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On the size-dependent behavior of functionally graded micro-beams with porosities

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Abstract. In this work, a new hyperbolic shear deformation beam theory is proposed based on a modified couple stress theory (MCST) to investigate the bending and free vibration responses of functionally graded (FG) micro beam made of porous material. This non-classical micro-beam model introduces the material length scale coefficient which can capture the size influence. The non-classical beam model reduces to the classical beam model when the material length scale coefficient is set to zero. The mechanical material properties of the FG micro-beam are assumed to vary in the thickness direction and are estimated through the classical rule of mixture which is modified to approximate the porous material properties with even and uneven distributions of porosities phases. Effects of several important parameters such as power-law exponents, porosity distributions, porosity volume fractions, the material length scale parameter and slenderness ratios on bending and dynamic responses of FG micro-beams are investigated and discussed in detail. It is concluded that these effects play significant role in the mechanical behavior of porous FG micro-beams.

Keywords: shear deformation theory; bending; vibration; micro beam; porosity; functionally graded material

1. Introduction

Functionally graded materials (FGMs) are inhomogeneous composites presenting a smooth and continuous variations in both compositional profile and material characteristics that allow them to be employed in a wide range of applications in many engineering devices (Shahrjerdi *et al.* 2011, Ait Amar Meziane *et al.* 2014, Kar and Panda 2015, Akbaş 2015, Houari *et al.* 2016, Bellifa *et al.* 2017a, Ait Atmane *et al.* 2017, Menasria *et al.* 2017, Zaoui *et al.* 2017, Abualnour *et al.* 2018). Using FGMs lead to uniform stress variation in the structures and overcome the problems such as jump in stress components between layers, interfacial debonding, matrix cracking, etc. In the last decade, the trend of employing beams and plates made of FGMs for engineering structures has considerably increased. Consequently, understanding the behavior of structures fabricated by porous FGMs under a variety of mechanical and thermal loadings is very important for their accurate design.

Many studies have been proposed by researchers on the bending and vibration response of functionally graded (FG) beams (Sankar 2001, Ying *et al.* 2008, Xiang and Yang 2008, Kapuria *et al.* 2008, Li 2008, Prakash *et al.* 2009, Jomehzadeh *et al.* 2009, Ould Larbi *et al.* 2013). Sankar (2001) proposed an elasticity solution for FG beams. Two-dimensional elasticity solution of an FG beam with simply supported edges is studied by Ying *et al.* (2008) and natural frequencies and mode shapes are presented by employing state space method. Xiang and Yang (2008) investigated free and forced vibration of a laminated FG beam of variable thickness under thermally induced initial stresses. The influence of various boundary conditions was examined and beam was considered to be subjected to one-dimensional steady heat conduction in the thickness direction before undergoing dynamic deformation. Kapuria *et al.* (2008) discussed the vibration behavior of laminated FG beams and results were compared with experimental results. Li (2008) presented a novel unified formulation for investigating the bending and vibration response of FG beams including rotary inertia and shear deformation. All of the reviewed studies are based on the classical theory of elasticity which has acceptable predictions for bending and vibration behavior of macro-scaled structures. According to the classical theory of elasticity, anticipated mechanical responses are absolutely independent of the structure size when they are stated in proper dimensionless forms.

FGMs are widely employed in micro- and nano-structures such as thin films in the form of shape memory alloys (Craciunescu and Wuttig 2003, Fu *et al.* 2003), micro- and nano-electromechanical systems (MEMS and NEMS) (Fu *et al.* 2004, Witvrouw and Mehta 2005, Lee *et*

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al. 2006), atomic force microscopes (AFMs) (Rahaeifard *et al.* 2009) and also FG nanostructures (Janghorbana and Zareb 2011, Belkorissat *et al.* 2015, Zemri *et al.* 2015, Karami and Janghorban 2016, Ahouel *et al.* 2016, Barati and Shahverdi 2016, Bounouara *et al.* 2016, Mouffoki *et al.* 2017, Besseghier *et al.* 2017, Bouafia *et al.* 2017, Karami *et al.* 2017a, b). Beams employed in MEMS, NEMS and AFMs, have the thickness in the order of microns and sub-microns. The size-dependent bending and dynamic response in micro scales are experimentally checked. For example in the micro-torsion test of thin copper wires, Fleck *et al.* (1992) demonstrated that decrease of wires diameter results in a noteworthy enhancement of the torsional hardening. Stolken and Evans (1998) indicated a considerable increase of plastic work hardening induced by the decrease of beam thickness in the micro bending test of thin nickel beams. Also, size-dependent behaviors are shown in some kinds of polymers. For instance, during micro bending tests of beams fabricated from epoxy polymers, Lam *et al.* (1999) observed a notable enhancement of bending rigidity induced by the beam thickness reduction. McFarland and Colton (2005) found an important difference between the stiffness values predicted by the classical beam theory and the stiffness values determined during a bending test of polypropylene micro-cantilever. According to the previous experimental outcomes, it can be concluded that size-dependent behavior is an inherent property of materials which appears for a beam when the characteristic size such as thickness or diameter is close to the internal material length scale parameter (Kong *et al.* 2008).

In 1960s some scientifics authors proposed the couple stress elasticity theory (Mindlin 1994, Mindlin and Tiersten 1962, Toupin 1962). In the constitutive equation of this theory, we find in addition to the two classical Lame constants, two higher-order material length scale parameters. Zhou and Li (2001) used this theory to study the bending and dynamic behavior of a micro-bar in torsional loading. Kang and Xi (2007) investigated the resonant frequencies of a micro-beam and indicated that these frequencies are size-dependent.

To reduce the problems encountered in determining length scale parameters of materials by experiments, Yang *et al.* (2002) used the modified couple stress theory, which in its constitutive equation only one material length scale parameter appears. Employing the modified couple stress theory, Park and Gao (2006) examined the static behavior of an Euler-Bernoulli beam and discussed the outcomes of an epoxy polymeric beam bending test. Kong *et al.* (2008) presented the governing equation, initial and boundary conditions of an Euler-Bernoulli beam via the modified coupled stress theory and the Hamilton principle. They showed that the natural frequencies of the beam are size-dependent. Also, the difference between the natural frequencies computed by the classical beam theory and those obtained by the modified couple stress theory is considerable when the beam property size is comparable to the internal material length scale parameter.

Because of the vast applications of functionally graded materials in MEMS and NEMS, and also the fact that the classical continuum theory is unable to predict size-dependent mechanical responses of microstructures, the use of the non-classical theory of elasticity to the microstructures made of FGMs seems to have great merits. Recently, Lü *et al.* (2009a) developed a generalized refined theory introducing surface effects in order to study the size-dependent elastic response of FGM ultra-thin films. Lü *et al.* (2009b) also assessed size-dependent elastic mechanical responses of nano-scaled FGM films by employing the Kirchhoff hypothesis and the continuum theory of surface elasticity. Not only FGM thin films but also FGM micro-beams are often used in MEMS and NEMS. More recently, Al-Basyouni *et al.* (2015) proposed a novel unified beam formulation and a modified couple stress theory (MCST) that considers a variable length scale parameter in conjunction with the neutral axis concept to study bending and dynamic behaviors of FG micro beam. Thus, a study on the size-dependent mechanical response of a micro-beam made of FGMs by employing an appropriate non-classical continuum theory capable of capturing small scale effects seems to be crucial.

With the rapid advancement in technology of structure components, structures with graded porosity can be considered as one of the latest developments in FGMs. The structures consider pores into microstructures by taking the local density into account. Moreover, a great opportunity in a wide range of engineering applications comes into result. Researchers have their eyes on development in preparation techniques of FGMs such as powder metallurgy, vapor deposition, self-propagation, centrifugal casting, and magnetic separation (Khor and Gu 2000, Seifried *et al.* 2001, Watanabe *et al.* 2001, Peng *et al.* 2007, Song *et al.* 2007). These techniques have some disadvantages such as high costs and complexity of the method. One of the simple and suitable ways to fabricate FGM is sintering process. During this process, because of the big difference in solidification between the material constituents, however, porosities or micro voids through material can be produced regularly (Zhu *et al.* 2001). A thorough study has been carried out on porosities appearing inside FGM samples fabricated by a multistep sequential infiltration method (Wattanasakulpong *et al.* 2012). Porosity may be modify the elastic and mechanical properties. Based on this information about porosities in FGMs, it is important to consider the porosity influence when designing FGM structures. However, researches on the mechanical response of porous FG structures, are still limited in number. The wave propagation of an infinite FG plate having porosities has been studied by Ait Yahia *et al.* (2015) using various simple higher-order shear deformation theories. Ait Atmane *et al.* (2015) presented a computational shear displacement model for vibrational analysis of FG beams with porosities. Wattanasakulpong and Ungbhakorn (2014) examined linear and nonlinear vibrations responses of porous Euler FG beams with elastically restrained ends. Material properties of FG beam have been described by a modified rule of mixture. Ebrahimi and Mokhtari (2015) investigated transverse vibration behavior of rotating Timoshenko FG beams with porosities. DTM was used to solve the equations of motion. It was demonstrated that porosity volume fractions play a considerable role in vibrations of porous FG beams. Moreover, Wattanasakulpong and Chaikittiratana (2015) discussed flexural vibration of porous FG beams via Timoshenko beam theory. Chebyshev collection technique was employed for solving equations. They expressed the porosities yield reduction in the mass and strength of FG beams. Ebrahimi and Zia (2015) studied the large amplitude nonlinear dynamic of porous FG Timoshenko beams. Galerkin and multiple scales techniques were used to solve motion equations. Ait Atmane *et al.* (2016) proposed an efficient beam theory to investigate static, dynamic, and buckling behavior of porous FG beams on elastic foundations. Most recently, Ebrahimi *et al.* (2016) investigated the dynamic response of porous FG Euler beams under thermal loading.

It should be signaled that in, the abovementioned work, there is no study on porous FG micro-beam. So, in the present work, a modified couple stress theory is proposed to investigate the static and dynamic behaviors of porous FG micro-beams on the basis on a hyperbolic shear deformation beam theory. The micro-scale beam model includes the material length scale parameter which can capture the size influence. The material properties of the FG micro-beams including the length scale parameter are supposed to vary in the thickness direction according to power-law distribution which is modified to approximate the porous material properties with even and uneven distributions of porosities phases. An analytical solution is employed to solve the governing equations derived from Hamilton's principle. Several numerical and illustrative results are presented to indicate the influences of the material length scale parameter, gradient index, and porosity parameters on the static and dynamic responses of porous FG micro-beams

2. Theory and formulation

2.1 Modified couple stress theory

The strain energy, U can be expressed using the modified couple stress theory (Yang *et al*. 2002) by

$$U=\frac{1}{2}\int\left(\sigma_{ij}\varepsilon_{ij}+m_{ij}\chi_{ij}\right)dV,\quad (i,j=1,2,3) \tag{1}$$

Where σ is the stress tensor, ε is the strain tensor, m is the deviatoric part of the couple stress tensor, and χ is the symmetric curvature .these tensors are given by

$$\varepsilon_{ij}=\frac{1}{2}\left(u_{i,j}+u_{i,i}\right) \tag{2}$$

$$\chi_{ij}=\frac{1}{2}\left(\theta_{i,j}+\theta_{i,i}\right) \tag{3}$$

where u_i is the displacement vector, and θ is the rotation vector that can be defined as

$$\theta=\frac{1}{2}e_{ijk}u_{k,j} \tag{4}$$

where e_{ijk} is the permutation symbol.

2.2 Kinematic relations and constitutive relations

The displacement field of the conventional HSDT is given as follows

$$u(x,z,t)=u_0(x,y)-z\frac{\partial w_b}{\partial x}-f(z)\frac{\partial w_s}{\partial x} \tag{5a}$$

$$v(x,z,t)=0 \tag{5b}$$

$$w(x,z,t)=w_b(x,t)+w_s(x,t) \tag{5c}$$

where u_0; w_b; w_s, are three unknown displacements of the mid-plane of the plate, such as w_b is the bending part and w_s is the shear one.

$f(z)$ represents shape function defining the variation of the transverse shear strains and stresses across the thickness.

In this article, the shape function is considered given by Nguyen (2015) as

$$f(z)=z-\sinh^{-1}\left(\frac{3z}{h}\right)+\frac{6z}{h\sqrt{13}}\quad\text{and}\qquad g(z)=1-\frac{df(z)}{dz} \tag{6}$$

The nonzero strains of the present refined beam theory are presented as

$$\varepsilon_x=\frac{\partial u_0}{\partial x}-z\frac{\partial^2 w_b}{\partial x^2}-f(z)\frac{\partial^2 w_s}{\partial x^2},\quad \varepsilon_y=\varepsilon_z=\gamma_{xy}=\gamma_{yz}=0,\quad \gamma_{xz}=2\varepsilon_{xz}=2g(z)\frac{\partial w_s}{\partial x} \tag{7}$$

In addition, Eqs. (5) and (4),the components of the rotation vector are obtained as

$$\theta_y=-\frac{\partial w_b}{\partial x}-\frac{1}{2}\psi(z)\frac{\partial w_s}{\partial x},\quad \theta_x=\theta_z=0 \tag{8}$$

With, $\psi(z)=1+f'(z)$

Substituting Eq. (8) into Eq. (3), the components of the curvature tensor take the form

$$\chi_{xy}=-\frac{1}{2}\frac{\partial^2 w_b}{\partial x^2}-\frac{1}{4}\psi(z)\frac{\partial^2 w_s}{\partial x^2},\quad \chi_{yz}=-\frac{1}{4}f''(z)\frac{\partial w_s}{\partial x},\quad \chi_{xx}=\chi_{yy}=\chi_{zz}=\chi_{xz}=0 \tag{9}$$

2.3 Constitutive relations

Consider a FG plate made of two constituent functionally graded materials, the material properties of the beam such as Young's modulus E, masse density ρ and the length scale parameter l are considered to change continuously across the thickness according to a power law distribution. The effective material properties of FG beams with two kinds of porosity distributions which are distributed identically in two phases of ceramic and metal can be expressed by using the modified rule of mixtures as

(Ait Yahia *et al.* 2015, Wattanasakulpong and Ungbhakorn 2014, Benferhat *et al.* 2016)

$$p = p_c\left(v_c - \frac{\alpha}{2}\right) + p_m\left(v_m - \frac{\alpha}{2}\right) \tag{10}$$

Where α is the volume fraction of porosity (α<<1), for perfect FGM, α is set to zero, P_c and P_m are the material properties of ceramic and metal and v_c and v_m are the volume fraction of ceramic and metal separately; the compositions are represented in relation to

$$v_c + v_m = 1 \tag{11}$$

In this project, imperfect FGM has been investigated witch two types of porosity distributions (even and uneven) through the micro-beam thickness due to defect during fabrication.

For the even distribution of porosities FGM-I, the effectives material properties are determined as follows

$$E(z) = (E_c - E_m)v_c + E_m - \frac{\alpha}{2}(E_c + E_m) \tag{12a}$$

$$l(z) = (l_c - l_m)v_c + l_m - \frac{\alpha}{2}(l_c + l_m) \tag{12b}$$

$$\rho(z) = (\rho_c - \rho_m)v_c + \rho_m - \frac{\alpha}{2}(\rho_c + \rho_m) \tag{12c}$$

Where v_c=$(0.5+z/h)^p$ is the volume fraction of ceramic.

For FGM-II defined as uneven porosities the effectives materials properties are replaced by following form (Wattanasakulpong and Ungbhakorn 2014)

$$E(z) = (E_c - E_m)v_c + E_m - \frac{\alpha}{2}(E_c + E_m)\left(1 - \frac{2|z|}{h}\right) \tag{13a}$$

$$l(z) = (l_c - l_m)v_c + l_m - \frac{\alpha}{2}(l_c + l_m)\left(1 - \frac{2|z|}{h}\right) \tag{13b}$$

$$\rho(z) = (\rho_c - \rho_m)v_c + \rho_m - \frac{\alpha}{2}(\rho_c + \rho_m)\left(1 - \frac{2|z|}{h}\right) \tag{13c}$$

The constitutive relations can be written as

$$m_{ij} = 2\mu(z)[l(z)]^2\chi_{ij} \tag{14a}$$

$$\sigma_{ij} = \lambda(z)\varepsilon_{kk}\delta_{ij} + 2\mu(z)\varepsilon_{ij} \tag{14b}$$

Where δ_{ij} is the Kronecker delta, l is the material length scale parameter which reflects the effect of couple stress, λ and μ are Lame's constants given by (Al-Basyouni *et al.* 2015)

$$\lambda(z) = \frac{E(z)\nu(z)}{[1+\nu(z)][1-2\nu(z)]} \quad \text{and} \quad \mu(z) = \frac{E(z)}{2[1+\nu(z)]} \tag{15}$$

2.4. Equations of motion

Hamilton's principle is employed in this work to determine the equations of motion. The principle can be expressed in analytical from as (Mahi *et al.* 2015, Attia *et al.* 2015, Adda Bedia *et al.* 2015, Bellifa *et al.* 2016, Boukhari *et al.* 2016, Meksi *et al.* 2017, Zidi *et al.* 2017)

$$\int_0^T (\delta U + \delta V - \delta K)dt = 0 \tag{16}$$

Where δU is the virtual strain energy, δV is the virtual work done by external loads, and δK is the virtual kinetic energy .The virtual strain energy is expressed by (see Eq. (1))

$$\begin{aligned}
\delta U &= \int\limits_0 \int_{-h/2}^{h/2} \left(\sigma_{ij}\delta\varepsilon_{ij} + m_{ij}\delta\chi_{ij}\right)dzdx \\
&+ \int_0^L \int_{-h/2}^{h/2} (\sigma_x\delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz} \\
&+ 2m_{xy}\delta\chi_{xy} + 2m_{yz}\delta\chi_{yz})dzdx \\
&= \int_0^L \left(N\frac{d\delta u_0}{dx} - (M_b + Y_1)\frac{d^2\delta w_b}{dx^2} \right. \\
&- \left(M_s + \frac{1}{2}Y_1 + \frac{1}{2}Y_2\right)\frac{d^2\delta w_s}{dx^2} \\
&\left. + \left(Q - \frac{1}{2}Y_3\right)\frac{d\delta w_s}{dx}\right)dx
\end{aligned} \tag{17}$$

Where L is the length of the micro-scale beam and the following stress resultants are expressed as

$$(N, M_b, M_s) = \int_{-h/2}^{h/2} (1, z, f)\sigma_x dz \tag{18a}$$

$$Q = \int_{-h/2}^{h/2} g\tau_{xz} dz \tag{18b}$$

$$(Y_1, Y_2) = \int_{-h/2}^{h/2} (1, f')m_{xy}dz \ , \ Y_3 = \int_{-h/2}^{h/2} f'' m_{yz}dz \tag{18c}$$

The variation of the work done by the external applied forces can be expressed as

$$\delta V = -\int_0^L q\delta(w_b + w_s)dx \tag{19}$$

Where q is the transverse load.

The variation of kinetic energy is expressed as

$$\begin{aligned}
\delta K &= \int_0^L \int_{-h/2}^{-h/2} \rho(z)\left[\dot{u}_x\delta\dot{u}_x + \dot{u}_y\delta\dot{u}_y\right]dzdx \\
&= \int_0^L \{ I_0\left[\dot{u}_0\delta\dot{u}_0 + (\dot{w}_b + \dot{w}_s)(\delta\dot{w}_b + \delta\dot{w}_s)\right] \\
&- I_1\left(\dot{u}_0\frac{d\delta\dot{w}_b}{dx} + \frac{d\dot{w}_b}{dx}\delta\dot{u}_0\right) \\
&+ I_2\left(\frac{d\dot{w}_b}{dx}\frac{d\delta\dot{w}_b}{dx}\right) - J_1\left(\dot{u}\frac{d\delta\dot{w}_s}{dx} + \frac{d\dot{w}_s}{dx}\delta\dot{u}\right) \\
&+ K_2\left(\frac{d\dot{w}_s}{dx}\frac{d\delta\dot{w}_s}{dx}\right) \\
&+ J_2\left(\frac{d\dot{w}_b}{dx}\frac{d\delta\dot{w}_s}{dx} + \frac{d\dot{w}_s}{dx}\frac{d\delta\dot{w}_b}{dx}\right)\}dx
\end{aligned} \tag{20}$$

Where dot-superscript convention denotes the differentiation with respect to the time variable t; $\rho(z)$ is the masse density; and (I_0, I_1, I_2, J_1, J_2, K_2) are the masse inertias defined as

$$\left(I_0, I_1, I_2, J_1, J_2, K_2\right) = \int_{-h/2}^{h/2} \left(1, z, z^2, f, zf, g\right)\rho(z)dz \quad (21)$$

Substituting Eqs. (17), (19) and (20) into Eq. (16) and integrating by parts, and collecting the coefficients of (δu_0, δw_b, δw_s), the following equations of motion are obtained

$$\delta u_0 : \frac{dN}{dx} = I_0 \ddot{u}_0 - I_1 \frac{d\ddot{w}_b}{dx} - J_1 \frac{d\ddot{w}_s}{dx} \quad (22a)$$

$$\begin{aligned} \delta w_b : & \frac{d^2 M_b}{dx^2} + \frac{d^2 Y_1}{dx^2} + q \\ & = I_0\left(\ddot{w}_b + \ddot{w}_s\right) + I_1 \frac{d\ddot{u}_0}{dx} \\ & - I_2 \frac{d^2 \ddot{w}_b}{dx^2} - J_2 \frac{d^2 \ddot{w}_s}{dx^2} \end{aligned} \quad (22b)$$

$$\begin{aligned} \delta w_s : & \frac{d^2 M_s}{dx^2} + \frac{1}{2}\frac{d^2 Y_1}{dx^2} + \frac{1}{2}\frac{d^2 Y_2}{dx^2} \\ & - \frac{1}{2}\frac{dY_3}{dx} + \frac{dQ}{dx} + q \\ & = I_0\left(\ddot{w}_b + \ddot{w}_s\right) + J_1 \frac{d\ddot{u}_0}{dx} \\ & - J_2 \frac{d^2 \ddot{w}_b}{dx^2} - K_2 \frac{d^2 \ddot{w}_s}{dx^2} \end{aligned} \quad (22c)$$

2.5 Equations of motion in terms of displacements

By employing Eqs. (18) and (22), the equations of motion in terms of the displacements are obtained as

$$\begin{aligned} \delta u_0 : & A_{11}\frac{d^2 u_0}{dx^2} - B^s{}_{11}\frac{\partial^3 w_s}{\partial x^3} \\ & = I_0 \ddot{u}_0 - I_1 \frac{d\ddot{w}_b}{dx} - J_1 \frac{d\ddot{w}_s}{dx} \end{aligned} \quad (23a)$$

$$\begin{aligned} \delta w_b : & -\left(D_{11} + A_{13}\right)\frac{d^4 w_b}{dx^4} \\ & - \left(D^s{}_{11} + \frac{1}{2}\left(A_{13} + B_{13}\right)\right)\frac{d^4 w_s}{dx^4} + q \\ & = I_0\left(\ddot{w}_b + \ddot{w}_s\right) + I_1 \frac{d\ddot{u}_0}{dx} \\ & - I_2 \frac{d^2 \ddot{w}_b}{dx^2} - J_2 \frac{d^2 \ddot{w}_s}{dx^2} \end{aligned} \quad (23b)$$

$$\begin{aligned} \delta w_S : & B^s{}_{11}\frac{d^3 u_0}{dx^3} - \left(D^s{}_{11} + \frac{1}{2}\left(A_{13} + B_{13}\right)\right)\frac{d^4 w_b}{dx^4} \\ & - \left(H^S{}_{11} + \frac{1}{4}\left(A_{13} + 2B_{13} + D_{13}\right)\right)\frac{d^4 w_S}{dx^4} \\ & + \left(A^S{}_{55} + \frac{1}{4}E_{13}\right)\frac{d^2 w_s}{dx^2} + q \\ & = I_0\left(\ddot{w}_b + \ddot{w}_s\right) + J_1 \frac{d\ddot{u}_0}{dx} \\ & - J_2 \frac{d^2 \ddot{w}_b}{dx^2} - K_2 \frac{d^2 w_s}{dx^2} \end{aligned} \quad (23c)$$

Where A_{11}, D_{11}, etc., are the beam stiffness, defined by

$$\begin{aligned} \left(A_{11}, D_{11}, B^s{}_{11}, D^s{}_{11}, H^s{}_{11}\right) = & \int_{-\frac{h}{2}}^{\frac{h}{2}} \lambda(z)\frac{1-\nu(z)}{\nu(z)} \\ & \times \left[1, z^2, f, z\, f, f^2\right] dz \end{aligned} \quad (29)$$

and

$$\begin{aligned} \left(A_{13}, B_{13}, D_{13}, E_{13}\right) = & \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[1, f', (f')^2, (f'')^2\right] \\ & \times \mu(z) l(z)^2 dz \end{aligned} \quad (30)$$

$$A^s{}_{55} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \mu(z) g(z)^2 dz \quad (31)$$

2.6 Analytical solutions

In this section, analytical solutions for bending and free vibration are presented for a simply supported rectangular beam under transverse load q. It is noted that other boundary conditions can be treated (Bennai *et al.* 2015, Bellifa *et al.* 2017b). Based on the Navier approach, the solutions are assumed as

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{n=1}^{\infty} \begin{Bmatrix} U_n \cos(\lambda x) e^{i\omega t} \\ W_{bn} \sin(\lambda x) e^{i\omega t} \\ W_{sn} \sin(\lambda x) e^{i\omega t} \end{Bmatrix} \quad (32)$$

where U_n, W_{bn} and W_{sn} are arbitrary parameters to be determined ω is the eigenfrequency associated with nth eigenmode, and $\lambda = n\pi/L$ the transverse load q is also expanded in Fourier series as

$$q(x, y) = \sum_{n=1}^{\infty} Q_n \sin(\lambda x) \quad (33)$$

Where Q_n is the load amplitude calculated from

$$Q_n = \frac{2}{L}\int_0^L q(x) \sin(\lambda x) dx \quad (34)$$

The coefficient Q_n are given below for some typical loads

$$Q_n = q_0, \quad n = 1 \text{ for sinusoidal load}, \quad (35a)$$

$$Q_n = \frac{4q_0}{n\pi}, \quad n = 1, 2, 3.... \text{ for point load } P \text{ at the midspan}, \quad (36b)$$

Substituting the expansions of U_0, W_b, W_s and q from Eqs. (27) and (28) into the equations of motion Eq. (25), the analytical solutions can be determined from the following equations

$$\left(\begin{bmatrix} s_{11} & 0 & s_{13} \\ 0 & s_{22} & s_{23} \\ s_{13} & s_{23} & s_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}\right) \times \begin{Bmatrix} U_n \\ W_{bn} \\ W_{sn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q_n \\ Q_n \end{Bmatrix} \quad (37)$$

Table 1 Dimensionless transverse deflection of the FG micro-beam for point load, l_m=15, h/l_m=2, b/h=1

l_c/l_m	α	Beam theory	L/h=10				L/h=100			
			k=0.3	k=1	k=3	k=10	k=0.3	k=1	k=3	k=10
1/3	0	CBT	0.3207	0.4081	0.5201	0.6054	0.3207	0.4081	0.5201	0.6054
	0.1		0.3395	0.4454	0.5901	0.6948	0.3395	0.4454	0.5901	0.6948
	0.2		0.3585	0.4863	0.6759	0.8070	0.3585	0.4863	0.6759	0.8070
	0	FSDT	0.3302	0.4199	0.5376	0.6329	0.3208	0.4083	0.5203	0.6057
	0.1		0.3495	0.4581	0.6095	0.7269	0.3396	0.4455	0.5903	0.6951
	0.2		0.3690	0.4999	0.6975	0.8454	0.3586	0.4865	0.6762	0.8074
	0	IHSDT	0.3281	0.4169	0.5331	0.6233	0.3208	0.4082	0.5202	0.6056
	0.1		0.3474	0.4551	0.6051	0.7173	0.3396	0.4455	0.5903	0.6951
	0.2		0.3670	0.4970	0.6936	0.8371	.3586	0.4864	0.6761	0.8074
1	0	CBT	0.1846	0.2649	0.3963	0.5380	0.1846	0.2649	0.3963	0.5380
	0.1		0.2006	0.2931	0.4481	0.6155	0.2006	0.2931	0.4481	0.6155
	0.2		0.2177	0.3242	0.5090	0.7080	0.2177	0.3242	0.5090	0.7080
	0	FSDT	0.1908	0.2736	0.4106	0.5631	0.1847	0.2651	0.3964	0.5383
	0.1		0.2071	0.3023	0.4639	0.6447	0.2007	0.2932	0.4483	0.6157
	0.2		0.2247	0.3342	0.5263	0.7425	0.2178	0.3243	0.5092	0.7083
	0	IHSDT	0.1877	0.2692	0.4032	0.5509	0.1847	0.2650	0.3963	0.5382
	0.1		0.2039	0.2977	0.4559	0.6312	0.2006	0.2931	0.4482	.6157
	0.2		0.2213	0.3292	0.5178	0.7279	0.2177	0.3242	0.5091	.7082
3/2	0	CBT	0.1203	0.1876	0.3123	0.4789	0.1203	0.1876	0.3123	0.4789
	0.1		0.1318	0.2081	0.3508	0.5445	0.1318	0.2081	0.3508	0.5445
	0.2		0.1443	0.2307	0.3945	0.6196	0.1443	0.2307	0.3945	0.6196
	0	FSDT	0.1252	0.1947	0.3247	0.5020	0.1204	0.1877	0.3124	0.4792
	0.1		0.1369	0.2157	0.3643	0.5712	0.1318	0.2081	0.3509	0.5448
	0.2		0.1498	0.2389	0.4093	0.6509	0.1444	0.2309	0.3947	0.6199
	0	IHSDT	0.1219	0.1899	0.3164	0.4883	0.1204	0.1876	0.3123	0.4790
	0.1		0.1335	0.2106	0.3552	0.5555	0.1318	0.2081	0.3508	0.5446
	0.2		0.1462	0.2335	0.3993	0.6329	0.1443	0.2308	0.3946	0.6198
2	0	CBT	0.0819	0.1356	0.2453	0.4200	0.0819	0.1357	0.2453	0.4200
	0.1		0.0900	0.1505	0.2736	0.4738	0.0900	0.1505	0.2736	0.4738
	0.2		0.0989	0.1669	0.3047	0.5329	0.0989	0.1669	0.3047	0.5328
	0	FSDT	0.0861	0.1418	0.2562	0.4412	0.0819	0.1357	0.2454	0.4203
	0.1		0.0944	0.1571	0.2854	0.4982	0.0901	0.1505	0.2737	0.4741
	0.2		0.1036	0.1739	0.3176	0.5611	0.0989	0.1669	0.3048	0.5332
	0	IHSDT	0.0829	0.1371	0.2477	0.4266	0.0819	0.1357	0.2453	0.4201
	0.1		0.0910	0.1519	0.2761	0.4812	0.0900	0.1505	0.2736	0.4739
	0.2		0.1000	0.1684	0.3073	0.5414	0.0989	0.1669	0.3047	0.5329
Classical theory	0	CBT	0.4081	0.5966	0.8295	0.9725	0.4081	0.5967	0.8295	0.9725
	0.1		0.4199	0.6289	0.9085	1.0655	0.4199	0.6289	0.9084	1.0655
	0.2		0.4319	0.6644	1.0059	1.1823	0.4319	0.6644	1.0059	1.1823
	0	FSDT	0.4203	0.6137	0.8568	1.0158	0.4083	0.5968	0.8298	0.9729
	0.1		0.4324	0.6469	0.9378	1.1144	0.4201	0.6292	0.9088	1.0660
	0.2		0.4448	0.6830	1.0377	1.2385	0.4321	0.6646	1.0063	1.1828
	0	IHSDT	0.4196	0.6133	0.8610	1.0259	0.4083	0.5968	0.8298	0.9730
	0.1		0.4318	0.6466	0.9441	1.1343	0.4201	0.6292	0.9088	1.0662
	0.2		0.4443	0.6831	1.0473	1.2813	0.4321	0.6646	1.0064	1.1834

Where

$$s_{11} = A_{11}\lambda^2,\ s_{22} = (D_{11} + A_{13})\lambda^4,$$

$$s_{13} = -B^s{}_{11}\lambda^3\ s_{23} = \left(D^s{}_{11} + \frac{1}{2}(A_{13} + B_{13})\right)\lambda^4$$

$$s_{33} = \left(H^s{}_{11} + \frac{1}{2}\left(\frac{1}{2}(A_{13} + D_{13}) + B_{13}\right)\right)\lambda^4 + \left(A^s{}_{55} + \frac{1}{4}E_{13}\right)\lambda^2$$

$$m_{11} = I_0,\ m_{12} = -I_1\lambda,\ m_{13} = -J_1\lambda,\ m_{22} = I_0 + I_2\lambda^2,\ m_{23} = I_0 + J_2\lambda^2,\ m_{33} = I_0 + K_2\lambda^2 \tag{31}$$

Table 2 Dimensionless transverse deflection of the FG micro-beam for uniform load, l_m=15, h/l_m=2, b/h=1

l_c/l_m	α	Beam theory	L/h=10				L/h=100			
			k=0.3	k=1	k=3	k=10	k=0.3	k=1	k=3	k=10
	0		0.2004	0.2551	0.3251	0.3784	0.2004	0.2551	0.3251	0.3784
	0.1	CBT	0.2122	0.2784	0.3688	0.4343	0.2122	0.2784	0.3688	0.4343
	0.2		0.2241	0.3039	0.4225	0.5044	0.2241	0.3039	0.4225	0.5044
	0		0.2053	0.2612	0.3341	0.3926	0.2005	0.2551	0.3252	0.3785
1/3	0.1	FSDT	0.2173	0.2849	0.3788	0.4509	0.2122	0.2784	0.3689	0.4344
	0.2		0.2294	0.3109	0.4336	0.5242	0.2241	0.3040	0.4226	0.5046
	0		0.2043	0.2596	0.3318	0.3877	0.2005	0.2551	0.3251	0.3785
	0.1	IHSDT	0.2163	0.2834	0.3766	0.4460	0.2122	0.2784	0.3689	0.4344
	0.2		0.2284	0.3095	0.4316	0.5202	0.2241	0.3040	0.4226	0.5046
	0		0.1154	0.1656	0.2477	0.3363	0.1154	0.1656	0.2477	0.3363
	0.1	CBT	0.1254	0.1832	0.2801	0.3847	0.1254	0.1832	0.2801	0.3847
	0.2		0.1361	0.2026	0.3181	0.4425	0.1361	0.2026	0.3181	0.4425
	0		0.1186	0.1708	0.2551	0.3493	0.1154	0.1657	0.2477	0.3364
1	0.1	FSDT	0.1288	0.1879	0.2882	0.3998	0.1254	0.1832	0.2802	0.3848
	0.2		0.1397	0.2077	0.3271	0.4604	0.1361	0.2027	0.3182	0.4427
	0		0.1169	0.1678	0.2513	0.3430	0.1154	0.1656	0.2477	0.3363
	0.1	IHSDT	0.1271	0.1856	0.2842	0.3929	0.1254	0.1832	0.2801	0.3848
	0.2		0.1379	0.2052	0.3227	0.4530	0.1361	0.2026	0.3182	0.4426
	0		0.0752	0.1172	0.1952	0.2993	0.0752	0.1172	0.1952	0.2993
	0.1	CBT	0.0824	0.1300	0.2192	0.3403	0.0824	0.1300	0.2192	0.3403
	0.2		0.0902	0.1442	0.2466	0.3873	0.0902	0.1442	0.2466	0.3873
	0		0.0778	0.1209	0.2016	0.3114	0.0752	0.1173	0.1952	0.2995
3/2	0.1	FSDT	0.0851	0.1339	0.2263	0.3542	0.0824	0.1301	0.2193	0.3405
	0.2		0.0930	0.1484	0.2542	0.4035	0.0902	0.1442	0.2467	0.3874
	0		0.0761	0.1185	0.1973	0.3043	0.0752	0.1172	0.1952	0.2994
	0.1	IHSDT	0.0833	0.1314	0.2216	0.3461	0.0824	0.1300	0.2193	0.3404
	0.2		0.0912	0.1456	0.2491	0.3943	0.0902	0.1442	0.2466	0.3873
	0		0.0512	0.0848	0.1533	0.2625	0.0512	0.0848	0.1539	0.2625
	0.1	CBT	0.0563	0.0941	0.1709	0.2961	0.0563	0.0941	0.1709	0.2961
	0.2		0.0618	0.1043	0.1904	0.3331	0.0618	0.1043	0.1904	0.3331
	0		0.0534	0.0880	0.1590	0.2736	0.0512	0.0848	0.1534	0.2626
2	0.1	FSDT	0.0585	0.0975	0.1772	0.3088	0.0563	0.0941	0.1710	0.2963
	0.2		0.0642	0.1079	0.1972	0.3478	0.0619	0.1043	0.1905	0.3332
	0		0.0517	0.0855	0.1546	0.2659	0.0512	0.0848	0.1533	0.2626
	0.1	IHSDT	0.0568	0.0948	0.1723	0.3001	0.0563	0.0941	0.1709	0.2962
	0.2		0.0624	0.1051	0.1918	0.3376	0.0618	0.1043	0.1905	0.3331
	0		0.2551	0.3729	0.5184	0.6078	0.2551	0.3729	0.5184	0.6078
	0.1	CBT	0.2625	0.3931	0.5678	0.6659	0.2625	0.3931	0.5678	0.6659
	0.2		0.2699	0.4152	0.6287	0.7389	0.2699	0.4152	0.6287	0.7389
	0		0.2612	0.3815	0.5321	0.6295	0.2551	0.3729	0.5186	0.6080
Classical theory	0.1	FSDT	0.2687	0.4021	0.5825	0.6904	0.2625	0.3932	0.5679	0.6662
	0.2		0.2764	0.4246	0.6446	0.7671	0.2701	0.4153	0.6289	0.7392
	0		0.2609	0.3814	0.5345	0.6351	0.2551	0.3729	0.5186	0.6081
	0.1	IHSDT	0.2685	0.4021	0.5859	0.7011	0.2625	0.3932	0.5679	0.6663
	0.2		0.2762	0.4247	0.6498	0.7897	0.2701	0.4153	0.6289	0.7395

3. Numerical results and discussion

3.1 Verification studies

In this section, static bending and dynamic of FG micro-beam are proposed based on the modified couple stress theory and porosity distribution.

The FG micro-beams are composed of metal (*Al*: E_m=70 GPa, ρ_m=2702 kg/m^3, ν_m=0.3) and ceramic (*SiC*: E_m=427 GPa, ρ_c=3100 kg/m^3, ν_m=0.17) (Ansari *et al.* 2011).

The employed non-dimensional quantities are:

Non-dimensional transverse deflection:

$$\overline{w} = 100 w \frac{E_m I}{P L^3} \quad \text{for point load}$$

$$\overline{w} = 100 w \frac{E_m I}{q_0 L^4} \quad \text{for uniform load}$$

Table 3 Dimensionless fundamental frequency of the FG micro-beam, l_m=15, h/l_m=2, b/h=1

l_c/l_m	α	Beam theory	L/h=10				L/h=100			
			k=0.3	k=1	k=3	k=10	k=0.3	k=1	k=3	k=10
1/3	0	CBT	6.8537	6.1844	5.5717	5.2252	6.8818	6.2121	5.5998	5.2498
	0.1		6.8271	6.0727	5.3697	5.0105	6.8559	6.1009	5.3986	5.0357
	0.2		6.8179	5.9696	5.1579	4.7826	6.8475	5.9986	5.1878	4.8089
	0	FSDT	6.7726	6.1127	5.4969	5.1305	6.8819	6.2113	5.5989	5.2488
	0.1		6.7467	6.0035	5.2996	4.9179	6.8559	6.1013	5.3989	5.0358
	0.2		6.7378	5.9026	5.0926	4.6919	6.8474	5.9988	5.1882	4.8089
	0	IHSDT	6.7888	6.1295	5.5149	5.1619	6.8811	6.2115	5.5992	5.2491
	0.1		6.7618	6.0185	5.3139	4.9439	6.8553	6.1004	5.3980	5.0350
	0.2		6.7518	5.9159	5.1031	4.7096	6.8469	5.9980	5.1873	4.8081
1	0	CBT	9.0328	7.6751	6.3832	5.5429	9.0699	7.7095	6.4155	5.5691
	0.1		8.8818	7.4865	6.1621	5.3235	8.9193	7.5214	6.1953	5.3504
	0.2		8.7497	7.3118	5.9437	5.1061	8.7876	7.3473	5.9782	5.1342
	0	FSDT	8.9127	7.5760	6.2914	5.4397	9.0716	7.7111	6.4165	5.5694
	0.1		8.7662	7.3926	6.0762	5.2228	8.9208	7.5228	6.1963	5.3506
	0.2		8.6377	7.2225	5.8636	5.0070	8.7890	7.3486	5.9792	5.1344
	0	IHSDT	8.9712	7.6247	6.3369	5.4882	9.0693	7.7090	6.4149	5.5685
	0.1		8.8213	7.4379	6.1179	5.2675	8.9186	7.5209	6.1948	5.3498
	0.2		8.6899	7.2649	5.9017	5.0469	8.7870	7.3468	5.9778	5.1336
3/2	0	CBT	11.1883	9.1220	7.1907	5.8749	11.2343	9.1630	7.2271	5.9026
	0.1		10.9577	8.8852	6.9647	5.6598	11.0039	8.9267	7.0023	5.6883
	0.2		10.7465	8.6668	6.7514	5.4581	10.7932	8.7090	6.7907	5.4882
	0	FSDT	11.0087	8.9853	7.0777	5.7619	11.2371	9.1653	7.2285	5.9031
	0.1		10.7878	8.7569	6.8587	5.5492	11.0067	8.9289	7.0037	5.6888
	0.2		10.5850	8.5460	6.6518	5.3487	10.7957	8.7111	6.7920	5.4885
	0	IHSDT	11.1261	9.0744	7.1519	5.8274	11.2337	9.1625	7.2267	5.9021
	0.1		10.8978	8.8402	6.9286	5.6125	11.0033	8.9262	7.0019	5.6878
	0.2		10.6886	8.6243	6.7179	5.4097	10.7926	8.7086	6.7903	5.4876
2	0	CBT	13.5599	10.7269	8.1133	6.2732	13.6157	10.7751	8.1534	6.3028
	0.1		13.2593	10.4474	7.8866	6.0673	13.3153	10.4963	7.9292	6.0979
	0.2		12.9791	10.1913	7.6821	5.8855	13.0355	10.2410	7.7269	5.9180
	0	FSDT	13.2865	10.5339	7.9702	6.1470	13.6192	10.7780	8.1562	6.3035
	0.1		13.0036	10.2676	7.7517	5.9433	13.3188	10.4991	7.9311	6.0986
	0.2		12.7392	10.0229	7.5544	5.7619	13.0389	10.2438	7.7287	5.9186
	0	IHSDT	13.4945	10.6807	8.0804	6.2328	13.6150	10.7746	8.1539	6.3023
	0.1		13.1973	10.4046	7.8568	6.0281	13.3146	10.4958	7.9289	6.0975
	0.2		12.9202	10.1517	7.6552	5.8468	13.0348	10.2406	7.7266	5.9176
Classical theory	0	CBT	6.0755	5.1151	4.4118	4.1229	6.1004	5.1380	4.4341	4.1423
	0.1		6.1387	5.1101	4.3278	4.0461	6.1645	5.1339	4.3511	4.0664
	0.2		6.2110	5.1075	4.2281	3.9513	6.2379	5.1323	4.2525	3.9730
	0	FSDT	6.0037	5.0566	4.3544	4.0507	6.0997	5.1374	4.4335	4.1415
	0.1		6.0663	5.0525	4.2726	3.9733	6.1638	5.1333	4.3505	4.0657
	0.2		6.1378	5.0505	4.1754	3.8778	6.2372	5.1317	4.2520	3.9723
	0	IHSDT	6.0066	5.0572	4.3446	4.0325	6.0997	5.1374	4.4334	4.1413
	0.1		6.0688	5.0524	4.2599	3.9427	6.1639	5.1333	4.3504	4.0653
	0.2		6.1399	5.0495	4.1587	3.8217	6.2372	5.1317	4.2518	3.9716

Non-dimensional stresses:

$$(\bar{\sigma}_x, \bar{\tau}_{xz}) = \left(\frac{\sigma_x A}{P}, \frac{\tau_{xz} A}{P}\right) \text{ for point load}$$

Non-dimensional frequency:

$$\varpi = \frac{\omega L^2}{h}\sqrt{\frac{\rho_m}{E_m}}$$

Results tabulated in Tables 1 and 2, show the non-dimensionless deflections of the porous FG micro-beam (FGM-I) for the point load and uniform load respectively. It can be seen increasing the power law index leads to an increase of the non-dimensional deflection for two different values of the aspect ratio (L/h=10, 100). The same remark is observed for the porosity parameter (α). Consequently, these two parameters make the micro beam more flexible. The ratio (l_c/l_m) represents the degree of length scale

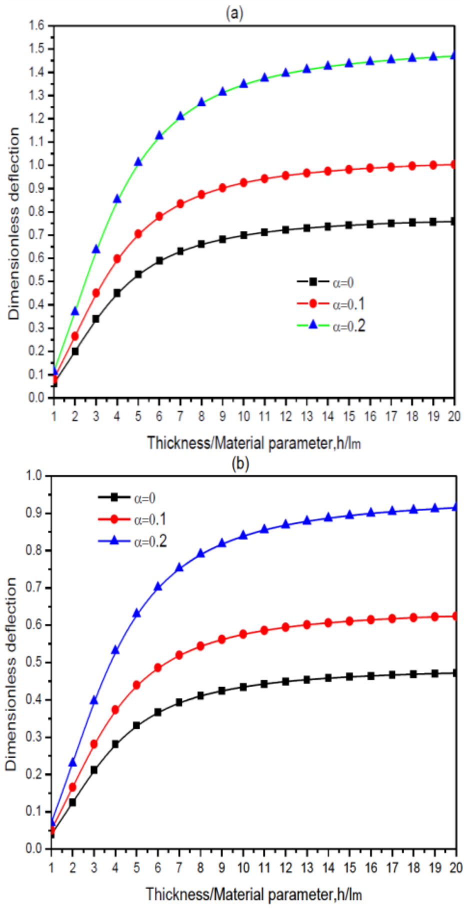

Fig. 1 Variation of the dimensionless transverse deflections of the FG micro-beam for different values of the parameter porosity l_c/l_m=2, L/h=10, l_m=15 μm, b/h=1, k=2: (a) point load, (b) uniform load

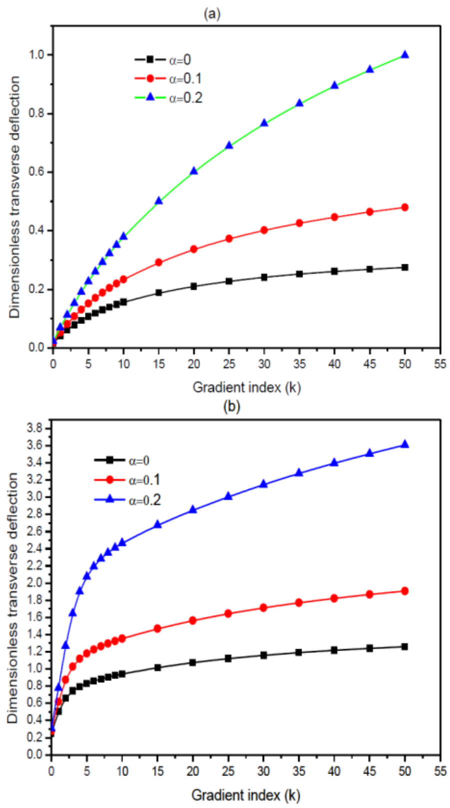

Fig. 2 Variation of the dimensionless transverse deflections of the FG micro-beam subjected to a point load exponent for L/h=10, l_m=15 μm, l_c/l_m=2, (a) h/l_m=1, (b) h/l_m=8

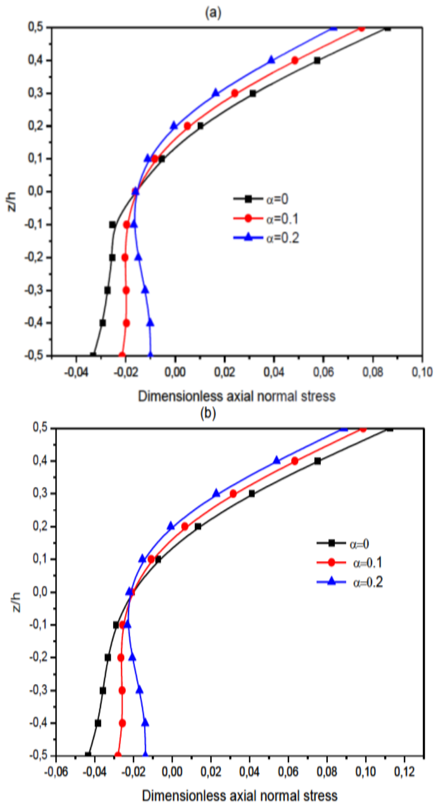

Fig. 3 Variation of the normal stress across the thickness of the FG micro-beam for different values of the parameter porosity with l_c/l_m=2, L/h=10, l_m=15 μm, b=2h, k=2 (a) h/l_m=1 (b) h/l_m=8: (a) point load, (b) uniform load

parameter variation within on the beam. It is observed that increase of (l_c/l_m) leads to a reduction deflection and the results are significantly different to the case where the length scale parameter is considered to be a constant (l_c/l_m=1). This remark is also a confirmation that the variation of the length scale parameter needs to be taken into account in to investigation of FG micro-beam. In addition, it can be concluded that the consideration of the variation of ratio (l_c/l_m) make the micro beam more stiffer when increasing (l_c/l_m). It is found that the CBT underestimates the velour of deflections because this theory neglects the shear deformation effect. Also the introduction of the length scale parameter leads to increase of deflection comparatively to a classical theory.

In Table 3, the dimensionless frequencies corresponding to the transverse deformation mode calculated for various values of the gradient index (k), the length scale parameter (l_c/l_m) and the porosity parameter (α) are presented. It can be observed that the influence of k, l_c/l_m and α are significant. Increasing the porosity parameter leads to a reduction of the non-dimensional frequencies. For each value of the gradient index k, the non-dimensional frequency diminishes considerably as the length scale parameter l_c/l_m is reduced.

In Fig. 1, the non-dimensional transverse deflection are presented versus the ratio (h/l_m) for different porosity

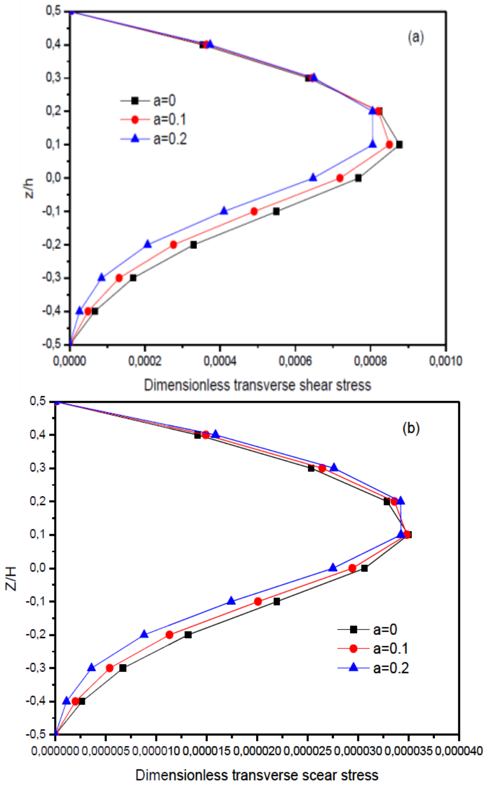

Fig. 4 Variation of the transverse stress across the thickness of the FG micro-beam under point load for different values of the parameter porosity with l_c/l_m=2, L/h=10, l_m=15 μm, b=2h , k=2 (a) h/l_m=1, (b) h/l_m=8

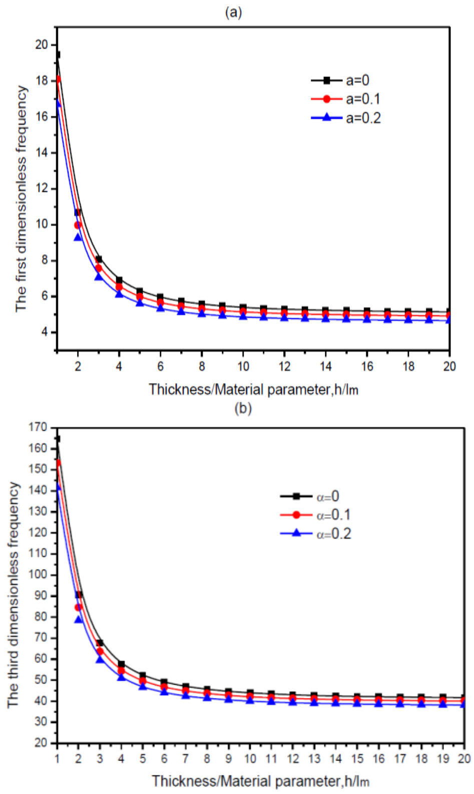

Fig. 5 Variation of the dimensionless frequencies the thickness of the FG micro-beam for different values of the parameter porosity with l_c/l_m=2, L/h=10, l_m=15 μm, b/h=1, k=1 (a) the first frequency, (b) the third frequency

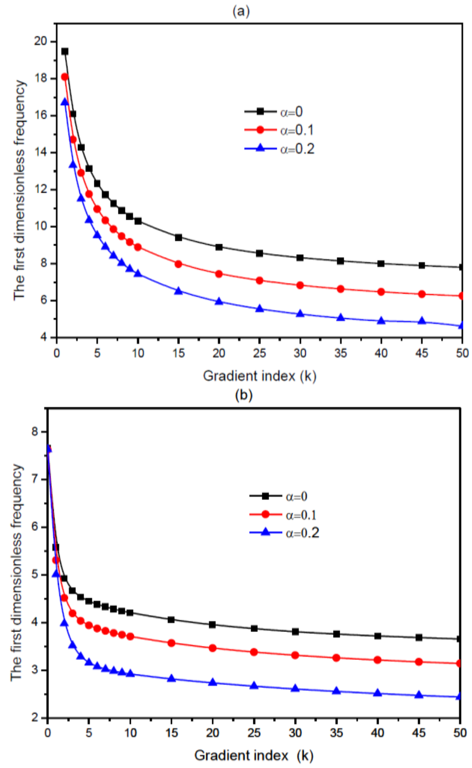

Fig. 6 Variation of the first dimensionless frequency of the FG micro-beam for different values of the parameter porosity versus the volume fraction exponent for l_c/l_m=2, L/h=10, (a) h/l_m=1, (b) h/l_m=8

parameter (α) with considering L/h=10, k=2 and the length scale parameter l_c/l_m=2. It can be found that decreasing the porosity parameter leads to a reduction of deflection for both the point load and uniform load. In addition, increasing the ratio h/l_m makes the micro-beam more flexible.

Fig. 2 presents the variation of the non-dimensional deflection with the gradient index k for different porosity parameter (α) and with considering two different values of h/l_m. It can be observed that the increase of the gradient index leads to an increase of the deflection. The introduction of the porosity parameter makes the micro-beam more flexible.

Fig. 3 presents the normal stress of the FG micro-beam with L/h=10 across the thickness for different values of the porosity parameter (α) and with considering h/l_m=1 and 8. It can be seen that the normal stress $\bar{\sigma}_x\left(L/2, z\right)$ decreases as the parameter porosity is increased from 0 to 0.2.

Fig. 4 presents the variation of the transverse stress across the thickness of FG micro-beam for different values of porosity parameter (α) and for h/l_m=1 and 8. It can be observed that the transverse stress increases with decreasing the porosity parameter (α).

In Fig. 5, both the first and the third frequency are plotted versus the ratio h/l_m for different porosity parameter (α) with considering L/h=10 and k=1. It can be shown that the reduction of the porosity parameter (α) leads to increases of dimensionless frequencies.

Fig 6 illustrates the variation of non-dimensional first frequency against the gradient index (k) and the porosity

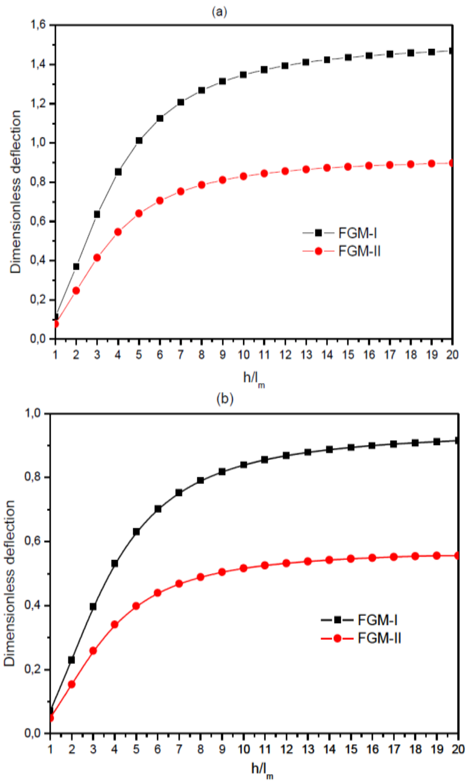

Fig. 7 Variation of the dimensionless transverse deflection of the FG micro-beam for two porosity distribution (FGM-I,FGM-II) with l_c/l_m=2, L/h=10, l_m=15 μm, b/h=1, k=2, α=0.2, (a) point load, (b) uniform load

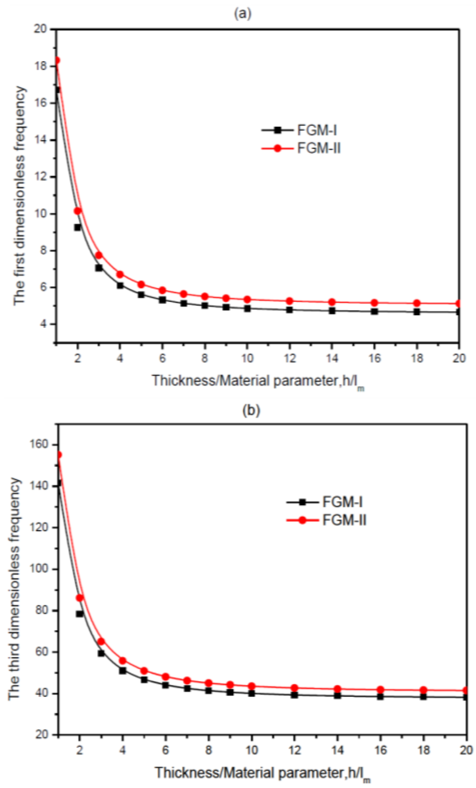

Fig. 9 Variation of the dimensionless frequencies of the FG micro-beam for two porosity distribution (FGM-I,FGM-II) with l_c/l_m=2, L/h=10, l_m=15 μm, b/h=1, k=1, α=0.2 (a) the first frequency.(b)the third frequency

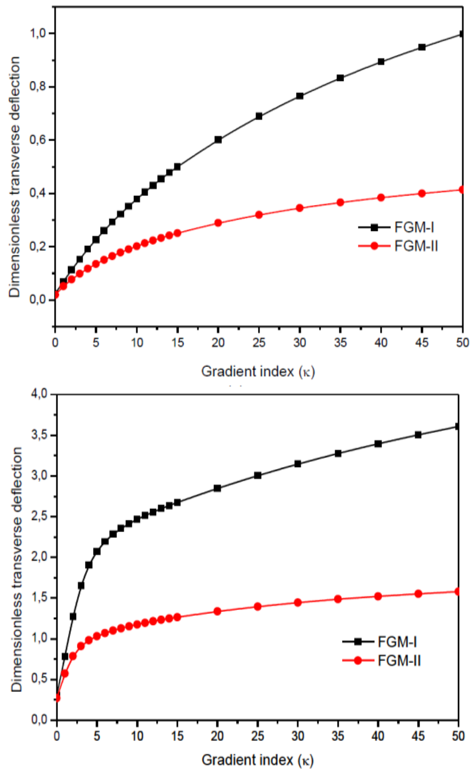

Fig. 8 Variation of the dimensionless transverse deflection of the FG micro-beam subjected to a point load with the volume fraction exponent for two porosity distribution with l_c/l_m=2, L/h=10, l_m=15 μm, b/h=1, k=2, α=0.2, (a) h/l_m=1, (b) h/l_m=8

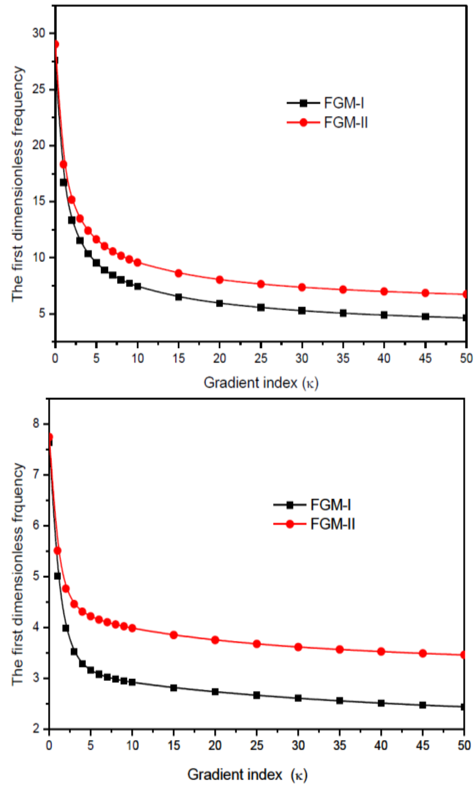

Fig. 10 Variation of the first dimensionless frequencies of the FG micro-beam versus the volume fraction exponent for two porosity distribution (FGM-I,FGM-II) with l_c/l_m=2, L/h=10, l_m=15 μm, b/h=1, k=1, α=0.2 (a) h/l_m=1 (b) h/l_m=8

parameter (α) for two different values of the non-dimensional material parameter h/l_m. It can be seen that the increase of gradient index leads to a reduction of the frequency. In addition, it can be shown that the increase of the porosity parameter (α) makes the micro-beams more flexible. The comparison between the values of the deflection for FG micro-beams with even and uneven porosity distribution is shown in Figs. 7 and 8. It can be observed that the deflections of the even porosity are higher and than uneven ones.

Figs. 9 and 10 present a comparison between the values of the frequencies for FG micro-beams with even and uneven porosity distribution. It can be remarked that the frequency of the even porosity are lower and than uneven ones.

5. Conclusions

In this research, a novel size -dependent FG porous micro-beams with two porosity distributions is presented for bending and vibration analysis. The equations of motion are obtained based on a new hyperbolic shear deformation theory and a modified couple stress theory. The present formulation considers a variable length scale parameter. The findings of this study demonstrated that the inclusion of couple stress influence and the porosity parameter makes a micro-beam stiffer and thus leads to a decrease of the deflection and an increase of the frequency. It is concluded that various factors such as porosity parameter, porosity distribution and gradient index have a considerable effect on the non-dimensional deflection and frequency of FG micro-beams with porosity which emphasizes the importance of inspected porosity volume fraction influence. An improvement of present formulation will be included in the future investigation to consider the thickness stretching effect by using quasi-3D shear deformation models (Bessaim *et al.* 2013, Bousahla *et al.* 2014, Bourada *et al.* 2015, Belabed *et al.* 2014, Fekrar *et al.* 2014, Hebali *et al.* 2014, Larbi Chaht *et al.* 2014, Bennai *et al.* 2015, Meradjah *et al.* 2015, Hamidi *et al.* 2015, Draiche *et al.* 2016, Bennoun *et al.* 2016, Benahmed *et al.* 2017) and to include the thermo-mechanical effects (Bouderba *et al.* 2013, Tounsi *et al.* 2013, Zidi *et al.* 2014, Taibi *et al.* 2015, Beldjelili *et al.* 2016, Bouderba *et al.* 2016, Bousahla *et al.* 2016, Chikh *et al.* 2017, El-Haina *et al.* 2017, Fahsi *et al.* 2017).

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