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Rotating effects on hygro-mechanical vibration analysis of FG beams based on Euler-Bernoulli beam theory

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Abstract. This paper investigates free vibration characteristics of a rotating functionally graded (FG) beam in hygro environments. In the present study, material properties of the FG beam vary continuously through thickness direction according to the power-law which approximates material properties of FG beam. The governing differential equations of motion are derived based on Euler-Bernoulli beam theory and using the Hamilton's principle which solved utilizing a semi-analytical technique called the Differential Transform Method (DTM). In order to verify the competency and accuracy of the current analysis, a comparative study with previous researches are performed and good agreement is observed. Influences of Several important parameters such as power-law exponent, hygro environment, rotational speed and slenderness ratio on natural frequencies are investigated and discussed in detail. It is concluded that these effects play significant role on dynamic behavior of rotating FG beam in the hygro environments. Numerical results are tabulated in several tables and figures that can be serving as benchmarks for future analyses of rotating FG beams in the hygro environments.

Keywords: functionally graded beams; vibration analysis; hygro-mechanical systems

1. Introduction

Hygro stresses arising from moisture variations can affect the mechanical performance of engineering structures even at rotating structures (Mahato and Maiti 2012). Also, advanced structural components composed of functionally graded materials (FGMs) may experience intense hygro environments which show an adverse influence on the stiffness and safety of such structures. Therefore, an accurate evaluation of environmental exposure is necessitated to find the essence of their detrimental influence on the FG structures (Kocaturk and Akbas 2012, Bouiadjra *et al*. 2013, Nguyen *et al*. 2014). Thus, it is of utmost significance to investigate hygro induced mechanical behavior of these structural elements. To this end, hygro stress analysis of one-dimensional functionally graded piezoelectric media via analytical solutions is carried out by Akbarzadeh and Chen (Akbarzadeh and Chen 2013). Post buckling of functionally graded (FG) plates under hygrothermal environments is studied by Lee and Kim (Lee and Kim 2013). The static behaviors of exponentially inhomogeneous plates subjected to a transverse uniform loading and hygro-thermal conditions are studied by Zenkour (2013). Khateeb and Zenkour (2014) presented a refined four-variable plate model for bending analysis of advanced plates embedded on elastic foundations in hygro-thermal environments. In another study, Zenkour *et al*. (2014) researched the influence of temperature and moisture on the mechanical behavior of shear deformable composite functionally graded material (FGM) plates resting on elastic foundations. Biaxial thermal buckling behavior of auxetic FG plates with temperature dependent and moisture-dependent material properties on elastic foundations is investigated by Mansouri and Shariyat (2015).

FG materials which are introduced by Japanese scientists in mid-1980s possess various advantages in comparison with traditional composites, for instance, multi-functionality, ability to control deformation, corrosion and dynamic response, minimizations or remove stress concentrations, smoothing the transition of thermal stress, resistance to oxidation. Hence FGMs have received wide engineering applications in modern industries including aerospace, nuclear energy applications, turbine components, rocket nozzles, chemical reactor tubes, batteries/fuel cells, critical furnace parts, etc. during the past two decades. These wide engineering applications is cause that researchers attracted to FGMs, and study their vibration, static and dynamic's behavior of the FG structures (Ebrahimi *et al*. 2009, Ebrahimi *et al*. 2009).

Many investigations are reported in literature to study the dynamic and static behavior of functionally graded beams, here some of these disquisitions are mentioned briefly. Aydogdu and taskin (Aydogdu and Taskin 2007) discussed free vibration analysis beam with power-low and exponential material graduation with simply supported boundary conditions based on different higher order shear deformation and classical beam theory, they understand by increasing mode number the difference between CBT and higher order theories is increase. Simsek (2010) investigated the vibration analysis of FGM beams by classical, the first-order and different higher order shear deformation beam theories under different type of boundary

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conditions, also in another paper, non-linear vibration of FG Timoshenko beam subjected to a moving harmonic loading has been studied by him (Şimşek 2010). Sina *et al*. (2009) analyze the free vibration of FG beams by developing a new beam theory for laminated composite beams, which has a little different with first-order shear deformation beam theory. Pradhan and chakraverty (2013) have presented free vibration characteristics FG beams based on Euler and Timoshenko beam theory with various boundary conditions by using Rayleigh-Ritz method. Rotating beams have wide usage in the modeling of engineering applications such as wind turbines and helicopter blades, airplane propellers and robot manipulators. For instance, the dynamic modeling and analysis of rotating blades made of FGMs has become a topic of considerable research over the last decade. Ramesh and Rao (2014) investigated Free Vibration Analysis of Rotating FG Cantilever Beams using the Rayleigh-Ritz method. Attarnejad and Shahba (2011) examined the free vibration of non-prismatic beams by introducing the basic displacement functions for deriving shape functions in the finite element method. This methodology also has been used to study the rotating, axially FG tapered beams by Zarrinzadeh *et al*. (2011) Furthermore, Shahba and Rajasekaran (2012) have investigated the free vibration of centrifugally stiffened tapered FG beams. Li *et al*. (2014) has introduced the free vibration analysis of a rotating hub-FGM beam system with the dynamic stiffening effect based on a rigid–flexible coupled dynamics theory. In the Most recently work (Ebrahimi and Mokhtari 2015) presented free vibration analysis of rotating functionally graded thick beams within the context of Timoshenko beam theory using differential transform method. In another attempt (Ebrahimi and Mokhtari 2015) also used this method to study the free vibration analysis of a rotating Mori-Tanaka-based FG beam. A new shear and normal deformations theory for FG beams were presented by (Bourada *et al*. 2015). A year later a new three-unknown sinusoidal shear deformation theory were declared for FG plates by (Houari *et al*. 2016). A quasi-3D hyperbolic shear deformation theory offered for the static and free vibration analysis of FG plates (Hebali *et al*. 2014). In the same year an efficient and simple higher order shear and normal deformation theory were proposed for FGM plates by (Belabed *et al*. 2014). Wave propagation on FG plates were studied by (Yahia *et al*. 2015) using various HOSDT. Also (Mahi and Tounsi 2015) suggested a new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic FG sandwich composite plates. Recently (Tounsi *et al*. 2016) presented a new 3-unknowns non-polynomial plate theory for buckling and vibration of FG sandwich plate. Also a novel five variable refined plate theory for vibration analysis of FG sandwich plates were studied by (Bennoun *et al*. 2016). (Draiche *et al*. 2016) offered to use a refined theory with stretching effect for the flexure analysis of laminated composite plates. Additionally bending and free vibration analysis of FG plates using a simple shear deformation theory and the concept the neutral surface position were proposed firstly by (Bellifa *et al*. 2016).

Many researchers studied the thermo-mechanical effect on beams and plates recently. Tounsi *et al*. (2013) offered a refined trigonometric shear deformation theory for thermoelastic bending of FG sandwich plates. The same year thermo-mechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations were given by (Bouderba *et al*. 2013). Afterwards (Zidi *et al*. 2014) introduced a four variable refined plate theory to analyze the bending of FGM plates under hygro-thermo-mechanical loading. Later, various four variable refined plate theories used by (Attia *et al*. 2015) to analyze free vibration of FG plates with temperature-dependent properties. The thermo-mechanical bending of FG sandwich plates with a sinusoidal plate theory with 5-unknowns and stretching effect were studied by (Hamidi *et al*. 2015). Recently (Bousahla *et al*. 2016) researched about thermal stability of plates with FG coefficient of thermal expansion. Also (Beldjelili *et al*. 2016) used a four-variable trigonometric plate theory for hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations. At last, (Bouderba *et al*. 2016) suggested to use a simple shear deformation theory for thermal stability of FG sandwich plates.

DTM was first introduced by Zhou (1986) in solving linear and non-linear initial value problems in electrical circuit analysis with simplicity and good precision. This method is a semi-analytical-numerical technique based on Taylor series expansion developed for various types of differential equations. DTM make possible to obtain highly convergent and accurate results and exact solutions for differential or integral-differential equations. By using this method, the governing differential equations can be reduced to recurrence relations and the boundary conditions may be transformed into a set of algebraic equations. DTM do not pose any restrictions on both the type of material gradation and the variation of the cross section profile; hence it could cover most of the engineering problems dealing with the mechanical behavior of non-uniform and non-homogenous structures.

As seen, there is no study consider moisture Effects on mechanical vibration analysis of functionally graded beams based on Euler-Bernoulli beam theory. Actually for the simplicity of calculations and to get focused on the effects of moisture we use Euler-Bernoulli beam theory. There is strong scientific need to understand the moisture effect on vibration behavior of rotating FG beams. It is assumed that material properties of the beam vary continuously through the thickness of beam according to power-law form. Governing equations and boundary conditions for the free vibration of cantilever FG beam have been derived via Hamilton's principle and DTM is utilized to obtain a semi-analytical solution for free transverse vibration problem. The detailed mathematical derivations are presented while the emphasis is placed on investigating the effect of several parameters such as moisture effect, power-law exponent, rotational speed and slenderness ratios on the vibration characteristics of rotating FG beams. To validate the present analysis, the results of this study are compared with the existing literature and the good agreement is observed. New numerical results can also be useful as valuable sources for other researches.

Some novelties of the present study are stated as follows:

• There is no work that considers moisture environments with a rotating functionally graded beam until this paper was written. Moisture distribution is considered as linear moisture rise.
• It is for the first time that a semi-analytical technique is used to solve the differential equations. This method which previously stated as DTM reduces the computational difficulties of the other traditional methods and all the calculations can be made simple manipulations. In many cases, the series solutions obtained with DTM can be written in exact closed form. And even comparison between the results of DT and analytical methods reveals the accuracy of DT method.
• The use of DTM to solve the differential equations of rotating functionally graded beam in moisture environments is for the first time.

2. Theory and formulation

The system examined, shown schematically in Fig. 1 is a beam of variable cross section, carrying a so called heavy tip mass M. Its mass moment of inertia with respect to the perpendicular axis at the centroid S is denoted by JS. The publications (Abolghasemi and Jalali 2003, Younesian and Esmailzadeh 2010, Arvin and Bakhtiari-Nejad 2011) are considered also with rotating beams in which nonlinear oscillations are investigated. Analytical and experimental investigations on vibrating frames carrying concentrated masses with characteristics of frames have been studied by using analytical solutions and the finite element method (Cheng *et al*. 2013a, b).

2.1 Power-law functionally graded beams

A FGM beam made by ceramic–metal with rectangular cross-section is considered in this paper. As shown in Fig. 1 a cantilever beam with length L is attached to periphery of a rigid hub of radius R and the hub rotates about vertical z axis in a fixed coordinate system with a constant angular velocity Ω.

Top surface of FG beam (z=h/2) is assumed to be pure ceramics and it varies continuously to the metal-rich surface at the bottom surface (z=−h/2). A *cartesian* coordinate system $o(x,y,z)$ is attached on the central axis of the beam, where x-axis is matched with neutral axis of the beam in the

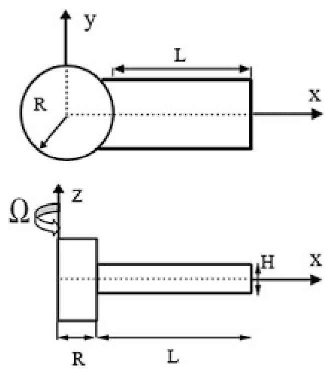

Fig. 1 Geometry of the rotating FG beam system

undeflected position, the y-axis in the width direction, and the z-axis in the thickness direction. The beam is made of homogeneous and isotropic functionally graded materials which the volume fraction and micro-structural morphology of the material compositions are varying continuously in the thickness direction only. Functionally graded materials are the new generation of composite materials which are usually produced from two or multi different materials. In this study FG material is made from a mixture of ceramic and metal and the material properties of FG beam are supposed to vary through thickness direction of the constitutes according to power-law distribution. The effective material properties of FG beam that distributed identical in two phases of ceramic and metal can be expressed by u

$$P = P_m(v_m) + P_c(v_c) \tag{1}$$

Where P_c and P_m are the material properties of ceramic and metal v_c and v_m are the volume fraction of ceramic and metal that are attached as

$$v_m + v_c = 1 \tag{2}$$

The power-law volume fraction of the ceramics constituents of the beam is assumed to be given by

$$v_c = (\frac{z}{h} + \frac{1}{2})^p \tag{3}$$

Here z is the distance from the mid-plane of the FGM beam and p is the non-negative variable parameter (power-law exponent) which determines the material distribution through the thickness of the beam. According to this distribution we have a fully metal beam for large value of p and when p equal to zero a fully ceramic beam remain. Material properties of FG beam are supposed to vary through thickness direction of the constitutes, according to power-law distribution. P-FGM is one of the most favorable models for FGMs. Effective material properties such as Young's modulus (E), Poissons' ratio (υ) and mass density (ρ) are assumed to vary continuously in the thickness direction according to power-law. The effective material properties of FG beam can be expressed by using the modified rule of mixture as

$$E(z) = (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + E_m \tag{4a}$$

$$\rho(z) = (\rho_c - \rho_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \rho_m \tag{4b}$$

$$v(z) = (v_c - v_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + v_m \tag{4c}$$

$$\kappa(z) = (\kappa_c - \kappa_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \kappa_m \tag{4d}$$

2.2 Kinematic relations

Based on the Euler–Bernoulli beam theory, axial displacement u_x, and transverse displacement of an arbitrary point in the beam along x and z axes are as follows

$$u_x(x,z,t) = u(x,t) - z\frac{\partial w(x,t)}{\partial x} \tag{5a}$$

$$u_z(x,z,t) = w(x,t) \tag{5b}$$

In a Cartesian coordinate system, x is the distance of the point from the hub edge parallel to beam length and u and w are the axial and transverse displacement components at any point on the mid-plane. Therefore, according to (EBT), every elements of strain tensor vanish except normal strain in the x-direction. Thus, the only nonzero strain is

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = \frac{\partial u(x,t)}{\partial x} - z\frac{\partial^2 w(x,t)}{\partial x^2} \tag{6}$$

By assuming that the material of FGM beam obeys Hooke's law, the normal stress in the beam becomes

$$\sigma_{xx} = E(z)\varepsilon_{xx} \tag{7}$$

Based on the Hamilton's principle, which states that the motion of an elastic structure during the time interval $t_1<t<t_2$ is such that the time integral of the total dynamics potential is extremum (Tauchert 1974)

$$\int_{t_1}^{t_2} \delta(T - U + V)dt = 0 \tag{8}$$

Here U is strain energy, T is kinetic energy and W is work done by external forces. The strain energy can be expressed as

$$U = \frac{1}{2}\int_v \sigma_{ij}\varepsilon_{ij}dV = \frac{1}{2}\int_v \sigma_{xx}\varepsilon_{xx}dV \tag{9}$$

Substituting Eqs. (6) and (7) into Eq. (9) yields

$$U = \frac{1}{2}\int_0^L A_{xx}(\frac{\partial u}{\partial x})^2 + D_{xx}(\frac{\partial^2 w}{\partial x^2})^2 - 2B_{xx}(\frac{\partial u}{\partial x})(\frac{\partial^2 w}{\partial x^2})dx \tag{10}$$

The kinetic energy for Euler-Bernoulli beam can be written as

$$T = \frac{1}{2}\int_0^L \int_A \rho(z)((\frac{\partial u_x}{\partial t})^2 + (\frac{\partial u_z}{\partial t})^2)dAdx \tag{11}$$

By using Eq. (5a) and Eq. (5b)

$$T = \frac{1}{2}\int_0^L I_A((\frac{\partial u}{\partial t})^2 + (\frac{\partial w}{\partial t})^2) - 2I_B(\frac{\partial u}{\partial t})(\frac{\partial^2 w}{\partial x \partial t}) + I_D(\frac{\partial^2 W}{\partial x \partial t}) \tag{12}$$

The centrifugal force $P(x)$ which varies along the x direction of the beam is given by

$$P(x) = \int_x^L I_A \Omega^2 (R + x)dx \tag{13}$$

Where in the Eqs. (10)-(13) the mass moment of inertias and cross-sectional rigidities for a FGM beam can written as follows

$$(I_A, I_B, I_D) = \int_A \rho(z)(1, z, z^2)dA \tag{14a}$$

$$(A_{xx}, B_{xx}, D_{xx}) = \int_A E(z)(1, z, z^2)dA \tag{14b}$$

The variation of work done by external forces as an axial force can be expressed as

$$\delta V = \int_0^L \left[f(x)\delta u + q(x)\delta w + \overline{N}\frac{\partial w}{\partial x}\frac{\partial(\delta w)}{\partial x} \right]dx \tag{15a}$$

Where $f(x)$, $q(x)$ are the axial and transverse loading that in this investigate equal to zero and $\overline{N}$ is external loading due to thermal or moisture environment, elastic foundation, centrifugal force, etc.

In this study for analyzing vibration of FG nanobeams in hygro environment, moisture distribution is considered vary through thickness direction in the case of as: linear moisture distribution.

The first variation of external loadings due to hygro change and rotating can be written in the form as

$$\delta v = \int_0^L \overline{N}\frac{\partial w}{\partial x}\frac{\partial(\delta w)}{\partial x}dx \tag{15b}$$

$$N^h = \int_{-h/2}^{h/2} \beta(z,T)E(z,T)(C - C_0)dz \tag{16}$$

Where N^h is obtained as:

In which $\beta_{\exp}$ is the coefficient of moisture dilatation that is typically positive and very small. $(0 \prec \alpha, \alpha \;1)$ According to Hamilton's principle, the governing equations of rotary functionally graded beam in in hygro environment can be obtained as

$$-I_A\frac{\partial^2 u}{\partial t^2} + I_B\frac{\partial^3 w}{\partial t^2 \partial x} + \frac{\partial}{\partial x}(A_{xx}\frac{\partial u}{\partial x}) - \frac{\partial}{\partial x}(B_{xx}\frac{\partial^2 w}{\partial x^2}) = 0 \tag{17}$$

$$\begin{aligned} &-I_A\frac{\partial^2 w}{\partial t^2} + \frac{\partial}{\partial x}(I_D\frac{\partial^3 w}{\partial t^2 \partial x}) - \frac{\partial}{\partial x}(I_B\frac{\partial^2 u}{\partial t^2}) - \\ &\frac{\partial^2}{\partial x^2}(D_{xx}\frac{\partial^2 w}{\partial x^2}) + \frac{\partial^2}{\partial x^2}(B_{xx}\frac{\partial u}{\partial x}) + \\ &\frac{\partial}{\partial x}([P(x) + N^h]\frac{\partial w}{\partial x}) = 0 \end{aligned} \tag{18}$$

Further, the two ends of the cantilever or clamped-free beam (clamped at x=0 and free at x=L) are subjected to the following boundary conditions

$$at \quad x = 0\,; \quad w = \frac{\partial w}{\partial x} = 0\,,\, u = 0 \tag{19a}$$

$$and\ at \quad x = L\,; \quad \frac{\partial^2 w}{\partial x^2} = \frac{\partial^3 w}{\partial x^3} = 0\,,\ \frac{\partial u}{\partial x} = 0 \tag{19b}$$

Assuming simple harmonic oscillation, w and u can be written as

$$w(x,t) = \overline{w}(x)e^{i\omega t} \tag{20a}$$

$$u(x,t) = \overline{u}(x)e^{i\omega t} \tag{20b}$$

Substituting Eqs. (20a)-(20b) into Eqs. (17)-(18), the governing equations are as follows

$$\omega^2 I_A \bar{u} - \omega^2 I_B \frac{\partial \bar{w}}{\partial x} + \frac{\partial}{\partial x}(A_{xx}\frac{\partial \bar{u}}{\partial x}) - \frac{\partial}{\partial x}(B_{xx}\frac{\partial^2 \bar{w}}{\partial x^2}) = 0 \quad (21)$$

$$\begin{aligned}&\omega^2 I_A \bar{w} - \omega^2 \frac{\partial}{\partial x}(I_D \frac{\partial \bar{w}}{\partial x}) + \omega^2 \frac{\partial}{\partial x}(I_B \bar{u}) - \\ &\frac{\partial^2}{\partial x^2}(D_{xx}\frac{\partial^2 \bar{w}}{\partial x^2}) + \frac{\partial^2}{\partial x^2}(B_{xx}\frac{\partial \bar{u}}{\partial x}) + \\ &\frac{\partial}{\partial x}([P(x) + N^h]\frac{\partial \bar{w}}{\partial x}) = 0\end{aligned} \quad (22)$$

2.3 Linear moisture rise (LMR)

For a FG beam for which the plate thickness is thin enough, the moisture distributions are linearly variable through the thickness as follows

$$c = c_m + \Delta c\left(\frac{1}{2} + \frac{z}{h}\right) \quad (23)$$

And Δc changings moisture should be defined as

$$\Delta c = c_t = c_b \quad (24)$$

3. The differential transform method (DTM)

$$Y(k) = \frac{1}{k!}\left[\frac{d^k}{dx^k} y(x)\right]_{x=0} \quad (25)$$

$$y(x) = \sum_{k=0}^{\infty} x^k Y(k) \quad (26)$$

Where $y(x)$ is the original function and $Y(k)$ is the transformed function. From Eqs. (25)-(26) we can obtain

$$y(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}\left[\frac{d^k}{dx^k} y(x)\right]_{x=0} \quad (27)$$

Eq. (27) reveals that the concept of the differential transformation is derived from Taylor's series expansion. In real applications the function $y(x)$ in Eq. (27) can be written in a finite form as

$$y(x) = \sum_{k=0}^{N} x^k Y(k) \quad (28)$$

$$\begin{aligned}&A_{xx}(k+1)(k+2)U[k+2] - B_{xx}(k+1)(k+2)(k+3)W[k+3] + \\ &I_A\omega^2 U[k] - I_B\omega^2(k+1)W[k+1] = 0\end{aligned} \quad (29)$$

$$\begin{aligned}&B_{xx}(k+1)(k+2)(k+3)U[k+3] - \\ &D_{xx}(k+1)(k+2)(k+3)(k+4)W[k+4] - \\ &(N^h + \frac{I_A\Omega^2 L(2R+1)}{2} - I_B\omega^2 - I_D\omega^2)(k+1)(k+2)w[k+2] - \\ &(I_A R\Omega^2)(k)(k+1)w[k+1]((-\frac{I_A\Omega^2}{2})(k-1)(k) + I_A\omega^2)w[k] \\ &+ I_B\omega^2(k+1)U[k+1] = 0\end{aligned} \quad (30)$$

In this calculations $y(x) = \sum_{n+1}^{\infty} x^k Y(k)$ is small enough to be neglected, and N is determined by the convergence of the eigenvalues. From the definitions of DTM in Eqs. (25)-(27), the fundamental theorems of differential transforms method can be performed that are listed in Table 1 while Table 2 presents the differential transformation of conventional boundary conditions. According to the basic transformation operations introduced in Table 1, the transformed form of the governing Eqs. (21)-(22) around x_0=0 may be obtained. In the Eqs. (29)-(30), $U[k]$ and $W[k]$ are the transformed functions of $\bar{u}$ and $\bar{w}$ respectively. Additionally, the differential transform method is applied to boundary conditions by using the theorems introduced in Table 2 and the following transformed boundary conditions are obtained.

Table 1 Some of the transformation rules of the one-dimensional DTM (Ju 2004)

Original function	Transformed function
$f(x) = g(x) \pm h(x)$	$F(K) = G(K) \pm H(K)$
$f(x) = \lambda g(x)$	$F(K) = \lambda G(K)$
$f(x) = g(x)h(x)$	$F(K) = \sum_{l=0}^{K} G(K-l)H(l)$
$f(x) = \frac{d^n g(x)}{dx^n}$	$F(K) = \frac{(k+n)!}{k!} G(K+n)$
$f(x) = x^n$	$F(K) = \delta(K-n) = \begin{cases}1 & k = n \\ 0 & k \neq n\end{cases}$

Table 2 Transformed boundary conditions (B.C.) based on DTM (Ju 2004)

x=0		x=**L**	
Original B.C.	Transformed B.C.	Original B.C.	Transformed B.C.
$f(0) = 0$	$F[0] = 0$	$f(L) = 0$	$\sum_{k=0}^{\infty} F[k] = 0$
$\frac{df(0)}{dx} = 0$	$F[1] = 0$	$\frac{df(L)}{dx} = 0$	$\sum_{k=0}^{\infty} k F[k] = 0$
$\frac{d^2 f(0)}{dx^2} = 0$	$F[2] = 0$	$\frac{d^2 f(L)}{dx^2} = 0$	$\sum_{k=0}^{\infty} k(k-1) F[k] = 0$
$\frac{d^3 f(0)}{dx^3} = 0$	$F[3] = 0$	$\frac{d^3 f(L)}{dx^3} = 0$	$\sum_{k=0}^{\infty} k(k-1)(k-2) F[k] = 0$

Clamped-Free

$$W[0] = 0, W[1] = 0, U[0] = 0 \quad (31)$$

$$\begin{aligned}&\sum_{k=0}^{\infty} k(k-1)L^{(k-2)} W[k] = 0, \\ &\sum_{k=0}^{\infty} k(k-1)(k-2)L^{(k-3)} W[k] = 0, \\ &\sum_{k=0}^{\infty} kL^{(k-1)} U[k] = 0\end{aligned} \quad (32)$$

Using Eqs. (33)-(34) together with the transformed boundary conditions one arrives at the following eigenvalue problem

$$\begin{bmatrix} M_{11}(\omega) & M_{12}(\omega) & M_{13}(\omega) \\ M_{21}(\omega) & M_{22}(\omega) & M_{23}(\omega) \\ M_{31}(\omega) & M_{32}(\omega) & M_{33}(\omega) \end{bmatrix}[C] = 0 \quad (33)$$

Table 3 Temperature dependent coefficients of Young's modulus, thermal expansion coefficient, mass density and Poisson's ratio for Si_3N_4 and SUS304

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
Si_3N_4	E (Pa)	348.43e+9	0	-3.070e-4	2.160e-7	-8.946e-11
	α (K^{-1})	5.8723e-6	0	9.095e-4	0	0
	ρ (Kg/m^3)	2370	0	0	0	0
	v	0.24	0	0	0	0
SUS304	E (Pa)	201.04e+9	0	3.079e-4	-6.534e-7	0
	α (K^{-1})	12.330e-6	0	8.086e-4	0	0
	ρ (Kg/m^3)	8166	0	0	0	0
	v	0.3262	0	-2.002e-4	3.797e-7	0

Where [C] correspond to the missing boundary conditions at x=0. For the non-trivial solutions of Eq. (36), it is necessary that the determinant of the coefficient matrix set equal to zero

$$\begin{vmatrix} M_{11}(\omega) & M_{12}(\omega) & M_{13}(\omega) \\ M_{21}(\omega) & M_{22}(\omega) & M_{23}(\omega) \\ M_{31}(\omega) & M_{32}(\omega) & M_{33}(\omega) \end{vmatrix} = 0 \tag{34}$$

Solution of Eq. (34) is simply a polynomial root finding problem. Solving equation (34), the i^{th} estimated eigenvalue for n^{th} iteration ($\omega = \omega_i^{(n)}$) may be obtained and the total number of iterations is related to the accuracy of calculations which can be determined by the following equation

$$\left|\omega_i^{(n)} - \omega_i^{(n-1)}\right| \prec \varepsilon \tag{35}$$

In this study ε=0.0001 considered in procedure of finding eigenvalues which results in 4 digit precision in estimated eigenvalues. Further a Mathematica program has been developed according to DTM rule stated above, in order to find eigenvalues.

The non-dimensional natural frequencies (λ) can be calculated by relations in Eq. (36).

$$\lambda = \omega \frac{L^2}{h}\sqrt{\frac{\rho_m}{e_m}} \tag{36}$$

4. Numerical results and discussions

In this section a numerical testing of the procedure as well as parametric studies are carried out in order to show the reliability and usefulness of the DTM approach and the influence of different beam parameters such as different moisture changings, power-law exponents, slenderness ratio and rotational speed on the natural frequencies of the rotating FG beam is examined. The functionally graded beam with made of Si_3N_4/SUS304 are considered throughout this paper where its properties are presented in Table 3. Table 4 shows the convergence study of differential transform method (DTM) for first two frequencies.

Table 4 Convergence study for first two frequencies of rotating FGM beam in linear temperature rise
(L/h_0=20, n=1, ΔC=40, Ω=5, R=0)

k	λ_1	λ_2
12	1.48343	7.50028
13	1.48327	7.96697
14	1.48319	9.43085
15	1.48320	8.49514
16	1.48320	8.39981
17	1.48320	8.43010
18	1.48320	8.44913
19	1.48320	8.44354
20	1.48320	8.44249
21	1.48320	8.44276
22	1.48320	8.44287
23	1.48320	8.44284
24	1.48320	8.44284

It is observed that after a certain number of iterations, the eigenvalues converged to a value with good precision, hence the number of iterations is important in DTM method. From the result of Table 4, high convergence rate of the method can be easily observed.

To verify the accuracy of the present method, the numerical results obtained will be compared with those available in the literature.

Table 5 compares the semi-analytical results of the proposed mathematical model and the results obtained for the nonrotating FG beam and not any moisture loadings with various constituent volume fraction exponents presented by Simsek (2010)which has been obtained by using Lagrange's equations and the results presented by Pradhan and Chakraverty (2013) which has been obtained by using Rayleigh-Ritz method, hereupon natural frequencies of FG beams combined of alumina and aluminium with following material and beam properties (E_{Al}=70 GPa, ρ_{Al}=2702 kg/m^3, v_{Al}=0.3, $E_{Al_2o_3} = 380GPa$, $\rho_{Al_2o_3} = 3960kg/m^3$, $v_{Al_2o_3} = 0.3$) for L/h=5, 20 and various gradient indexes with simply-simply boundary conditions are obtained by DTM.

It is observed that the fundamental frequency parameters obtained in the present research are in approximately enough to the results provided in the study that is used for comparison and validate the proposed method of solutions.

It is showed that DTM is an effective and reliable tool for the solution of system of ordinary differential equations. DT method implies an iterative procedure to obtain the high-order Taylor series solution of differential equations. The Taylor series method requires a long computational time for large orders, whereas one advantage of employing DTM in solving differential equations is a fast convergence rate and small calculation error. This method reduces the computational difficulties of the other traditional methods and all the calculations can be made simple manipulations. In many cases, the series solutions obtained with DTM can be written in exact closed form. And even comparison between the results of DT and analytical methods reveals

Table 5 Variation of the fundamental frequencies with respect to various temperature changings and thermal loading for rotating FG beam (Ω=2, R=0, L/h_0=10)

ΔC	Gradient index					
	0	1	2	3	4	5
0	2.34259	1.42067	1.27572	1.21601	1.18193	1.15912
1	2.11902	1.13928	1.00372	0.9217	0.87123	0.682541
2	1.9830	0.92186	0.8318	0.6452	0.52723	0.42134

Table 6 Variation of the first dimensionless natural frequencies with various rotational speed for two thermal loadings (LTR and NLTR) and different temperature changings (n=2, R=0, L/h_0=10)

$\Omega\ (\frac{Rad}{S}\times 10^2)$	LTR			
	ΔC			
	0	1	2	4
0	1.4338	1.2214	1.1801	1.1567
1	1.4590	1.2790	1.2078	1.1832
2	1.5320	1.4372	1.3397	1.2076
3	1.6461	1.5483	1.41873	1.3128
4	1.7931	1.6688	1.5791	1.47124
5	1.9650	1.8750	1.6103	1.5190
6	2.1554	1.9274	1.8210	1.6721
7	2.3593	2.1176	2.0085	1.8329
8	2.5731	2.3199	2.18910	1.9624
9	2.7944	2.6109	2.3810	2.1679
10	3.0231	2.9912	2.6621	2.4329

Table 7 Variation of the fundamental natural frequencies with various slenderness ratios and LTR (Ω=5, ΔC=2, R=0)

L/h_0	n		
	0	2	5
5	1.79055	1.0037	0.7862
10	1.92291	1.21794	1.0824
20	2.12872	1.4952	1.2823

Table 8 Variation of the fundamental frequencies with respect to various hub radius parameters for different value of rotational speed (n=2, LTR, ΔC=20, L/h_0=10)

R	$\Omega\ (\frac{Rad}{s}\times 10^2)$	Fundamental frequency
0	1	1.18423
	5	1.3837
0.2	1	1.2742
	5	1.6418
0.5	1	1.2312
	5	1.8542

the accuracy of DT method.

The fundamental frequency parameters obtained in the present investigation are in approximately close enough to the results provided in these literatures and thus validates the proposed method of solution. Table 5 show the influence of the moisture and power law index on the vibrational behavior of the rotating FG beams. The variation of the first dimensionless frequencies of FG rotating beam against the different values of moisture is investigated in this table at constant value of rotation speed and (Ω=2, R=0, L/h_0=10)

The results of this table show that increasing the power-law exponent (n) leads to reduction in frequency results of the beam. The highest frequencies are obtained for full ceramic beam (n=0) while the lowest frequencies are obtained for full metal beam ($n\rightarrow\infty$). This is due to this fact that, increasing the power-law exponent leads to decrease in the elasticity modulus and bending rigidity. In other words, the beam becomes more flexible as the power law exponent increases. Thus, as also known from mechanical vibrations, natural frequencies decrease as the stiffness of a structure decreases. The non-dimensional natural frequencies will be decreased by increasing of moisture value for all gradient indexes. So it is shown the importance of moisture changings on the mechanical vibration of rotating beams.

In Table 6, rotational effect on the fundamental frequencies of FG beams subjected to linear moisture changings as LTR is presented. Another important parameter in vibration behavior of rotating FG beam is its rotational speed. By looking at the result of this table it's concluded that, the effect of rotational speed on the first natural frequency of FGM beams is considerable. Increasing the rotational speed increases the fundamental frequency of rotating FG beam in a constant value of power-law exponent and moisture value. The reason attributed to this observation is that rotational speed is directly proportional to the centrifugal force according, which causes stiffening effect on vibration characteristic of the rotating beam. And also it's evident that temperature rising causes in the decreasing of natural frequency.

The effect of slenderness ratio (L/h_0) on the natural frequencies for different power-law exponents is presented in Table 7. It can be concluded that by increasing the slenderness ratio, the natural frequencies increase.

In Table 8 the effect of hub radius on natural frequencies of FG beams subjected to LTR moisture distributions with various rotational speeds has been investigated. For this purpose, the hub radius parameter has different values. It is observed that all frequencies have increased with increasing the hub radius as expected due to increase in centrifugal force and its stiffening effect on the beam. Also this rising rate increases for higher rotational speed because of this fact that, the centrifugal force is directly proportional to both of these parameters.

Figs. 2-3 presented to show the convergence trend of DTM for the 1st and 2nd frequencies. As seen in Fig. 2, 1st natural frequency converged after 16 iterations with 4-digit precision while according to Fig. 3 the 2nd natural frequency converged after 20 iterations.

The variations of the non-dimensional frequency of the rotating FG beam versus power law exponents and for different moisture values at (Ω=2, R=0, L/h=10) are displayed in Fig. 4. It is found that moisture values and power law exponents decreasing the natural frequencies of FG beam when their values changes from zero to positive one at a constant value of slenderness which highlights the notability of the moisture effect on the vibration of rotating

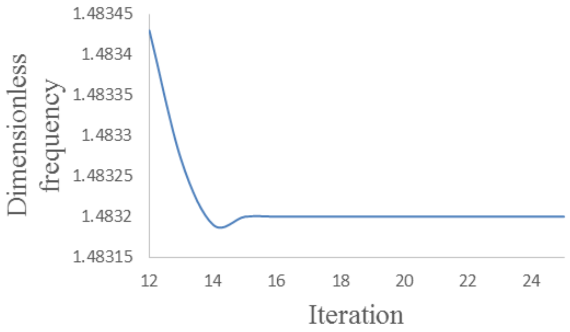

Fig. 2 Convergence study of first dimensionless frequency (ΔC=40, n=1, Ω=5, R=0, L/h=20)

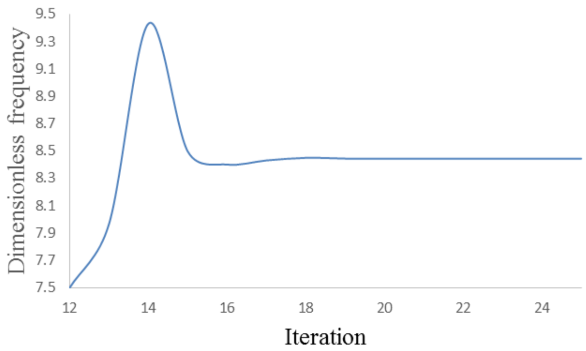

Fig. 3 Convergence study of second dimensionless frequency (ΔC=40, n=1, Ω=5, R=0, L/h=20)

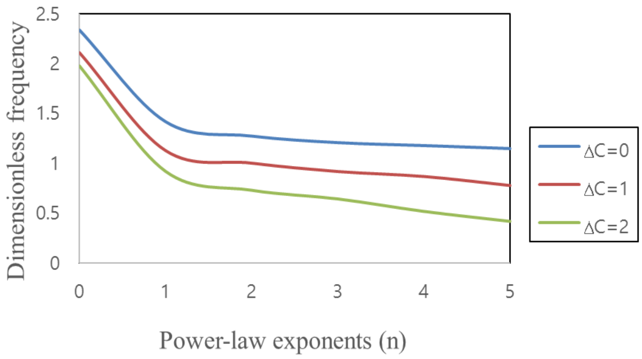

Fig. 4 Effect of hygro environment and power-law exponents on the dimensionless frequency of the rotational FG beam in the moisture environment (Ω=2, R=0, L/h=10)

FG beam. Furthermore, according to Fig. 5, the non-dimensional frequency increase as the rotation speed value increase with variation of material graduation. It is seen that the FG beam with higher value of rotation speed value provide larger values of the frequency results. However, this behavior is opposite by increasing of power-law exponents.

Fig. 6 shows the dimensionless frequency of rotating FG beam as function of various rotational speed values for different values of moisture concentration at (n=1, R=0, L/h=10). It is found that by increasing of the rotational speed, the fundamental frequency of the FG beam in hygro environment will be increased and also it is shown that the moisture effect has a notable effect on the dynamic behavior of the FG beam.

Also the effect of slenderness ratio on the fundamental

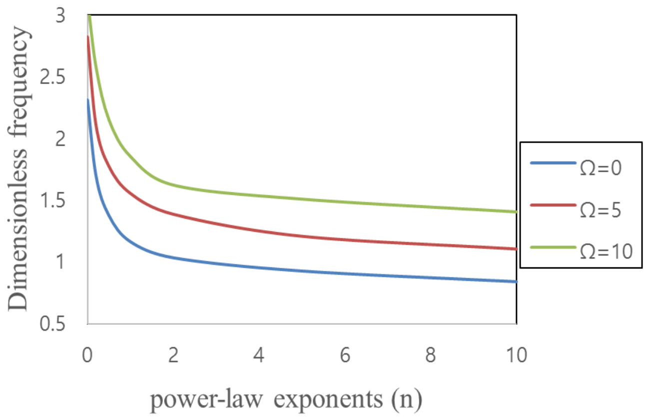

Fig. 5 Effect of rotational speed and material graduation exponents on the dimensionless frequency of the rotational FG beam in the moisture environment (ΔC=0, R=0, L/h=10)

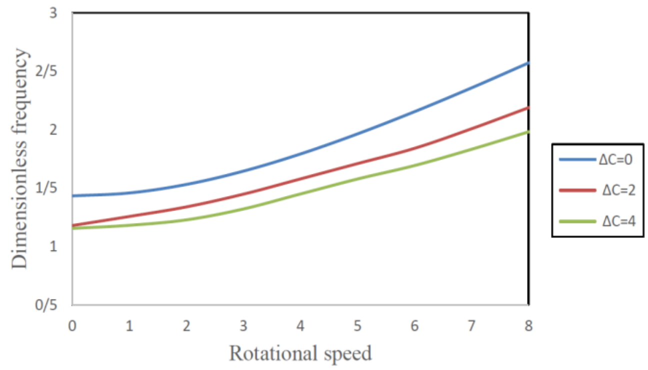

Fig. 6 Effect of rotational speed and moisture environment on the dimensionless frequency of the rotational FG beam in the moisture environment (n=1, R=0, L/h=10)

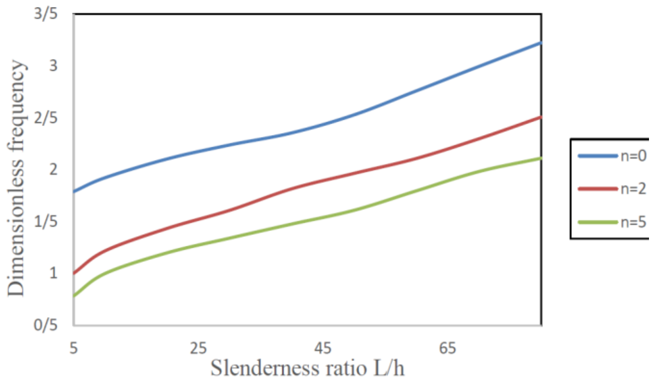

Fig. 7 Effect of slenderness ratio and material graduation exponents on the dimensionless frequency of the rotational FG beam in the moisture environment (ΔC=0, R=0, Ω=2)

frequencies of FG beam is plotted in Fig. 7 for three different power law indexes at (ΔC=0, R=0, Ω=2). It can be concluded that by increasing the slenderness ratio, the natural frequencies increase.

Fig. 6 shows the dimensionless frequency of rotating FG beam as function of various rotational speed values for different values of moisture concentration at (n=1, R=0, L/h=10). It is found that by increasing of the rotational speed, the fundamental frequency of the FG beam in hygro environment will be increased and also it is shown that the moisture effect has a notable effect on the dynamic behavior of the FG beam.

Also the effect of slenderness ratio on the fundamental frequencies of FG beam is plotted in Fig. 7 for three different power law indexes at (ΔC=0, R=0, Ω=2). It can be concluded that by increasing the slenderness ratio, the natural frequencies increase.

5. Conclusions

This paper investigates free vibration characteristics of a rotating functionally graded (FG) beam in hygro environments. Since the rising of moisture changings cause the reduction in mass and strength of FG beams, so it is important to consider the moisture effect. In the present study, Material properties of the FG beam vary continuously through thickness direction according to the power-law which approximates material properties of FG beam. The governing differential equations of motion are derived based on Euler-Bernoulli beam theory and using the Hamilton's principle which solved utilizing a semi-analytical technique called the Differential Transform Method (DTM). In order to verify the competency and accuracy of the current analysis, a comparative study with previous researches are performed and good agreement is observed. Accuracy of the results verified using available data in the literature. Influences of several important parameters such as power-law exponent, hygro environment, rotational speed and slenderness ratio on natural frequencies are investigated and discussed in detail. It is concluded that these effects play significant role on dynamic behavior of rotating FG beam in the hygro environments.

Numerical results are tabulated in several tables and figures that can be serving as benchmarks for future analyses of rotating FG beams in the hygro environments. According to numerical experiments, the effect of moisture distribution on non-dimensional frequency of rotating FG beams is considerable. Increasing the moisture changings leads to decreasing the frequency. Furthermore, numerical results indicate that the flexural natural frequencies of FG beam are increased with rotational speed and hub radius while they are decreased with the power-law exponent.

Numerical results show that:

- By increasing the gradient index value, the non-dimensional frequencies are found to be decreased.
- Fundamental frequencies decreased by increasing the moisture rising changings.
- Flexural natural frequencies of FG beam increase with rotational speed and hub radius.
- It is seen that the influence of moisture or humidity is significant for higher values of gradient index and slenderness ratio.

It is concluded that various factors such as moisture effect, hygro distribution, rotation speed, hub radius and power-law index have a notable effect on the non-dimensional frequencies of FG beams, Which emphasizes on the importance of inspected rotations effect in moisture environment. Therefore, the rotation and hygro or moisture effects should be considered in the analysis of vibration behavior of FG structures.

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