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# A new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates

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**Abstract.** In this paper, a new nonlocal trigonometric shear deformation theory is proposed for thermal buckling response of nanosize functionally graded (FG) nano-plates resting on two-parameter elastic foundation under various types of thermal environments. This theory uses for the first time, undetermined integral variables and it contains only four unknowns, that is even less than the first shear deformation theory (FSDT). It is considered that the FG nano-plate is exposed to uniform, linear and sinusoidal temperature rises. Mori-Tanaka model is utilized to define the gradually variation of material properties along the plate thickness. Nonlocal elasticity theory of Eringen is employed to capture the size influences. Through the stationary potential energy the governing equations are derived for a refined nonlocal four-variable shear deformation plate theory and then solved analytically. A variety of examples is proposed to demonstrate the importance of elastic foundation parameters, various temperature fields, nonlocality, material composition, aspect and side-to-thickness ratios on critical stability temperatures of FG nano-plate.

**Keywords:** nonlocal elasticity theory; FG nanoplate; thermal buckling refined theory; elastic foundation

## 1. Introduction

In recent years, materials with designed mechanical characteristics have attracted extensive attention. Functionally graded materials (FGMs) with continuous variation in composition and characteristics in desired directions are utilized to improve the materials performance with sudden distributions in material characteristics at the interfaces of multilayered structures. Having different advantageous characteristics, FGMs are suitable for various engineering applications and gained intense interest by several researchers (Alshorbagy *et al.* 2011, Chakraborty *et al.* 2003, Ait Amar Meziane *et al.* 2014, Pradhan and Chakraverty 2015, Wattanasakulpong and Chaikittiratana 2015, Ait Yahia *et al.* 2015, Ait Atmane *et al.* 2015, Darılmaz 2015, Mahi *et al.* 2015, Taibi *et al.* 2015, Kar and Panda 2015, 2016, El-Haina *et al.* 2017, Menasria *et al.* 2017, Ait Atmane *et al.* 2017). Many investigations are reported in literature to investigate the vibration and bending behavior of FG structures, here some of these disquisitions are mentioned briefly (Tornabene *et al.* 2014, 2016, Bellifa *et al.* 2016, Houari *et al.* 2016, Boukhari *et al.* 2016, Tounsi *et al.* 2016, Zidi *et al.* 2017, Abualnour *et al.* 2018). Moreover, structural elements such as beams, plates, and membranes in micro or nano-length scale are commonly used as components in micro/nano electro-mechanical systems (MEMS/NEMS).

Therefore, understanding the mechanical and physical properties of nanostructures is necessary for its practical applications.

Nano-plates are one of the most important types of nanostructures which can be used as building blocks for the fabrication of nano-electro-mechanical systems (NEMs). Therefore, it is essential to consider the small scale influences

in their mechanical investigation. Avoid the scale parameter in the classical continuum model makes it impossible to describe the size influences. Therefore, size dependent continuum models such as nonlocal elasticity theory of Eringen (1972, 1983) and strain gradient theory (Li *et al.* 2015) are proposed to take into account the small scale effects. Lots of works have been realized according to Eringen's nonlocal elasticity theory to examine the size-dependent behavior of structural systems (Aghababaei and Reddy 2009, Natarajan *et al.* 2012, Nami and Janghorban 2013, Ebrahimi and Nasirzadeh 2015, Zemri *et al.* 2015, Adda Bedia *et al.* 2015, Barati *et al.* 2016, Mouffoki *et al.* 2017, Karami *et al.* 2017). They demonstrated that nonlocal elastic theories can only provide softening stiffness with

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increase of nonlocal parameter. Alshorbagy *et al*. (2013) studied the bending response of nanobeams using nonlocal FEM. For investigation of FGM nano-structures, nonlocal elasticity model of Eringen is used in many studies. Vibration and stability of nonlocal Euler-Bernoulli FG nano-beams implementing finite element method is investigated by Eltaher *et al*. (2012, 2013a). Eltaher *et al*. (2013b) presented the effects of neutral axis position on natural frequencies of FG macro/nanobeams. Eltaher *et al*. (2014a) examined the vibration behavior of nonlinear graduation of nano-Timoshenko beam by considering the neutral axis position. By employing the nonlocal Timoshenko beam theory, Eltaher *et al*. (2014b) analyzed the static and buckling behaviors of FG nanobeam. Rahmani and Jandaghian (2015) discussed stability analysis of FG nano-beams using nonlocal third-order shear deformable beam model. Hosseini-Hashemi *et al*. (2013) proposed an exact analytical solution for dynamic of FG circular/annular Mindlin nano-plates according to a nonlocal elasticity. Nami and Janghorban (2014) studied the resonance behaviors of FG micro/nano-plates by employing Kirchhoff plate theory. Daneshmehr and Rajabpoor (2014) employed a nonlocal higher order plate model for buckling analysis of FG nano-plates subjected to biaxial in-plane loadings using generalized differential quadrature (GDQ). Belkorissat *et al*. (2015) analyzed the dynamic properties of FG nano-plates via a novel nonlocal hyperbolic refined plate model. Zare *et al*. (2015) investigated the natural frequencies of a FG nano-plate for different combinations of boundary conditions. Recently, Bounouara *et al*. (2016) developed a nonlocal zeroth-order shear deformation theory for free vibration of FG nanoscale plates resting on elastic foundation. Hamed *et al*. (2016) investigated the free vibration of symmetric and sigmoid functionally graded nanobeams. Thermal stability can have a destructive effect on the safety of structures and thus it is considered as an undesired phenomenon in several works (Bachir Bouiadjra *et al.* 2013). Recently, Barati *et al*. (2016) analyzed the thermal buckling behavior of size-dependent FG nano-plates resting on two-parameter elastic foundation under various types of thermal environments based on a new refined trigonometric shear deformation theory. More reports on the nanomechanics theories may be also found in the open literature (see, e.g., Mahmoud *et al.* 2012, Eltaher *et al*. 2013c, d, Khater *et al*. 2014, Eltaher *et al*. 2014c, d, Bouafia *et al*. 2017, Ebrahimi and Salari 2015, Ebrahimi *et al*. 2015, Larbi Chaht *et al.* 2015, Ebrahimi and Barati 2016ab, Eltaher *et al*. 2016a, b, c, d, Ahouel *et al*. 2016, Besseghier *et al*. 2017).

Structures are often exposed to heat and moisture during manufacturing or use. The variation of temperature and moisture leads to a reverse impact on the stiffness and strength of the composite materials. The influences of thermal or hygrothermal conditions on the FGM structures have been examined in many works (see, e.g., Beldjelili *et al*. 2016, Bouderba *et al*. 2016, Bousahla *et al*. 2016, Hamidi *et al*. 2015, Tounsi *et al*. 2013, Zidi *et al*. 2014, Attia *et al*. 2015, Bouderba *et al*. 2013).

In this article, a novel nonlocal trigonometric shear deformation theory is proposed for the thermal stability analysis of simply supported FG nano-plates on elastic foundation subjected to three kinds of thermal loading. The consideration of the integral term in the displacement field leads to a reduction in the number of variables and governing equations. Implementing the stationary potential energy, the nonlocal governing equations are derived and they are solved via Navier solution technique. Various cases of thermal loading such as uniform, linear and sinusoidal temperature rises are considered in this study and applied in the analysis of FG nano-plates. Finally, the effects of the elastic foundation, different thermal loads, gradient index, nonlocal parameter, aspect and thickness ratios on the thermal stability of embedded FG nano-plates is explored.

## 2. Theory and formulation

### *2.1 Mori-Tanaka FGM plate model*

By employing the Mori-Tanaka homogenization method the local effective material characteristics of the FG nano-plate such as effective local bulk modulus $K_e$ and shear modulus $\mu_e$ can be computed

$$\frac{K_e - K_m}{K_c - K_m} = \frac{V_c}{1 + V_m\left(K_c - K_m\right)/\left(K_m + 4\mu_m/3\right)} \tag{1}$$

$$\frac{\mu_e - \mu_m}{\mu_c - \mu_m} = \frac{V_c}{1 + V_m\left(\mu_c - \mu_m\right)/\left[\mu_m + \mu_m\left(9K_m + 8\mu_m\right)/\left(6\left(K_m + 2\mu_m\right)\right)\right]} \tag{2}$$

Where subscripts *m* and *c* represent metal and ceramic, respectively and the relation between the volume fraction of the ceramic and the metal is defined by

$$V_c + V_m = 1 \tag{3}$$

The volume fraction of the ceramic constituent of the FG nano-plate is considered to be expressed by

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^p \tag{4}$$

Here $P$ is the power law index which defines the material variation across the thickness of the plate and $Z$ is the distance from the mid-surface of the FG nano-plate. Thus, the effective Young's modulus ($E$), via the Mori-Tanaka technique can be written by

$$E(z) = \frac{9K_e\mu_e}{3K_e + \mu_e} \tag{5}$$

The thermal expansion coefficient ($\alpha$) may be given by

$$\frac{\alpha_e - \alpha_m}{\alpha_c - \alpha_m} = \frac{\dfrac{1}{K_e} - \dfrac{1}{K_m}}{\dfrac{1}{K_c} - \dfrac{1}{K_m}} \tag{6}$$

The material composition of FG nano-plate at the top surface ($z$=+$h$/2) is considered to be the pure ceramic and it vary continuously to the bottom surface ($z$=−$h$/2) which is pure metal as presented in

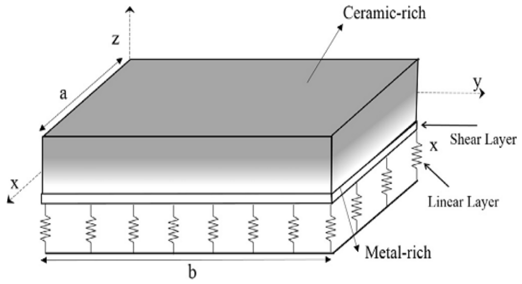



Fig. 1 Geometry and coordinates of embedded functionally graded nano-plate

### *2.2 Displacement base field*

The displacement field of the novel theory is expressed as follows (Chikh *et al.* 2017, Bourada *et al.* 2016, Merdaci *et al.* 2016, Hebali *et al.* 2016, Meftah *et al.* 2017, Meksi *et al.* 2017, Fahsi *et al.* 2017)

$$u(x,y,z)=u_0(x,y)-z\frac{\partial w_0}{\partial x}+k_1\,f(z)\int\theta(x,y)\,dx \tag{7a}$$

$$v(x,y,z)=v_0(x,y)-z\frac{\partial w_0}{\partial y}+k_2\,f(z)\int\theta(x,y)\,dy \tag{7b}$$

$$w(x,y,z)=w_0(x,y) \tag{7c}$$

where $u_0$, $v_0$, $w_0$, and $\theta$ are the four unknown displacement functions of the mid-surface of the plate. The last variable is a mathematical term that allows obtaining the rotations of the normal to the mid-plate about the *x* and *y* axes (as in the ordinary HSDT). The coefficients $k_1$ and $k_2$ depends on the geometry. The integrals employed are undetermined.

$f(z)$ represents a shape function determining the transverse shear strain variations and the stress distribution within the thickness. The kinematic of the classical plate theory (CPT) can be easily obtained, if $f(z)$=0. In this article, the proposed HSDT has a trigonometric function in the form

$$f(z)=\frac{z\left(\pi+2\cos\left(\frac{\pi z}{h}\right)\right)}{(2+\pi)} \tag{8}$$

The nonzero strains of the proposed plate theory are expressed as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \tag{9}$$

Where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix} \tag{10a}$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}=\begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\displaystyle\int\theta\,dx+k_2\dfrac{\partial}{\partial x}\displaystyle\int\theta\,dy\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}=\begin{Bmatrix}k_1\displaystyle\int\theta\,dy\\ k_2\displaystyle\int\theta\,dx\end{Bmatrix} \tag{10b}$$

and

$$g(z)=\frac{df(z)}{dz} \tag{10c}$$

The integrals employed in the above equations shall be resolved by a Navier type technique and can be written as follows

$$\begin{gathered}\frac{\partial}{\partial y}\int\theta\,dx=A'\frac{\partial^2\theta}{\partial x\partial y},\quad \frac{\partial}{\partial x}\int\theta\,dy=B'\frac{\partial^2\theta}{\partial x\partial y},\\ \int\theta\,dx=A'\frac{\partial\theta}{\partial x}\quad,\quad \int\theta\,dy=B'\frac{\partial\theta}{\partial y}\end{gathered} \tag{11}$$

where the coefficients $A'$ and $B'$ are expressed according to the type of solution employed, in this case via Navier. Therefore, $A'$, $B'$, $k_1$ and $k_2$ are expressed as follows

$$A'=-\frac{1}{\alpha^2}\quad,\quad B'=-\frac{1}{\beta^2}\quad,\quad k_1=\alpha^2\quad,\quad k_2=\beta^2 \tag{12}$$

where $\alpha$ and $\beta$ are used in expression (26).

It should be noted that unlike the FSDT, this theory does not require shear correction coefficients.

The stability equations of FG plates under thermal loadings may be obtained on the basis of the stationary potential energy (Reddy 1984, Klouche *et al.* 2017). The stability equations are expressed as

$$\begin{aligned}&\delta\, u_0:\quad \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\\ &\delta\, v_0:\quad \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0\\ &\delta\, w_0:\quad \frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}+\frac{\partial^2 M_y^b}{\partial y^2}+N_x^0\frac{\partial^2 w_0}{\partial x^2}+\\ &2N_{xy}^0\frac{\partial^2 w_0}{\partial x\partial y}+N_y^0\frac{\partial^2 w_0}{\partial y^2}-k_w w_0+k_s\nabla^2 w_0=0\\ &\delta\,\theta:\quad -k_1 M_x^s-k_2 M_y^s-\left(k_1A'+k_2B'\right)\frac{\partial^2 M_{xy}^s}{\partial x\partial y}+\\ &k_1A'\frac{\partial S_{xz}^s}{\partial x}+k_2B'\frac{\partial S_{yz}^s}{\partial y}=0\end{aligned} \tag{13}$$

where $N_x^0$, $N_y^0$, $N_{xy}^0$ are in-plane applied loads and $k_w$, $k_s$ are elastic foundation parameters.

Using constitutive relations, the stress and moment resultants are defined by

$$\begin{gathered}\left(N_i,M_i^b,M_i^s\right)=\int_{-h/2}^{h/2}(1,z,f)\sigma_i dz\qquad (i=x,y,xy)\\ \left(S_{xz}^s,S_{yz}^s\right)=\int_{-h/2}^{h/2}g\left(\tau_{xz},\tau_{yz}\right)dz\end{gathered} \tag{14}$$

and in this work it is supposed that the nano-plate is under a biaxial thermo-mechanical loading and the shear loading is ignored ( $N_x^0 = N_y^0 = N^T$ , $N_{xy}^0 = 0$ ) and thermal resultant can be expressed as

$$N^T = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu} \alpha(z) T dz \tag{15}$$

where $T(x,y,z)$ is the temperature rise through-the-thickness.

### *2.3 The nonlocal elasticity model for FG nano-plate*

Based on Eringen's nonlocal elasticity theory (Eringen, 1972), the stress state at a point inside a body is considered to be a function of strains of all points in the neighbor regions. For homogeneous elastic solids, the nonlocal stress-tensor components $\sigma_{ij}$ at each point $x$ in the solid can be expressed as

$$\sigma_{ij}(x) = \int_{\Omega} \alpha\left(\left|x'-x\right|, \tau\right) t_{ij}(x') d\Omega(x') \tag{16}$$

where $t_{ij}(x')$ are the components available in local stress tensor at point $x$ which are related to the strain tensor components $\varepsilon_{kl}$ as

$$t_{ij} = C_{ijkl}\varepsilon_{kl} \tag{17}$$

The concept of Eq. (16) is that the nonlocal stress at any point is a weighting average of the local stress of all near points, and the nonlocal kernel $\alpha(|x'-x|, \tau)$ considers the effect of the strain at the point $x'$ on the stress at the point $x$ in the elastic body. The parameter $\alpha$ is an internal characteristic length (e.g., lattice parameter, granular distance, the length of C-C bonds). Also $|x'-x|$ is Euclidean distance and $\tau$ is a constant value as follows

$$\tau = \frac{e_0 a}{l} \tag{18}$$

which defines the relation of a characteristic internal length, and a characteristic external length, $l$ (e.g., crack length and wavelength) by employing a constant, $e_0$, dependent on each material. The value of $e_0$ is experimentally determined by comparing the scattering curves of plane waves with those of atomistic dynamics.

In the nonlocal elasticity theory, the points undergo translational motion as in the classical case, but the stress at a point depends on the strain in a region near that point. As for physical interpretation, the nonlocal model incorporates long range interactions between points in a continuum model.

Such long range interactions occur between charged atoms or molecules in a solid. Eringen (1972, 1983) numerically obtain the functional form of the kernel. By appropriate selection of the kernel function, Eringen shown that the nonlocal constitutive equation given in integral form (see Eq. (19)) can be represented in an equivalent differential form as

$$\left(1-(e_0 a)\nabla^2\right)\sigma_{kl} = t_{kl} \tag{19}$$

In which $\nabla^2$ is the Laplacian operator. Hence, the scale length $e_0 a$ considers the effects of small size on the behavior of nanostructures. Thus, the constitutive relations of nonlocal theory for a FG nano-plate can be written as

$$\left(1-\mu\nabla^2\right)\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{20}$$

In which $\mu = (e_0 a)^2$ and the stiffness coefficients, $C_{ij}$, can be defined as

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu(z)^2},\ C_{12} = \frac{\nu\, E(z)}{1-\nu(z)^2},\ C_{44} = C_{55} = C_{66} = \frac{E(z)}{2\left[1+\nu(z)\right]} \tag{21}$$

Integrating Eq. (20) over the plate's cross-section area yields the force-strain and the moment-strain of the nonlocal refined FG nano-plates as follows

$$\left(1-\mu\nabla^2\right)\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & D_{66}^s \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x^b \\ k_y^b \\ k_{xy}^b \\ k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \tag{22a}$$

$$\left(1-\mu\nabla^2\right)\begin{Bmatrix} S_{xz}^s \\ S_{yz}^s \end{Bmatrix} = \begin{bmatrix} A_{55}^s & 0 \\ 0 & A_{44}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \tag{22b}$$

Where the cross-sectional rigidities are defined as follows

$$\left(A_{ij}, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right) = \int_{-h/2}^{h/2} C_{ij}\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) dz, \quad \left(i, j = 1,2,6\right) \tag{23a}$$

$$A_{ij}^s = \int_{-h/2}^{h/2} C_{ij}\left[g(z)\right]^2 dz, \quad \left(i, j = 4,5\right) \tag{23b}$$

The nonlocal equations of stability of FG nano-plates in terms of the displacement can be obtained by substituting Eq. (22), into Eq. (13) as follows

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + \left(A_{12}+A_{66}\right)\frac{\partial^2 v_0}{\partial x\,\partial y} - B_{11}\frac{\partial^3 w_0}{\partial x^3} - \left(B_{12}+2B_{66}\right)\frac{\partial^3 w_0}{\partial x\,\partial y^2} + \left(B_{66}^s\left(k_1 A' + k_2\, B'\right)\right)\frac{\partial^3 \theta}{\partial x\,\partial y^2} + \left(B_{11}^s k_1 + B_{12}^s\, k_2\right)\frac{\partial \theta}{\partial x} = 0 \tag{24a}$$

$$A_{22}\frac{\partial^2 v_0}{\partial y^2} + A_{66}\frac{\partial^2 v_0}{\partial x^2} + \left(A_{12}+A_{66}\right)\frac{\partial^2 u_0}{\partial x\,\partial y} - B_{22}\frac{\partial^3 w_0}{\partial y^3} - \left(B_{12}+2B_{66}\right)\frac{\partial^3 w_0}{\partial x^2 \partial y} + \left(B_{66}^s\left(k_1 A' + k_2\, B'\right)\right)\frac{\partial^3 \theta}{\partial x^2 \partial y} + \left(B_{22}^s k_2 + B_{12}^s k_1\right)\frac{\partial \theta}{\partial y} = 0 \tag{24b}$$

$$B_{11}\frac{\partial^3 u_0}{\partial x^3}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 u_0}{\partial x\,\partial y^2}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 v_0}{\partial x^2\,\partial y}+$$
$$B_{22}\frac{\partial^3 v_0}{\partial y^3}-D_{11}\frac{\partial^4 w_0}{\partial x^4}-2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_0}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 w_0}{\partial y^4}$$
$$+\left(D_{11}^s k_1+D_{12}^s k_2\right)\frac{\partial^2\theta}{\partial x^2}+2\left(D_{66}^s\left(k_1A'+k_2B'\right)\right)\frac{\partial^4\theta}{\partial x^2\partial y^2}+ \tag{24c}$$
$$\left(D_{12}^s k_1+D_{22}^s k_2\right)\frac{\partial^2\theta}{\partial y^2}+$$
$$\left(1-\mu\nabla^2\right)\left(-k_w w_0+\left(N^T+k_s\right)\nabla^2 w_0\right)=0$$

$$-\left(B_{11}^s k_1+B_{12}^s k_2\right)\frac{\partial u_0}{\partial x}-\left(B_{66}^s\left(k_1A'+k_2B'\right)\right)\frac{\partial^3 u_0}{\partial x\partial y^2}$$
$$-\left(B_{66}^s\left(k_1A'+k_2B'\right)\right)\frac{\partial^3 v_0}{\partial x^2\partial y}-\left(B_{12}^s k_1+B_{22}^s k_2\right)\frac{\partial v_0}{\partial y}$$
$$+\left(D_{11}^s k_1+D_{12}^s k_2\right)\frac{\partial^2 w_0}{\partial x^2}+2\left(D_{66}^s\left(k_1A'+k_2B'\right)\right)\frac{\partial^4 w_0}{\partial x^2\partial y^2} \tag{24d}$$
$$+\left(D_{12}^s k_1+D_{22}^s k_2\right)\frac{\partial^2 w_0}{\partial y^2}-H_{11}^s k_1^2\,\theta-H_{22}^s\,k_2^2\,\theta-2H_{12}^s k_1 k_2\theta$$
$$-\left(\left(k_1A'+k_2B'\right)^2 H_{66}^s\right)\frac{\partial^4\theta}{\partial x^2\partial y^2}+A_{44}^s\left(k_2B'\right)^2\frac{\partial^2\theta}{\partial y^2}+A_{55}^s\left(k_1A'\right)^2\frac{\partial^2\theta}{\partial x^2}=0$$

## 3. Solution procedures

In this section, an analytical solution based on the Navier method is utilized to solve the nonlocal governing equations of a simply supported FG nano-plate. To satisfy governing equations of stability and the simply supported boundary condition, the displacement variables are adopted to be of the form

$$\begin{Bmatrix} u_0\\ v_0\\ w_0\\ \theta\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}\cos(\alpha\,x)\sin(\beta\,y)\\ V_{mn}\sin(\alpha\,x)\cos(\beta\,y)\\ W_{mn}\sin(\alpha\,x)\sin(\beta\,y)\\ X_{mn}\sin(\alpha\,x)\sin(\beta\,y)\end{Bmatrix} \tag{25}$$

where ($U_{mn}$, $V_{mn}$, $W_{mn}$, $X_{mn}$) are the unknown Fourier coefficients.

With

$$\alpha=m\pi/a \quad\text{and}\quad \beta=n\pi/b \tag{26}$$

Inserting Eq. (25) into Eq. (24), leads to

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14}\\ S_{12} & S_{22} & S_{23} & S_{24}\\ S_{13} & S_{23} & S_{33} & S_{34}\\ S_{14} & S_{24} & S_{34} & S_{44}\end{bmatrix}\begin{Bmatrix} U_{mn}\\ V_{mn}\\ W_{mn}\\ X_{mn}\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\\0\end{Bmatrix} \tag{27}$$

Where

$$S_{11}=-\left(A_{11}\alpha^2+A_{66}\beta^2\right),\quad S_{12}=-\alpha\beta\left(A_{12}+A_{66}\right),$$
$$S_{13}=\alpha\left(B_{11}\alpha^2+B_{12}\beta^2+2B_{66}\beta^2\right)\quad S_{22}=-\left(A_{66}\alpha^2+A_{22}\beta^2\right)$$
$$S_{14}=\alpha\left(k_1B_{11}^s+k_2B_{12}^s-\left(k_1A'+k_2B'\right)B_{66}^s\beta^2\right)$$
$$S_{23}=\beta\left(B_{22}\beta^2+B_{12}\alpha^2+2B_{66}\alpha^2\right)$$
$$S_{24}=\beta\left(k_2B_{22}^s+k_1B_{12}^s-\left(k_1A'+k_2B'\right)B_{66}^s\alpha^2\right)$$
$$S_{33}=-\left(D_{11}\alpha^4+2(D_{12}+2D_{66})\alpha^2\beta^2+D_{22}\beta^4\right)+\lambda\left(k_w+\left(N^T+k_s\right)\left(\alpha^2+\beta^2\right)\right)$$

Table 1 Material properties of metal and ceramic phases

| Properties | Metal | Ceramic |
|---|---|---|
| $E$ (Pa) | 70 | 380 |
| $\alpha$ (K-1) | 2310-6 | 7.4 10-6 |
| $\nu$ | 0.3 | 0.3 |

$$S_{34}=-k_1\left(D_{11}^s\alpha^2+D_{12}^s\beta^2\right)+2(k_1A'+k_2B')\,D_{66}^s\alpha^2\beta^2-k_2\left(D_{22}^s\beta^2+D_{12}^s\alpha^2\right)$$
$$S_{44}=-k_1\left(H_{11}^s k_1+H_{12}^s k_2\right)-(k_1A'+k_2B')^2\,H_{66}^s\alpha^2\beta^2-k_2\left(H_{12}^s k_1+H_{22}^s k_2\right)-$$
$$\left(k_1A'\right)^2A_{55}^s\alpha^2-\left(k_2B'\right)^2A_{44}^s\beta^2$$
$$\lambda=1+\mu\left(\alpha^2+\beta^2\right) \tag{28}$$

## 4. Types of thermal loading

### 4.1 Uniform temperature rise (UTR)

For a FG nanoscale plate at reference temperature $T_0$, the temperature is uniformly raised to a final value $T$ which the temperature variation is $\Delta T=T-T_0$.

### 4.2 Linear temperature rise (LTR)

For a FG nanoscale plate for which the plate thickness is thin enough, the temperature variation is considered to be changed linearly within the thickness as follows (Barati *et al.* 2016)

$$T=T_m+\Delta T\left(\frac{1}{2}+\frac{z}{h}\right) \tag{29}$$

where the buckling temperature difference in Eq. (29) is $\Delta T=T_c-T_m$ and $T_c$ and $T_m$ are the temperature of the top surface which is ceramic-rich and the bottom surface which is metal-rich, respectively (see Table 1).

### 4.3 Sinusoidal temperature rise (STR)

The temperature field when FG nano-plate is exposed to sinusoidal temperature rise across the thickness can be defined as (Na and Kim 2004, Barati *et al.* 2016)

$$T=T_m+\Delta T\left[1-\cos\frac{\pi}{2}\left(\frac{1}{2}+\frac{z}{h}\right)\right] \tag{30}$$

where $\Delta T=T_c-T_m$ is temperature change.

## 5. Numerical results and discussions

In this section, illustrative results are presented to examine the thermal stability of embedded FG nano-plates modeled based on a new nonlocal trigonometric shear deformation theory. The material properties of the FG nano-plate change across the thickness direction according to Mori-Tanaka homogenization method. The effects of gradient index, scale parameter, various thermal forces, elastic foundation parameters and aspect ratio on the critical buckling temperatures of the FG nano-plate are

Table 2 Minimum critical temperature parameter $\alpha T_{cr}$ of the simply supported isotropic plate ($a/b$=1, $\alpha_0$=1.0×$10^{-6}$ /K, $E$=1.0×$10^6$ N/m$^2$, $\nu$=0.3)

| $a/h$ | Present theory | Noor and Burton (1992) | Matsunaga (2005) | Kettaf (2013) |
|---|---|---|---|---|
| 10 | 0.1198×$10^{-1}$ | 0.1183×$10^{-1}$ | 0.1183×$10^{-1}$ | 0.1198×$10^{-1}$ |
| 20 | 0.3120×$10^{-2}$ | 0.3109×$10^{-2}$ | 0.3109×$10^{-2}$ | 0.3119×$10^{-2}$ |
| 100 | 0.1265×$10^{-3}$ | 0.1264×$10^{-3}$ | 0.1264×$10^{-3}$ | 0.1265×$10^{-3}$ |

investigated. A 5 K increase in metal surface to reference temperature $T_0$ of FG nano-plate is considered, i.e., $T_m-T_0$=5 K (Barati *et al.* 2016). The following dimensionless of Winkler's and Pasternak's elastic foundation parameters are used in the present analysis

$$K_w = k_w \frac{a^4}{D_c}, \quad K_s = k_s \frac{a^2}{D_c}, \quad D_c = \frac{E_c h^3}{12\left(1-\nu^2\right)} \tag{31}$$

In order to verify the correctness of the present theory, a comparison has been made with the results reported by Matsunaga (2005), Kettaf *et al.* (2013) and Noor and Burton (1992) for homogeneous isotropic plates subjected to uniform temperature load. The critical buckling temperature difference is tabulated in Table 2. From these results, it can be confirmed that the present theory is in excellent agreement with those obtained by Kettaf *et al.* (2013).

In the second verification, the results for thermal buckling of FG plates under uniform and linear thermal loading through the thickness are computed and compared with those obtained by CPT, FSDT, TSDT and SSDT as shown in Table 3.

It is clear that the results present considerable differences between the shear deformation plate theories and the CPT one, indicating the shear deformation influence. In addition, a good agreement is obtained between the present theory and other HSDTs for all values the side-to-thickness ratio $a/h$ and the elastic foundations parameters ($K_w$, $K_s$).

Another verification is carried out in this work by comparing the obtained results with those computed with

Table 3 Comparison of critical buckling temperature of simply-supported power-law FG plates ($p$=1)

| | Theory | $K_w$=0 $K_s$=0 | | | $K_w$=10 $K_s$=0 | | | $K_w$=0 $K_s$=0 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $a/h$ | | | $a/h$ | | | $a/h$ | | |
| | | 5 | 10 | 50 | 5 | 10 | 50 | 5 | 10 | 50 |
| UTR | Present | 2.67426 | 0.75858 | 0.03171 | 2.83787 | 0.79949 | 0.03335 | 6.06743 | 1.60688 | 0.06565 |
| | SSDT (a) | 2.67241 | 0.75845 | 0.03171 | 2.83602 | 0.79935 | 0.03335 | 6.06558 | 1.60674 | 0.06565 |
| | TSDT (a) | 2.67153 | 0.75840 | 0.03171 | 2.83514 | 0.79930 | 0.03335 | 6.06470 | 1.60669 | 0.06565 |
| | FSDT | 2.67039 | 0.75837 | 0.03171 | 2.83400 | 0.79928 | 0.03335 | 6.06356 | 1.60667 | 0.06565 |
| | CPT | 3.17751 | 0.79438 | 0.03178 | 3.34112 | 0.83528 | 0.03341 | 6.57068 | 1.64267 | 0.06571 |
| LTR | Present | 5.00611 | 1.41332 | 0.05010 | 5.31296 | 1.49003 | 0.05317 | 11.36989 | 3.00427 | 0.11374 |
| | SSDT (a) | 5.00264 | 1.41307 | 0.05010 | 5.30948 | 1.48978 | 0.05317 | 11.36642 | 3.00402 | 0.11374 |
| | TSDT (a) | 5.00099 | 1.41297 | 0.05010 | 5.30784 | 1.48968 | 0.05317 | 11.36477 | 3.00391 | 0.11374 |
| | FSDT | 4.99885 | 1.41292 | 0.05010 | 5.30570 | 1.48964 | 0.05317 | 11.36263 | 3.00387 | 0.11374 |
| | CPT | 5.94993 | 1.48045 | 0.05022 | 6.25678 | 1.55716 | 0.05328 | 12.31372 | 3.07140 | 0.11385 |

Table 4 Critical buckling temperature of simply-supported FG nano-plates for various shear deformation theories ($a/b$=1, $p$=1, $K_w$=$K_s$=0)

| loading | MODEL | $\mu$=0 nm$^2$ | | | $\mu$=2 nm$^2$ | | |
|---|---|---|---|---|---|---|---|
| | | $a/h$ | | | $a/h$ | | |
| | | 5 | 10 | 20 | 5 | 10 | 20 |
| UTR | CPT[(a)] | 3199.97 | 897.771 | 231.517 | 2294.24 | 643.663 | 165.988 |
| | HPT[(a)] | 3077.83 | 887.779 | 230.845 | 2206.67 | 636.499 | 165.506 |
| | SSDT[(a)] | 3077.60 | 887.735 | 230.842 | 2206.51 | 636.468 | 165.504 |
| | Present | 3078.95 | 887.823 | 230.848 | 2207.47 | 636.531 | 8165.50 |
| LTR | CPT[(a)] | 5874.65 | 1641.56 | 416.502 | 4209.27 | 1174.32 | 296.012 |
| | HPT[(a)] | 5650.08 | 1623.18 | 415.266 | 4048.26 | 1161.15 | 295.126 |
| | SSDT[(a)] | 5649.66 | 1623.10 | 415.260 | 4047.96 | 1161.09 | 295.121 |
| | Present | 5874.65 | 1641.56 | 416.502 | 4209.27 | 1174.32 | 296.012 |
| STR | CPT[(a)] | 7773.35 | 2172.11 | 551.116 | 5569.71 | 1553.87 | 391.683 |
| | HPT[(a)] | 7476.19 | 2147.80 | 549.481 | 5356.67 | 1536.44 | 390.511 |
| | SSDT[(a)] | 7475.64 | 2147.69 | 549.473 | 5356.27 | 1536.36 | 390.505 |
| | Present | 7478.90 | 2147.91 | 549.486 | 5358.61 | 1536.52 | 390.515 |

(a) Results reported from Barati *et al.* (2016)

Table 5 Critical buckling temperature of simply-supported FG nano-plates under various temperature rises ($p$=1, $K_w$=25, $K_s$=5)

| Loading | $\mu$ | $a/b$=1 | | | $a/b$=2 | | | $a/b$=3 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $a/h$ | | | $a/h$ | | | $a/h$ | | |
| | | 5 | 10 | 20 | 5 | 10 | 20 | 5 | 10 | 20 |
| UTR | 0 | 6059.95 | 1633.07 | 417.160 | 8712.56 | 2716.06 | 729.501 | 11585.3 | 4310.76 | 1251.92 |
| | 1 | 5552.38 | 1486.72 | 379.105 | 8043.41 | 2489.70 | 667.366 | 10687.3 | 3946.49 | 1143.63 |
| | 2 | 5188.48 | 1381.78 | 351.821 | 7506.70 | 2308.14 | 617.528 | 9950.91 | 3647.74 | 1054.82 |
| | 3 | 4914.80 | 1302.87 | 331.301 | 7066.63 | 2159.27 | 576.665 | 9336.01 | 3398.30 | 980.671 |
| LTR | 0 | 11133.4 | 2993.57 | 757.847 | 16010.8 | 4984.88 | 1332.15 | 21292.9 | 7917.01 | 2292.73 |
| | 1 | 10200.1 | 2724.46 | 687.874 | 14780.4 | 4568.67 | 1217.91 | 19641.8 | 7247.30 | 2093.62 |
| | 2 | 9530.96 | 2531.52 | 637.706 | 13793.5 | 4234.82 | 1126.27 | 18287.7 | 6697.99 | 1930.33 |
| | 3 | 9027.75 | 2386.41 | 599.977 | 12984.4 | 3961.10 | 1051.13 | 17157.1 | 6239.33 | 1793.99 |
| STR | 0 | 14731.7 | 3961.10 | 1002.78 | 21185.5 | 6596.00 | 1762.71 | 28174.8 | 10475.9 | 3033.74 |
| | 1 | 13496.8 | 3605.01 | 910.196 | 19557.4 | 6045.27 | 1611.53 | 25990.1 | 9589.64 | 2770.28 |
| | 2 | 12611.4 | 3349.70 | 843.813 | 18251.6 | 5603.52 | 1490.28 | 24198.4 | 8862.79 | 2554.22 |
| | 3 | 11945.5 | 3157.71 | 793.890 | 17180.9 | 5241.33 | 1390.86 | 22702.3 | 8255.89 | 2373.81 |

various plate theories (Barati *et al.* 2016) such as CPT, higher order plate theory (HPT) and SSDT in buckling analysis of square FG nano-plates under thermal environments when $p$=1 and ($K_w$=$K_s$=0).

The results of this comparison are presented in Table. 4.

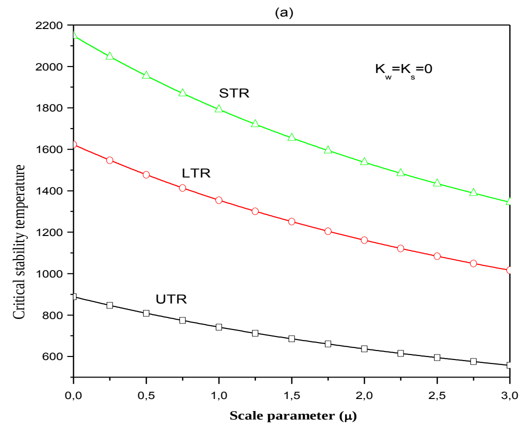
It can be seen from these results that that the obtained critical buckling temperatures using by higher order shear deformation models are very close together. But, the CPT by ignoring shear deformation influence gives larger values for critical temperature. This difference between CPT and HSDTs becomes more important for lower values of *a/h*.

Table 5 presents the critical buckling temperature of FG nano-plate supported by elastic foundation. In this Table, we present also the influences of elastic foundation parameters, the thermal loads types (UTR, LTR and STR), scale parameter ($\mu$) and geometric parameters (*a/h* and *a/b*) on critical temperature. Compared to the results presented by Barati *et al*. (2016), we can see that Table 5 gives almost the same results.

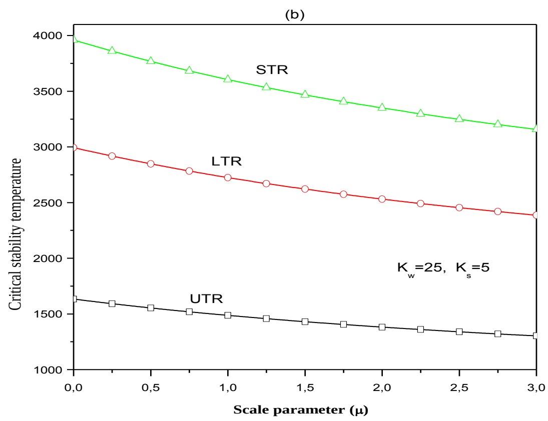
According to these results, it is clear that for all types of thermal loads because of the reducing effect of scale parameter on the plate stiffness increasing their values leads to a decrease in critical stability temperatures of FG nano-plates.

Contrary to scale parameter, Winkler and Pasternak parameters have an increasing influence on the both plate stiffness and critical stability temperature. In addition, it is concluded that for all values of foundation parameters, regardless of thermal load type increasing in the ratio *a/b* and the ratio *a/h* respectively increases and reduces the buckling temperatures ($\Delta T_{cr}$) of FG nano-plates. Finally, it must be indicated that sinusoidal thermal load (STR) produces higher values of critical temperature difference compared to those of UTR and LTR, while uniform temperature rise provides the lower ones.

Fig. 2 presents the effect of scale parameter on the critical stability temperature ($\Delta T_{cr}$) of square FG nano-plates with and without elastic foundation for various temperature fields. It can be observed that for all type of temperature distribution the critical stability temperature of FG nano-plate decreases when the scale parameter increases. This is due to the fact that presence of nonlocality makes the plate structure more flexible.




$K_w$=$K_s$=0




$K_w$=25, $K_s$=5

Fig. 2 Effect of scale parameter on the critical buckling temperature of simply-supported square FG nano-plate ($a/h$=10; $p$=1)

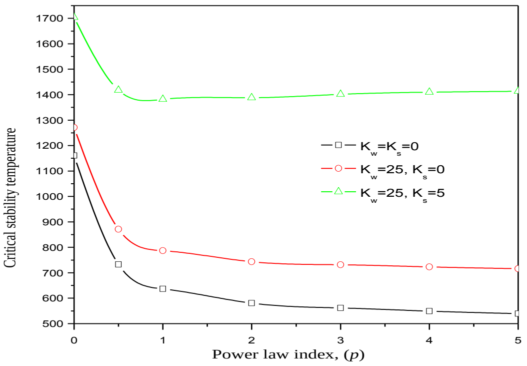



Fig. 3 Effect of power law index on the critical stability temperature of simply supported square FG nano-plate under uniform temperature rise ($a/h$=10, $\mu$=2)

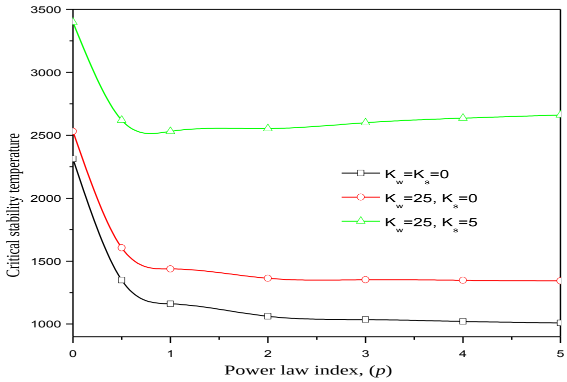



Fig. 4 Effect of power law index on the critical stability temperature of simply supported square FG nano-plate under linear temperature rise ($a/h$=10; $\mu$=2)

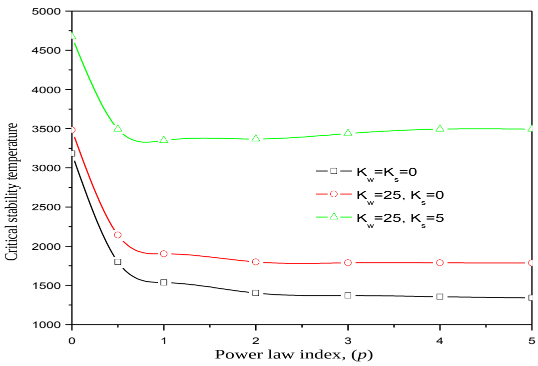
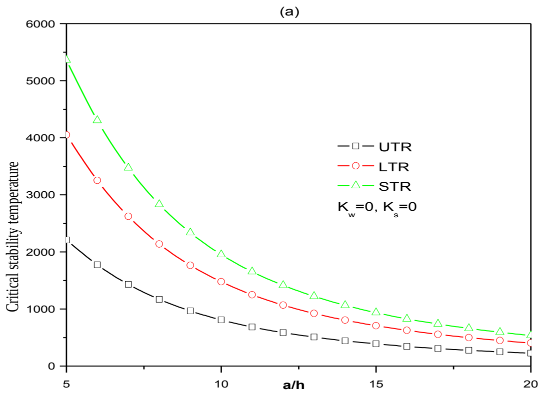
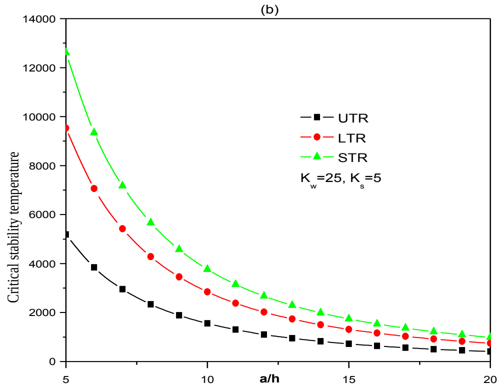



Fig. 5 Effect of power law index on the critical stability temperature of simply supported square FG nano-plate under sinusoidal temperature rise ($a/h$=10; $\mu$=2)




$K_w$=$K_s$=0




$K_w$=25, $K_s$=5

Fig. 6 Influence of side-to-thickness ratio on the critical stability temperature of square simply-supported FG nano-plate ($p$=1, $\mu$=2)

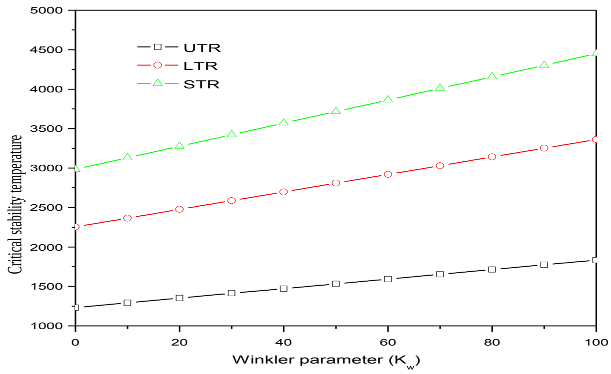



Fig. 7 Influence of Winkler parameter on the critical stability temperature of simply supported square FG nano-plate under various temperature rises ($a/h$=10, $p$=1, $\mu$=2, $K_s$=25)

Thus, nonlocal plate theory provides lower buckling results compared to local plate theory.

The influences of power law index ($p$) on variations of the critical stability temperature of simply supported FG nano-plates under uniform, linear and sinusoidal temperature distributions at $a/h$=10 and $\mu$=2 are demonstrated in Figs. 3-5, respectively.

It can be observed that for all values of elastic foundation parameters the critical stability temperature decreases with

Increasing the power law index, where this decrease is more sensible according to the lower values of power law index. In addition, it is shown that the Pasternak parameter has a more considerable effect on the critical stability temperature than Winkler parameter.

Therefore, with an increase of Pasternak constant the critical stability temperature increases considerably.

Fig. 6 presents the variation of the critical stability

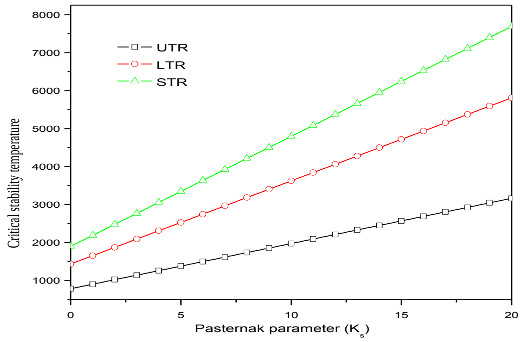



Fig. 8 Influence of Pasternak parameter on the critical stability temperature of simply supported square FG nano-plate under various temperature rises ($a/h$=10, $p$=1, $\mu$=2, $K_s$=25)

temperature difference of FG nano-plate versus side-to-thickness ratio $a/h$. The FG nano-plate is assumed to be exposed to uniform, linear and sinusoidal temperature loads.

It can be observed that as the plate side-to-thickness ratio $a/h$ increases the critical stability temperature decreases with a severe rate, especially for the case of sinusoidal temperature load. To demonstrate the effects of elastic foundation coefficients on the thermal stability response of FG nano-plate individually, Figs. 7 and 8 show the variations of the critical stability temperature difference versus the Winkler and Pasternak constants, respectively.

It is seen from these results that regardless of the type of the thermal load, the critical stability temperature arises with the increase of Winkler and Pasternak coefficients, because of the increment in stiffness of the FG nano-plate. Moreover, according to these results, it is clear that sinusoidal temperature load (STR) gives larger values of $\Delta T_{cr}$ than UTR and LTR, while UTR provides the lower values for critical temperature. Also, the differences between the buckling results of various thermal loads become more important for larger values of elastic foundation coefficients.

## 6. Conclusions

In this article, thermal stability response of the FG nano-plates resting on two-parameter elastic foundation subjected to various thermal loads is investigated within a new nonlocal trigonometric shear deformation theory. By proposing further simplifying suppositions to the existing HSDTs and with the incorporation of an undetermined integral term, the number of unknowns and governing equations of the proposed HSDT are reduced by one, and thus, make this theory simple and efficient to use. Three types of thermal loads including uniform, linear and sinusoidal temperature variations are considered in this study. Material properties of the FG nano-plates vary gradually according to Mori-Tanaka model. Via the stationary potential energy and nonlocal constitutive relations of Eringen, the nonlocal governing differential equations are deduced. Then, these equations are solved by employing Navier analytical procedure. Finally, it is demonstrated that buckling behaviors of FG nano-plates are affected by various parameters such as elastic foundation parameters, scale parameter, power law index, thermal loadings, and side-to-thickness ratio. It is found that the presence of nonlocality reduces the plate stiffness and diminishes the critical stability temperature of FG nano-plates. Contrary to the scale parameter, Winkler and Pasternak coefficients enhance the plate structure and increase the buckling temperatures. Moreover, it is concluded that sinusoidal temperature load provides higher critical stability temperatures than uniform and linear temperature loads. An improvement of present formulation will be considered in the future work to account for the thickness stretching effect by using quasi-3D shear deformation models (Bessaim *et al*. 2013, Bousahla *et al*. 2014, Swaminathan and Naveenkumar 2014, Sayyad and Ghugal 2014, Belabed *et al*. 2014, Fekrar *et al*. 2014, Hebali *et al*. 2014, Meradjah *et al*. 2015, Hamidi *et al*. 2015, Bourada *et al*. 2015, Bennoun *et al*. 2016, Draiche *et al*. 2016).

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