Structural Engineering and Mechanics, *Vol. 63, No. 3 (2017) 281-292*
DOI: https://doi.org/10.12989/sem.2017.63.3.281

Distribution of strength and stiffness in asymmetric wall type system buildings considering foundation flexibility

Gh.R. Atefatdoost[*1], H. Shakib[2a] and B. JavidSharifi[3b]

[1]Department of Civil Engineering, Estahban Branch, Islamic Azad University, Estahban, Iran
[2]Tarbiat Modares University, Tehran, Iran
[3]Fars Regional Electrical Company (FREC), Shiraz, Iran

(Received October 8, 2016, Revised January 27, 2017, Accepted February 24, 2017)

Abstract. Architecture constraints in buildings may typically cause irregularities in the distribution of stiffness and mass and consequently causes non-compliance of centers of mass, stiffness and strength. Such buildings are known as asymmetric buildings the distribution of strength and stiffness is one of whose main challenges. This distribution is more complicated for concrete buildings with RC shear walls in which stiffness and strength are interdependent parameters. The flexibility under the foundation is another subject that can affect this distribution due to the variation of dynamic properties of the structure and its constituting elements. In this paper, it is attempted to achieve an appropriate distribution pattern by expressing the effects of foundation flexibility on the seismic demand of concrete shear walls and also evaluate the effects of this issue on strength and stiffness distribution among lateral force resistant elements. In order to understand the importance of flexibility in strength and stiffness distribution for an asymmetric building in different conditions of under-foundation flexibility, the assigned value to each of the walls is numerically calculated and eventually a procedure for strength and stiffness distribution dependencies on flexibility is provided.

Keywords: foundation flexibility; asymmetric buildings; strength and stiffness distribution; strength and stiffness interdependency; yield displacement distribution; wall type system

1. Introduction

Practically, there are several cases in which centers of mass and stiffness of a structure do not match as a result of architectural constraints. This non-compliance may cause floor rotation and consequently additional deformations, which in turn may increase the risk of damage in structural and non-structural elements (Mysilmaj and Tso 2004, Mysilmaj and Tso 2002, Tso and Myslimaj 2003, Mysilmaj and Tso 2001, Mysilmaj and Tso 2005, Aziminejad and Moghadam 2009, Shakib and Ghasemi 2007). For asymmetric buildings in order to consider rotation-caused damages, most references have presented patterns for distribution of story shear which, in general, are categorized into two approaches. The traditional approach is that the stiffness and strength of lateral force-resisting elements (LFREs) be assumed independent and distribution is supposed to be performed based on elements stiffness. This viewpoint is still seen in numerous seismic codes. The more recent approach is based on the dependency of stiffness on strength and holds about an element such as a concrete shear wall. In this approach, the basis of strength distribution is yield displacement of the wall. This is called yield displacement distribution-based (YDDB) strength assignment approach in the literature. Unlike the traditional method, the wall's strength assignment is first found and then the required stiffness of the element is determined by dividing the strength assignment by yield displacement. Taking this attitude in extensive studies, such researchers as Myslimaj & Tso proposed an appropriate algorithm for strength distribution, and analytically as well as numerically confirmed this algorithm and interpreted the behavior of concrete buildings with shear wall systems for different distributions of stiffness and strength. They found out that what is obvious in this approach is the simultaneous effect of strength and stiffness centers on the behavior of asymmetric structures including shear wall systems, so that to express a criterion for the minimum rotation of the floor, it is essential to simultaneously consider such effects on (Mysilmaj and Tso 2004, Mysilmaj and Tso 2002, Tso and Myslimaj 2003, Mysilmaj and Tso 2001, Mysilmaj and Tso 2005, Shakib and Ghasemi 2007).

Most engineering structures are built on underlying soils. When an earthquake happens, due to the induced deformations in the sub-soil, displacements of the building basemat relative to free-field displacements are different when the latter is measured far away from the structure's place. As a result, the structure’s response will differ due to soil existence and based on its physical properties. This difference in responses is the very well recognized soil-structure interaction (SSI). Considering SSI due to the

∗Corresponding author, Assistant Professor
E-mail: gh_atefatdoost@iauest.ac.ir
[a]Professor
E-mail: shakib@modares.ac.ir
[b]Construction Projects Superintendent
E-mail: b.javidsharifi@sutech.ac.ir

http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

variation of structure's dynamic properties such as increased period and damping leads to variation of structure's seismic demands compared to the case in which this interaction is not taken into account (Wolf 1985, NEHRP 2012). In asymmetric buildings with reinforced concrete shear wall systems this issue may affect structural responses during earthquake depending on wall yield displacement as well as general properties of the structure. According to the new perspective, it is expected that the distribution of strength and stiffness of the elements vary along with wall yield displacement.

Most researches on effects of flexibility on asymmetric buildings are performed in order to estimate the seismic demands of such structures and this is what the rest of this section introduces to an extent. SSI in asymmetric buildings is holistically observed in (Shakib and Fuladgar 2004), where an idealized SDOF building is experimented and analyzed while placed on different soil conditions so as to focus on different flexibility states along with effects on structural responses, which is partly of interest in the present study. In a different aspect, effects of soil flexibility on inelastic seismic demands of low-rise buildings were studied by Roy and Dutta (Roy and Dutta 2010). The response reduction factor (R) was addressed by the researchers and questioned for low-rise, stiff structures. Considering elasto-plastic and hysteresis behavior for lateral load-resisting structural elements and idealizing the sub-soil as linear and elasto-plastic behaving, it was found that short period systems are very sensitive to R and may be phenomenally amplified even for small R due to SSI, which would practically render restrictions on the dual-design philosophy. At the same time, Roy and Dutta showed that asymmetric low-rise buildings are not appreciably affected by SSI relative to their symmetric counterparts (Roy and Dutta 2010). Effect of SSI on dynamic behavior of building frames on raft foundation was studied by Bhattacharya *et al.*, where an investigation was made into SSI effects on torsional-to-lateral period ration of building frames on raft foundations. Different soil conditions, number of stories, number of bays, column-beam flexural stiffness ratio, excitation frequency, etc. were seen in the analyses and curves and tables were provided presenting changes in lateral natural period and torsional-to-lateral period ratio due to effect of soil-flexibility of various building frames (Bhattacharya and Dutta 2004). Asymmetric structures have also been of vast interest in multi-story buildings subject to seismic loading and SSI, where P-Δ may always be an issue that cannot simply be neglected. Sivakumaran and Balendra (2004) presented a method of analysis of three dimensional such buildings placed on flexible foundations. Therein, the soil is considered of linear-elastic idealized nature modeled so as to take into account the structure's underlying half space simply and effectively. The main point of this study was to have considered P-Δ effects through applying fictitious lateral forces and torques. Sivakumaran and Balendra uncoupled the equations in terms of footing displacements and then placed structural deformations in combination of SSI force-displacement relationships, resulting in integro-differential equations for footing displacements to be then solved by numerical step-by-step time history analysis. Next, a 10-story asymmetric building on soft soil was subjected to an earthquake to account for P-Δ and eccentricity effects. It was finally concluded that 'soft soil conditions increase lateral deflections, but reduce the twists, story shears and torques'. Also, it was found out that floor twists and story torques increase with eccentricity burdening no considerable effect on lateral deflections at the mass center or total story shears.

Asymmetric buildings, with regard to their caution-needing nature, have been the subject of other studies especially those regarding seismic loading and earthquake effects. A study on vertical component of earthquake and its effect on response of such systems in the base-isolated state was presented by Shakib and Fuladgar (2003). This is while the study of torsionally asymmetric buildings subject to SSI dates back to years before when Sikaroudi and Chandler studied the response of an idealized elastic structure-foundation system subjected to free-field earthquake input motions represented by design response spectra. Structures with different periods standing for their heights and with different types of eccentricity were addressed and their lateral-torsional responses were monitored (Sikaroudi and Chandler 1992). In addition, suggestions have been proposed based on findings of researches and experiments in related fields to eccentricity of asymmetric structures with and without SSI having conducted analyses on 2- and 3-dimensional models (e.g., Shakib 2004, De-la-Colina *et al*. 2013, Shakib and Atefatdoost 2018). Certain earthquake record-based studies have been carried out one distinguishing of which was considering 2015 Nepal earthquake which looked through asymmetrical buildings supported on piled rafts where SSI was a major issue of interest (Badry and Satyam 2015). Also, Eccentricity-caused rotation of the structure could be addressed in the foundation of the structure, the way assumed by De-la-Colina *et al*. (2013), based on which 'an unanticipated high torsional response can be obtained when the flexibility of the foundation-soil system is neglected at the building design stage'.

As is perceived from the literature, considering flexibility in the process of strength and stiffness distribution among LFREs is an issue which is not addressed much before. The current paper, bearing in mind that seismic responses of asymmetric SSI-prone buildings could be inspected in quite a number of aspects, looks into strength and stiffness assignments to lateral force-resisting elements in the system, as will be discussed in details shortly. Herein, an existing strength distribution algorithm for asymmetric structures with wall type systems based on a new approach (Mysilmaj and Tso 2004, Mysilmaj and Tso 2002, Tso and Myslimaj 2003, Mysilmaj and Tso 2001, Mysilmaj and Tso 2005) is modified considering foundation flexibility and it is then applied to an ideal one-story structure considering different conditions of flexibility so that the effects of flexibility on strength and stiffness assignments of structural elements can be observed. Finally, possible effects of foundation flexibility on strength and stiffness locations are addressed, and whether or not SSI-prone structures are subject to changes in strength and stiffness centers locations is examine. It will be shown by

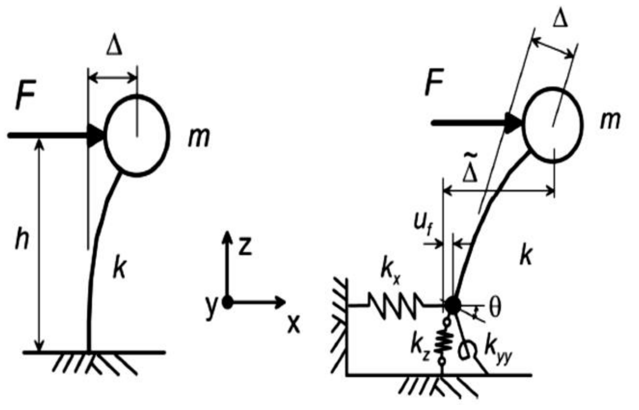

Fig. 1 A schematic view of force and displacement of a one-degree-of-freedom system under lateral loads with and without SSI (Wolf 1985)

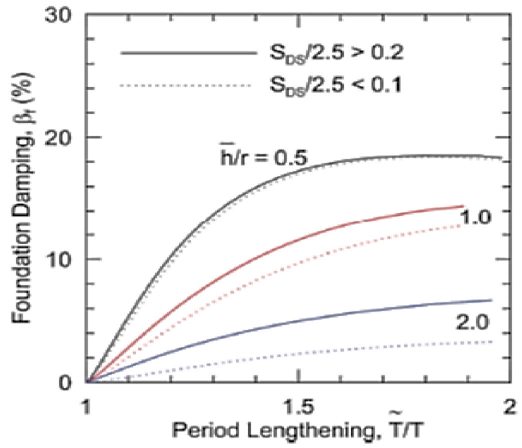

Fig. 2 Diagram for extraction of soil-structure system foundation damping (ASCE 2010)

the results that while foundation flexibility leads to reduction of assignments to LFREs from total shear, the pattern of strength and stiffness variation is not affected by SSI.

2. Soil-structure interaction

Soil is eventually the support of most structures and can play the role of an energy damper in earthquakes through interaction with the structure due to its significant non-linear deformation. This has been taken into account as soil-structure interaction (SSI) in seismic researches for many years. In fact, the deformation of the sub-soil changes structural responses in comparison with the state in which soil is without flexibility. Deformations of the soil under the foundation cause the reactions to be different from the case in which no flexibility is assumed in this sub-structure. This is primarily because the input motions to the foundation are different for the two cases of fixed-base structure and that resting on an underlying soil. This is known as the Kinematic Effect. Moreover, as a result of inertial forces induced in the structure, it tends to rotate/rock in the foundation in its interface with the soil the resulting deformations of which increase the fundamental period and damping of the system. This effect is called Inertial Effect since it is caused by the inertia in the structure. According to these definitions it can be understood that what can change the dynamic properties of a structure is the inertial part of SSI. To check out the effects of foundation flexibility on seismic demands of a structure, a single-degree-of-freedom (SDOF) system in two states of fixed-support and flexible-foundation are addressed as seen in Fig. 1. Based on these two systems, effects of flexibility on seismic parameters of interest of the structure are presented in the following (Wolf 1985).

2.1 Effect of foundation flexibility on displacement

According to Fig. 1 and structural dynamics basic equations, in case of fixed supports, the system displacements can be calculated by (Wolf, 1985; Chopra, 1995)

$$\Delta = F/K \tag{1}$$

If the system is located on a rigid foundation with flexible sub-soil, the effect of flexibility can be assumed by linear and torsional springs in the foundation. In this case, the total displacement of the soil-structure system ($\bar{\Delta}$) includes horizontal displacements caused by slippage (Δ_{HF}) and rotations of the foundation (θ) in addition to non-interactional displacements (Δ) which can be mathematically stated as below (Wolf 1985)

$$\bar{\Delta} = \Delta + \Delta_{HF} + \theta \times h = \Delta + \Delta_f \tag{2}$$

in which (Δ_f) is the displacement caused by rotation and horizontal displacement of the foundation due to foundation flexibility and is added to the structure's displacement when ignoring the interaction. This explains that foundation flexibility increases the displacements of the structural system.

2.2 Effect of foundation flexibility on period

According to basic equations of structural dynamics, the natural period (T) of a SDOF system in cases of with and without SSI may be calculated as below (Wolf 1985)

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m\Delta}{F}} \tag{3}$$

$$\bar{T} = 2\pi\sqrt{\frac{m}{\bar{k}}} = 2\pi\sqrt{\frac{m\bar{\Delta}}{F}} \tag{4}$$

based on which the following equation can be easily deduced

$$\bar{T} = T\sqrt{\frac{\bar{\Delta}}{\Delta}} = T\sqrt{1 + \frac{\Delta_f}{\Delta}} \tag{5}$$

that expresses the relationship between flexibility and period of a SDOF system. This equation indicates that as flexibility of the foundation increases, the period of the system goes up.

2.3 Effect of foundation flexibility on base shear

The decreasing effect of foundation flexibility on base shears of a structure is another issue that is addressed in most engineering codifications. For example, NEHRP instructions regard the equation of decreasing base shear considering SSI as below (ASCE/SEI7-10)

$$\Delta V = \left[C_S - \bar{C}_S\left(\frac{0.05}{\beta_0}\right)^{0.4}\right]\bar{W} \tag{6}$$

where the base shear factors corresponding to period for cases of with or without SSI are C_s and $\bar{C}_s$, respectively. $\bar{w}$ is equal to 70 percent of the total weight of the structure and β_0 parameter represents radiation damping, soil hysteresis and structural damping and can be calculated as (ASCE/SEI7-10)

$$\beta_0 = \beta_f + \frac{\beta_i}{\left(\frac{\bar{T}}{T}\right)^3} \tag{7}$$

β_i indicates the damping ratio of the structure which is assumed 5% of the critical damping. β_f stands for foundation damping and can be calculated as obtained from Fig. 2 using $\frac{h}{r}$ ratio, in which h is effective height and r is the radius of an equivalent circular foundation which is assumed 5% of the critical damping. β_f stands for foundation damping and can be calculated as obtained from Fig. 2 using $\frac{h}{r}$ ratio, in which h is effective height and r is the radius of an equivalent circular foundation.

2.4 Effect of foundation flexibility on yield displacements of shear walls

A shear wall is a common LFRE in reinforced concrete structures. The force-displacement plot of such elements under lateral loading is shown in Fig. 3, where stiffness, strength and wall yield displacement are the three main parameters (Priestley and Kowalsky 2007). For a flexible foundation in an SDOF reinforced concrete structure involving shear wall(s), the force-displacement plot compared with the case without interaction is illustrated in Fig. 3. As mentioned in Eq. (2), the relation of wall yield displacement in the case of with SSI $(\bar{\Delta}_y)$ with that of without SSI (Δ_y) is as below

$$\bar{\Delta}_y = \Delta_y + \Delta_f \tag{8}$$

The equation above can be rewritten like:

$$\bar{\Delta}_y = \Delta_y\left(1 + \frac{\Delta_f}{\Delta_y}\right) \tag{9}$$

Assuming $\alpha = \frac{\Delta_f}{\Delta_y}$, the equation will be written as

$$\bar{\Delta}_y = \Delta_y(1 + \alpha) \tag{10}$$

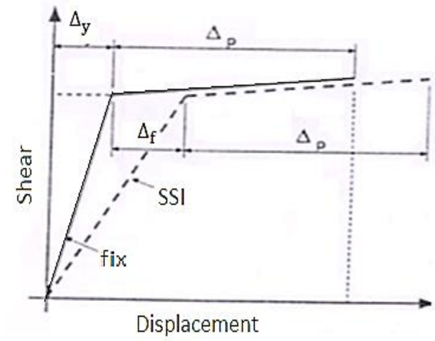

Fig. 3 Diagram of force versus concrete shear wall displacement under lateral load with and without SSI (Priestley and Kowalsky 2007)

α, which will be referred to as flexibility factor from this point on, has a direct relation with Δ_f which is generally a function of foundation dimensions, soil type and structure height (Wolf 1985, Priestley and Kowalsky 2007). It is obvious that $\alpha = 0$ corresponds to the case of no SSI and increasing in its amount causes increase in foundation flexibility.

For many years the process of strength distribution in LFRE has been considered by earthquake-resistant design codes to take into account torsional responses. This distribution can be done according to two approaches: the traditional and the new. In the traditional approach, yield displacement is similar for all walls of a same story and also stiffness is assumed independent from strength, which is the main point of difference between the two attitudes. In the traditional method, story shear is distributed due to stiffness ratio. With regard to previous researches, the dependence of stiffness and strength in a reinforced concrete shear wall is proven and it is also proven analytically that walls with different lengths have different yield displacements (Pauley 2001, Pauley 2002, Pauley 2002, Pauley 1992, Pauley 1997). Therefore, special strength distribution methods have been suggested by several researchers regarding the new approach for reinforced concrete structures including shear walls. Because of the inter-dependency of stiffness and strength in such walls, the new method is different from the traditional one. In this approach, a base function is suggested to distribute the strength based on yield displacement. Due to specific assumptions this base function depends on the strength distribution function. Given the distribution function, the strength assignment of each element can be calculated by dividing the value of the function by yield displacement of element strength (Mysilmaj and Tso 2004, Mysilmaj and Tso 2002, Tso and Myslimaj 2003, Mysilmaj and Tso 2001, Mysilmaj and Tso 2005, Shakib and Ghasemi 2007).

As expressed in previous sections, considering the flexibility can affect period, damping, and structure's base shear as well as yield displacement of the shear wall. Considering this multiple effect of SSI on strength distribution, it is expected that elements assignments of story shear vary proportional to the case without interaction. Regarding this approach, firstly the strength distribution algorithm will be re-stated and then the effect of foundation

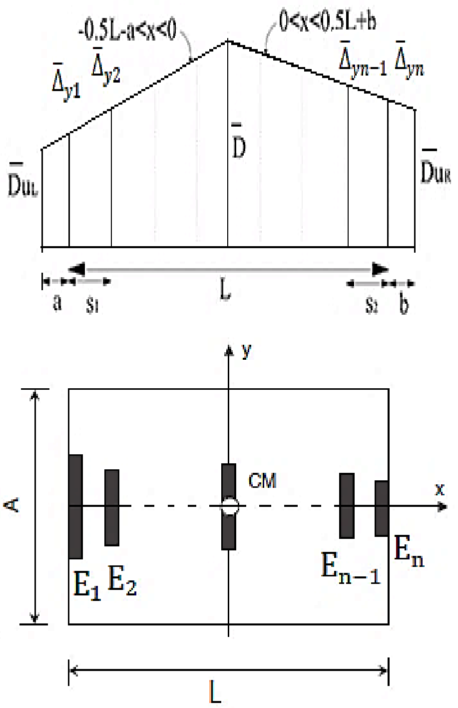

Fig. 4 Plan of the asymmetric structure investigated for estimating the discrete yield displacement of the wall with a continuous linear function (Tso and Myslimaj 2003)

flexibility is considered in the algorithm.

3. Formulation of strength and stiffness distribution based on the YDDB approach considering foundation flexibility

In the new theory of strength distribution, displacement is the parameter which correlates stiffness and strength. For each element, the yield displacement can be calculated considering its known dimensions using base equations. For instance, some references (e.g., Pauley 1997, 2003) suggest yield displacement of a concrete shear wall to be calculated as below

$$\Delta_{yi}=\left(\frac{2\xi_y h_w^2}{\eta}\right)\frac{1}{l_{wi}} \tag{11}$$

in which l_{wi} and h_w are length and height of the wall, ξ_y is steel yield strain and η is the function of lateral load distribution whose value is suggested to be taken 3 (Paulay, 1992). Thus, for each story in addition to eccentricity of stiffness (e_v) and strength (e_r), the eccentricity of yield displacement (e_D) will be defined. As mentioned in the previous section, foundation flexibility may increase yield displacements. As an example, if there exist *N* reinforced concrete shear walls in the plan of a story of an asymmetric building (Fig. 4), for the *i*'th wall foundation flexibility changes the yield displacement from Δ_{yi} to $\bar{\Delta}_{yi}$, so that:

$$\bar{\Delta}_{yi}=\Delta_{yi}+\Delta_f=\Delta_{yi}(1+\alpha) \tag{12}$$

Considering the above mentioned measures, the algorithm proposed by Tso, & Myslimaj (2003) is modified as in the following to more accurately account for strength and stiffness distribution.

3.1 Yield displacement function

Yield displacement approximation of the walls of a story is done using a continuous function. This continuous distribution function can be assumed linear as

$$\bar{u}(x)=\begin{cases}\dfrac{\bar{D}(1-\bar{u}_L)}{0.5L+a}x+\bar{D} & -0.5L-a\le x\le 0\\ \dfrac{\bar{D}(\bar{u}_R-1)}{0.5L+b}x+\bar{D} & 0\le x\le 0.5L+b\end{cases} \tag{13}$$

where $\bar{D}$ is the yield displacement of the center of mass and is equal to $\bar{u}_L$ and $\bar{u}_R$ for the right and left ends of the plan. The method of approximation is illustrated in Fig. 4. As shown, two linear functions are used in the left and right sides of the center of mass. To calibrate the mentioned linear function, the three values $\bar{u}_L$, $\bar{u}_R$, and $\bar{D}$ will suffice. Note that the values of S1 and S2 correspond to the laterally loaded area of elements E_1 and E_n. Lengths *a* and *b* are considered to convert the discrete system to a continuous one.

Equalization requires a continuous function with discrete points in left and right sides of which an extra length will be considered so that the integration lead to the yield displacement of the first and last elements $(\bar{\Delta}_{yn},\ \bar{\Delta}_{y1})$. Hence, the function has to cover extra lengths *a* and *b*.

First, it is required to calculate the values of $\bar{u}_L$ and $\bar{u}_R$ considering elements distribution and yield displacement. It can be assumed that the yield displacement of the first element on the left is equal to the area under the corresponding curve in the continuous approximation function to calculate $\bar{u}_L$. This can be stated as below

$$\bar{\Delta}_{y1}=\int_{-0.5L-a}^{-0.5L+S_1}\bar{u}(x)dx \tag{14}$$

Substituting Eq. (13) in Eq. (14), it is resulted

$$\bar{u}_L=1+\left[1-\frac{\bar{\Delta}_{y1}}{\bar{D}(a+S_1)}\right]\frac{(L+2a)}{(S_1-a-L)} \tag{15}$$

The following equation must hold to calculate $\bar{u}_R$ in the same way

$$\bar{\Delta}_{yn}=\int_{0.5L-s_2}^{0.5L+b}\bar{u}(x)dx \tag{16}$$

Using Eq. (13) in Eq. (16), the following equation is obtained

$$\bar{u}_R=1+\frac{L+2b}{b+L-S_2}\left[\frac{\bar{\Delta}_{yn}}{\bar{D}(b+S_2)}-1\right] \tag{17}$$

To determine the mass center yield displacement parameter $(\bar{D})$, the important term of approximation can be used which states that the area under the curve of the continuous system must be equal to the total sum of displacement in the system. This term can be mathematically stated as below

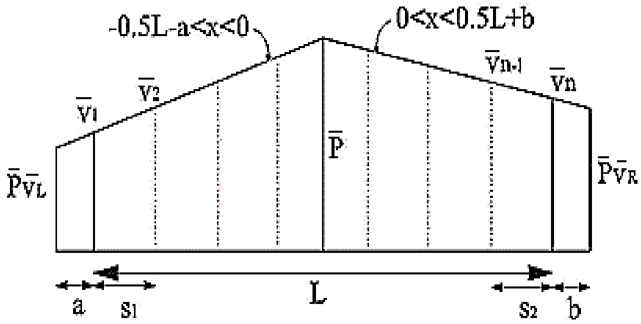

Fig. 5 Approximating wall discrete yield strength using a linear function

$$\int_{-0.5L-a}^{0} \bar{u}(x)dx + \int_{0}^{0.5L+b} \bar{u}(x)dx = \sum_{i=1}^{N} \bar{\Delta}_{yi} \tag{18}$$

Replacing the continuous function which was defined before, the following equation can be obtained

$$\left[\left(\frac{L+2a}{4}\right)\left(1-\frac{s_1-a}{L+a-s_1}\right) + \left(\frac{L+2b}{4}\right)\left(1-\frac{s_2+b}{L+b-s_2}\right)\right]\bar{D} = \sum_{i=1}^{N} \bar{\Delta}_{yi} - T \tag{19}$$

where

$$T = \frac{(0.5L+a)^2}{(S_1+a)(L+a-S_1)}(\bar{\Delta}_{y1}) + \frac{(0.5L+b)^2}{(S_2+b)(L+b-S_2)}(\bar{\Delta}_{yn}) \tag{20}$$

The yield displacement of the mass center ($\bar{D}$) can be obtained by Eq. (19). Achieving this value and the two previous ones, the approximation function will be practically calibrated.

3.2 Strength distribution function

Knowing that one of the main objectives of this algorithm is to distribute strength among the elements, it is next required to well determine a strength distribution function. To achieve this goal, a distribution function similar to that of yield distribution is assumed and shown in Fig. 5. In this continuous function, strength is $\bar{P}$ in the mass center and is respectively $\bar{V}_L$ and $\bar{V}_R$ in the left and right terminal layers. Therefore, the strength distribution function will be of a form as in the following

$$\bar{v}(x) = \begin{cases} \dfrac{\bar{P}(1-\bar{V}_L)}{0.5L+a}x + \bar{P} & -0.5L-a < x < 0 \\ \dfrac{\bar{P}(\bar{V}_R-1)}{0.5L+b}x + \bar{P} & 0 < x < 0.5L+b \end{cases} \tag{21}$$

As in the previous case, the unknowns $\bar{V}_L$, $\bar{V}_R$ and $\bar{P}$ must be determined. For this purpose, the following assumptions have been considered:

1) Strength eccentricity (e_V) is bound to yield function eccentricity (e_D) with a β factor, as in the following

$$e_V = \beta e_D \tag{22}$$

2) The yield function radius of gyration (r_D) is equal to the strength radius of gyration (r_V)

$$r_D = r_V \tag{23}$$

In the equations above, β is a key parameter in LFREs strength and stiffness distribution. Substituting the strength and yield displacement functions, the following results can be achieved

$$\bar{V}_R(I_1) + \bar{V}_L(I_2) = I_3 \tag{24}$$

$$\bar{V}_L(X) + Y = 0 \tag{25}$$

In the equations above, the values of X and Y depend on I_1 and I_2; and I_1 and I_2 are in turn dependent on $\bar{u}_L$ and $\bar{u}_R$. The complete formulae of these parameters are stated in the appendix. Solving the two latter equations gives the values of $\bar{V}_L$ and $\bar{V}_R$. To calculate the strength of the mass center ($\bar{P}$), it should be known that the total assignment of strength to the wall must be equal to the area under the curve of the continuous strength function. This can be stated mathematically as below

$$\int_{-0.5L-a}^{0} \bar{v}(x)dx + \int_{0}^{0.5L+b} \bar{v}(x)dx = \sum_{i=1}^{N} \bar{V}_i = \bar{V} \tag{26}$$

Using the strength function in the previous equation, the following result is obtained

$$\bar{P} = \frac{2\bar{V}}{(0.5L+a)(1+\bar{V}_L) + (0.5L+b)(1+\bar{V}_R)} \tag{27}$$

where $\bar{V}$ is the story shear force considering foundation flexibility.

3.3 LFREs strength and stiffness assignment

After the strength distribution function unknowns are achieved, the form of the function is practically known and can be used to calculate the strength of each element ($\bar{v}_i$) as below

$$\bar{v}_i = \int_{C_1}^{C_2} \bar{v}(x)dx \tag{28}$$

where c_1 and c_2 are the top and bottom limits of the integral whose difference states the lateral loaded area of element i (i.e., coordinates of average distances of the walls on each side of the i'th wall). After determining the strength assignment to each LFRE, each stiffness can be eventually calculated using the base equation given in the following

$$\bar{K}_i = \frac{\bar{v}_i}{\bar{\Delta}_{yi}} \tag{29}$$

The proposed modified algorithm was implemented in the framework of MATLAB considering all of the above mentioned steps .In the following, to check the efficiency of the proposed algorithm, for a specific asymmetric building subject to different flexibility conditions under the foundation, strength and stiffness assigned to the LFRE are

Table 1 Comparison of strength results by the suggested algorithm and results by (Tso & Myslimaj, 2003)

	β=0.0			β=0.5			β=1		
	Reference*	Proposed algorithm	Error (%)	Reference*	Proposed algorithm	Error (%)	Reference*	Proposed algorithm	Error (%)
1	50.6	50.0	1.19	44.5	43.2	2.84	38.5	37.7	2.19
2	50.6	50.0	1.19	47.6	46.9	1.43	44.5	43.6	1.96
3	50.6	51.0	0.79	50.6	50.6	0.00	50.6	50.6	0.00
4	50.6	51.0	0.79	53.6	54.3	1.27	56.7	57.6	1.54
5	50.6	51.0	0.79	56.7	58.0	2.23	62.7	64.5	2.94

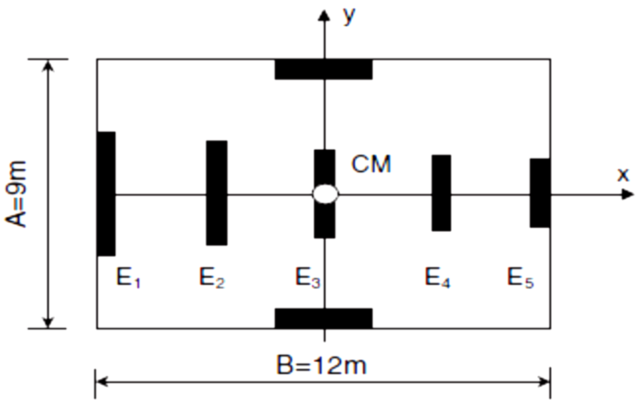

Fig. 6 Plan of the investigated asymmetric structure (Tso and Myslimaj 2003)

numerically calculated.

4. Distribution of strength and stiffness for an asymmetric concrete building

In this section, in order to achieve a tangible process of the effect of foundation flexibility on strength and stiffness distribution and their assignment to each wall, a specific asymmetric building is considered. Before addressing the main issue, the accuracy and precision of the suggested algorithm is proven by rechecking a structure from previous researches with no SSI.

4.1 Validating the suggested distribution algorithm in the absence of SSI (α=0)

To verify the suggested distribution algorithm in the absence of SSI, the strength distribution checked in (Tso and Myslimaj 2003), for various values of β is repeated through the given algorithm. The results of this distribution which are in fact strength assignment to each element are given in Table 1. Errors of the suggested algorithm for each quantity of β and for strength assignment to each wall are reported in this table. The maximum error is about 3% which indicates sufficient accuracy of the algorithm.

4.2 Properties of the model

The investigated asymmetric structure with one direction asymmetry and dimensions of 9×12 square meters as in Tso and Myslimaj (2003) includes five walls in the plan. These five walls are named E_5, E_4, E_3, E_2, E_1 and their heights are equal to 7.5 meters. The lengths of the walls are respectively 3.70, 3.20, 2.8, 2.51 and 2.27 meters and they are all placed with a distance of 3 meters from one another. Assuming the yield stress of wall reinforcements to be 300 MPa and with a Young modulus of 2×105 MPa, using Eq. (11) to calculate the yield displacement, the values of wall yield displacements will respectively be δ_1=1.53 cm , δ_2=1.76 cm , δ_3=2.0 cm , δ_4=2.24 cm and δ_5=2.48 cm (Fig. 6). The effective weight of the structure is assumed to be W=1265-KN and according to UBC 97 provisions, in the absence of SSI base shear is calculated equal to V=253-KN. In order to investigate the effect of foundation flexibility on strength and stiffness distribution, in the absence of SSI (fixed-base state) as well as with four flexibility cases including α=0.25, 0.50, 0.75, 1, strength and stiffness assignment to the five walls are calculated using the suggested algorithm.

Table 2 General properties of the structure with and without foundation flexibility

α	T_{fix}	T_{SSI}	T_{SSI}/T_{fix}	β_0	ΔV	V_{SSI}
0(*fix*)	0.62	0.62	1.00	0.05	0	253
0.25	0.62	0.69	1.12	0.08	38	215
0.5	0.62	0.76	1.22	0.09	56	197
0.75	0.62	0.82	1.32	0.10	71	182
1	0.62	0.88	1.41	0.12	82	171

4.3 Effect of flexibility on general structural demands

Flexibility of the foundation can change the structural properties. In the investigated structure, the values of period, damping and base shear of the structure vary for different conditions of flexibility. Table 2 gives a conclusion of the mentioned parameters. As is obvious, the effect of flexibility on damping and period is increasing while it is decreasing on the base shear. An increase in the period equal to 41% for the most extreme case of flexibility (α = 1) is observable. According to Table 2, damping of the SSI-prone structure with the highest flexibility possible is 2.4 times the corresponding fixed-base structure. Increase in the period and damping can cause a 33% reduction in the base shear compared to the SSI-free case.

4.4 Effect of flexibility on approximation function of wall yield displacement

Changes of the yield displacement function in the plan

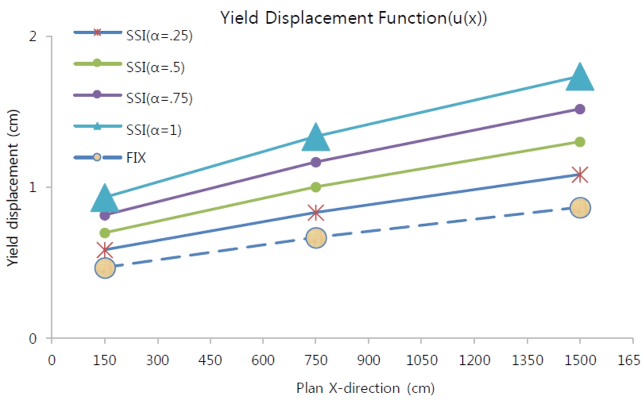

Fig. 7 Yield displacement function for different conditions of foundation flexibility

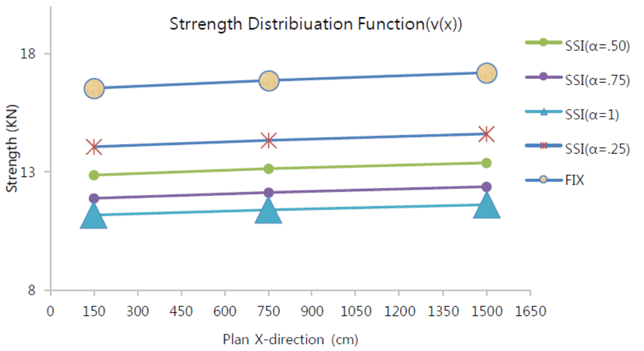

Fig. 8 Strength distribution function for different conditions of foundation flexibility and $\beta = 0$

for different types of under-foundation flexibility can be observed in Fig. 7. It is seen that as under-foundation flexibility gets more, yield function $(u(x))$ increases compared to the interaction-free case with regard to which it is expected for the pattern of strength distribution function $(v(x))$ to change accordingly.

4.5 Effect of foundation flexibility on strength distribution function

As discussed in the previous section, the strength distribution function depends on β, a good example of which is given by Fig. 8 for $\beta = 0$ and considering different types of flexibility, where different cases of flexibility and their effects on strength distribution function are compared. The mentioned diagram reveals that increasing flexibility causes the strength distribution function area to reduce, which in turn reduces the strength assignment to all walls.

4.6 Effect of foundation flexibility on strength and stiffness assignments to LFREs

The goal of strength distribution algorithm in asymmetric structures is to determine LFREs stiffness and strength assignments. In the studied structure, for the condition of foundation flexibility assignments are calculated using the algorithm suggested in the previous section. In the following, for different conditions the walls stiffness and strength assignments are discussed.

For $\beta = 1$ (corresponding to $e_V > 0$ and $e_r = 0$), the five walls' strength and stiffness assignments are given in Fig. 9. If all LFREs keep behaving linearly, this condition can be a criterion for the minimum floor rotation in the asymmetric structure since the center of stiffness is determinative in torsional response (Tso and Myslimaj 2003). As shown in the diagrams, increase in foundation flexibility leads to reduction of all stiffness assignments to walls. This reduction is equal for all walls and can be observed to rise by 33% in the maximum limit of flexibility

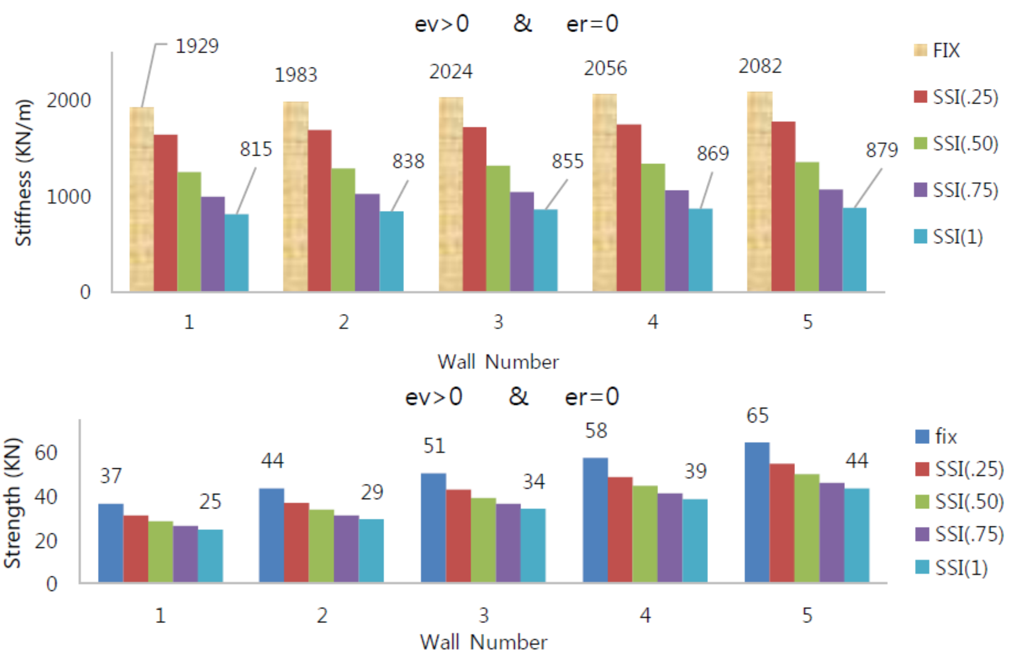

Fig. 9 Strength and stiffness distribution for different conditions of foundation flexibility and $\beta = 1$

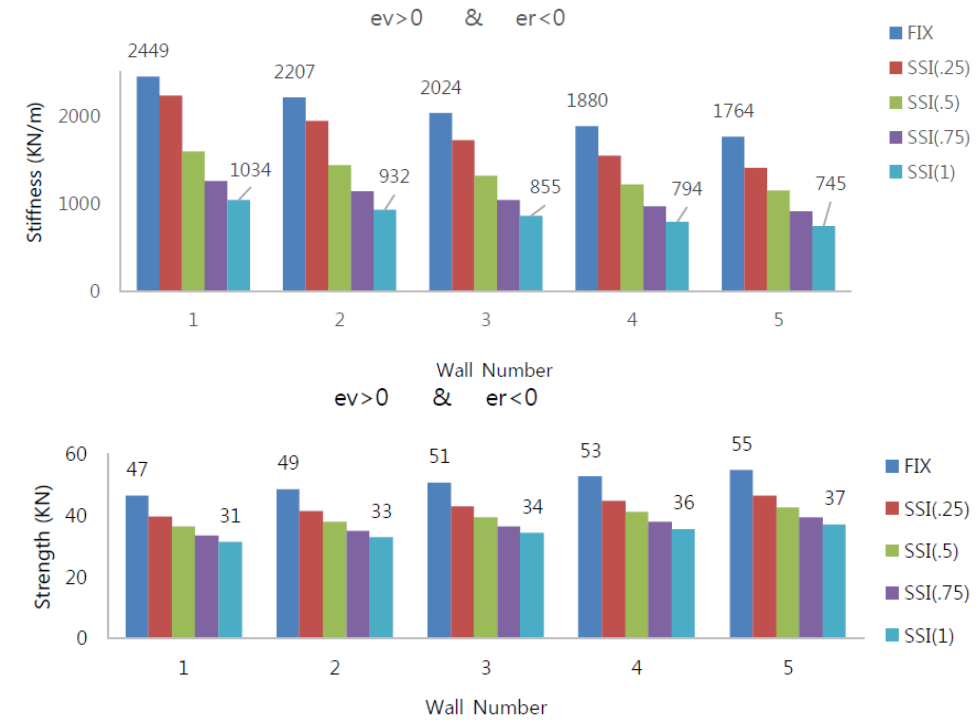

Fig. 10 Strength and stiffness distribution for different conditions of foundation flexibility and $\beta = 0.50$

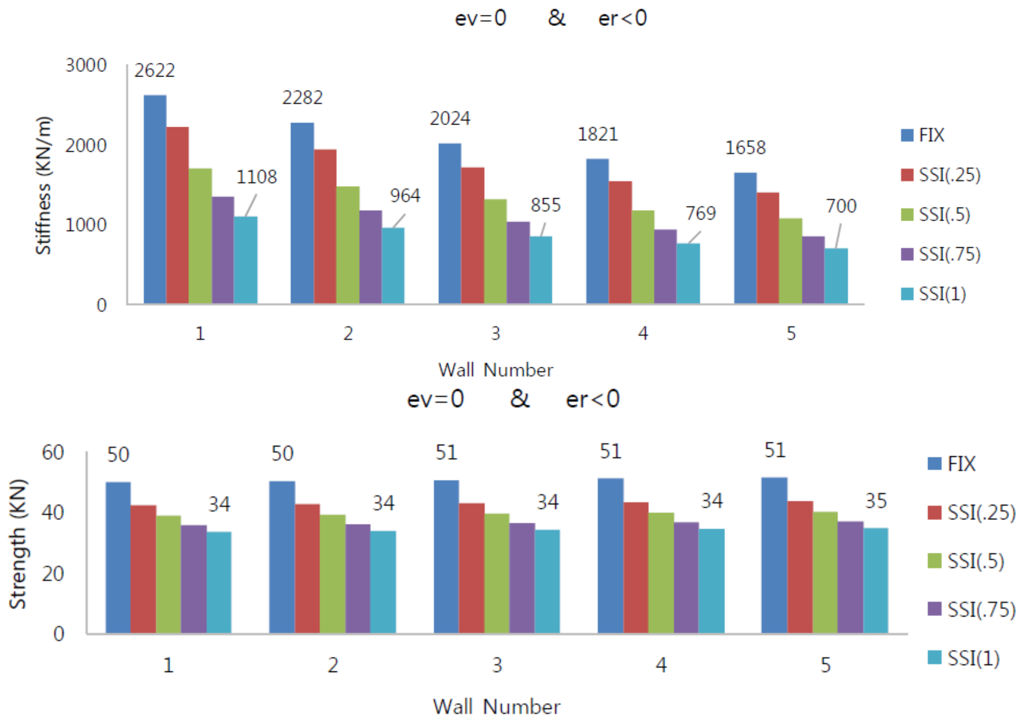

Fig. 11 Strength and stiffness distributions for different conditions of foundation flexibility and $\beta = 0$

(α=1). Similarly, this reduction of the walls' strength assignments can be observed in Fig. 9, which is equal to 58% for all walls even though this percentage of reduction is more than that of stiffness. It should be noted that in this condition of strength and stiffness distribution, the centers of stiffness and strength are constant in all conditions of flexibility so that eccentricity remains 6% of the plan length for strength center and zero for stiffness center.

For $\beta = 0.5$ (corresponding to $e_V > 0$ and $e_r < 0$) the five walls' strength and stiffness assignments are given in Fig. 10. If one part of the LFREs is continuing to behave in the linear range and another in the non-linear, since both centers of stiffness and strength are determinative in torsional response, this condition will be the criterion for minimum floor rotation in the asymmetric structure (Tso and Myslimaj 2003). As shown on the plots, increase in

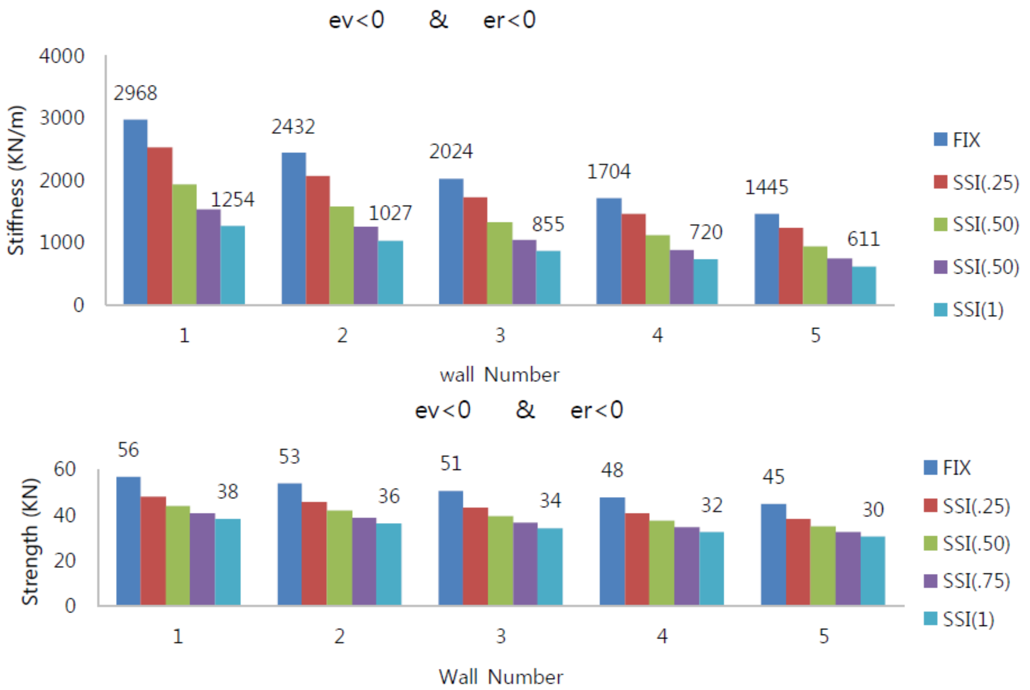

Fig. 12 Strength and stiffness distributions for different foundation flexibility conditions and $\beta = -0.50$

foundation flexibility results in reduction of stiffness assignments of all walls. This reduction is similar for all walls and is observed to be up to 35% in the maximum limit of flexibility (α=1). Similarly, the reduction in the strength assignments to walls can be seen in Fig. 10, which is equal to 57% for all walls even though this percentage is more than that of stiffness. It should be noted that in this condition of strength and stiffness distributions, the centers of stiffness and strength are constant in all flexibility conditions so that eccentricity is 4% of the plan length for strength center of and -3% of the plan length for stiffness center.

For $\beta = 0$ (corresponding to $e_V = 0$ and $e_r < 0$) the five walls' strength and stiffness assignments are given in Fig. 11. If all LFREs rest in the non-linear behavior range, because the center of strength is determinative in torsional response, this condition will be the criterion for the minimum floor rotation in the asymmetric structure (Tso and Myslimaj 2003). As shown on the plots, increase in the foundation flexibility causes stiffness assignments of all walls to reduce. This reduction is similar for all walls and is observed to be up to 58% in the maximum limit of flexibility (α=1). Similarly, the reduction of strength assignments of the walls can be observed in Fig. 11. This reduction is equal to 32% for all walls even though it is less than that of stiffness. It should be noted that in such a state of strength and stiffness distribution, the centers of stiffness and strength are constant in all conditions of flexibility so that eccentricity is 0% of the plan length for center of strength and -6% of the plan length for center of stiffness. For $\beta = -0.50$ (corresponding to $e_V < 0$ and $e_r < 0$), the five walls' strength and stiffness assignments are given in Fig. 12. Under no circumstances can this condition be the criterion for minimum floor rotation in an asymmetric structure (Tso and Myslimaj 2003). As shown in the diagrams, increasing the flexibility causes stiffness assignments to all walls to decrease. This reduction is similar for all walls and is observable up to 58% in the maximum limit of flexibility (α=1). Similarly, a reduction in the wall's strength assignment can be observed in Fig. 12, which is equal to 32% for all walls although it is less than that of stiffness. It should be noted that in this condition of strength and stiffness distributions the centers of stiffness and strength are constant in all conditions of flexibility so that eccentricity is -3% of the plan length for strength center and -9% of the plan length for stiffness center.

5. Conclusions

For an idealized single-story asymmetric wall type system building, the strength and stiffness distribution algorithm presented in previous researches based on inter-dependency of stiffness and strength is modified taking foundation flexibility into account. Considering the flexibility in reviewing seismic parameters and also strength distribution process of this kind of structure leads to the following results:

• Foundation flexibility causes the period and damping of all models to increase significantly which in turn results in reduction of structure's base shear.

• Wall yield displacement and its continuous approximation function increases with flexibility. Due to the effect of this parameter on strength and stiffness distributions, the strength distribution function will change compared to the SSI-free case. Unlike the yield displacement function, this change is decreasing.

• Strength and stiffness distribution using the suggested modified algorithm for different cases of flexibility suggests that considering such flexibility generally leads to reduction in the walls' strength assignment and consequently the stiffness of the story. Despite this reduction, in all cases of flexibility, distributions of strength and stiffness are such that the positions of centers of strength and stiffness do not change compared to the SSI-free case.

• With regard to the fact that the position of the centers of stiffness and strength do not change, when the flexibility of the structure increases, it can be expected that the criteria of floor rotation minimization (as the favorable criteria for torsional behavior) investigated in previous researches take no effect from foundation flexibility.

The strength and stiffness distribution procedure which was proposed and assumed in this research is usable for an idealised single-story building as has been the benchmark structure for many of the aforementioned researches. However, one cannot be certain that the results could be readily usable for multi-story buildings. Further future studies shall be necessary to determine the implementation of this and such algorithms for the sake of high-rise structures.

References

ASCE-2010, "Minimum Design Loads for Buildings and Other Structures", ASCE/SEI7-10, American Society of Civil Engineers, Reston, Virginia.

Aziminejad, A. and Moghadam, A.S. (2009), "Performance of asymmetric multi story shear buildings with different strength distributions", *J. Appl. Sci.*, **9**(6), 1082-1089.

Badry, P. and Satyam, N. (2016), "Seismic soil structure interaction analysis for asymmetrical buildings supported on piled raft for the 2015 Nepal earthquake", *J. Asian Earth Sci.*, **133**, 102-113.

Bhattacharya, K., Dutta, S.C. and Dasgupta, S. (2004), "Effect of soil-flexibility on dynamic behaviour of building frames on raft foundation", *J. Sound Vib.*, **274**(6), 111-135.

Chopra, A.K. (1995), *Dynamics of Structures: Theory and Applications to Earthquake Engineering*, Prentice-Hall, Englewood Cliffs, NJ.

De-la-Colina, J., Valdés-González, J. and González-Pérez, C.A. (2013), "Experiments to study the effect of foundation rotation on the seismic building torsional, response of a reinforced concrete space frame", *Eng. Struct.*, **56**, 1154-1163.

Mysilmaj, B. and Tso, W.K. (2004), "Desirable strength distribution for asymmetric structure with strength-stiffness dependent element", *J. Earthq. Eng.*, **8**(2), 231-248.

Myslimaj, B. and Tso, W.K. (2001), "A strength distribution criterion for minimizing torsional response of asymmetric wall-type systems", *Earthq. Eng. Struct. Dyn.*, **29**, 182-192.

Myslimaj, B. and Tso, W.K. (2002), "A strength distribution criterion for minimizing torsional response of asymmetric wall-type systems", *Earthq. Eng. Struct. Dyn.*, **31**, 99-120.

NEHRP Consultants Joint Venture (2012), "Soil-Structure Interaction for Building Structures".

Paulay, T. (1986), "The design of ductile reinforced concrete structural walls for earthquake resistance", *Earthq. Spectra*, EERI, **2**(4), 783-823.

Paulay, T. (1997), "Seismic torsional effects on ductile structural wall systems", *J. Earthq. Eng.*, **1**(4), 721-745.

Paulay, T. (2002), "A displacement-focused seismic design of mixed building systems", *Earthq. Spectra*, **18**(4), 689-718.

Paulay, T. (2002), "An estimation of displacement limits for ductile systems", *Earthq. Eng. Struct. Dyn.*, **31**(3), 583-99.

Paulay, T. (2003), "Seismic displacement capacity of ductile reinforced concrete building systems", *Bull. Soc. Earthq. Eng.*, **36**(1), 147-164.

Paulay, T. and Priestley, M.J.N. (1992), *Seismic Design of Reinforced Concrete and Masonry Buildings*, John Wiley & Sons.

Pauley, T. (2001), "Some design principles relevant to torsional phenomena in ductile buildings", *J. Earthq. Eng.*, **5**, 104-120.

Priestley, N. and Kowalsky, M.J. (2007), "Displacement based seismic design of structures", IUSS.

Roy, R. and Dutta, S.C. (2010), "Inelastic seismic demand of low-rise buildings with soil-flexibility", *Int. J. Nonlin. Mech.*, **45**(4), 419-432.

Shakib, H. (2004), "Evaluation of dynamic eccentricity by considering soil-structure interaction: a proposal for seismic design codes", *Soil Dyn. Earthq. Eng.*, **24**(5), 369-378.

Shakib, H. and Atefatdoost, G.R. (2011), "Effect of soil-structure interaction on torsional response of asymmetric wall type systems", *The Proceedings of the Twelfth East Asia-Pacific Conference on Structural Engineering and Construction-EASEC12*, *Procedia Eng.*, **14**, 1729-1736.

Shakib, H. and Fuladgar, A. (2003), "Effect of vertical component of earthquake on the response of pure-friction base-isolated asymmetric buildings", *Eng. Struct.*, **25**(14), 1841-1850.

Shakib, H. and Fuladgar, A. (2004), "Dynamic soil-structure interaction effects on the seismic response of asymmetric buildings", *Soil Dyn. Earthq. Eng.*, **24**(5), 379-388.

Shakib, H. and Ghasemi, A. (2007), "Considering different criteria for minimizing torsional response of asymmetric structures under near-fault and far-fault excitations", *Int. J. Civil Eng.*, **5**(4), 247-265.

Sikaroudi, H. and Chandler, A.M. (1992), "Structure-foundation interaction in the earthquake response of torsionally asymmetric buildings", *Soil Dyn. Earthq. Eng.*, **11**(1), 1-16.

Sivakumaran, K.S. and Balendra, T. (1994), "Seismic analysis of asymmetric multistorey buildings including foundation interaction and P-Δ effects", *Eng. Struct.*, **16**(8), 609-624.

Tso, W.K. and Myslimaj, B. (2003), "A yield displacement distribution-based approach for strength assignment to lateral force-resisting elements having strength dependent stiffness", *Earthq. Eng. Struct. Dyn.*, **32**, 2319-351.

UBC (1997), "Uniform Building Code", International Conference of Building Officials, Whittier, California, USA.

Wolf, J.P. (1985), *Dynamic Soil-structure Interaction*, Prentice-Hall. Inc., Englewood Cliffs, N. J.

CC

Appendix

The parameters presented in Section 3 (I1, I2, I3. X and Y) are introduced as in the following. The parameters found in the formulations are all the same as those defined in the manuscript.

$$I_1 = 4 + \frac{32b}{L} - 12\frac{a}{L}u_L + 12\frac{b}{L}u_L$$
$$I_2 = 4 + \frac{32a}{L} - 12\frac{a}{L}u_R - 12\frac{b}{L}u_R$$
$$I_3 = 4u_L + 4u_R + \frac{32a}{L}u_L + \frac{32b}{L}u_R$$

$$X = -4\frac{I_2}{I_1} - \frac{2I_2}{I_1}u_L - \frac{2I_2}{I_1}u_R - 4 - 2u_L - 2u_R - 2\beta u_R + 2\beta\frac{I_2}{I_1}u_R +$$
$$2\beta u_L - 2\beta\frac{I_2}{I_1}u_L - 2\frac{a}{L}\frac{I_2}{I_1} - 2\frac{a}{L}\frac{I_2}{I_1}u_L - 16\frac{a}{L} - 12\frac{a}{L}u_L - 8\frac{a}{L}u_R$$
$$-4\frac{a}{L}\beta u_R + \frac{4a}{L}\beta + 12\frac{a}{L}\beta u_L - 4\frac{a}{L}\beta\frac{I_2}{I_1} - 8\frac{a}{L}\frac{I_2}{I_1}\beta u_L - 8\frac{b}{L}\frac{I_2}{I_1}u_L$$
$$-20\frac{b}{L}\frac{I_2}{I_1} - 12\frac{b}{L}\frac{I_2}{I_1}u_R - 4\frac{b}{L} - 4\frac{b}{L}u_R - 4\frac{b}{L}\beta - 8\frac{b}{L}\beta u_R + 6\frac{b}{L}\beta\frac{I_2}{I_1}$$
$$+12\frac{b}{L}\frac{I_2}{I_1}\beta u_R - 2\frac{b}{L}\beta\frac{I_2}{I_1} - 4\frac{b}{L}\beta\frac{I_2}{I_1}u_L$$

$$Y = 4\frac{I_3}{I_1} + \frac{2(1+\beta)I_3}{I_1}u_L + \frac{2(1-\beta)I_3}{I_1}u_R - 4\beta u_R + 4\beta u_L + 8\frac{a}{L}u_L +$$
$$2\frac{a}{L}\frac{I_3}{I_1} + 2\frac{a}{L}\frac{I_3}{I_1}u_L - 8\frac{a}{L} - 8\frac{a}{L}u_R + 8\frac{a}{L}\beta + 4\frac{a}{L}\beta\frac{I_3}{I_1} + 8\frac{a}{L}\beta u_L$$
$$+8\frac{a}{L}\beta\frac{I_3}{I_1}u_L + 8\frac{b}{L} + 4\frac{b}{L}u_L + 8\frac{b}{L}\frac{I_3}{I_1}u_L + 4\frac{b}{L}u_R + 20\frac{b}{L}\frac{I_3}{I_1}$$
$$+12\frac{b}{L}\frac{I_3}{I_1}u_R - 20\frac{b}{L}\beta u_R - 8\frac{b}{L}\beta - 4\frac{b}{L}\frac{I_3}{I_1}\beta - 12\frac{b}{L}\frac{I_3}{I_1}\beta u_R + 4\frac{b}{L}\beta u_L$$
$$+4\frac{b}{L}\frac{I_3}{I_1}\beta u_L$$