denoting the value of the moment that cause flexural cracking at the section, the nominal ultimate moment resistance of the cross-section, capacity reduction factor taken as 0.9 for PPC beams, the depth of neutral axis at ultimate strength of the beam and the distance from the extreme compression fiber to the centroid of the tensile force resultant of the tensile reinforcement, respectively.

3.4.5 Shear strength constraint

The maximum factored shear force (V_{max}) is checked by using the nominal shear strength of the cross-section (V_c) which is given in Eq. (18).

$$g_{10} = \frac{V_{max}}{V_c} - 1 \tag{18}$$

3.4.6 Flexural ultimate strength constraint

The maximum moment due to externally applied loads (M_{max}) is controlled with the nominal ultimate moment capacity of the cross-section (M_n) given in Eq. (19).

$$g_{11} = \frac{M_{max}}{M_n} - 1 \tag{19}$$

In Eq. (19), the nominal ultimate moment capacity of the cross-section (M_n) is determined by using the formulae given in Naaman (1992).

3.4.7 Beam geometrical constraints

The geometrical constraints of the beam are related with the size and the adequate thickness for the prestressing and ordinary reinforcing steel to fit to the cross-section. In Eq. (20), the width of the top flange is controlled with the width of the bottom flange.

$$g_{12} = \frac{b_b}{b_t} - 1 \tag{20}$$

In Eq. (20), the width of the top flange (b_t) should be larger than the width of the bottom flange (b_b). Also, the width of bottom flange (b_b) should be larger than the width of the web (b_w) given with Eq. (21).

$$g_{13} = \frac{b_w}{b_b} - 1 \tag{21}$$

In the tension region of the beam (*i.e., the bottom of the PPC beam*), the minimum thickness of the bottom flange should be adequate in order to place concrete among prestressing and ordinary reinforcing steels homogeneously. This can be achieved by using Eq. (22).

$$g_{14} = \frac{t_{b\,min}}{t_b} - 1 \tag{22}$$

In this study, the net concrete cover and the center to center spacing of the both reinforcements (*conventional and prestressed*) are used as 38 mm. and 50 mm., respectively.

3.4.8 Ordinary reinforcing steel constraints (fitting to beam cross-section)

In this study, the number of rows that ordinary reinforcing steels placed in tension (SSs) and compression (SSs2) region of the PPC beam are controlled with Eq. (23) and Eq. (24), respectively.

$$g_{15} = \frac{SSs}{SSs_{max}} - 1 \tag{23}$$

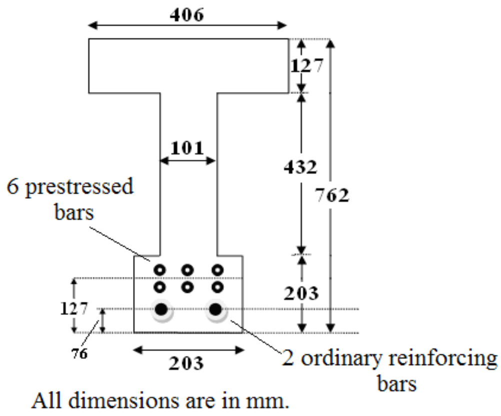

Fig. 2 Typical cross-section of the PPC beam

$$g_{16} = \frac{SSs2}{SSs_{max}2} - 1 \tag{24}$$

In Eq. (23) and Eq. (24), SSs_{max} and $SSs_{max}2$ are denoting the maximum number of rows that ordinary reinforcing steels placed in tension and compression region of the PPC beam and taken as 2.0.

4. Numerical examples

In order to demonstrate the performance of GA on the optimum design of I cross sectioned PPC beams, two design problems were selected from the technical literature (Nilson 1976, Saouma 1984). The problems selected seem to be outdated but in technical literature, it is difficult to obtain a PPC beam problem that includes all the relevant data used in the optimization process. The objective functions, design parameters, constraints and design variables were determined separately for the two numerical problems. Also, a software package was developed in MATLAB (2008) to perform the optimization of the selected PPC beams by using GA (Türkeli 2016).

4.1 Numerical example 1

The first PPC beam design problem is selected from the exercise problems of the study of Nilson (1976). In this problem, ordinary reinforcing steel is placed only in the tension region of the beam. In Fig. 2, the typical cross-section of the PPC beam represented from the Nilson (1976) is given.

In Eq. (7), the objective function of the first PPC beam design problem is given. Also, the first fifteen constraints given in the preceding parts of the study are used as the constraints of the first PPC beam design problem due to the reason that there is no ordinary reinforcing steel in the compression region of the beam. Moreover, the design parameters (*unchanged throughout the optimum design process*) used in the study of Nilson (1976) are given below.

Beam span length (L) :11.4 m

Table 1 The design variables for the first PPC design problem

Design Variable	Values	Number of Value
Upper flange thickness (t_t) (mm)	80; 100; 120; 150; 180; 200; 220; 250; 300; 350; 400; 450; 500	13
Width of upper flange (b_t) (mm)	200; 250; 300; 350; 400; 450; 500; 550; 600; 650	10
Bottom flange thickness (t_b) (mm)	50; 100; 150; 200; 220; 250; 280; 300; 350; 400; 450; 500; 550; 600; 650	15
Width of bottom flange (b_b) (mm)	50; 100; 150; 200; 250; 300; 350; 400; 450; 500; 550; 600; 650; 700; 750; 800	16
Web width (b_w) (mm)	100; 110; 120; 130; 140; 150; 160; 170; 180; 190; 200; 210; 220; 230; 240; 250; 260; 270; 280; 290; 300	21
Total height of the PPC beam (h_b) (mm)	400; 450; 500; 550; 600; 650; 700; 750; 800; 850; 900; 950; 1000; 1050; 1100; 1150; 1200; 1250; 1300; 1350; 1400	20
Number of prestressing steel bars (n_{ps})	2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 13; 14; 15; 16; 17; 18; 19; 20; 21; 22; 23; 24; 25; 26; 27; 28; 29; 30; 31; 32; 33	32
Number of ordinary reinforcing steel bar in tension region of the PPC beam (n_{st})	2; 4; 6; 8; 10; 12; 14; 16;	8

Concrete cover (t_{cov}) : 38 mm
Unit weight of ordinary reinforcing steel (γ_s) : 7.85 tons/m^3
Initial prestress force (P_i) : 683.75 kN
Compressive strength of concrete (f_c'): 35 MPa
Modulus of rupture of concrete (f_r) : 3.5 MPa
The diameter of ordinary reinforcing steel (d_{ste}) : 25.4 mm
Capacity reduction factor ($\emptyset$) : 0.9
Modulus of elasticity of prestressed steel (E_{ps}) : 186000 MPa
Factor for normal weight concrete (λ) : 1.0
Diameter of prestressed steel (d_{pre}) : 12.7 mm
Unit weight of concrete (γ_c) : 25 kN/m^3
Modulus of elasticity of concrete (E_c) : 24900 MPa
Modulus of elasticity of ordinary reinforcing steel (E_s) : 200000 MPa
Ultimate strength of prestressed steel (σ_{pu}) : 1725 MPa
Yield strength of ordinary reinforcing steel (f_y) : 414 MPa
Creep coefficient (C_u) : 1.8
Moment due to superimposed dead load (M_{SD}) : 52 kN.m
Moment due to live load (M_L) : 259 kN.m
Ratio of prestress losses occurred in prestressed steel (R) : 0.8

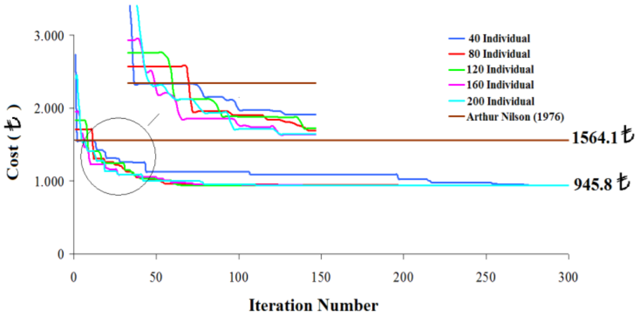

Fig. 3 The variation of the cost of the first PPC design problem with iterations

Also, the unit cost of the labor and material costs (for year 2014) are determined as follows in Turkish Liras (₺) (Currency in 2017: 1 \$ ≈ 3.5 ₺).

Unit cost of ordinary reinforcing steel: 2,226.24 ₺/tons
Unit cost of labor of prestressed steel : 6,416.99 ₺/tons
Unit cost of high strength prestressed steel : 2,812.50 ₺/tons
Unit cost of cover of prestressed steel : 1.96 ₺/m
Unit cost of prestressed concrete : 309.66 ₺/m^3
Unit cost of placement of PPC beams to the bridge : 27.13 ₺/tons

The design variables shown in Fig. 1 is valid for the first PPC beam design problem except the ordinary reinforcing steel found in the compression region. The design variables for the first PPC beam design problem are given in Table 1.

As cited before, the GA optimization of the first PPC beam design problem was performed by developing a software package problem in MATLAB (2008). In this problem, self-adaptive penalty function method was utilized in processing the constraints (Toğan and Daloğlu 2006). In order to determine the effect of number of individuals in generation in GA on optimum solution, the generations that include 40, 80, 120, 160 and 200 individuals were used for each generation, 100 independent runs of the program were carried out. After approximately 300 iterations, all generations that include 40, 80, 120, 160 and 200 individuals converge to the same optimum solution. For every generation considered in GA, the convergences of the algorithm to the optimum solution were given in Fig. 3.

From Fig. 3, it can be clearly seen that the generation that includes 40 individuals converges slowly to the optimum solution compared to the other generations that include different sizes of individuals. Also, the speed of convergence of generations that include 80, 120, 160 and 200 individuals is nearly the same with each other. Moreover, by interpreting the obtained results, the generations that include 40, 80, 120, 160 and 200 individuals reached to the same optimum cost result i.e., 945.8 ₺. This optimum cost is 39 % economical compared to the cost of PPC beam designed by Nilson (1976) which is 1564.1 ₺. This dramatic decrease in the cost is obtained by changing the dimensions of the beam cross-section and decreasing the number of prestressed steel without violating

Table 2 Comparison of the results of this study with Nilson (1976)

Design Variables	t_t	b_t	t_b	b_b	b_w	h_b	n_{ps}	n_{st}
Nilson (1976)	127	406	203	203	101	760	6	2
Optimum Design (This Study)	80	250	200	150	100	900	2	2

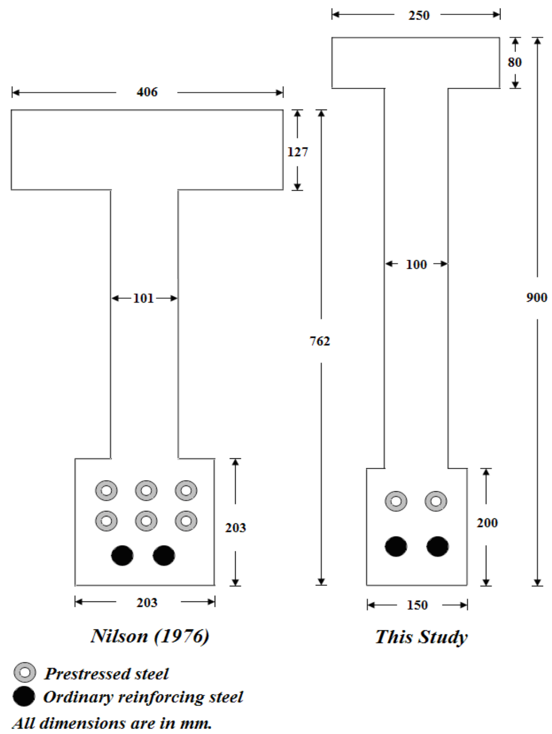

Fig. 4 The schematically view of the compared results

Table 3 Mean, standard deviation and optimum results of the first PPC beam design

The number of individuals	40	80	120	160	200
Mean (₺)	982,05	952,09	947,97	947,71	947,27
Standart Deviation (₺)	47,42	21,04	8,22	10,05	10,03
Optimum Result (₺)	945,84	945,84	945,84	945,84	945,84

any constraint. In Table 2, the results obtained at the end of the optimum design process are compared with the ones provided by Nilson (1976).

The compared results given in Table 2 are given in Fig. 4 schematically.

On the other hand, after 100 runs with 800 iterations, the mean, standard deviation and the optimum results are given in Table 3.

From Table 3, it can be clearly seen that the mean values of the iterations converge to the optimum solution as the number of individuals in the generations increase. In another words, the convergence of the mean of the generations to the optimum solution increase as the number individuals increase from 40 to 200. The other parameter that should be evaluated is the standard deviation of the generations. As the mean values, the standard deviations of the generations decrease from 40 to 200. This shows that as the number of individuals increase in the generation, the probability of the generations converge to the optimum solution increase. Although the probability to the convergence to optimum solution increase as the number of individuals in the generation increase, the total time of the run increase also which can be accepted as the disadvantage of the GA. Moreover, the partially prestressed ratio (PPR), a parameter utilized in the decision of the behavior of the PPC beam, found as 0.446 for the first PPC beam design problem. Moreover, the value of PPR is varying from 0 to 1.0. The values of PPR near to zero means that the beam shows a structural behavior that resembles to the behavior of RC beams. For the first PPC problem, by interpreting the result of PPR, the behavior of the first PPC beam design problem is nearly same with the RC beam.

4.2 Numerical example 2

The optimized PPC I cross-sectioned beam design problem in the numerical examples of the study of Saouma and Murad (1984) by using interior penalty function method is selected as the second PPC beam design problem for optimization. Contrary to first numerical example, the optimized PPC beam contains ordinary reinforcing steel in both tension and compression region. Also, the top and bottom slopped thickness are taken into account as design variables. In Eq. (8), the objective function of the second PPC beam design problem is given with the ordinary reinforcing steel in compression region of the beam. Also, in this example, the sixteen constraints given in the preceding parts of the study are valid and used as the constraints. Moreover, the design parameters used in the study of Saouma and Murad (1984) are given below.

Beam span length (L) : 24.400 m
Concrete cover (t_{cov}) : 38 mm
Unit weight of ordinary reinforcing steel (γ_s) : 7.85 tons/m^3
Compressive strength of concrete (f_c') : 34.5 MPa
Modulus of rupture of concrete (f_r) : 3.5 MPa
The diameter of ordinary reinforcing steel (d_{ste}) : 12.7 mm
Capacity reduction factor ($\emptyset$) : 0.9
Factor for normal weight concrete (λ) : 1.0
Diameter of prestressed steel (d_{pre}) : 15.24 mm
Unit weight of concrete (γ_c) : 20 kN/m^3
Modulus of elasticity of concrete (E_c) : 27169 MPa
Modulus of elasticity of prestressed steel (E_{ps}) : 207300 MPa
Modulus of elasticity of ordinary reinforcing steel (E_s) : 200000 MPa
Ultimate strength of prestressed steel (σ_{pu}) : 1725 MPa
Yield strength of ordinary reinforcing steel (f_y) : 414 MPa
Creep coefficient (C_u) : 1.0
Uniform superimposed dead load (w_{SD}): 5.4 N/mm
Uniform live load (w_L) : 17.5 N/mm
Ratio of prestress losses occurred in prestressed steel (R) : 0.85

Due to the reason that the fabrication of the ordinary reinforcing steel cannot be longer than 12.0 meters, for the

Table 4 The design variables for the second PPC design problem

Design Variable	Values	Number of Value
Upper flange thickness (t_t) (mm)	100; 110; 120; 130; 140; 150; 160; 170; 180; 190; 200	11
Top slopped thickness (p_t) (mm)	50; 60; 70; 80; 90; 100; 125; 150	8
Bottom flange thickness (t_b) (mm)	100; 110; 120; 130; 140; 150; 160; 170; 180; 190; 200	11
Bottom slopped thickness (p_b) (mm)	50; 60; 70; 80; 90; 100; 125; 150	8
Width of bottom flange (b_b) (mm)	150; 200; 250; 300; 350; 400; 450; 500; 550; 600; 650; 700	12
Web width (b_w) (mm)	150; 160; 170; 180; 190; 200; 225; 250; 300; 350; 400; 450	12
Total height of the PPC beam (h_b) (mm)	500; 550; 600; 650; 700; 750; 800; 850; 900; 950; 1000; 1050; 1100; 1150; 1200; 1250; 1300; 1350; 1400; 1450; 1500	21
Number of prestressing steel bars (n_{ps})	2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 13; 14; 15; 16; 17; 18; 19; 20; 21; 22; 23; 24; 25; 26; 27; 28; 29; 30; 31; 32; 33	32
Number of ordinary reinforcing steel bar in tension region of the PPC beam (n_{st})	2; 4; 6; 8; 10; 12; 14; 16; 18; 20; 22; 24; 26; 28; 30; 32	16
Number of ordinary reinforcing steel bar in compression region of the PPC beam (n_{stb})	2; 4; 6; 8; 10; 12; 14; 16; 18; 20; 22; 24; 26; 28; 30; 32	16
Width of upper flange (b_t) (mm)	1000; 1050; 1100; 1150; 1200; 1250; 1300; 1350; 1400; 1450; 1500; 1550; 1600; 1650; 1700; 1750; 1800; 1850; 1900; 1950; 2000	21

beams that have the span longer than 12.0 meters, ordinary reinforcing steel in tension and compression region of the beams should be in two pieces. Therefore, these two pieces of ordinary reinforcing steel should be clamped to each other at enough length which can be determined according to Eq. (25).

$$l_b = 0.12 \frac{f_{yd}}{f_{ctd}} \phi \geq 20\phi \qquad (25)$$

In Eq. (25), f_{yd}, f_{ctd} and $Ø$ are denoting the design tensile strength of ordinary reinforcing steel, the design tensile strength of concrete and diameter of ordinary reinforcing steel, respectively (TS 2000).

The design variables shown in Fig. 1 are valid for the second PPC beam design problem. Also, the design variables for the second PPC beam design problem are given in Table 4.

Table 5 The results of this study and comparison with Saouma and Murad (1984)

Design Variables	t_t	p_t	t_b	p_b	b_b	b_w	h_b	n_{ps}	n_{st}	n_{stb}	b_t
Saouma and Murad (Initial Design) (1984)	152	50	152	-	356	356	1219	17	10	3	1778
Saouma and Murad (Optimum Design) (1984)	160	50	154	50	354	295	1231	16	1	0	1782
Optimum Design (This Study)	100	50	200	150	200	150	1450	10	2	2	1000

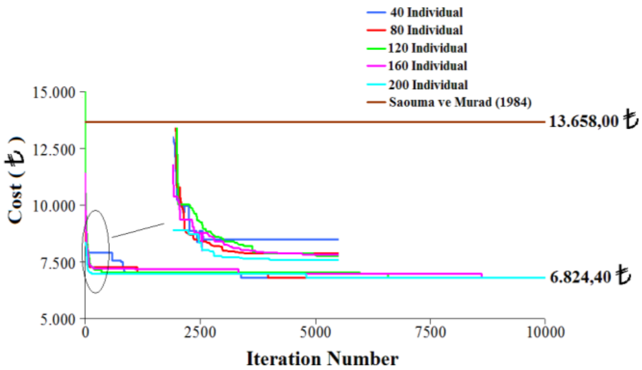

Fig. 5 The convergence to the optimum solution of the second PPC design problem with iterations

Table 6 Mean, standard deviation and optimum results of the second PPC beam design

The number of individuals	40	80	120	160	200
Mean (₺)	7.061,27	7.012,48	6.970,10	6.874,69	6.862,30
Standart Deviation (₺)	144,33	71,84	67,97	63,76	54,75
Optimum Result (₺)	6.824,40	6.824,40	6.824,40	6.824,40	6.824,40

In this numerical example, as in numerical example 1, a software package problem in MATLAB (2008) was developed in order to perform the GA optimization. Also, in processing the constraints, self-adaptive penalty function method developed by Toğan and Daloğlu (2006) was utilized. For each generation, 100 independent runs of the program were carried out.

By interpreting the obtained results, it can be clearly identified from Fig. 5 that the generations that include 40, 80, 120, 160 and 200 individuals converged to the same optimum cost result which is 6824.40 ₺. This obtained optimum result is 50 % economical compared to the cost of PPC beam i.e., 13658.00 ₺ provided by Saouma and Murad (1984). Different from Numerical Example 1, this PPC beam contains ordinary reinforcing steel in the compression region of the beam. This can be counted as the another reason of the dramatic decrease in the cost besides the reasons of the decrease which are changing the dimensions of the beam cross-section and decreasing the number of prestressed steel without violating any constraint.

The effect of number of individuals in generation in GA on optimum solution was tried to be determined by using generations that include 40, 80, 120, 160 and 200

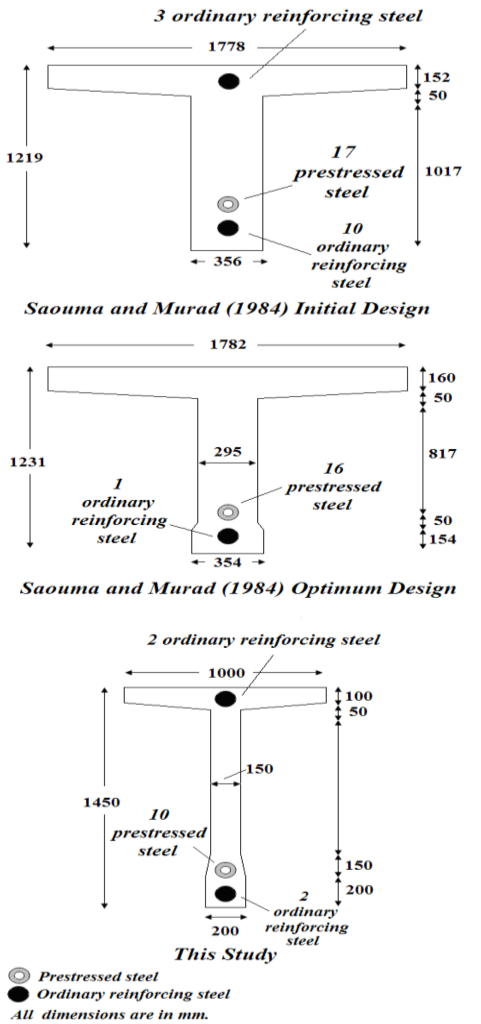

Fig. 6 The comparison of the obtained results with the results of Saouma and Murad (1984)

individuals. By iterating approximately 10000 times, all generations that include 40, 80, 120, 160 and 200 individuals converge to the same optimum solution. The convergences of the generations to the optimum solution with the iteration number are given in Fig. 5.

In Table 5, the results obtained at the end of the optimum design process are compared with the ones provided by Saouma and Murad (1984).

The compared results given in Table 5 are given in Fig. 6 schematically.

On the other hand, after 100 runs with 10000 iterations, the mean, standard deviation and the optimum results are given in Table 6.

From Table 6, it can be clearly seen that the convergence of the mean of the generations to the optimum solution increase as the number individuals increase from 40 to 200. Also, as expected, the standard deviations of the generations decrease from 40 to 200. This shows the effect of the number of individuals on the probability of the generations to converge to the optimum solution. In another words, the probability to the convergence to optimum solution increase as the number of individuals in the generation increase. Moreover, the partially prestressed ratio (PPR) was found as 0.9638 for the second PPC beam design problem which shows that the behavior of the second PPC beam design problem is nearly same with the prestressed beam.

5. Conclusions

The aim of this study is to minimize the cost of PPC I cross sectioned beams by using the global optimization algorithm i.e., GA. For this purpose, two example PPC beam problems that have different behavioral characteristics are selected from the literature. Then, a software package program is constituted in MATLAB with the objective function, constraints, design variables and design parameters. The software package was run many times and some results were obtained. The main conclusions and suggestions derived from this study were summarized as follows.

• By using the proposed GA on the two example PPC beam problems selected from the technical literature, 39 % and 50 % more economical results were obtained for Numerical Example 1 and 2 without violating any constraints, respectively. This shows that GA can be effectively used in the cost optimization of PPC beams.

• The main advantage of PPC beams is to decrease the amount of prestressed steel and its labor by replacing with the ordinary reinforcing steel considering them in the load carrying capacity of the beam which is the main reason of this dramatic cost decrease without violating any constraint.

• The partially prestressed ratios (PPR), a parameter utilized in the decision of the behavior of the PPC beam, were found as 0.446 and 0.9638 for the first and second numerical examples, respectively. This shows that the PPR ratio is depending on the type of the PPC problem. Therefore, the use of PPR ratio as the design variables can lead to uneconomical solutions.

• The effect of number of individuals in the generations on the optimum solution is tried to be identified by selecting the generations that include 40, 80, 120, 160 and 200 individuals. In every run of the program, the probability of the generation that has more individuals than the others to converge to the optimum solution is greater compared with the others. Therefore, the convergence of the mean of the generations to the optimum solution increase as the number individuals increases from 40 to 200. As expected, contrary to the mean values of the generations, the standard deviation is greater in the generation that has lower individuals among others. Although the probability to converge to the optimum solution increase as the number of individuals increase, the total time elapsed for the completion of the run increase also. This can be counted as the disadvantage of the GA.

• The solutions obtained from the GA can be used without any modification because of the use of discrete design variables and their probable value sets.

• GA can be used effectively in the optimum cost design I-crossectioned PPC beams.

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This study is dedicated to the venerable memory of our light, great scientist Prof. Dr. Ing. Ahmet DURMUŞ whom we lost in 07.03.2017.

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