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A comprehensive evaluation method study for dam safety

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Abstract. According to the multi-index system of dam safety assessment and the standard of safety, a comprehensive evaluation model for dam safety based on a cloud model is established to determine the basic probability assignment of the Dempster-Shafer theory. The Dempster-Shafer theory is improved to solve the high conflict problems via fusion calculation. Compared with the traditional Dempster-Shafer theory, the application is more extensive and the result is more reasonable. The uncertainty model of dam safety multi-index comprehensive evaluation is applied according to the two theories above. The rationality and feasibility of the model are verified through application to the safety evaluation of a practical arch dam.

Keywords: dam safety; comprehensive evaluation; cloud model; Dempster-Shafer theory

1. Introduction

Hydropower engineering projects have substantially benefitted social and economic development. However, a significant proportion of dams have been damaged to varying degrees at the beginning of this century (Mirzabozorg *et al*. 2009, François *et al*. 2015). Reasonable assessment of dam safety is critical to the detection of a project's risks and implementation of the dam's protection measures (Bayagoob *et al*. 2010, Hu *et al*. 2017). The comprehensive assessment of the security situation of a dam is a typical uncertainty problem. One reason is the numerous factors that can affect the performance of a dam; in addition, the complex characteristics of the attributes, such as monitoring errors in the data acquisition process, the uncertainty of the indices and the randomness of each index weight, lead to further uncertainty (Su *et al*. 2015). Many models have been developed to assess the safety of dams: a fuzzy extension evaluation model of dam behaviour based on rhombic thinking mode and the extension of matter elements has been developed (Yang *et al*. 2017, Zhu *et al*. 2010). An improved analytic hierarchy process method was employed to analyse the influences of the basic events of dam breakage (Chen *et al*. 2016). Using a logistic model, Zhang obtained a precise quantitative evaluation model, which accurately fit the quantitative evaluation parameters (Zhang *et al*. 2013). These methods played an important role in enhancing the rationality and efficiency of dam safety assessment. However, the existing methods have focused more on the fuzziness in uncertainty, whereas a comprehensive study of fuzziness and randomness has yet to be undertaken.

Integrated with fuzziness and randomness, a cloud model can use the natural language value to achieve the mapping between a qualitative concept and a quantitative concept, taking full account of the randomness of each element (Li *et al*. 2009). The Dempster-Shafer theory is an information-fusion method that can address “uncertain, unknown” problems without any prior information. Synthetic results with uncertain correlations among the evidence were obtained by the synthesis rule (Guan *et al*. 1991; Su *et al*. 2016). However, the acquisition of the basic probability assignment (BPA) is one of the difficulties in establishing the theoretical model of evidence. In this study, the membership obtained from the normal cloud model is used to determine the basic probability assignment. The traditional Dempster-Shafer theory is improved to address high conflict problems in fusion calculation. The uncertainty model of multi-index comprehensive evaluation of dam safety is applied according to the two theories above. The rationality and feasibility of the model are verified through application to the safety evaluation of a practical arch dam.

In our research, the membership obtained from the normal cloud model by considering the uncertainties in the assessment process is used to determine basic probability assignment, which is an important part of evidence theory. The degree of support between the evidence and the importance of the evidence is fully considered by introducing the dynamic weight coefficient and the static weight coefficient, which can solve the high conflict in the fusion calculation. The combination of the cloud model and the improved evidence theory make the fusion calculation more scientific and convincing.

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2. Multi-index comprehensive evaluation model of dam safety

A multi-index comprehensive evaluation model of dam safety is applied to evaluate the status of dam safety. The model is based on the cloud model and Dempster-Shafer theory. The cloud model is used to calculate the membership of each index at each security level. The Dempster-Shafer theory is improved to solve high conflict problems of fusion calculations.

The basic concept of the dam safety comprehensive evaluation model based on the cloud model and Dempster-Shafer theory is as follows: 1) set up a dam safety evaluation index system and divide the safety levels; 2) calculate the membership of each index at each security level from the evaluation of the measured value using the cloud model; 3) revise the membership of each index at each security level and transform it into a basic probability assignment according to Dempster-Shafer theory; and 4) develop fusion calculation of the data and complete the multi-index comprehensive evaluation of dam safety by introducing the dynamic weight coefficient and the static weight coefficient, aiming at the conflict problem of traditional Dempster-Shafer theory in fusion.

2.1 Determination of the membership matrix of dam safety assessment based on the cloud model

The cloud model is an uncertainty measurement model that can be transformed between a qualitative concept and quantitative values (Li *et al*. 1998, Shariatmadar *et al*. 2011). Using the three characteristics of cloud expectation E_x, entropy E_n and super entropy H_e, this paper reveals a great deal of randomness, ambiguity and the correlation between them. Expectation is the standard value of an indicator under the corresponding security level, reflecting the most representative point of a qualitative concept in numerical space. Entropy refers to the discrete degree of cloud droplets, which represents the uncertainty of the qualitative concept. This uncertainty consists of three aspects: 1) The magnitude of the entropy represents the range of acceptable cloud droplets, which reflects the fuzzy degree of the qualitative concept; 2) the entropy reflects the probable density of the cloud droplet group, which can represent the qualitative concept in the domain space, and reveals the randomness of the cloud droplets, which can represent a qualitative concept; and 3) the entropy demonstrates the relationship between fuzziness and randomness. Super entropy, which is the entropy of the entropy, reflects the coherence of the uncertainty of all the points of the qualitative concept in numerical space and indirectly reflects the thickness of the cloud (Liu *et al*. 2005, Rezaiee-Pajand *et al*. 2013).

Definition 1: U is the time series of the dam safety monitoring effect, and C is the qualitative judgment of dam safety status on U. For a random number x on U, if $x \sim N(E_x, E_n'^2)$ can be satisfied and the certainty of the qualitative concept C can be represented by $\mu = e^{-(x-E_x)^2/(2E_n'^2)}$, the random number x is a normal cloud on U.

Definition 2: A qualitative concept can be transformed into a quantitative value by a forward cloud generator. The specific steps are as follows:

(1) Generate a normal random number E_n' using MATLAB, with E_n as the expectation and H_e as the standard deviation;

(2) Generate a normal random number x using MATLAB, with E_x as the expectation and E_n' as the standard deviation;

(3) Calculate E_x according to E_n'; x: $\mu = e^{-(x-E_x)^2/(2E_n'^2)}$, where μ is the degree of certainty that x is qualitative concept A. $\{x, \mu\}$ is a cloud droplet that satisfies the condition, which fully reflects the transformation of the qualitative concept to quantitative values.

(4) Repeat step 1 to step 3 until N drops are generated.

The normal cloud model of dam safety comprehensive evaluation is applied by determining expectation E_x, entropy E_n and excess entropy H_e, according to the definition of the normal cloud. Assuming that $\left(x_{ij}^1, x_{ij}^2\right)$ is the value range of an evaluation index corresponding to a certain security level, the characteristic parameter E_x can be determined according to the upper and lower limits of the value range

$$E_{x_{ij}} = \left(x_{ij}^1 + x_{ij}^2\right)/2 \tag{1}$$

Because x_{ij}^1 is a bounded value of the adjacent security level, x_{ij}^1 has the same membership for both security classes: $\mu = \exp\left(-\frac{(x_{ij}^1 - E_{x_{ij}})^2}{2E_{n_{ij}}{}^2}\right) = \exp\left(-\frac{(x_{ij}^1 - x_{ij}^2)^2}{8E_{n_{ij}}{}^2}\right) = 0.5$.

Thus

$$E_{n_{ij}} = \left(x_{ij}^2 - x_{ij}^1\right)/2.355 \tag{2}$$

The super entropy $H_{e_{ij}}$ can be obtained according to the fuzziness and randomness of a specific index. After the eigenvalues of the cloud model are determined, the membership of each evaluation index under the corresponding security level can be shown through the determination degree, which is calculated by the measured value x of each index and the above two definitions. According to its definition, super entropy H_e can be understood as the uncertainty measure of x for a certain security level. Thus, the membership of the index of each security level is the average of N cloud drops; the specific formulas are as follows

$$\mu_{ij}(k) = \exp\left(-\frac{\left(x_k - E_{x_{ij}}\right)^2}{2E_{n_{ij}}'}\right) \tag{3}$$

$$\mu_{ij} = \sum_{k=1}^{N} \frac{\mu_{ij}(k)}{N} \tag{4}$$

where E_n' is a normal random number, for which E_n is the expectation, and H_e is the standard deviation.

The normal cloud model of dam safety comprehensive evaluation is applied by determining expectation E_x, entropy E_n and super entropy H_e, according to the definition of the normal cloud. And the membership of each evaluation index under the corresponding security level can be shown through the determination degree which is calculated by the measured value x of each index and the above two definitions. In this process, there is no requirement that the evaluation index should be subject to a normal distribution.

2.2 Multi-index comprehensive evaluation model of dam safety based on the improved Dempster-Shafer theory

The evidence theory was first introduced by Arthur P. Dempster (Dempster 1967) and developed by Glenn Shafer (Shafer 1976) into a general framework for modelling epistemic uncertainty as a mathematical theory of evidence. It is an information fusion method expanded from probability theory, which is suitable for expert systems, artificial intelligence and pattern recognition and other fields. It allows people to model and reason for imprecise, uncertain or ambiguous problems, providing a new concept for the fusion calculation of uncertain information. Since the high conflict problem in fusion calculation is difficult to solve by the traditional Dempster-Shafer theory, the idea of dynamic weight and static weight is introduced in this study to improve the fusion rule.

2.2.1 Determination of basic probability assignment

Definition 3 If Θ is the universe set of all evaluation results, and each element in Θ is mutually exclusive, then Θ is called the recognition frame. Basic probability assignment function $m:2^{\Theta}\to[0,1]$ (2^{Θ} is all subsets of Θ) is defined as the confidence level that each possible result is assigned between [0,1], which satisfies $m(\varnothing)=0$ and $\sum_{A\subset\Theta} m(A)=1$.

According to this definition, it can be concluded that the security level of dam safety assessment can constitute an identification framework. The membership of the evaluation index c_i (i=1,2,…n), which belongs to the security level H_j (j=1,2,…q), basically satisfies the definition of the basic probability assignment. Because $\sum_{j=1}^{q}\mu_{ij}$ is not equal to 1, additional definitions are required:

$$\begin{cases}\theta_i = 1-\max\left(\mu_{i1},\mu_{i2},\cdots,\mu_{iq}\right)\\ m_i\left(X\right)=\theta_i, i=1,2,\cdots,n\\ m_i\left(A_j\right)=\left(1-\theta_i\right)\mu_{ij}/\sum_{j=1}^{q}\mu_{ij}, j=1,2,\cdots q, i=1,2,\cdots,n\end{cases} \tag{5}$$

where, θ indicates the randomness of the monitoring data or the uncertainty of the measured signal caused by the errors in detection tools or detection methods, and $m_i(X)$ is the probability that the evaluation result of the indicator is uncertain. The basic probability assignment matrix is obtained as

$$\mathbf{M}_{n\times(q+1)}=\begin{pmatrix} m_1(A_1) & \cdots & m_1(A_q) & \theta_1\\ \vdots & \ddots & \vdots & \vdots\\ m_n(A_1) & \cdots & m_n(A_q) & \theta_n\end{pmatrix} \tag{6}$$

The basic probability assignment matrix M is calculated from the membership matrix L, which is obtained by substituting the measured value into the cloud model. So the basic probability assignment matrix M is basically objective. While the range of evaluation grade of each evaluation index is partly subjective which is based on the summary and feedback of the long-term observation data, the analogy of similar projects and the advice of experts. The cloud model is an effective tool in dealing with the fuzziness and randomness of the data, which can greatly reduce the subjective factors. The dynamic weight coefficient and the static weight coefficient are both dependent on the measured data without any subjective factor.

2.2.2 Determination of the weight coefficient

For the two evaluation indexes m_1 and m_2 of the basic probability assignment, the synthesis rule of $A\in\Theta$ is proposed by evidence theory

$$m(A)=K^{-1}\sum_{A_i\cap B_j=A} m_1(A_i)m_2(B_j) \tag{7}$$

$$K=\sum_{A_i\cap B_j\neq\varnothing} m_1(A_i)m_2(B_j) \tag{8}$$

Because K is the degree of conflict between the evidence, higher K^{-1} values indicate a greater degree of conflict. For the evidence fusion problem with strong conflict, the combination rule has a serious flaw and will reach a conclusion that is contrary to common sense (Diao *et al*. 2011) largely because the evidence in the fusion process has the same importance. In this study, the concept of dynamic weight and static weight is introduced to improve the fusion rule (Liu *et al*. 2016).

Dynamic weight coefficient ω_{i2}

The n evidence vectors are defined as $\boldsymbol{p}_i=\left(m_i(A_1),m_i(A_2),\cdots,m_i(A_q),m_i(X)\right)^{\mathrm{T}}\left(1\le i\le n\right)$. The compatibility coefficient between any two evidence vectors $\boldsymbol{p_i}$ and $\boldsymbol{p_j}$ is

$$R_{ij}=\cos\left(\boldsymbol{p}_i,\boldsymbol{p}_j\right)=\frac{\boldsymbol{p}_i^{\mathrm{T}}\boldsymbol{p}_j}{\left[\left(\boldsymbol{p}_i^{\mathrm{T}}\boldsymbol{p}_i\right)\left(\boldsymbol{p}_j^{\mathrm{T}}\boldsymbol{p}_j\right)\right]^{1/2}} \tag{9}$$

where $\boldsymbol{p}_i^{\mathrm{T}}\boldsymbol{p}_j=\sum_{k=1}^{q+1}m_i(A_k)m_j(A_k)(i,j=1,\cdots n)$. The formula suggests that at lower angles of the two evidence vectors, where the cosine is greater, the compatibility coefficient between the two is higher. Hence, the absolute consistency of evidence m_i can be calculated by

$$R_i=\sum_{j=1,j\neq i}^{n}R_{ij} \tag{10}$$

The credibility of the evidence can be obtained by normalizing the absolute degree of consistency

$$\omega_{i1} = \frac{R_i}{\sum_{i=1}^{n} R_i} \tag{11}$$

The confidence of the weight coefficient level changes with the evidence, so the coefficient is referred to as the dynamic weight coefficient.

Static weight coefficient ω_{i2}

Because different effects have different influences on the evaluation results, the static weight coefficient ω_{i2} is introduced as the importance coefficient of each effect factor, which can be obtained by the entropy method. Suppose that x_{ij} is the measured value j of the evaluation index i, and y_{ij} is the normalized value of x_{ij}, then the static weight coefficient of the index i is

$$\omega_{i2} = (1-e_i)/(N - \sum_{i=1}^{N} e_i) \tag{12}$$

where $e_i = -k\sum_{j=1}^{n} y_{ij} \ln y_{ij}$.

Final weight coefficient α_i

Define the final weight coefficient α_i of the evidence m_i

$$\alpha_i = \beta Crd_i + (1-\beta)\omega_i \tag{13}$$

where β is the correlation coefficient between the evidence. If the degree of support between the evidence has a greater effect than the importance of each effect on the final evaluation, $\beta>0.5$; otherwise, $\beta<0.5$. If the two factors have the same effect on the final evaluation results, then $\beta=0.5$. *Crd* is credibility extent supported by other evidences.

2.2.3 Fusion calculation of evidence theory

After obtaining the weight coefficient α_i, the mean evidence of the qualitative concept k can be obtained by calculating the sequence weighted mean of the evidence m_i

$$m_{ak} = \sum_{i=1}^{n} \alpha_i m_{ik}, k = 1, 2, \cdots q \tag{14}$$

The probability function of the qualitative concept k can be obtained by n-1 times the fusion calculation using the joint formula.

3. Implementation process of dam safety evaluation based on the cloud model and evidence theory

The implementation process of dam safety evaluation based on the cloud model and evidence theory is as follows:

Step 1: Set up the dam safety evaluation index, i.e., $C = \{c_1, c_2, \cdots c_n\}$, and determine the division of dam safety levels, i.e., $\Theta = \{A_1, A_2, \cdots, A_q\}$;

Step 2: Generate the cloud model using a forward cloud generator after determining the parameters E_x,E_n and H_e of the cloud model. Then, input the measured values into the cloud model and calculate the membership matrix;

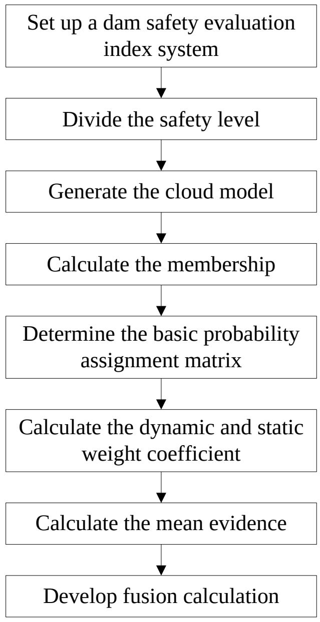

Fig. 1 Implementation process of dam safety evaluation

Step 3: Transform the membership matrix into the basic probability assignment matrix by formula (5);

Step 4: Calculate the dynamic weight coefficient and static weight coefficient, and define the final weighting coefficient according to the correlation coefficient;

Step 5: Calculate the mean evidence based on the weight coefficient and basic probability assignment matrix. Perform the fusion calculation of the final basic probability of each security level according to the combination rule. According to the maximum attribute principle given by Agouzal *et al*. (2008), the security level of the maximum basic probability value is selected as the safety evaluation result.

4. Application of the model in dam engineering

In this section, the deformation of an arch dam is taken as an example to demonstrate the above theory.

4.1 Project Profile

The arch dam is the first one of the series of dams from Kala to the estuary on the Yalong River, which is located in the Liangshan Yi Autonomous Prefecture in Sichuan Province. The dam is the critical project in the middle and lower reaches of the Yalong River hydropower development planning and plays an important role in the “linking up and turning on” during the Yalong River cascade development. The project's purpose is to supply energy, improve flood protection, and prevent erosion. It has a normal water level of 1880 m and a dead water level of 1800 m. The 305-m tall and 568-m long arch dam supplies the power station with water from a 7.7 billion m^3 reservoir, of which 4.9 billion m^3 is active or usable storage. The dam crest width is 16 m,

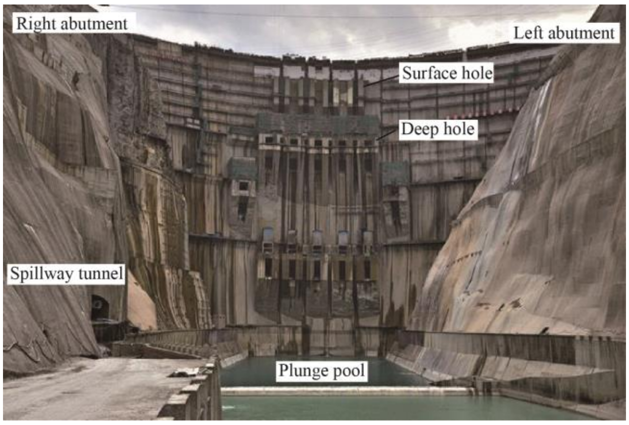

Fig. 2 Downstream elevation of the dam

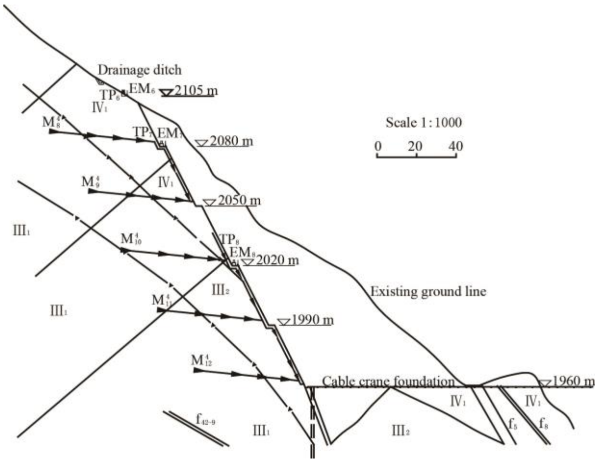

Fig. 3 Abutment section of left abutment

and the dam foundation's thickness is 63 m. The thickness and height ratio is 0.207. The project achieved river closure on December 4, 2006, and it achieved the normal water level on August 24, 2014.

The dam is equipped with a comparatively perfect deformation, seepage and stress-strain system, which can monitor the safety of the dam in real time. To monitor the dam deformation, an observation system composed of the vertical and horizontal displacements was installed. The multi-point displacement meter is used to monitor the deformation of the dam foundation and the slope (see in

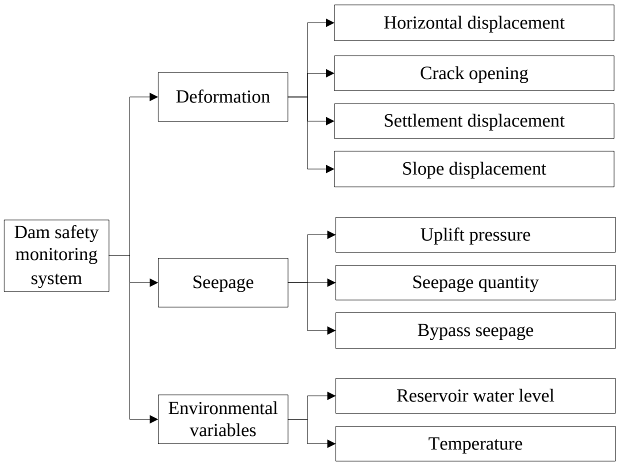

Fig. 4 Dam safety monitoring system

Fig. 3), as well as the cracking in the tensile stress area of the dam heel. A joint meter was installed to oversee the crack opening between the slope and the foundation.

The dam foundation leakage monitoring includes drainage hole monitoring, water catchment monitoring of grouting holes and drainage holes in the dam abutment. To monitor the seepage around the dam, a measuring weir was arranged in the resistance block.

The environmental variables are considered as well. The thermometers were arranged in the No. 9, 13 and 19 dam sections, corresponding to the deformation monitoring sections. The reservoir water level was monitored since it achieved the four water reservoir stages on November 30, 2012.

4.2 Application example

The multi-index comprehensive evaluation model of dam safety is used to analyse the operating condition of the arch dam.

Step **1**. Divide the security level into {Normal, Basically normal, Mild abnormality, Moderate abnormality, Severe abnormality}, according to the existing dam safety evaluation results (Hartford *et al*. 2004). Based on the analysis of the prototype observation data and the characteristics of the hydropower station, nine factors are selected from the three aspects of deformation, seepage and

Table 1 Dam safety evaluation index system and standard

Class	Parameter	Normal	Basically normal	Mild abnormality	Moderate abnormality	Severe abnormality
Deformation	Horizontal displacement C_1/mm	(0,30)	(30,60)	(60,100)	(100,140)	(140,200)
	Crack opening C_2/mm	(0,0.4)	(0.4,1.0)	(1.0,1.6)	(1.6,4.0)	(4.0,10.0)
	Settlement displacement C_3/mm	(0,10)	(10,30)	(30,80)	(80,130)	(130,200)
	Slope displacement C_4/mm	(0,20)	(20,80)	(80,160)	(160,300)	(300,500)
Seepage	Uplift pressure C_5	(0,0.2)	(0.2,0.4)	(0.4,0.8)	(0.8,1.2)	(1.2,1.6)
	Seepage quantity $C_6/L\cdot s^{-1}$	(0,3)	(3,10)	(10,20)	(20,50)	(50,100)
	Bypass seepage C_7/L•s^{-1}	(-0.04,0)	(-0.1,-0.04)	(-0.2,-0.1)	(-1,-0.2)	(-10,-1)
Environmental variables	Reservoir water level C_8/m	(-40,-10)	(-10,2)	(2,4)	(4,6)	(6,8)
	Temperature C_9/℃	(0,5)	(5,10)	(10,20)	(20,30)	(30,50)

Table 2 Cloud model parameters for dam safety evaluation

Index	Normal	Basically normal	Mild abnormality	Moderate abnormality	Severe abnormality
C_1	(15,12.739,1)	(45,12.739,1)	(80,16.985,1)	(120,16.985,1)	(170,25.478,1)
C_2	(0.2,0.170,0.1)	(0.7,0.255,0.1)	(1.3,0.255,0.1)	(2.8,1.020,0.1)	(7.0,2.548,0.1)
C_3	(5,4.246,1)	(20,8.493,1)	(55,21.23,1)	(105,21.23,1)	(165,29.724,1)
C_4	(10,8.493,1)	(50,25.478,1)	(120,33.97,1)	(230,59.448,1)	(400,84.926,1)
C_5	(0.1,0.085,0.5)	(0.3,0.085,0.5)	(0.6,0.17,0.5)	(1.0,0.17,0.5)	(1.4,0.17,0.5)
C_6	(1.5,0.142,0.5)	(6.5,2.972,0.5)	(15,4.246,0.5)	(35,12.739,0.5)	(75,21.231,0.5)
C_7	(-0.02,-0.017,1)	(-0.07,-0.025,1)	(-0.15,-0.1,1)	(-0.6,-0.4,1)	(-5.5,-4.5,1)
C_8	(-25,12.739,1)	(-4,5.096,1)	(3,0.849,1)	(5,0.849,1)	(7,0.849,1)
C_9	(2.5,2.123,0.1,)	(7.5,2.123,0.1)	(15,4.246,0.1)	(25,4.246,0.1)	(40,8.493,0.1)

Table 3 Fusion evaluation results of dam safety

Security level	Normal	Basically normal	Mild abnormality	Moderate abnormality	Severe abnormality	Uncertainty	Sum
Basic probability	0.7230	0.2668	0.0098	0	0	1.2944×10^{-5}	1.0

environment as the dam safety evaluation index. Table 1 lists the range of evaluation grade of each evaluation index, which is mainly based on the summary and feedback of long-term observations, the analogy of similar projects, and the advice of experts. The specific implementation method is as follows. According to the specific technical parameters, engineering experience and specifications, et al., experts quantify the criteria of the indicators anonymously and then gradually achieve a consensus through the feedbacks and exchanges. Finally, quantify the data according to the criteria.

The horizontal displacement in Table 1 refers to the radial displacements of the dam body. The settlement displacement index refers to the settlement of the middle dam section, as the settlement trend is sinking in the middle section of the dam and slightly increasing on both sides. The uplift pressure refers to the curtain reduction factor because the factor can fully reflect the effect of the anti-seepage curtain and the uplift pressure (Bernstone *et al*. 2009, Hu and Ma 2016). The reservoir water level refers to the difference between the actual operating water level and the normal water level of 1880 m. The temperature refers to the difference between the actual temperature and the annual average temperature. The standard values of the evaluation indexes in the table are not fixed, mainly based on the summary and feedback of the monitoring data, engineering practice and an expert's suggestion. The use of a cloud model in computing is helpful to address the uncertainty of the standard values.

Step **2**. Building the cloud model

The cloud model of each evaluation index corresponding to each security level is generated by formulas (1) and (2) in the forward cloud generator after determining the parameters of the cloud model. Table 2 lists the parameters of the cloud model for dam safety evaluation.

Table 2 lists the eigenvalues of an indicator at the corresponding security level, $\left(E_{x_{ij}}, E_{n_{ij}}, H_{e_{ij}}\right)$. Certainty of x belonging to a certain qualitative concept μ can be obtained by substituting the measured data x and the eigenvalues above into Eq. (3). $\{x, \mu\}$ is a cloud droplet that satisfies the condition, which fully reflects the transformation of the qualitative concept to quantitative values. The membership of the index of each security level can be calculated as the average of N cloud drops by Eq. (4).

The basic probability assignment matrix M can be transformed from the membership matrix L. First, normalize the data of each column in the membership matrix. Then calculate the uncertainty of the measured data caused by the errors in the detection tool and the detection method θ, and the probability that the evaluation result of the indicator is uncertain $m_i(X)$. The basic probability assignment matrix M is consisted of the above two parameters.

Select 600 measured values randomly for each index and input these values into the cloud model. Then, calculate the membership matrix using formulas (3) and (4):

$$\mathbf{L} = \begin{bmatrix} 0.698 & 0.421 & 0.004 & 0 & 0 \\ 0.802 & 0.256 & 0 & 0 & 0 \\ 0.801 & 0.286 & 0 & 0 & 0 \\ 0.512 & 0.463 & 0.128 & 0 & 0 \\ 0.789 & 0.304 & 0 & 0 & 0 \\ 0.786 & 0.298 & 0 & 0 & 0 \\ 0.754 & 0.321 & 0 & 0 & 0 \\ 0.885 & 0.106 & 0 & 0 & 0 \\ 0.802 & 0.199 & 0 & 0 & 0 \end{bmatrix}$$

Step **3**. Calculate the basic probability assignment matrix using the membership matrix according to Equation (14):

$$\mathbf{M} = \begin{bmatrix} 0.434 & 0.262 & 0.003 & 0 & 0 & 0.302 \\ 0.608 & 0.194 & 0 & 0 & 0 & 0.198 \\ 0.590 & 0.211 & 0 & 0 & 0 & 0.199 \\ 0.238 & 0.215 & 0.059 & 0 & 0 & 0.488 \\ 0.570 & 0.220 & 0 & 0 & 0 & 0.211 \\ 0.570 & 0.216 & 0 & 0 & 0 & 0.214 \\ 0.523 & 0.225 & 0 & 0 & 0 & 0.246 \\ 0.790 & 0.095 & 0 & 0 & 0 & 0.115 \\ 0.643 & 0.159 & 0 & 0 & 0 & 0.198 \end{bmatrix}$$

Step **4**. Calculate the dynamic weight coefficient by formulas (8) and (9):

$$\omega_{i1} = (0.1128, 0.1148, 0.1151, 0.0891, 0.1153, 0.1154, 0.1156, 0.1078, 0.1141)^{\mathrm{T}}$$

Calculate the static weight coefficient by formula (10):

$$\omega_{i2} = (0.1558, 0.1251, 0.1241, 0.1315, 0.1087, 0.0863, 0.0767, 0.1211, 0.0707)^{\mathrm{T}}$$

In this example, the importance of each effect is greater than the support between the evidence for the final evaluation, so β=0.3, and the final weight coefficient is

$$\alpha = (0.1429, 0.1220, 0.1214, 0.1188, 0.1107, 0.0950, 0.0884, 0.1171, 0.0837)^{\mathrm{T}}$$

Step **5**. Calculate the mean evidence according to the weight coefficient and the basic probability assignment matrix:

$$m_a = \begin{array}{cccccc} \textit{Normal} & \textit{Basically normal} & \textit{Mild abnormality} & \textit{Moderate abnormality} & \textit{Severe abnormality} & \textit{Uncertainty} \\ (0.5459 & 0.2015 & 0.0074 & 0 & 0 & 0.2449) \end{array}$$

Table 3 lists the safety assessment results of the dam obtained by synthesizing the mean evidence eight times according to the fusion rule.

Table 3 shows how the improved evidence theory gradually assigns the uncertain basic probability assignment to the other focal elements in the recognition framework, leaving 1.2944×10^{-5} to remain. The safety level of the dam is determined to be "normal" according to the final basic probability assignment of the security level and the "maximum membership principle". The impact on the dam by external load and environmental factors is the limit, since the dam was completed in August 2014. There is no abnormal deformation displacement, cracks and unexpected water seepage in the arch dam, so the dam is operating in a normal state. Therefore, the multi-index comprehensive evaluation model of dam safety applied in the study is reasonable and feasible.

5. Conclusions

A multi-index comprehensive evaluation model for dam safety is proposed in this paper, which is based on the cloud model and Dempster-Shafer theory. The conclusions are as follows:

- The membership obtained from the normal cloud model and the measured data is used to determine the basic probability assignment. The uncertainties in the assessment process are fully considered in the model. The membership is obtained from the measured data, which is more objective and reliable than the traditional one based on subjective assignment.
- The improved Dempster-Shafer theory can solve the high conflict problem in the fusion calculation. Compared with the traditional Dempster-Shafer theory, the application is more extensive and the result is more reasonable.
- Applying the model to a practical dam, the calculate results are consistent with the actual operation of the dam. The example shows that the dam safety assessment method described in this paper is reasonable and feasible, providing a new concept for comprehensive dam safety evaluation. And the evaluation model could be more widely applied in solving high conflict problems via fusion calculation by integrating other methods.

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