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Study of modified Westergaard formula based on dynamic model test on shaking table

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Abstract. The dynamic model test of dam-reservoir coupling system for a 203m high gravity dam is performed to investigate effects of reservoir water on dynamic responses of dam during earthquake. The hydrodynamic pressure under condition of full reservoir, natural frequencies and acceleration amplification factors along the dam height under conditions of full and empty reservoir are obtained from the test. The results indicate that the reservoir water have a stronger influence on the dynamic responses of dam. The measured natural frequency of the dam model under full reservoir is 21.7% lower than that of empty reservoir, and the acceleration amplification factor at dam crest under full reservoir is 18% larger than that under empty reservoir. Seismic dynamic analysis of the gravity dams with five different heights is performed with the Fluid-Structure Coupling Model (FSCM). The hydrodynamic pressures from Westergaard formula are overestimated in the lower part of the dam body and underestimated in its upper part to compare with those from the FSCM. The underestimation and overestimation are more significance with the increase of the dam height. The position of the maximum hydrodynamic pressure from the FSCM is raised with the increase of dam height. In view of the above, the Westergaard formula is modified with consideration in the influence of the height of dam, the elasticity of dam on the hydrodynamic pressure. The solutions of modified Westergaard formula are quite coincident with the hydrodynamic pressures in the model test and the previous report.

Keywords: dynamic model test; Fluid-Structure Coupling Model; hydrodynamic pressure; modified Westergaard formula; shaking table

1. Introduction

The hydrodynamic pressure of reservoir water on upstream surface of dam has a great effect on the dynamic response of the dam under the earthquake. Therefore, the dam-reservoir interaction must be taken into account in the analysis of the seismic response of the dam.

Since the Westergaard formula (Westergaard 1933) to calculate the hydrodynamic pressure was published in 1933, the dynamic effect of reservoir water on the dam during earthquake has been widely concerned by dam engineering scholars. However, Westergaard's study on the hydrodynamic pressure is based on the assumptions that the reservoir water is non rotating, non stickiness, can be slightly compressed liquid, and the effect of surface wave of reservoir is ignored, and the reservoir boundary is infinite in the direction of the upstream, and the reservoir bottom is a rigid non energy absorbing plane, and the dam is rigid and the upstream surface of dam is vertical. Although the Westergaard formula for hydrodynamic pressure can reflect the some essential characteristics of the effect of reservoir water on dam body, the formula is too idealistic because of too many assumptions.

Many scholars have conducted the researches in Many scholars have done a great deal of research on the dam-reservoir interaction in order to get rid of the adverse influences of the assumptions. Altunisik and Sesli (2015) applied different mathematical and analytical modelling approaches to analysis the hydrodynamic pressure effect on the dam body. It is aimed to determine the dynamic response of concrete gravity dams using different water modelling approaches such as Westergaard, Lagrange and Euler. They found that the finite element model based on Lagrange and Euler approaches were closer to the real situation of Dam-reservoir interaction during the earthquake. Lotfi and Samii (2012) proposed a referred to as the wavenumber approach for dynamic analysis of dam-reservoir systems in the context of pure finite element programming. The response of an idealized triangular dam-reservoir system is obtained by this approach, and the results are compared against the exact response. It is concluded that the approach can be envisaged as a great substitute for the mathematical formulation. Vahid Lotfi (2005) carried out dynamic analysis of dam-reservoir-foundation system by employing a simplified and approximate one-dimensional model to account for fluid-foundation interaction. The approximation introduced on this basis is examined thoroughly by comparing the method with the rigorous approach. It is concluded that the errors due to approximate method could be very significant both for horizontal and vertical ground motions. Ziaolhagh and

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Goudarzi *et al*. (2016) conducted the free vibration analysis of a coupling system of flexible gravity dam-reservoir of compressible water. The dam-reservoir system was modeled by solely one 21-node element. It is clearly concluded that the one high-order element treats more precisely than the eight-node elements, since the first one utilizes fifth-degree polynomials to construct the shape functions and the second implements polynomials of degree two. Akpinar and Binici *et al*. (2014) adopted frequency domain methods that rigorously incorporate dam-reservoir-foundation interaction and time domain methods with approximate hydrodynamic foundation interaction effects to investigate earthquake induced stresses and effective damping on concrete gravity dam. They proposed a new effective damping prediction equation to estimate earthquake stresses accurately with the approximate time domain approach. Shariatmadar and Mirhaj (2011) explained briefly the basic equation involved the water-structure-foundation interaction and the effective factors for concrete dams. The finite element modeling of gravity dam-reservoir-foundation coupled system with 5 m, 150 m height were investigated on modal characteristic of dams. The analytical results obtained from numerical studies and modal analysis show that the accurate modeling of dam-reservoir-foundation and their interaction considerably affects the modal periods, mode shapes and modal hydrodynamic pressure distribution. Haciefendioglu and Bayraktar (2010) examined the ice cover effects on the seismic response of gravity dam-reservoir-foundation interaction systems subjected to a horizontal earthquake ground motion by using the Lagrangian fluid and solid-quadrilateral-isoparametric finite elements for *Sarıyar* gravity dam in Turkey.

There are a lot of controversy about whether the influence of reservoir water compressibility on the dynamic response of dam during earthquake can be neglected in dam engineering. When Permumalswami and Kar (1973) studied the hydrodynamic pressure on arch dam, they concluded that ignoring the compressibility of the reservoir water would underestimate the hydrodynamic pressure. Through the experimental study, Selby and Seven (1972) found that the hydrodynamic pressure without considering reservoir water compressibility was consistent with the analytical solution of the Laplace equation, so the water compressibility may be not considered in the dynamic analysis of dam-reservoir coupling system. When the Nath and Potamitis (1982) studied the hydrodynamic pressure on arch dam, they believed that ignoring water compressibility did not influence the dynamic response of the dam too much. In the joint project of China and the United States, the researchers carried out field excitation tests for many arch dams in China, but there were still no definite conclusion about whether the compressibility of water should be considered in the dynamic analysis of dam-reservoir coupling system. Chopra (1985, 2010) applied substructure method to establish the associated substructure model of the dam-reservoir-foundation coupling system, in the mothed, the compressibility of water and the absorption characteristic of reservoir bottom both can be considered at the same time. Millán and Young (2007) investigated the influence of the reservoir geometry, the water compressibility and the absorption characteristic of the reservoir bottom on the dynamic response of dam with using Boundary Element Method (BEM), the results showed that the reservoir shape influenced the dynamic response of the dam, making it necessary to account for 3-D effects in order to obtain accurate results.

In the paper, the dynamic model test of dam-reservoir coupling system for a 203m high gravity dam on a shaking table is carried out to investigate effects of reservoir water on dynamic responses of dam during earthquake. The dynamic responses (hydrodynamic pressure, natural frequencies and acceleration amplification factors along the dam height) of dam under conditions of full and empty reservoir are obtained in earthquake. The dynamic responses of dam under conditions of full and empty reservoir is compared to clarify the influence of reservoirs water on dam body. The Fluid-Structure Coupling Model (FSCM) based on Lagrange equation (Millán *et al*. 2007) is established to investigate the hydrodynamic pressure on the upstream surface of gravity dams with 5 different height in earthquake. By comparison with the results of the FSCM, the Westergaard formula is modified based on the differences between hydrodynamic pressures of the FSCM and solutions of Westergaard formula. The modified Westergaard formula for hydrodynamic pressure includes the correction equation of dam height, correction equation of dam flexibility.

2. Model test research for dynamic response of dam-reservoir system

2.1 Design of dynamic model test

2.1.1 Brief introduction of prototype dam

A 203 m-high gravity dam (retaining dam section) is taken as the object of the dynamic model test. The dam material is the roller compacted concrete, its elastic modulus is 2.55×104 MPa, and its Poisson ratio is 0.167, and its density is 2400 kg/m^3. The design water level before dam is 197 m. The horizontal design peak acceleration is 0.251 g. The dynamic model test is proceeded in the Research Laboratory of earthquake engineering of Dalian University of Technology in China, and the main test equipment in the test included large simulation system of underwater earthquake, acquisition and processing system of digital signal (DSPS), water pressure sensor and acceleration sensor and so on. The DSPS, which is developed by the 49th Research Institute of China Aerospace group, has 64 measurement channels, and can simultaneously capture datum of water pressure and acceleration.

2.1.2 Similarity theory and model material

The prototype dam section is reduced at a scale to perform the dynamic model test on the shaking table. Based on the size of the prototype dam section and the loading capabilities of the shaking table as well as properties of model material, the scale for the model is a 1:100 geometric scale. In order to accurately reproduce the prototype dam,

Table 1 Similarity requirements and model material properties

Physical parameter	Scale factor	Ratio	Prototype value	Model value
Length	L_r	100	——	——
Dam density	ρ^d_r	0.857	2400 kg/m^3	2800 kg/m^3
Dynamic elastic modulus	$E^d_r=\rho^d_r L_r$	85.7	33.15 GPa	0.38 GPa
Poisson ratio	μ_r	1	0.16	0.2
Water density	ρ^w_r	1	1000 kg/m^3	1000 kg/m^3
Time	$T_r=\sqrt{L_r}$	10	——	——
Acceleration	$A_r=1$	1	0.251 g	0.253 g
Strain	ε_r	1	——	——
Force	$F_r=\rho^d_r L^3_r A_r$	857000	——	——

the model must follow certain similitude laws. Besides the three well-known basic similarity laws (Donlon 1989, Ghobarah and Ghaemian 1998, Ghaemmaghami and Ghaemian 2008, Ghaemmaghami and Ghaemian 2010, Wang *et al*. 2014), the reservoir water density scale and force scale are established from the similarity theory for the investigation (Wang *et al*. 2014). According to dimensional analysis, dimensionless ratios that relate the behavior of model and prototype are given in following equations

$$T_r = \sqrt{L_r} \tag{1}$$

$$S_r = \rho^d_r L_r \tag{2}$$

$$A_r = 1 \tag{3}$$

$$\rho^w_r = \rho^d_r \tag{4}$$

$$F_r = \rho^d_r L^3_r A_r \tag{5}$$

where T, L, S, A, F and ρ represent respectively time, length, stress, acceleration, force and mass density, respectively; Subscript r is the ratio of these parameters in model and prototype dam. Superscript w and d represent respectively the reservoir water and dam in modeling system.

To satisfy the similitude requirements of spatial, physical, boundary, and moving conditions between the scaled model and those of its prototype, the model material is a low-strength and fine aggregate model concrete made of cement, water, barite, barite sand, and ore powder. The sizes of aggregates are calculated according to the length scale. The particles sizes of barite, barite sand, and ore powder are 0.05~2 mm. The concrete-like model material is used to construct the model of the dam section. The model material contains only small quantities of cement to reduce the strength. The static elastic modulus of the model material can be controlled in 50~500 MPa and its density is about 2800 kg/m^3, and similar to the that of concrete. The all similarity scales can be acquired according to the similarity theory calculation in the test and the results are shown in Table 1. The strain scale between model materials used in the test and prototype concrete is not strictly equal to 1, but the existing research result (Xia *et al*. 1980) shows that the strain scale in the linear elastic model test is not equal to 1, and has no effect on the experimental results. In order to apply linear similarity theory more conveniently, take the A in Table 1 as 1.

(a) The model of empty reservoir

(b) The model of full reservoir

Fig. 1 Dam model cured for 24 h in the test

2.1.3 Model design, construction and instrumentation

The dynamic model of dam section is 203 cm high, 25 cm in the thickness, and weighs about 3.5 tons for the 1/100 scale. The foundation of the model are constructed with the model material which is the different from the physical and mechanical properties of the model material for dam section. An all-thread rods are imbedded in the foundation to stabilize the model on the shake table. A tank with a size of 6.0×0.8×2.2 m is instilled on the water side of the model. The length of reservoir simulated with the water tank is 3 times the dam height. The tank is filled with water to the designed height in the testing. The water body vibrates together with the dam model to simulate the dam-reservoir interaction in the testing. To avoid water impulsive wave reflection from the back wall of the tank to the upstream face of the model, an energy dissipater, which is a cage made of fiber nets and battens, is installed at the other end away from the dam model in the tank. The cage is a device for simulating the infinite reservoir area. The model cured for 24 h is shown in Fig. 1.

The sensor layout on model is shown in Fig. 2. The test employees the water pressure sensors developed from Tianjin Harbor Engineering Research Institute in china. The sensor, which is waterproof, can measure both the soil pressure and the water pressure. It has the advantages of high precision, small size, convenient installation. Ten water pressure sensors are placed every 25 cm along height of the dam model to record the hydrodynamic pressure on the upstream face of the dam model, and the specific installation method of water pressure sensors is shown in Fig. 3. Seven accelerometers are placed every 30cm along height of the model to record the accelerations in horizontal

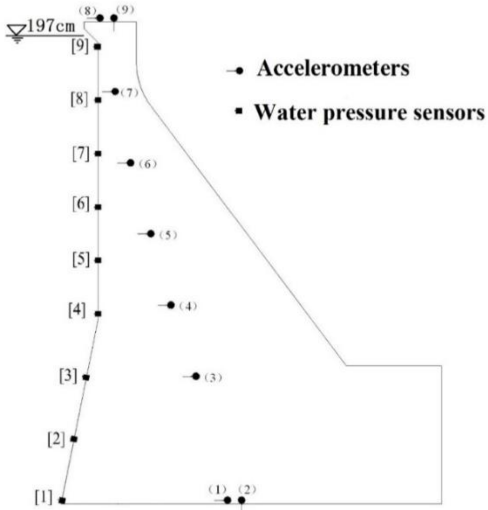
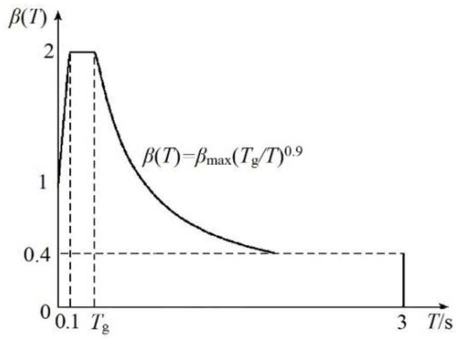


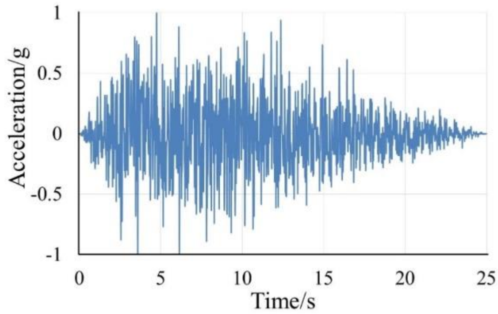
Fig. 2 Sensor layout on model

Fig. 3 The specific installation method of water pressure sensors

Fig. 4 The specific installation method of the accelerometers



(a) Seismic response spectrum



(b) Seismic wave

Fig. 5 Seismic response spectrum and earthquake wave as dynamic input

along the stream direction, and the specific installation method of the accelerometers is shown in Fig. 4. Two are placed on the crest of model and the supporting platform respectively, to record the accelerations in the horizontal stream and the vertical directions.

2.1.4 Loading

Each earthquake accelerogram duration is reduced to 1:10 of the original duration from Table 1. In the test, the artificial seismic wave generated according to the design response spectrum of the Category I site specified in the Code for Seismic Design of Hydraulic structures in China is selected as dynamic input the shaking table for the model. The peak ground acceleration (PGA) of the seismic wave is 0.251 g. The seismic wave and its response spectrum is shown in Fig. 5. To measure the natural frequency of dam model before inputting seismic wave for studying dam-reservoir interaction, using experimental white noise (sine-sweep) tests on the model under the condition of full reservoir and empty reservoir, respectively. A series of excitations are applied, as listed in Table 2. To facilitate comparison with the solution of Westergaard formula, which does not consider vertical seismic loading, the vertical seismic action is not considered in model test and numerical analyse.

Table 2 Sequence of input excitations

Series	Test conditions	Target PGA	Recorded PGA (platform)	Remarks
W0	White noise	——	0.051g	Empty reservoir
E1	Seismic wave	0.251	0.252	Empty reservoir
W1	White noise	——	0.05g	Full reservoir
E2	Seismic wave	0.251g	0.253	Full reservoir
W2	White noise	——	0.052g	Full reservoir

2.2 Analysis of experimental results

2.2.1 Natural frequency analysis of dam

According to transfer function from the platform to the

Table 3 Measured and predicted fundamental frequencies of model

	Cases	Fundamental frequencies
Test results	Empty reservoir	23.5Hz
	Full reservoir	18.4Hz

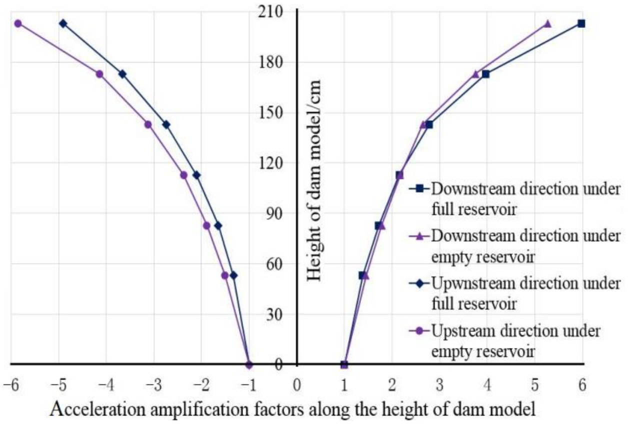


Fig. 6 The distributions of acceleration amplification factors along the height of dam model

model top with the using white noise sweeping for the model, the first two natural frequencies of the dam section model in cases of full and empty reservoirs are obtained.

Under empty and full reservoir cases, the natural frequencies of model dam in the test are shown in Table 3. Can be seen from the Table 3 that the natural frequencies of dam models under empty and full reservoir are respectively 23.5Hz and 18.12Hz from the measured test. The measured natural frequency of the dam model under full reservoir is 21.7% lower than that of empty reservoir, and it indicates that reservoir water has obvious influence on the natural frequency of gravity dam in earthquake.

2.2.2 Acceleration distribution analysis

The distributions of acceleration amplification factors along the height of dam model from measured in cases of full and empty reservoirs are shown in Fig. 6. In the figure, it is obvious that the acceleration amplification factors are all increased obviously along height of the model, especially in the dam neck and above part, and the amplification factors under condition of full reservoir is larger than those under condition of empty reservoir, and the amplification factors to the downstream is larger than those to upstream under the same condition. The amplification factor to downstream at dam crest is 5.97 under condition of full reservoir, and that of empty reservoir is 5.26. It can be seen from this that the effect of reservoir water increases the acceleration response of the dam body, especially the upper part of the dam body.

2.2.3 Hydrodynamic pressure analysis of dam

All the time, because the Westergaard formula for hydrodynamic pressure is simple, practical, easy to calculate, and the calculation results of the formula is partial to safety, so far the formula is still accepted and adopted by the dam engineering community. At present, the

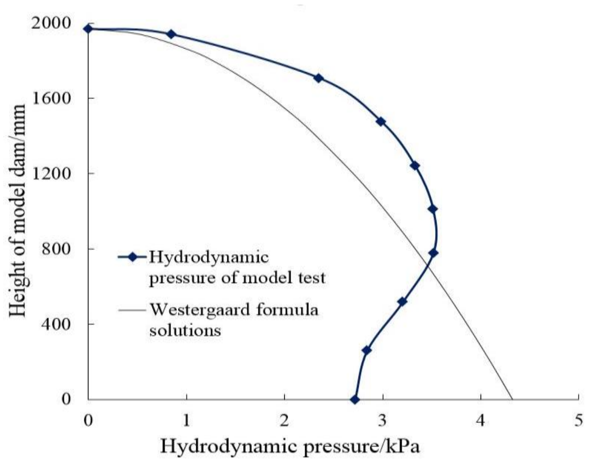


Fig. 7 The comparison of the measured hydrodynamic pressure and solution of Westergaard formula

Westergaard formula is still applied to calculate hydrodynamic pressure for seismic design of gravity dam or arch dam in the Code for Seismic Design of Hydraulic Structures in China.

The comparison of the measured hydrodynamic pressure and solution of Westergaard formula is shown the Fig. 7. From the figure, can be seen that the distribution of hydrodynamic pressures along the dam height from the test are very different from that of the formula. The hydrodynamic pressures on the upstream surfaces of the dams from Westergaard formula are overestimated in the lower part of the dam body and underestimated in its upper part to compare with those from the test. The maximum hydrodynamic pressure from the test is located at the center of the upstream surface of dam body, not at the dam heel like the solution of the Westergaard formula. The maximum hydrodynamic pressures from the test and the formula are 3.516kPa and 4.236kPa, respectively. The Measured maximum hydrodynamic pressures is are lower than the calculated result by 18.7%. According to the above results, the Westergaard formula should be modified to discard the adverse effects of those assumptions.

3. Research of modified Westergaard formula

3.1 Westergaard formula of hydrodynamic pressure

In 1933, Westergaard studied the hydrodynamic pressure of the vertical upstream surface of rigid dam under horizontal earthquake, and the approximate formula of hydrodynamic pressure was given by

$$P_{\max} = \frac{7}{8}\gamma\alpha\sqrt{H_0 h} \tag{6}$$

where $P_{\max}$ is the maximum hydrodynamic pressure of the calculated point on upstream surface of dam, H_0 is the maximum water depth of upstream of dam, h is the water depth of the calculated point, γ is the bulk density of reservoir water, α is the maximum seismic acceleration coefficient.

3.2 Fluid-solid coupling model based on Lagrangian formulation

Based on Lagrangian formulation, the displacements are selected as the variables in both fluid and structure domains (Calayir 1996, Olson 1983, Calayir 2005). The fluid is assumed to be linear elastic, inviscid and irrotational. The motion equation of reservoir water is given by

$$\nabla^2 P - \frac{1}{C^2} P = 0 \tag{7}$$

where ∇^2 and P are the laplacian operator and hydrodynamic pressure (tension has a positive sign), respectively; $C = \sqrt{\frac{K}{\rho}}$, K and ρ are the bulk modulus density of water, respectively.

The boundary conditions satisfying the formula (7) are as follows

(1) The interface of dam-reservoir system should be satisfied by $\frac{\partial P}{\partial n} = -\rho \ddot{u}_n$, where n stands for the normal direction of the interface of dam-reservoir system, $\ddot{u}_n$ is the acceleration of the normal direction.

(2) The boundary of reservoir bottom should be satisfied by $\frac{\partial P}{\partial m} = \frac{r-1}{c} \frac{p}{t}$, where r is the absorption coefficient of the reservoir bottom ($0 \le r \le 1$), m stands for the normal direction of the reservoir bottom surface, c is wave velocity in water, t is the movement time of system;

(3) The reservoir surface should be satisfied by $\frac{\partial P}{\partial z} = -\frac{1}{g} \ddot{P}$, where g is gravity acceleration, z stands for the vertical direction. Usually, the effect of surface wave of reservoir are neglected, so P=0;

(4) The boundary of the end of reservoir should be satisfied by $\frac{\partial P}{\partial h} = -\frac{1}{c} \frac{\partial P}{\partial t}$, where h stands for the normal direction of boundary of the end of reservoir.

After four boundary conditions of reservoir have been determined, the hydrodynamic pressure at any point in reservoir in two dimensional can be expressed as

$$P(x, y, t) = \sum_1^n N_n(x, y) P_n(t) \tag{8}$$

where shape function is $N = \begin{bmatrix} N_1(x,y) \\ N_2(x,y) \\ \cdot \\ \cdot \\ N_n(x,y) \end{bmatrix}$, the hydrodynamic pressure vector is $P = \begin{bmatrix} P_1(t) \\ P_2(t) \\ \cdot \\ \cdot \\ P_n(t) \end{bmatrix}$.

Solving the differential of Eq. (8), according to the Galerkin method, the discrete equations of reservoir water motion are obtained as

$$HP + A\dot{P} + E\ddot{P} + \rho B\ddot{d} + r = 0 \tag{9}$$

where $H = \iint_\Omega \nabla N \nabla N^T d\Omega$, $A = \frac{1}{C} \int_S NN^T dS$, $E = \frac{1}{C^2} \iint_\Omega NN^T d\Omega + \frac{1}{g} \int_{S_F} NN^T dS_F$, $B = (\int_{S_l} NN_S^T dS_l)\Lambda$, r is the vector of inputting excitation, d is the vector of displacement, Λ is the transformation matrix of coordinates.

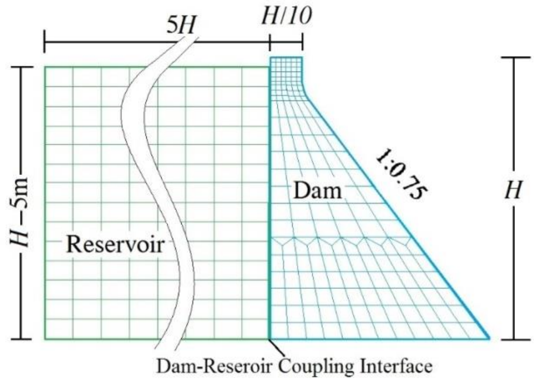


Fig. 8 The Finite Element Model for the dam-reservoir interaction system, enlarges or reduces the size of the elements by scale for dams with different heights

3.3 Analysis and comparison of calculation results

The vertical upstream surface gravity dams with heights of 70 m, 100 m, 130 m 160 m and 200 m are chosen as the numerical analysis models in the research, and the normal water level of the dams is 65 m, 95 m, 125 m, 155 m and 195 m, respectively. The Finite Element Model for the dam-reservoir interaction system is shown in Fig. 8. The Elastic modulus of the concrete of dam body is 2.5×10^4 MPa, and its Poisson ratio is 0.167, and its density is 2400 kg/m^3. The length of reservoir area is five times the dam height, the water density is 1000 kg/m^3, the bulk modulus of the water is 2.3×10^3 MPa. In the analysis, the seismic wave in the Fig. 5 is selected as dynamic input. The peak ground acceleration (PGA) of the seismic wave is 0.2 g.

By comparing the results of FSCM and the solutions of Westergaard formula, it is found that the distributions of hydrodynamic pressures on the upstream surfaces of the dams from the two methods are different along the dam height. The difference is more obvious with the increase of dam height. In order to facilitate the comparison, the hydrodynamic pressures on the upstream surfaces of the dams with 5 heights from the FSCM and Westergaard formula are normalized and shown in Fig. 9. The maximum hydrodynamic pressures of the FSCM and their positions on the upstream surfaces are compared with those of the Westergaard formula, as shown in Table 4.

As can be seen from Fig. 9 and Table 4 that the hydrodynamic pressures from the FSCM are larger in the upper parts on upstream surfaces of the dams and smaller in their lower parts than the solutions of the Westergaard

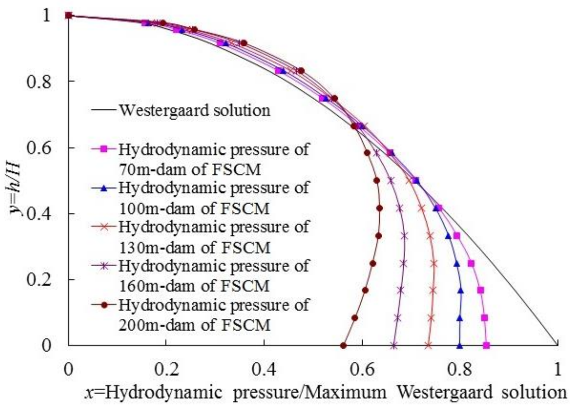


Fig. 9 The normalized hydrodynamic pressures on the upstream surface of the dams with 5 heights

Table 4 Comparison of the maximum hydrodynamic pressures from the FSCM and Westergaard formula

Dam height/m	The maximum hydrodynamic pressures of the FSCM and their positions			P_{max}^{W} /KPa	x /%
	P_{max}^{FSCM} /KPa	The positions of the P_{max}^{FSCM}	y /%		
70	94.87	5.4 m above the dam heel	7.7	114.7	14.9
100	133.51	12.8 m above the dam heel	12.8	162.9	19.1
130	160.10	31.5 m above the dam heel	24.2	214.3	25.4
160	182.45	55.6 m above the dam heel	34.7	265.8	31.9
200	206.69	91.5 m above the dam heel	45.7	334.4	38.2

formula. The trend is more pronounced with the increase of dam height. The above analysis is consistent with the dynamic model test results of Li *et al*. (2003) The maximum hydrodynamic pressure on the 70 m-high dam from the FSCM is reduced by 14.8% compared with that from Westergaard formula, and that on a grand 200 m-high dam is also reduced by 38.2%. Compared to the maximum solution of Westergaard formula, the magnitude of the maximum hydrodynamic pressure of the FSCM reduced is linearly increased with the increase of the dam height, as shown in Fig. 10. The linear relationship can be expressed as

$$H = 535.37x - 6.5533, \quad R^2 = 0.995 \tag{10}$$

$$x = \frac{P_{max}^{W} - P_{max}^{FSCM}}{P_{max}^{W}} \tag{11}$$

where H is the dam height, x is the percentage which the maximum hydrodynamic pressure of the FSCM is lower than that of Westergaard formula, P_{max}^{FSCM} is the maximum hydrodynamic pressure from the FSCM, P_{max}^{W} is the maximum solution of the Westergaard formula, R^2 is the decision coefficient of the linear fitting function.

The position of the maximum hydrodynamic pressure on the upstream surface from the FSCM has been raised

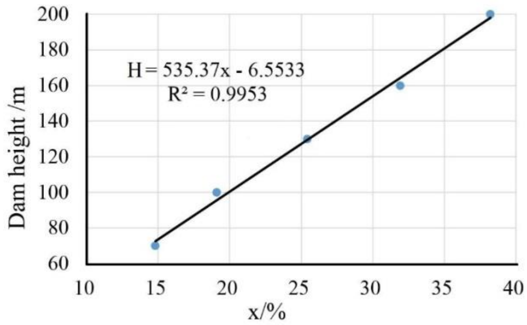


Fig. 10 Relationship between x and dam heights

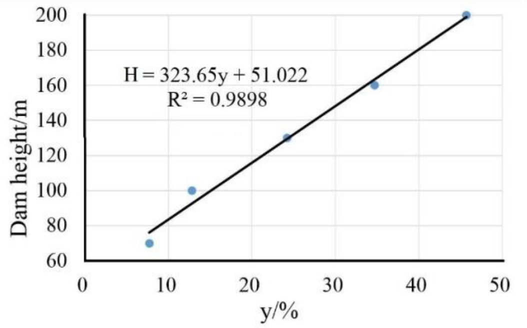


Fig. 11 Relationship between y and dam heights

with the increase of the dam height. The maximum hydrodynamic pressure on the 70m-high dam appears at the 5.4m above dam heel, and those of the 160m-high and 200m-high dams are located at around 1/2 water head of dam upstream (45%~55% of the dam height). The relationship between the position of the maximum hydrodynamic pressure and dam height can also be used to fit the linear function (shown in Fig. 11), which is expressed as,

$$H = 323.65y + 51.022, \quad R^2 = 0.989 \tag{12}$$

$$y = h_{max}/H \tag{13}$$

where y is the ratio of the relative height of the position of the maximum hydrodynamic pressure to dam heel to the total height of dam, h_{max} is height of the position of the maximum hydrodynamic pressure relative to dam heel.

3.4 Development of modified Westergaard formula

3.4.1 Research of correction model of Westergaard formula

According to the comparison and analysis between the distributions of hydrodynamic pressures on the upstream surfaces of dams with 5 different heights from the FSCM and the Westergaard formula in the Section 2 in the paper, it can be drawn the conclusion that the influence of dam height on hydrodynamic pressure should be taken into account in the modified Westergaard formula. The assumption that the dam is rigid makes a great difference between Westergaard formula solution and actual hydrodynamic pressure because the dynamic response along the height of elastic dam body in the earthquake becomes more severe with the increase of dam height, especially for

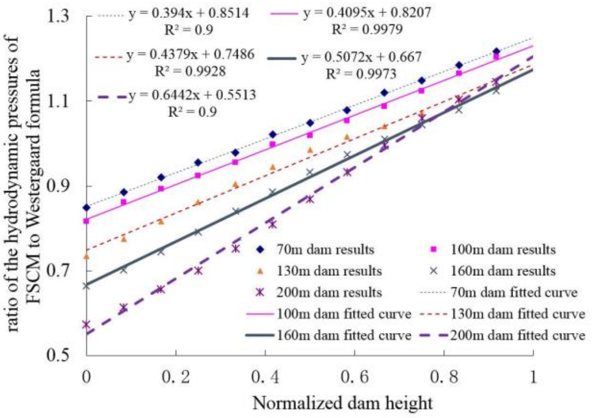


Fig. 12 The ratio of the hydrodynamic pressures of FSCM to Westergaard formula along dam height normalized

200 m-high or above dam. Therefore, the correction terms about height and elasticity of dam should be introduced into the modified Westergaard formula, when the Westergaard formula is modified.

The hydrodynamic pressures on upstream surfaces of dams of 5 different heights from the FSCM and Westergaard formula are compared in the Fig. 9 (All datum of the hydrodynamic pressures and the dam heights are normalized). According to the datum in the Fig. 9, the ratios of the hydrodynamic pressures of FSCM to Westergaard formula are linear along dam height, as shown in Fig. 12. The hydrodynamic pressures from FSCM and Westergaard formula at free surface of water on upstream surface are 0, the 0/0 is meaningless. Based on the linear fitting equations between the dam heights and the ratios, the slopes and intercepts of the 5 linear fitting equations are also linear with increase of dam height. Accordingly, the Westergaard formula should be introduced into the correction term about dam height to obtain more accurate the hydrodynamic pressure, and the modified Westergaard formula should be obtained as

$$P_{\max}^{M} = P_{\max}(\alpha, H_0, h) \times G(H, H_0, h) \tag{14}$$

$$\begin{aligned} G(H, H_0, h) = {} & (0.0043H - 0.1161)E(H, e)\frac{h}{H_0} \\ & + (0.00065H + 0.9203)[E(H, e) - 0.3747] \end{aligned} \tag{15}$$

$$E(H, e) = (-0.0029H + 1.4788)e \tag{16}$$

where $P_{\max}^{M}$ is the maximum hydrodynamic pressure of the modified Westergaard formula, $P_{\max}(\alpha, H_0, h)$ is the maximum hydrodynamic pressure of the Westergaard formula; $G(H, H_0, h)$ and $E(H, e)$ are respectively the correction equations about dam height and elasticity in the modified Westergaard formula; H is the dam height; e is the correlation coefficient of elasticity modulus of dam. The hydrodynamic pressures on 100 m-high dam with elasticity modulus of 15 GPa, 20 GPa and 25 GPa from the FSCM are respectively fitted by the modified Westergaard formula (14), and it is determined that the e values are 1.03, 1.00 and 0.982, respectively.

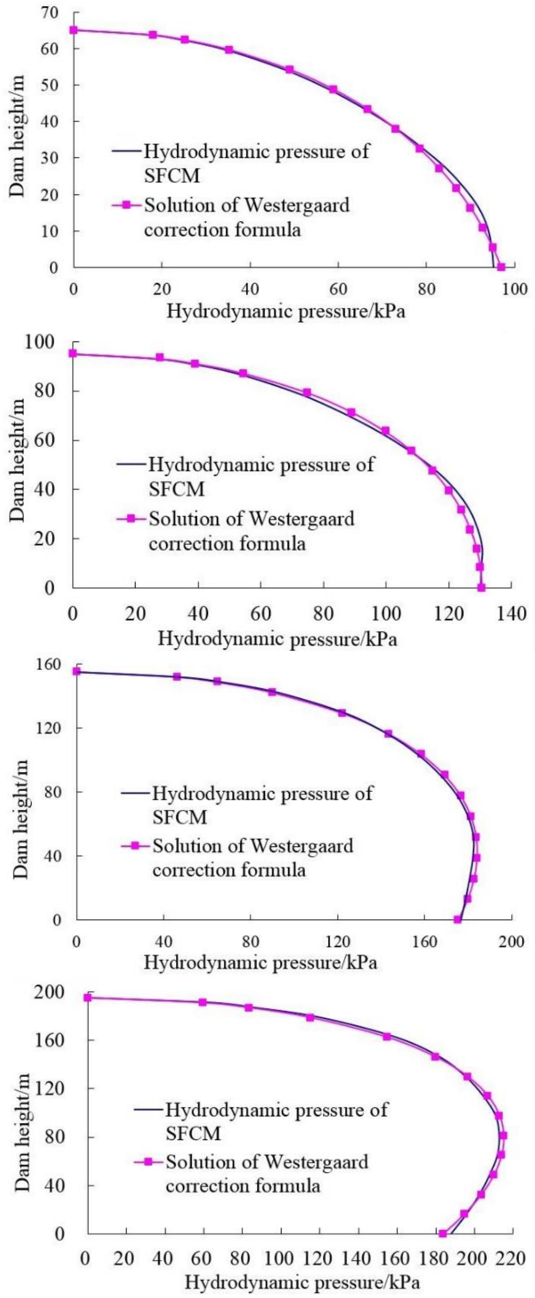


Fig. 13 Hydrodynamic pressures of the modified Westergaard formula (14) are in good agreement with those of the FSCM.

The modified Westergaard formula (14) includes a correction Eq. (15) about dam height, and the Eq. (15) includes the participation correction Eq. (16) about dam elasticity. It is demonstrated that the modified Westergaard formula (14) fully considers the relation between the hydrodynamic pressure and dam height, which the ratio of maximum hydrodynamic pressure of the FSCM to the maximum solution of Westergaard formula is reduced with increase of the dam height and the position of the maximum hydrodynamic pressure is relatively raised. Moreover, the dam flexibility on the influence of the hydrodynamic pressure distribution is also considered in the modified Westergaard formula (14). The seismic dynamic response of

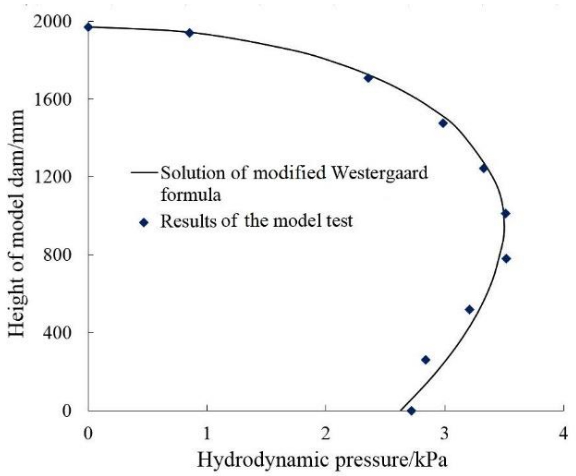


Fig. 14 Comparison of hydrodynamic pressures of the modified Westergaard formula (14) and model test in the paper

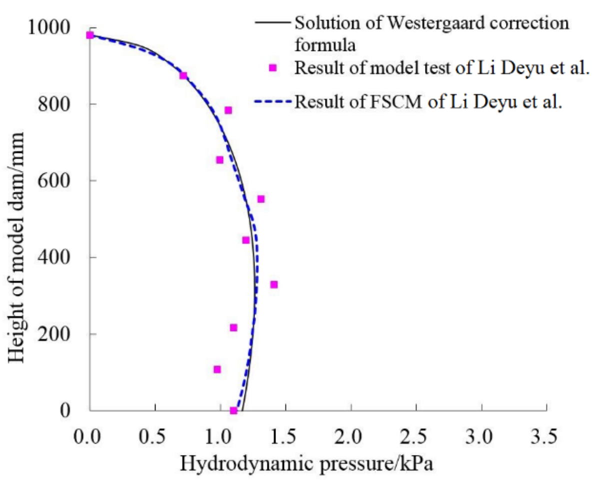


Fig. 15 Comparison of hydrodynamic pressures of the modified Westergaard formula (14) and model tests of Li *et al*.

upper part of dam with lower elastic modulus is more severe during earthquake, so the hydrodynamic pressure of upper part on upstream surface of the dam will be magnified.

3.4.2 Verification of Westergaard modified formula

The distributions of hydrodynamic pressures on upstream surfaces of the dams with 5 different heights from the modified Westergaard formula (14) are compared with those of the FSCM, as shown in Fig. 13. As can be seen from the Fig. 13 that the hydrodynamic pressures of the modified Westergaard formula (14) are in good agreement with those of the FSCM for the 70 m~200 m-high dams.

To verify the accuracy of the modified Westergaard formula, the distributions of the hydrodynamic pressures on the gravity dam in the test in the Section 2 of the paper are calculated by adopting the modified Westergaard formula (14). The calculated results of modified Westergaard formula (14) are in agreement with the hydrodynamic pressure distributions obtained from the test, as shown in Fig. 14.

Moreover, the hydrodynamic pressures of the modified Westergaard formula (14) and the previous test results of Li *et al*. (2003) are also very consistent, as shown in Fig. 15. From the two comparisons in the Figs. 14 and 15, the distributions of hydrodynamic pressures along the dam height from modified Westergaard formula (14) are in good agreement with those of test in the paper and the literature (Li *et al*. 2003), except for the slight difference in the dam heels. It is because the upstream slopes at the dam heels in the two tests have the influence on the hydrodynamic pressure in the earthquake, but the influence is not very obvious.

4. Conclusions

The dynamic model test of dam-reservoir coupling system for a 203 m high gravity dam on the shaking table is performed to study effects of reservoir water on dynamic responses of dam during earthquake, and Seismic dynamic analysis of the gravity dams with five different heights is performed with the Fluid-Structure Coupling Model (FSCM). The hydrodynamic pressures distributions on the upstream surfaces of the dams from the test and numerical analysis are compared with those of the classical Westergaard formula, and the Westergaard formula is revised accordingly to calculate more accurate the result. The primary conclusion is summarized as follows:

1. The results of the test indicate that the reservoir water have a stronger influence on the dynamic responses of dam. The measured natural frequency of the dam model under full reservoir is 21.7% lower than that of empty reservoir, and the acceleration amplification factor at dam crest under full reservoir is 18% larger than that under empty reservoir.
2. The hydrodynamic pressures on the upstream surfaces of the dams from Westergaard formula are overestimated in the lower part of the dam body and underestimated in its upper part to compare with those from the test. The maximum hydrodynamic pressure from the test is located at the center of the upstream surface of dam body, not at the dam heel like the solution of the Westergaard formula. The Measured maximum hydrodynamic pressures is are lower than the calculated result by 18.7%.
3. By comparing the results of FSCM and solutions of Westergaard formula, it can be found that the maximum hydrodynamic pressure on the upstream surface of the dam from the FSCM is significantly smaller than that from Westergaard formula with the increase of dam height, and the position of maximum hydrodynamic pressure is obviously raised along the dam height. The results are consistent with the previous results of dynamic model test reported in the literature.
4. The classical Westergaard formula is modified with considering the influences of the height and flexibility of dam and the compressibility of reservoir water on the hydrodynamic pressure. The modified Westergaard formula includes a correction equations about dam

height and dam elasticity to takes into account the influences of dam height and dam elasticity on hydrodynamic pressure. The distribution of hydrodynamic pressure on the upstream surface along the dam height from the modified Westergaard formula is in good agreement with that of the test and FSCM reported in the literature.

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