Structural Engineering and Mechanics, *Vol. 64, No. 2 (2017) 271-278*
DOI: https://doi.org/10.12989/sem.2017.64.2.271

A new conjugate gradient algorithm for solving dynamic load identification

Lin J. Wang[*1,2], Qi C. Deng[1a] and You X. Xie[3b]

[1]*Hubei Key Laboratory of Hydroelectric Machinery Design and Maintenance, College of Mechanical and Power Engineering, China Three Gorges University, Yichang, Hubei 443002, PR China*
[2]*School of Chemistry, Physics and Mechanical Engineering, Queensland University of Technology, Brisbane, QLD 4001, Australia*
[3]*College of Science Technology, China Three Gorges University, Yichang, Hubei 443002, PR China*

(Received January 6, 2017, Revised September 15, 2017, Accepted September 18, 2017)

Abstract. In this paper, we propose a new conjugate gradient method which possesses the global convergence and apply it to solve inverse problems of the dynamic loads identification. Moreover, we strictly prove the stability and convergence of the proposed method. Two engineering numerical examples are presented to demonstrate the effectiveness and speediness of the present method which is superior to the Landweber iteration method. The results of numerical simulations indicate that the proposed method is stable and effective in solving the multi-source dynamic loads identification problems of practical engineering.

Keywords: inverse problems; multi-source dynamic loads identification; finite element method; conjugate gradient method

1. Introduction

Along with the rapid development of science and technology, it is often required to obtain the exact value of the expected load acting on the engineering structure. For instance, we need to know it when carrying the experimental modal analysis and structural health monitoring. However, it is difficult or impossible to directly measure the dynamic load in many practical engineering problems considering the economic condition, environment factors and technical limitations. In addition, the determination of dynamic loads acting on the structure is a very important task in the fields of civil engineering, aerospace engineering, mechanical engineering, etc. For instance, aerodynamic load influences flight; the tall building is subjected to wind load; the bridge suffers exciting forces from the vehicles and pedestrian, etc.

Theoretically, the dynamic load identification is an important inverse problem in the field of structural dynamics considering that these loads play important role in the safety of structure, so many researchers have developed lots of indirect methods to identify the dynamic loads acting on the practical structure by the measured dynamic responses (Zhu *et al*. 2014, Wang *et al*. 2011a, Wang *et al*. 2012, Chan *et al*. 2008a, Chan *et al*. 2008b). Bartlett and Flannelly exploited modal verification of force determination to identify vibration loads acting on the structure of helicopter spindle (Bartlett and Flannelly 1979). Giansante *et al*. calculated the magnitudes and phases of the external loads through the acceleration response measured on the AH-IG helicopter airframe in flight (Giansante *et al*. 1982). Simonian predicted wind loads on a structure by using a dynamic programming filter (Simonian 1981a, Simonian 1981b). Wang *et al*. presented an improved iteration regularization method for solving linear inverse problems (Wang *et al*. 2011b). Xu *et al*. proposed an improved force identification model based on modal filter and identified the dynamic load acting on the structure of cantilever (Xu *et al*. 2002). Zhang and Zhu proposed a dynamic load identification method based on neural network method and obtained well results (Zhang and Zhu 1997). Liu *et al*. proposed a computational inverse scheme based on the Gegenbauer polynomial expansion theory and regularization method to identify dynamic loads acting on stochastic structures (Liu *et al*. 2015a). Emilio Turco proposed an inverse strategy to identify the external static loads and external dynamic loads acting on the structure of pin-jointed truss by the stress or strains responses data (Turco 1998, Turco 2005a, Turco 2005b).

Some researchers developed lots of computational inverse methods in time domain mainly depending on the relation between loads and system responses (Liu *et al*. 2011b, Lu and Law 2007, Liu *et al*. 2002, Ronasi *et al*. 2011, Liu *et al*. 2014). Gunawan exploited B-splines functions, quadratic splines functions and harmonic functions as the basis functions to identify the impact forces acting on the structure (Gunawan and Homma 2008a, Gunawan and Homma 2008b, Gunawan *et al*. 2006). Amiri used Derivation of a new parametric impulse response matrix to identify the nodal wind load by response measurement (Amiri and Bucher 2015). Zhang and Qin exploited orthogonal polynomial fitting technique to

∗Corresponding author, Professor
E-mail: ljwang2006@126.com
[a]M.S. Student
E-mail: 578420767@qq.com
[b]Lecturer
E-mail: xieyouxiangxie@126.com

http://www.techno-press.com/journals/sem&subpage=7 ISSN: 1225-4568 (Print), 1598-6217 (Online)

identify the dynamic loads acting on the structure (Zhang *et al*. 2006, Qin *et al*. 2007). Xu and He proposed a time domain substructural identification approach for simultaneous identification of physical parameters of concerned substructures and unknown external excitations (Xu and He 2015).

In Reference (Neubauer 2000), Neubauer assured that Landweber iteration method is an iterative regularization method, and has good performances when solving the ill-posed problems, yet the rate of convergence of regularized solution by this method is very slow and inefficient. Moreover, most of these inverse problems mentioned above are ill-posed. However, there are few papers about regularization methods which are proved mathematically and suitable for solving the load identification problems, as far as we know. In the authors' previous work (Wang and Xie 2012), the simple ill-conditioned problem of dynamic load identification has been solved by the new conjugate gradient method which is based on the gradient operator $\beta_k = \|g_{k+1}\|^2 (g_{k+1} - \|d_k\|^2 g_k)^T d_k$. Due to that this operator is not very effective in revising unusually smaller singular values when solving the extremely ill-conditioned problems of dynamic load identification, it seems incapable to deal with the extremely ill-posed problem of dynamic load identification, especially in complex engineering structure.}} In order to break through these bottlenecks and solve these difficulties, we attempt to propose and create an efficient conjugate gradient method (MCG) based on the ideas of (Wei *et al*. 2006), and investigate the minimum of this minimization problem.

The rest of the paper is organized as follows. Section 2 briefly introduces the basic theories and the establishment of a new conjugate gradient method. In Section 3, the convergence of the proposed conjugate gradient method is provided and proved. Numerical simulations are given to demonstrate the effectiveness of the proposed method in Section 4. Finally Section 5 summarizes some conclusions.

2. The establishment of a new conjugate gradient algorithm

Conjugate gradient methods are very stable and efficient in solving the optimization problems, and they can also converge in a few steps and find the approximate solution of inverse problems stably and effectively.

We consider the following unconstrained optimization problem

$$\min_{x\in R^n} f(x), \tag{1}$$

where $f{:}R^n \to R$ is a continuously differentiable function, and $g(x_k)$ is the gradient of this function $f(x)$ at point x_k. The iterative form of this method is

$$x_{k+1} = x_k + \alpha_k d_k,\ k=0,1,2,\cdots, \tag{2}$$

where α_k is a steplength, and d_k is the search direction. This stepsize is often obtained by one dimensional search method. The exact line search is generally given as follows

$$f(x_k + \alpha_k d_k) = \min_{\alpha\geq 0} f(x_k + \alpha_k d_k) \tag{3}$$

The search direction is d_k, which is often defined by

$$d_k = \begin{cases} -g_k + \beta_k d_{k-1}, & k \geq 1 \\ -g_0, & k=0, \end{cases} \tag{4}$$

where β_k is a scalar, and different choices correspond to different conjugate methods. Next we propose a new formula

$$\beta_k^{WCX} = \begin{cases} \dfrac{\|g_{k+1}\|^2 - \|g_{k+1}\|}{\|g_{k+1}\|^\alpha \|d_k\|} (\alpha \geq \dfrac{1}{2}) & if\ \|g_{k+1}\| \geq 1 \\ 1 & otherwise \end{cases}. \tag{5}$$

The corresponding algorithm is defined as follows:

Algorithm 2.1. (MCG method)

Step 0: Initialization. Given $x_0 \in R^n$, set k=0.

Step 1: Compute β_k by the formula (5). \\

Step 2: Generate d_k based on (4). If $\|g_k\| < \varepsilon$, then stop.

Step 3: Compute α_k by (3).

Step 4: Updating the new point based on (2). If $f(x_{k+1}) < f(x_k)$ and $\|g_k\| < \varepsilon$, then stop. Otherwise go to Step 0 with k=k+1.

3. The convergence analysis of the proposed method

Next we will discuss and study the convergent properties of β_k^{WCX}. In order to validate the well definition of the proposed method, we should prove its global convergence properties and sufficient descent properties.

Theorem 3.1. Consider the CG method in the form (3) and (4) based on the formula (5), then the sufficient descent condition holds, i.e., there exists a positive constant C such that

$$g_k^T d_k \leq -C\|g_k\|^2 \tag{6}$$

for $k \geq 0$.

Proof. It is easy to check that the assertion holds for k=0. Next we will show that the sufficient condition also holds for $k \geq 1$. Exploiting (4), we can obtain

$$\begin{aligned} g_{k+1}^T d_{k+1} &= g_{k+1}^T(-g_{k+1} + \beta_{k+1} d_k) \\ &= -\|g_{k+1}\|^2 + \beta_{k+1} g_{k+1}^T d_k. \end{aligned} \tag{7}$$

We also know that $g_{k+1}^T d_k = 0$ in terms of the exact line search. Therefore we have

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2.$$

Then we can give a conclusion that d_{k+1} is a sufficient direction. Thus we complete the proof of Theorem 3.1.

Next we will show the global convergence properties of the proposed method.

(H1). f is bounded below on the level set R^n; f is continuously differentiable in a neighborhood N of the level set $\Gamma = \{x \in R^n | f(x) \leq f(x_0)\}$ at the initial point x_0.

(H2). The gradient $g(x)$ is Lipschitz continuous, i.e., there exists a constant L>0 such that

$$\|g(x) - g(y)\| \leq L\|x - y\|, \forall x, y \in N.$$

Based on the two assumptions above, we immediately have the following Lemma (Yuan *et al*. 2010, Zoutendijk 1970):

Lemma 3.1 Let (H1)-(H2) be satisfied. Consider any CG method by the formula (4), where α_k satisfies the exact minimization rule and d_k is a descent search direction. Then

$$\sum_{k=0}^{\infty}\frac{g_k^T d_k}{\|d_k\|^2}<\infty.$$

Under Lemma 3.1, we immediately have the following convergent theorem of the present method.

Theorem 3.2. Suppose that (H1)-(H2) and the descent condition hold true. Consider the CG method in the form of (2) and (4), where α_k is generated by the exact minimization rule. Then either

$$\lim_{k\to\infty}\|g_k\|=0$$

or

$$\sum_{k=0}^{\infty}\frac{(g_k^T d_k)^2}{\|d_k\|^2}<\infty.$$

Proof. In fact, we can prove it by contradiction. So there exists a constant $\varepsilon>0$ such that

$$\|g_k\|\geq\varepsilon. \tag{8}$$

Using the formula (4), we have

$$d_{k+1}+g_{k+1}=\beta_{k+1}d_k.$$

Then we can obtain

$$\|d_{k+1}\|^2=\beta_{k+1}^2\|d_k\|^2-2g_{k+1}^T d_{k+1}-\|g_{k+1}\|^2. \tag{9}$$

Then

$$\begin{aligned}\frac{\|d_{k+1}\|^2}{(g_{k+1}^T d_{k+1})^2}&=\frac{\|d_k\|^2}{(g_{k+1}^T d_{k+1})^2}\frac{(\|g_{k+1}\|^2-\|g_{k+1}\|)^2}{\|g_{k+1}\|^{2\alpha}\|d_k\|^2}-(\frac{1}{\|g_{k+1}\|}+\frac{\|g_{k+1}\|^2}{g_{k+1}^T d_{k+1}})^2\\&+\frac{1}{\|g_{k+1}\|^2}\\&\leq\frac{1}{\|g_{k+1}\|^2}+\frac{1}{\|g_{k+1}\|^2}\\&=\frac{2}{\|g_{k+1}\|^2}\end{aligned} \tag{10}$$

So

$$\frac{\|d_k\|^2}{(g_k^T d_k)^2}\leq\frac{2}{\|g_k\|^2}\leq\sum_{i=0}^{K}\frac{2}{\|g_i\|^2}. \tag{11}$$

Then

$$\frac{(g_k d_k)^2}{\|d_k\|^2}\geq\frac{\varepsilon^2}{2K}. \tag{12}$$

Exploiting (8) and (12), we can obtain

$$\sum_{k=0}^{\infty}\frac{g_k^T d_k}{\|d_k\|^2}=\infty,$$

which obviously contradicts Lemma 3.1. Then we complete the proof of the Theorem 3.2.

4. Numerical examples and discussion

In order to evaluate the effectiveness of the proposed method described in the previous sections, two practical engineering problems will be investigated. Numerical results performed by the proposed method and Landweber iteration regularization method will be contrasted under the same convergence criteria.

Herein we consider the multi-source dynamic loads identification problem for a linear and time-invariant dynamic system. The response at an arbitrary receiving point in a structure can be expressed as a convolution integral of the forcing time history and the corresponding Green's kernel in time domain (Liu *et al*. 2002, Liu *et al*. 2011)

$$y(t)=\int_0^t G(t-\tau)p(\tau)d\tau, \tag{13}$$

where $y(t)$ is the response which can be displacement, velocity, acceleration, strain, etc. $G(t)$ is the corresponding Green's function, which is the kernel of impulse response. $p(t)$ is the desired unknown dynamic load acting on the structure.

By discretizing this convolution integral, the whole concerned time period is separated into equally spaced intervals, and the Eq. (13) is transformed into the following system of algebraic equation

$$Y(t)=G(t)P(t), \tag{14}$$

or equivalently,

$$\begin{pmatrix}y_1\\y_2\\\vdots\\y_m\end{pmatrix}=\begin{pmatrix}g_1&&&\\g_2&g_1&&\\\vdots&\vdots&\ddots&\\g_m&g_{m-1}&\cdots&g_1\end{pmatrix}\begin{pmatrix}p_1\\p_2\\\vdots\\p_m\end{pmatrix}\Delta t,$$

where y_i, g_i and p_i are response,

Green's function matrix and input force at time $t=i\Delta t$, respectively. Δt is the discrete time interval. Since the structure without applied force is static before force is applied, y_0 and g_0 are equal to zero. All the elements in the upper triangular part of G are zeros and are not shown. The special form of the Green's function matrix reflects the characteristic of the convolution integral.

To recover the time history $P(t)$, the knowledge of $y(t)$ and $G(t)$ are required. In fact, the response at a receiving point and the numerical Green's function of a structure can be obtained by finite element method. However, the problem of identifying the dynamic load $P(t)$ by $y(t)$ and $G(t)$ is usually ill-posed, and cannot be solved by inverse matrix method. In the following, our method will be suggested to solve this problem.

In order to illustrate the accuracy of the present method, we consider the simulated exact concentrated loads as follows

$$\begin{aligned}F_1(t)&=\begin{cases}q_1\sin(\frac{2\pi t}{t_d}), & 0\leq t\leq 2t_d\\0, & t<0\ \ and\ \ t>2t_d\end{cases}\\F_2(t)&=\begin{cases}4q_2t/t_d, & 0\leq t\leq t_d/4\\2q_2-4q_2t/t_d, & t_d/4<t\leq 3t_d/4\\4q_2t/t_d-4q_2, & 3t_d/4<t\leq t_d\\0, & t>t_d\end{cases}\end{aligned} \tag{15}$$

where t_d is the time cycle of sine force, and q is a constant amplitude of the force. In order to compare the performances of two regularization methods mentioned above, we choose

t_d=0.004 s, q_1=1000 N, and q_2=800 N. Herein, the experimental data of response is simulated by the computed numerical solution, and the corresponding vertical displacement response can be obtained by finite element method. Furthermore, a noise is directly added to the computer-generated response to simulate the noise-contaminated measurement, and the noisy response is defined as follow

$$Y_{err} = Y_{cal} + l_{noise} \cdot std\,(Y_{cal}) \cdot rand\,(-1{,}1) \qquad (16)$$

where Y_{cal} is the computer-generated response; $std(Y_{cal})$ is the standard deviation of Y_{cal}; $rand(-1,1)$ denotes the random number between -1 and $+1$; l_{noise} is a parameter which controls the level of the noise contamination.

In order to investigate the effect of measurement error on the accuracy of the estimated values, we consider the case of noise level namely 5%, and the present method is adopted to determine the dynamic forces.

In the reference (Neubauer 2000), it is shown that Landweber iteration method can stably and effectively solve ill-posed problems. Therefore, the optimal solution obtained by the present method will be compared with the Landweber iterative solution. The comparison will be quantitatively made by way of the relative estimation error

$$\tilde{F} = \left| \frac{F_{\mathrm{Real}}(i) - F_{\mathrm{Identified}}(i)}{\max\{F_{\mathrm{j}}\}} \right| * 100. \qquad (17)$$

and the average error

$$F_{\mathrm{Average}} = \frac{1}{n}\sum_{i=1}^{n} \left| \frac{F_{\mathrm{Real}}(i) - F_{\mathrm{Identified}}(i)}{\max\{F_{\mathrm{j}}\}} \right| * 100, \qquad (18)$$

where i=1,2,…,n, j=1,2.

4.1 A stiffened plate

A practical engineering problem is to determine vertical forces of stiffened plate, as shown in Fig. 1. The main parameters of the model are as: It is 0.48 m long, 0.32 m wide, and the thickness is 0.002 m. The properties of the material used are listed as follows: E=210000 N/m^2, $v=0.33\$$, $\rho=7.8\times10^3$ kg/m^3. The vertical concentrated load is applied to the outside surface of stiffened plate and the measured response is the vertical displacement. The bottoms of the stiffened plate are fixed, and the other parts are free. We establish its finite element model as shown in Fig. 1. The arrow in Fig. 1 denotes the acting point of dynamic force.

In order to illustrate the accuracy of the present method, we also consider the simulated exact concentrated loads as the formula (15). Herein, the experimental data of response is simulated by the computed numerical solution, and the corresponding vertical displacement response at node 1013316 and 1015766 can be respectively obtained by finite element method, as shown in Figs. 2-3. Likewise, we simulate the noise-contaminated measurement in the form of (16). In order to investigate the effect of measurement error on the accuracy of the estimated values, we consider the case of noise level namely 5% and the present method is adopted to determine the dynamic forces. We also compare the regularized solution by two regularization methods and the true solution by the formulas (17) and (18).

To evaluate the effectiveness of two regularization methods mentioned above, five time points are selected, and the identified force for each point will be compared with the corresponding actual force. The results of numerical

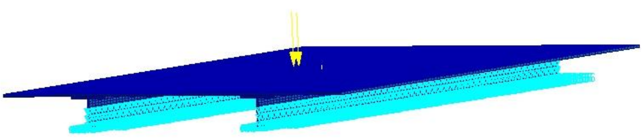

Fig. 1 The finite element model of stiffened plate

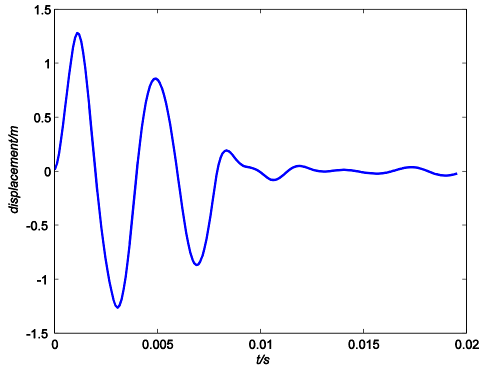

Fig. 2 The corresponding vertical displacement response at Point 1013316

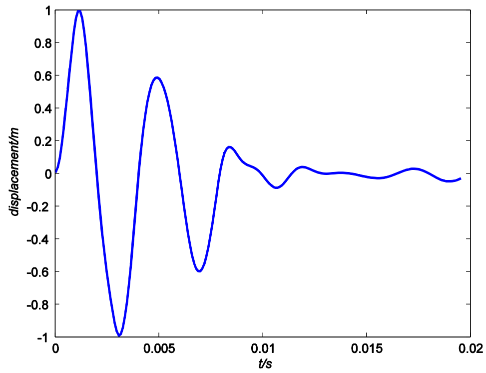

Fig. 3 The corresponding vertical displacement response at Point 1015766

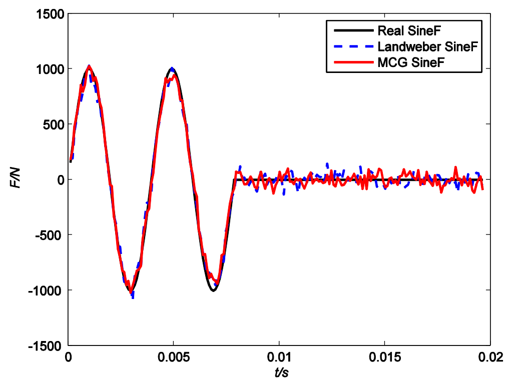

Fig. 4 The identified sine force at noise level 5%; the number of iterations: $N_{Landweber}$=400, N_{MCG}=42

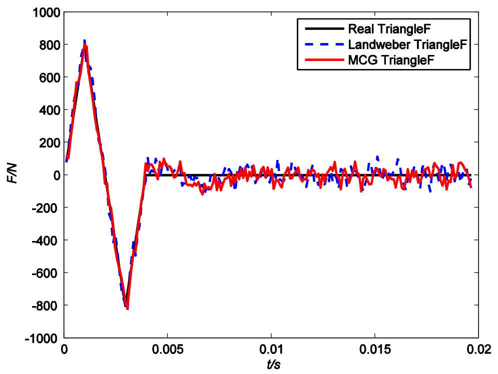

Fig. 5 The identified triangle force at noise level 5%; the number of iterations: $N_{Landweber}$=400, N_{MCG}=42

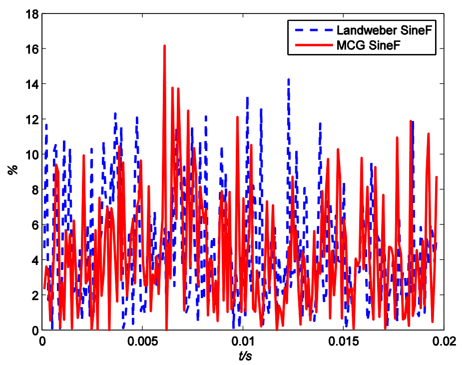

Fig. 6 The relative deviations for the identified sine force at noise level 5%

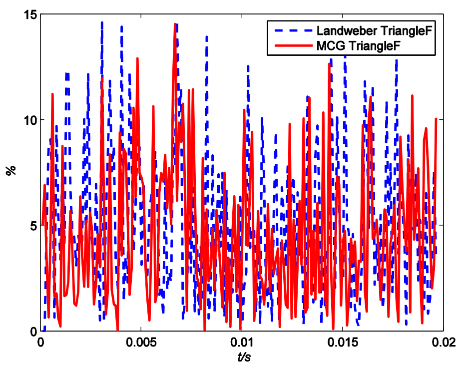

Fig. 7 The relative deviations for the identified triangle force at noise level 5%

simulations are as follows:

The proposed method is applied to the above engineering problem, and the corresponding analysis results are shown in Figs. 4-7 and listed in Table 1. Figs. 4 and 5 show the performances of two regularization methods in identifying the multi-source dynamic forces. In these two figures, it is clearly shown that these two methods are both efficient and stable in identifying the loads, yet the number of iterations by MCG method is 42, smaller than the Landweber iteration method whose iterative number is 400. Moreover, the detailed comparison results between the Landweber iteration method and the proposed method is also provided in Table 1 in terms of the relative estimation error which is computed by the

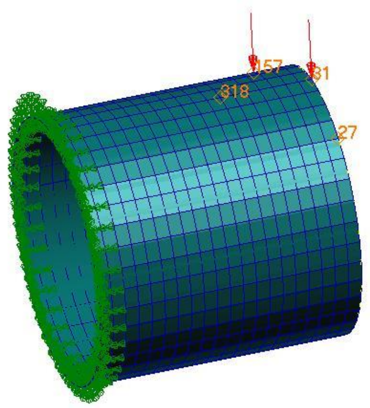

Fig. 8 The finite element model of thin-walled cylindrical shell

formulas (17) and (18). From Table 1, we can find that the maximal deviations and average deviations of the sine force and triangle force by the Landweber iteration method are 14.27%, 4.55%, 14.70%, and 5.08%, respectively. For the new conjugate gradient method, the maximal deviations and average deviations of the sine force and triangle force are 16.24%, 4.40%, 14.5%, and 4.5714.27%, 4.55%, 14.70%, respectively, most of which are smaller than the Landweber iteration method, respectively. It could be also found that a great amount of deviations by Landweber method and the proposed method converge the range in 14.7014.27%, 4.55%, 14.70%, and 16.2414.27%, 4.55%, 14.70%, respectively. This further indicates that the present method offers a better and more effective solution than the Landweber iteration method which is shown in Figs. 6 and 7. In one word, the improved algorithm achieves a prominent computation in the practical engineering structure.

4.2 A thin-walled cylindrical shell

A practical engineering problem is to determine vertical forces of thin-walled cylindrical shell, as shown in Fig. 8. The main parameters of the model are as: This thin-walled cylindrical shell is 190.0 mm in outside diameter, 180.0 mm in inner diameter and 180.0 mm in length. The properties of the material used are listed as follows: E=7.0×10^{10} N/m^2, v=0.33, ρ=2.8×10^3 kg/m^3. The radial concentrated load is applied to the outside surface of thin-walled cylindrical shell and the measured response is the radial displacement. One side of the shell is free, and the other is fixed. We establish its finite element model as shown in Fig. 8. The arrow in Fig. 8 denotes the acting point of dynamic force.

In order to illustrate the accuracy of the present method, we also consider the simulated exact concentrated loads as the formula (15). Herein, the experimental data of response is simulated by the computed numerical solution, and the corresponding vertical displacement response at node 27 and 318 can be respectively obtained by finite element method, as shown in Figs. 9-10. Likewise, we simulate the noise-contaminated measurement in the form of (16). In order to investigate the effect of measurement error on the accuracy of the estimated values, we consider the case of noise level namely 5% and the present method is adopted to determine the

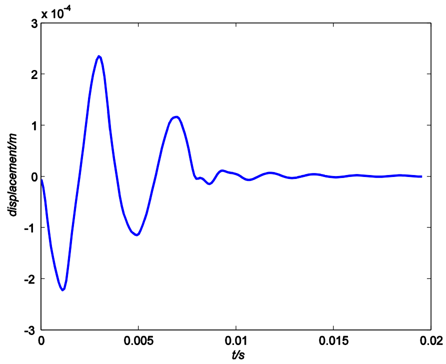

Fig. 9 The corresponding radial displacement response at Point 27

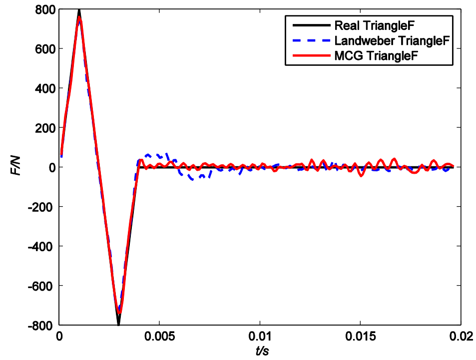

Fig. 12 The identified triangle force at noise level 5%; the number of iterations: $N_{Landweber}$=52, N_{MCG}=16

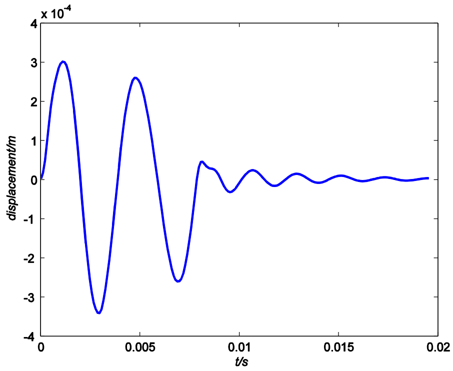

Fig. 10 The corresponding radial displacement response at Point 318

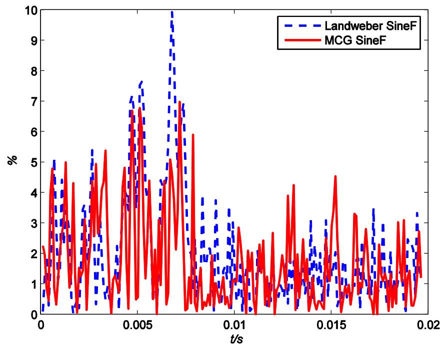

Fig. 13 The relative deviations for the identified sine force at noise level 5%

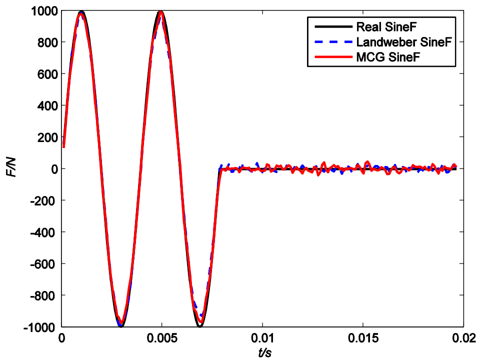

Fig. 11 The identified sine force at noise level 5%; the number of iterations: $N_{Landweber}$=52, N_{MCG}=16

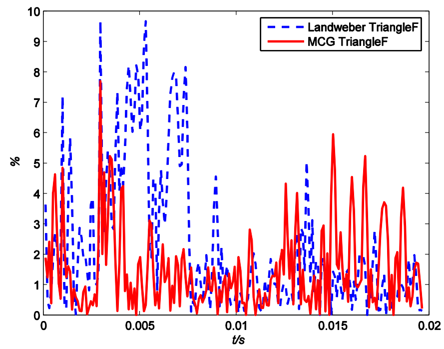

Fig. 14 The relative deviations for the identified triangle force at noise level 5%

dynamic forces. We also compare the regularized solution by two regularization methods and the true solution by the formulas (17) and (18). To evaluate the effectiveness of two regularization methods mentioned above, five time points are also selected, and the identified force for each point will be compared with the corresponding actual force.

The proposed method is applied to the above engineering problem, and the corresponding analysis results are shown in Figs. 11-14 and listed in Table 2. Figs. 11-12 show the performances of two regularization methods in identifying the multi-source dynamic forces. In these two figures, it is clearly shown that these two methods are both efficient and stable in identifying the loads, yet the number of iterations by MCG method is 16, smaller than the Landweber iteration method whose iterative number is 52. Moreover, the detailed comparison results between the Landweber iteration method and the proposed method is also provided in Table 2 in terms of the relative estimation error which is computed by the formulas (17) and (18). It can be found that at these five time points for noise level ±5%, the most deviations of the identified loads by the present method are smaller than Landweber iteration method due to its better efficient identification. It can be also found that the most deviations by Landweber method and the present method concentrate in the range of 9.93%, 7.71%, respectively. For the identification of sine force, the maximal deviations and average deviations by the present method and the Landweber iteration method are

Table 1 The identified force at five time points at noise level 5%

			Landweber method		MCG method	
	Time point	Real force	Identified force	Error (%)	Identified force	Error (%)
Sine	0.001	1000	1033.6	3.36	1027.5	2.75
Triangle	0.0006	480	534.26	6.78	569.84	11.23
Sine	0.003	-1000	-944.03	5.60	-1019.9	1.99
Triangle	0.001	800	827.2	3.4	801.77	0.22
Sine	0.0045	707.11	654.73	5.24	651.7	5.54
Triangle	0.0016	320	281.17	4.85	288.69	3.91
Sine	0.0063	-453.99	-438.52	1.55	-454.36	0.04
Triangle	0.0033	-560	-515.69	5.54	-566.42	0.80
Sine	0.0073	-891.01	-880.52	1.05	-842.55	4.85
Triangle	0.0038	-160	-185.43	3.18	-150.29	1.21
Error (%)			Maximum	Average	Maximum	Average
Sine			14.27	4.55	16.24	4.40
Triangle			14.70	5.08	14.55	4.57

Table 2 The identified force at five time points at noise level 5%

			Landweber method		MCG method	
	Time point	Real force	Identified force	Error (%)	Identified force	Error (%)
Sine	0.001	1000	985.37	1.46	978.66	2.13
Triangle	0.0006	480	458.48	2.69	442.84	4.65
Sine	0.003	-1000	-984.64	1.54	-973.32	2.67
Triangle	0.001	800	742.26	7.22	761.15	4.86
Sine	0.0045	707.11	667.93	3.92	689.29	1.78
Triangle	0.0016	320	314.42	0.70	327	0.88
Sine	0.0063	-453.99	-409.95	4.40	-444.16	0.98
Triangle	0.0033	-560	-524.43	4.45	-573.32	1.67
Sine	0.0073	-891.01	-832.92	5.81	-821.02	7.00
Triangle	0.0038	-160	-116.83	5.40	-146.01	1.75
Error (%)			Maximum	Average	Maximum	Average
Sine			9.93	2.03	7.00	1.76
Triangle			9.69	2.35	7.71	1.53

7.00%, 1.76%, 9.93%, 2.03%, respectively. For the identification of triangle force, the maximal deviation and average deviation by the Landweber iteration method is 9.69%, 2.35%, respectively, which are also respectively both larger than the MCG method whose maximal deviation and average deviation are 7.71%, 1.53%, respectively. This further shows that the proposed method provides a better and more accurate solution than the Landweber iteration method which is also shown from Figs. 13-14. However, these deviations are relatively small and it can fully satisfy the request of the practical structural analysis. Analyzing the results above, we can find that the proposed method has a quick convergence speed and relatively high computational efficiency, as only small numbers of iterative steps are required. In summary, the present algorithm is stable, effective and robust in identifying the multi-source dynamic loads acting on the practical engineering structure, and gives very good and satisfactory results.

5. Conclusions

In this paper, a new conjugate gradient method is proposed, strictly proved and suggested for solving the multi-source dynamic loads identification problems. To verify the feasibility and effectiveness of the proposed method, two engineering examples are performed by the proposed method and Landweber iteration method, and their computational results are also contrasted. The results of numerical simulations show that the proposed method provides more efficient and numerically stable approximation of the true loads than the traditional Landweber iteration method. In a word, the proposed method is stable, effective and robust in solving the multi-source dynamic loads identification problems of practical structural engineering.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (11202116), the Open Fund of Hubei key Laboratory of Hydroelectric Machinery Design and Maintenance (2016KJX01) and Hubei Chenguang Talented Youth Development Foundation. The authors thank the anonymous referees and the editor for carefully reading this paper and suggesting many helpful comments on improving the original manuscript.

References

Amiri, A.K. and Bucher, C. (2015), "Derivation of a new parametric impulse response matrix utilized for nodal wind load identification by response measurement", *J. Sound Vib*., **344**, 101-113.

Bartlett, F.D. and Flannelly, W.D. (1979), "Modal verification of force determination for measuring vibration Loads", *J. Am. Helicopter Soc*., **19**(6), 10-18.

Giansante, N., Jones, R. and Galapodas, N.J. (1982), "Determination of in-flight helicopter loads", *J. Am. Helicopter Soc*., **27**(3), 58-64.

Gunawan, F.E. and Homma, H. (2008a), "A solution of the ill-posed impact-force inverse problems by the weighted least squares method", *J. Mech. Phys. Solid*., **2**(2), 188-198.

Gunawan, F.E. and Homma, H. (2008b), "Impact-force estimation by quadratic spline approximation", *J. Mech. Phys. Solid*., **2**(8), 1092-1103.

Gunawan, F.E., Homma, H. and Kanto, Y. (2006), "Two-step b-splines regularization method for solving an ill-posed problem of impact-force reconstruction", *J. Sound Vib*., **297**(1), 200-214.

Liu, G.R., Ma, W.B. and Han, X. (2002), "An inverse procedure for identification of loads on composite laminates", *Compos. Part B: Eng*., **33**(6), 425-432.

Liu, J., Han, X., Jiang, C., Ning, H.M. and Bai, Y.C. (2011), "Dynamic load identification for uncertain structures based on interval analysis and regulation method", *Int. J. Comput. Meth.*

8(4), 667-683.
Liu, J., Sun, X., Han, X., Jiang, C. and Yu, D. (2014). "A novel computational inverse technique for load identification using the shape function method of moving least square fitting", *Comput. Struct*., **144**, 127-137.
Liu, J., Sun, X.S. and Han, X. (2015a), "Dynamic load identification for stochastic structures based on Gegenbauer polynomial approximation and regularization method", *J. Mech. Syst. Signal Pr*., **56-57**, 35-54.
Lu, Z.R. and Law, S.S. (2007), "Identification of system parameters and input force from output only", *Mech. Syst. Signal Pr*., **21**(5), 2099-2111.
Neubauer, A. (2000), "On Landweber iteration for nonlinear il1-posed problems in Hilbert scales", *Numer. Math*., **85**(2), 309-328.
Qin, Y.T., Zhang, F. and Chen, G.P. (2007), "A frequency method for two dimensional distributed load identification", *J. Vib. Eng*., **20**(5), 512-518.
Ronasi, H., Johansson, H. and Larsson, F. (2011), "A numerical framework for load identification and regularization with application to rolling disc problem", *Comput. Struct*., **89**(1), 38-47.
Simonian, S.S. (1981a), "Inverse problems in structural dynamics-I. Theory", *Int. J. Numer. Meth.Eng. A*, **17**(3), 357-365.
Simonian, S.S. (1981b), "Inverse problems in structural dynamics-II. Applications", *Int. J. Numer. Meth.Eng., B*, **17**(3), 367-386.
Turco, E. (1998), "Load distribution modelling for pin-jointed trusses by an inverse approach", *Comput. Meth. Appl. M*., **165**(1-4), 291-333.
Turco, E. (2005a), "Is the statistical approach suitable for identifying actions on structures?", *Comput. Struct*., **83**(25), 2112-2120.
Turco, E. (2005b), "A strategy to identify exciting forces acting on structures", *Int. J. Numer. Meth. Eng*., **64**(11), 1483-1508.
Wang, L.J., Han, X. and Xie, Y.X. (2012), "A new iterative regularization method for solving the dynamic load identification problem", *Comput. Mater. Continua*, **31**(2), 113-126.
Wang, L.J., Han, X., Liu, J. and Chen, J.J. (2011b), "An improved iteration regularization method and application to reconstruction of dynamic loads on a plate", *J. Comput. Appl. Math*., **235**(14, 15), 4083-4094.
Wang, L.J., Han, X., Liu, J., He, X.Q. and Huang, F. (2011a), "A new regularization method and application to dynamic load identification problems", *Inverse Probl. Sci. Eng*., **19**(6), 765-776.
Wang, L.J., Xie, Y.X. (2012), "A new conjugate gradient method for the solution of linear Ill-posed problem", *J. Appl. Computat. Math*., **1**, 108, doi: 10.4172/2168-9679.1000108.
Wei, Z., Yao, S. and Liu, L. (2006), "The convergence properties of some new conjugate gradient methods", *Appl. Math. Comput*., **183**(2), 1341-1350.
Xu, B., He, J. (2015), "Substructural parameters and dynamic loading identification with limited observations", *Smart Struct. Syst*., **15**(1), 169-189.
Xu, F., Chen, H.H. and Bao, M. (2002), "Force identification model improved based on modal filter", *J. Acta Aeronautica et Astronautica Sinica*, **23**, 51-54.
Yu, L., Chan, T.H. and Zhu, J.H. (2008a), "A MOM-based algorithm for moving force identification: Part I-Theory and numerical simulation", *Struct. Eng. Mech*., **29**(2), 135-154.
Yu, L., Chan, T.H. and Zhu, J.H. (2008b), "A MOM-based algorithm for moving force identification: Part II-Experiment and comparative studies", *J. Struct. Eng*., **29**(2), 155-169.
Yuan, G., Lu, S. and Wei, Z. (2010), "A line search algorithm for unconstrained optimization", *J. Softw. Eng. Appl*., **3**(5), 503-509.
Zhang, F. and Zhu, D.A. (1997), "The dynamic load identification research based on neural network model", *J. Vib. Eng*., **10**(2), 156-162.
Zhang, F., Qin, Y.T. and Deng J.H. (2006), "Research of identification technology of dynamic load distributed on the structure", *J. Vib. Eng*., **19**(1), 81-85.
Zhu, T., Xiao, S.N. and Yang, G.W. (2014), "Force identification in time domain based on dynamic programming", *J. Appl. Math. Comput*., **235**(25), 226-234.
Zoutendijk, G. (1970), *Nonlinear Programming Computational Methods*, Ed. J. Abadie, Integer and Nonlinear Programming, North Holland, Amsterdam.

CC