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Structural damage identification using incomplete static displacement measurement

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Abstract. A local damage identification method using measured structural static displacement is proposed in this study. Based on the residual force vector deduced from the static equilibrium equation, residual strain energy (RSE) is introduced, which can localize the damage in the element level. In the case of all the nodal displacements are used, the RSE can localize the true location of damage, while incomplete displacement measurements are used, some suspicious damaged elements can be found. A model updating method based on static displacement response sensitivity analysis is further utilized for accurate identification of damage location and extent. The proposed method is verified by two numerical examples. The results indicate that the proposed method is efficient for damage identification. The advantage of the proposed method is that only limited static displacement measurements are needed in the identification, thus it is easy for engineering application.

Keywords: damage identification; residual force vector; strain energy; incomplete static displacement; model updating

1. Introduction

It is well known that structures will accumulate local damages in their components because of earthquake, impact, fatigue, etc. These damages can lead to the failure of the structures. To assure the safety of structures the damage identification methods have become important topics of research in the past few decades. So far, the existed approaches for damage identification are generally classified into two major categories: methods based on static measurements and methods based on dynamic measurements. The latter methods identify the local damage based on the vibration characteristics, while the former ones use the static deformations for identification.

Comparing with the static methods the dynamic approaches have been developed more fully. Lots of approaches are studied in fields of frequency domain in the past few decades. For example, methods based on the changes in natural frequency are used to identify the local damage (Cawley and Admas 1979, Kim and Stubbs 2003). Majumdar *et al*. (2012) utilized the ant colony optimization to identify damages in truss structures using the natural frequencies only. Damage identification methods from mode shape changes are investigated by many researchers (Pandey *et al*. 1991, Rizos *et al*. 1990, Ratcliffe 1997). A damage index based on strain energy numerical technique is studied by Hu *et al*. (2012) to identify the structural local damage. And there are other approaches using modal flexibility (Pandey and Biswas 1994, Wu and Law 2004,) and frequency response function(Chatterjee 2010, Mohan *et al*. 2013) to identify structural damage. Yang and Liu (2007) proposed a method for structural damage identification based on residual force vector. Li *et al*. (2015) presented a method for simultaneous identification of stiffness and damping in discrete systems based on derivatives of eigen-parameters. Li and Lu (2015) presented a method for structural identification based on fruit fly optimization using natural frequencies and mode shapes. Vo-Duy *et al*. (2016) developed a two-step approach for damage detection in laminated composite structures using modal strain energy method and an improved differential evolution algorithm.

On the other hand damage identification methods in time domain have been developed rapidly in recent years. Cattarius and Inman (1997) used time histories of vibration responses for identification. Lu and Law (2007) developed the response-sensitivity method to detect the damage. Then He and Lu (2010) utilized the response-sensitivity-based method to identify multiple cracks in beam structure. Recently, Lu *et al*. (2013) used the modal curvature based method to develop the efficiency and accuracy of the response-sensitivity method in cracked beam. Li *et al*. (2016) proposed a method for structural damage identification based on residual force vector and response sensitivity analysis. Fu *et al*. (2016) proposed a two-step approach based on modal strain energy and dynamic response sensitivity analysis.

Making comparison between the static and dynamic approaches, the advantage of the static methods can be concluded as: (1) Equation of equilibrium is only related to

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stiffness. (2) Static response data are much easier to obtain. (3) The noise in static measurement is less than the dynamic one. But the researches in the static field are much less than that in the dynamic field because the information can be used in the static system is limited. By minimizing the difference between the applied and the internal forces, Sanayei and Scamploi (1991) identified the structural damage in static system. It should be pointed out that the method required the measured points and the location of the applied external force. Then Snanyei and Onipide (1991) improved the previous method by minimizing a condensation procedure. Hjelmstad *et al*. (1992) used the so call mutual residual energy to estimate the parameter of complex linear structures. Later, Banan and Hjelmstad (1994a, 1994b) proposed a set of parameter estimation methods based on the static system. Recently Nejad *et al*. (2005) presented a new structural damage detection algorithm by minimizing the difference between the load vector of measured and analytical load vector. Chen *et al*. (2005) defined a grey relation coefficient of displacement curvature and used it to locate the structural damage. Yang and Sun (2010) proposed a localization method using the flexibility disassembly technique. But this method requires the input data to contain all the deformation information. Abdo (2012) studied the influence of the parameters in the damage detection based on the changes in the static displacement curvatures.

In this paper a method for localization and quantification of structural damage using measured incomplete static displacement is proposed. A residual force vector is deduced based on the static equilibrium equations of the intact and damaged structure. And the residual strain energy (RSE) is derived from the residual force vector. In order to reduce the number of measurements in the identification, model reduction method is adopted to obtain a reduced finite element model. Two numerical simulations are studied to verify the proposed method. The results illustrate that the RSE can localize the damaged element exactly with full displacement measurement. When only a small number of static displacement data are used, some suspicious damaged elements will appear adjacent to the true damaged elements. The true locations and extents of damage can be obtained from a model updating method based on sensitivity analysis of the static displacement.

2. Methodology

2.1 Residual force vector in static system

The static equilibrium equation for an undamaged structural system with n_d degrees-of-freedom (DOFs) can be written as

$$\mathbf{KU} = \mathbf{F} \tag{1}$$

where $\mathbf{K}$ is the global stiffness matrix and $\mathbf{U}$ is the nodal displacement vector when a static load vector $\mathbf{F}$ is applied on the system. Then the displacement $\mathbf{U}$ can be calculated from Eq. (1)

$$\mathbf{U} = \mathbf{K}^{-1}\mathbf{F} \tag{2}$$

As the mass has no effect on the static system, structural damage only leads to the change of the stiffness parameters. If local damage leads to a perturbation in the structural stiffness matrix by $\boldsymbol{\delta}\mathbf{K}$, the equilibrium equation for the damaged system can be expressed as

$$(\mathbf{K} + \boldsymbol{\delta}\mathbf{K})\mathbf{U_d} = \mathbf{F} \tag{3}$$

where $\mathbf{U_d}$ is the displacement vector of the damaged system while the applied static force vector remain unchanged.

Subtracting Eq. (1) from Eq. (3), the residual force vector denoted by $\mathbf{Rf}$ is expressed as

$$\mathbf{Rf} = \boldsymbol{\delta}\mathbf{KU_d} = \mathbf{K}\boldsymbol{\delta}\mathbf{U} \tag{4}$$

where $\boldsymbol{\delta}\mathbf{U}$=$\mathbf{U}$−$\mathbf{U_d}$. It can be seen that the value of the residual force vector can be calculated by the right term of Eq. (4). And a detailed expression of the middle term in Eq. (4) can be written as

$$\begin{bmatrix} \boldsymbol{\delta}\mathbf{k}_1^{\mathbf{T}} \\ \boldsymbol{\delta}\mathbf{k}_2^{\mathbf{T}} \\ \vdots \\ \boldsymbol{\delta}\mathbf{k}_{\mathbf{n_d}}^{\mathbf{T}} \end{bmatrix} \mathbf{U_d} = \begin{Bmatrix} rf_1 \\ rf_2 \\ \vdots \\ rf_{n_d} \end{Bmatrix} \tag{5}$$

where $\boldsymbol{\delta}\mathbf{k_i}$ (i=1,2,…,n_d) is the stiffness change vector of the i th DOF. Eq. (5) shows that the residual force rf_i will be nonzero only if the element which contains the i th DOF is damaged.

2.2 Residual strain energy

It can be seen that residual force vector indicates the damage information in the DOF level. In order to reflect the damage information in the element level, residual strain energy (*RSE*) is defined based on the residual force vector $\mathbf{Rf}$. The residual strain energy can be written as

$$RSE_i = \boldsymbol{\delta}\mathbf{U}^{\mathbf{T}}\mathbf{K_i}\boldsymbol{\delta}\mathbf{U} \quad (i = 1, 2, \cdots, nel) \tag{6}$$

where *nel* denotes the total number of the elements, $\boldsymbol{\delta}\mathbf{U}$=$\mathbf{U}$−$\mathbf{U_d}$ and $\mathbf{K_i}$ represents the stiffness matrix of the i th element.

Consider the expression of residual force vector $\mathbf{Rf}$ in Eq. (4) and it can be re-written as

$$\mathbf{Rf} = \sum_{i=1}^{nel} \mathbf{K_i}(\mathbf{U} - \mathbf{U_d}) \tag{7}$$

Meanwhile the residual force vector $\mathbf{Rf}$ can be regarded as a summation of the elemental residual force vector $\mathbf{Rf^e}$

$$\mathbf{Rf} = \sum_{i=1}^{nel} \mathbf{Rf_i^e} \tag{8}$$

From Eq. (7) and Eq. (8), an elemental residual force can be expressed as

$$\mathbf{Rf_i^e} = \mathbf{K_i}\boldsymbol{\delta}\mathbf{U} \quad (i = 1, 2, \cdots, nel) \tag{9}$$

Eq. (5) shows that the component of *Rf* will be non-zero only if the related element is damaged. In other words, if the j th component of *Rf* is zero it can be expressed as following

$$\mathbf{Rf}(j)=\sum_{i=1}^{nel}\mathbf{Rf_i^e}(j)=0 \tag{10}$$

Then considering the definition of *RSE* in Eq. (6), it can be found that RSE could be obtained by multiplying $\boldsymbol{\delta}\mathbf{U^T}$ to Eq. (9). So another form of *RSE* is

$$RSE_i=\boldsymbol{\delta}\mathbf{U^T Rf_i^e}\ \ (i=1,2,\cdots,nel) \tag{11}$$

The *RSE* value of the damage element will be distinctive bigger than that of the others. Therefore value of *RSE* can be regarded as an index to determine which element is damaged. This will be verified in the numerical simulations.

2.3 Reduction of the finite element model

In practice it is impossible to measure all the DOFs of the system for damage identification. In order to make use of the incomplete measured data, a reduction method (Guyan 1965) is utilized to match the measured DOFs with the ones in FEM.

Assuming that the static force is applied only among the measured DOFs, the static equilibrium equation can be rewritten as

$$\begin{bmatrix}\mathbf{K_{mm}} & \mathbf{K_{ms}}\\ \mathbf{K_{sm}} & \mathbf{K_{ss}}\end{bmatrix}\begin{bmatrix}\mathbf{U_m}\\ \mathbf{U_s}\end{bmatrix}=\begin{bmatrix}\mathbf{F}\\ \mathbf{0}\end{bmatrix} \tag{12}$$

where m and s represents the measured and unmeasured DOFs respectively. Consider the second equation in Eq. (11), $\mathbf{U_s}$ can be represented by $\mathbf{U_m}$

$$\mathbf{U_s}=-\mathbf{K_{ss}^{-1}K_{sm}U_m} \tag{13}$$

Then a transform matrix is expressed as

$$\mathbf{T}=\begin{bmatrix}\mathbf{I}\\ \mathbf{-K_{ss}^{-1}K_{sm}}\end{bmatrix} \tag{14}$$

Multiplying Eq. (11) on both sides by $\mathbf{T^T}$, a reduced FEM model is obtained

$$\mathbf{K^R U_m}=\mathbf{F} \tag{15}$$

$$\mathbf{K^R}=\mathbf{T^T K T} \tag{16}$$

Based on the reduced model, *RSE* can be calculated as following

$$RSE_i^R=\boldsymbol{\delta}\mathbf{U_m^T K_i^R}\boldsymbol{\delta}\mathbf{U_m}\ \ (i=1,2,\cdots,nel) \tag{17}$$

$$\mathbf{K_i^R}=\mathbf{T^T K_i T}\ (i=1,2,\cdots,nel) \tag{18}$$

It should be pointed out that $RSE_i^R\neq RSE_i$ because of the incomplete displacement data. It means that RSE of the reduced model can only find out the suspicious damaged elements but not the true damaged elements. As the number of measurement increases, the number of false alarms will be reduced.

2.4 Selection of load case

Strain energy is utilized here to represent the contribution of an element. The strain energy of the j th element under the i th load case is calculated as

$$SE_{ij}=\frac{1}{2}\mathbf{U_i^T K_j U_i} \tag{19}$$

In the static method, a better load case should have less variation in contribution of elements. In order to find out the best load cases, deviation of the strain energy is used. The Deviation Index can be expressed as

$$D_i=\frac{1}{\frac{1}{nel}\sum_{j=1}^{nel}SE_{ij}}\sqrt{\sum_{j=1}^{nel}(SE_{ij}-\frac{1}{nel}\sum_{j=1}^{nel}SE_{ij})} \tag{20}$$

A smaller value of D_i indicates that the i th load case is better than other cases. This strategy was also adopted by Nejad *et al*. (2005).

2.5 Static displacement response sensitivity analysis

The equilibrium equation of the damaged system can be expressed simply as

$$\mathbf{K_d U_d}=\mathbf{F} \tag{21}$$

It is usually assumed that the local damage only relates to the stiffness parameter of the structure, the damage model can be expressed as follows

$$\mathbf{K_d}=\sum_{i=1}^{nel}\alpha_i\mathbf{K_i} \tag{22}$$

where α_i (i=1,2,…,nel) is the stiffness parameter whose value varies from 0 to 1. When the element is undamaged the value of α_i is 1, and 0 means the element is completely damaged.

Differentiating both sides of Eq. (21) with respect to the stiffness parameter, we have

$$\frac{\partial\mathbf{U}_d}{\partial\alpha_i}=-\mathbf{K_d^{-1}}\frac{\partial\mathbf{K_d}}{\partial\alpha_i}\mathbf{U_d} \tag{23}$$

Considering Eq. (22) the static displacement response sensitivity can be expressed in a simple form

$$\frac{\partial\mathbf{U}_d}{\partial\alpha_i}=-\mathbf{K_d^{-1}K_i U_d} \tag{24}$$

2.6 Identification of extent of damage

As the suspicious damaged elements are localized, the stiffness parameters of these elements are used for model updating. And the value of the parameter of the intact element will remain as 1 during the iteration.

For given stiffness parameters the system deformation $\mathbf{U}_{cal}$ can be obtained by Eq. (1). As the stiffness parameters of the undamaged system are known and all the initial values of the damaged parameters are set to be 1.

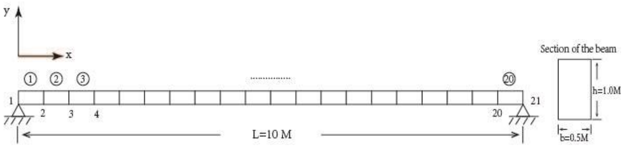

Fig. 1 Finite element model of a simply supported beam

Obviously, the initial damaged parameters are not the true values of the damaged system, $\mathbf{U}_{cal}$ generally deviates from the measured static response of the damage system. The difference between the measured and calculated displacement can be expressed as

$$\Delta\mathbf{U}=\mathbf{U}_{\mathbf{m}}-\mathbf{U}_{\mathbf{cal}} \tag{25}$$

where $\mathbf{U}_{\mathbf{m}}$ is the measured static response vector. The task of the model updating is to minimize the difference between the calculated and measured static displacement data $\Delta\mathbf{U}$. To this end, the damage parameter vector $\boldsymbol{\alpha}$ needs to be updated iteratively by solving the following equation

$$\Delta\mathbf{U}=\mathbf{S}\Delta\boldsymbol{\alpha} \tag{26}$$

where $\Delta\boldsymbol{\alpha}$ is the increment of the damage parameter vector $\boldsymbol{\alpha}$. And $\mathbf{S}$ is the static response sensitivity matrix whose entries can be obtained from Eq. (24). The detailed sensitivity matrix $\mathbf{S}$ can be written as

$$\mathbf{S}=\begin{bmatrix} \frac{\partial u_1}{\partial \alpha_1} & \frac{\partial u_1}{\partial \alpha_2} & \cdots & \frac{\partial u_1}{\partial \alpha_n} \\ \frac{\partial u_2}{\partial \alpha_1} & \frac{\partial u_2}{\partial \alpha_2} & \cdots & \frac{\partial u_2}{\partial \alpha_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial u_k}{\partial \alpha_1} & \frac{\partial u_k}{\partial \alpha_2} & \cdots & \frac{\partial u_k}{\partial \alpha_n} \end{bmatrix} \tag{27}$$

where the subscript n represents the number of suspicious elements determined from damage localization, and k represents the number of measured data. Generally the number of measured displacement will be larger than that of the damaged parameters. In this case, the over-determined Eq. (26) can be solved using the least-squared method, i.e.

$$\Delta\boldsymbol{\alpha}=\left[\mathbf{S}^{\mathbf{T}}\mathbf{S}\right]^{-1}\mathbf{S}^{\mathbf{T}}\Delta\mathbf{U} \tag{28}$$

Usually $\mathbf{S}^{\mathbf{T}}\mathbf{S}$ is ill conditioned and Eq. (28) is an ill-posed problem. So the Tikhonov regularization (1963) is utilized here to solve Eq. (26). Then solution of Eq. (26) can be expressed as

$$\Delta\boldsymbol{\alpha}=\left[\mathbf{S}^{\mathbf{T}}\mathbf{S}+\lambda\mathbf{I}\right]^{-1}\mathbf{S}^{\mathbf{T}}\Delta\mathbf{U} \tag{29}$$

where λ is a nonnegative regularization parameter determined by L-curve method (Hansen 1992) and $\mathbf{I}$ is an identity matrix.

During model updating, the damage parameter vector is updated as

$$\boldsymbol{\alpha}^{iter+1}=\boldsymbol{\alpha}^{iter}+\Delta\boldsymbol{\alpha}^{iter} \tag{30}$$

where the superscript *iter* denotes the number of iteration.

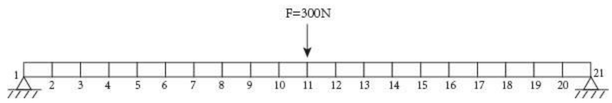

Fig. 2(a) Load case 1 for a simply supported beam

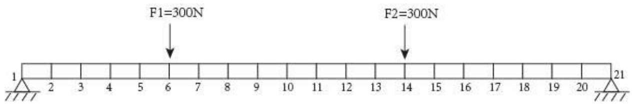

Fig. 2(b) Load case 2 for a simply supported beam

The convergence criterion is defined as

$$\frac{\left\|\boldsymbol{\alpha}^{\mathbf{iter+1}}-\boldsymbol{\alpha}^{\mathbf{iter}}\right\|}{\left\|\boldsymbol{\alpha}^{\mathbf{iter}}\right\|}\leq \text{tolerance} \tag{31}$$

3. Numerical simulation

3.1 A simply supported beam

In this case a simply supported beam shown in Fig. 1 is studied. The physical parameters of the beam are: E=3.0×10^{10} N/m^2, ρ=2800 kg/m^3. The beam is discretized into 20 Euler-Bernoulli beam elements. Finite element model of the beam are established using the Matlab software.

As shown in Fig. 2(a) and Fig. 2(b), there are two load cases for selection. Using Eq. (19) and Eq. (20), the criteria D of these load cases are D_1=3.9750 and D_2=2.9814. Since D_2 is smaller load Case 2 presented by Fig. 2(b) is chosen for analysis.

Case1: Single damage

In this section, a simply supported beam with a single damage is studied as a forward problem. Assuming the 4th element is damaged with a 15% reduction in the elemental stiffness. Three measurement scenarios are studied in the following: 1) The first measurement scenario deals with full measurement, i.e., the deflection and rotation of each node are measured. Fig. 3 shows result of damage localization. One can find that the proposed method can identify the location of damage precisely with full measurement. 2) The second scenario deals with the case when only the deflection of each node is measured, i.e., totally 21 measurements. Fig. 4 shows the localization result. The location of damage can also be identified under such a measurement scheme, but the RSE values of element 3 and element 5 are also relatively large and they are regarded as suspicious damaged element. 3) The third scenario further deals with incomplete measurement of nodal deflection. In this case, only the deflection data of nodes 4, 6, 8, 11, 14, 16 and 18 are used for damage localization. As shown in Fig. 5, elements 2, 3, 4 and 5 is identified as suspicious damaged ones according to their large RSE values while the other 16 elements can be regarded as undamaged ones.

These scenarios show that the damage can be localized from the elemental RSE value. Studies also show that the

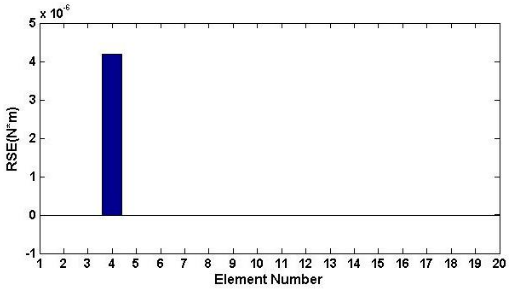

Fig. 3 Damage localization result of the single damaged beam with full deformation

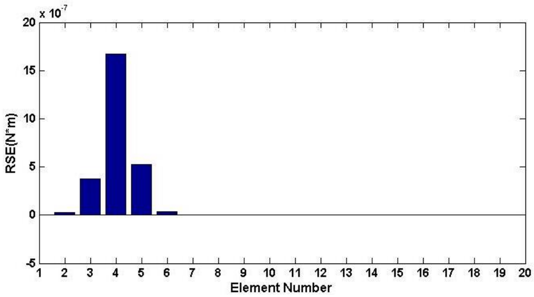

Fig. 4 Damage localization result of the single damaged beam with deflection data only

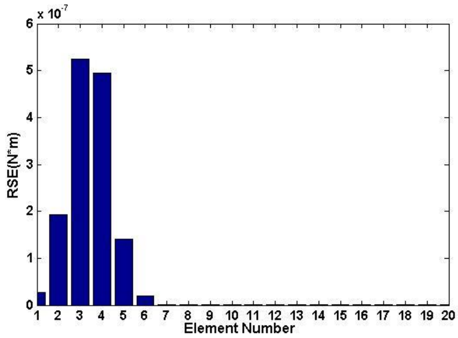

Fig. 5 Damage localization result of the single damaged beam with incomplete deflection data

incomplete measurement will leads to more suspicious damaged element, the less the measurement, the more suspicious damaged elements. The accurate location of damage and its extent will be identified from the finite element model updating using the measured static displacement.

Case2: Multiple damages

In this case two local damages locating at the 5th and 14th elements are studied. The stiffness parameters of these two elements are reduced by 15% and 20%, respectively. The same as 7 measurements as the third scenario in Case 1 are used to localize the damage. Fig. 6 shows the result of damage localization. Again the result is not the exact one due to the incomplete measurement data. Elements 5, 6, 12, 13, 14 and 15 are regarded as the suspicious ones according to their RSE values. The other 14 elements are excluded as undamaged elements.

As the suspicious damaged elements are determined, the

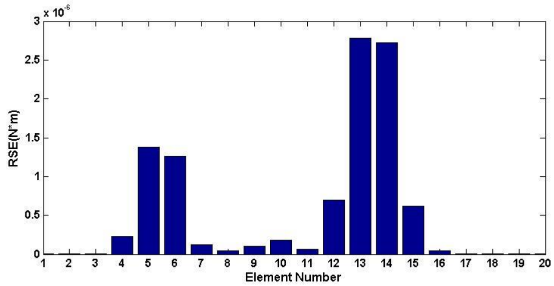

Fig. 6 Localization result of the multiple damaged beam with 7 deflection data

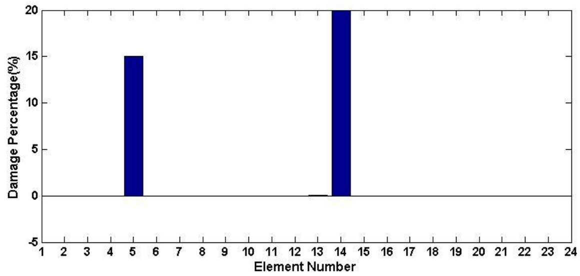

Fig. 7 Quantification of damage extents for a simply supported beam

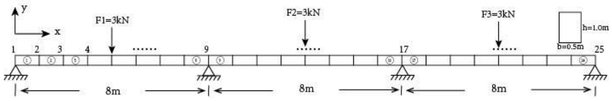

Fig. 8 A three-span continuous beam

true damage location and corresponding damage quantity will be accomplished from sensitivity based model updating. In order to assure the identification equation, i.e., Eq. (26) is over-determined, multiple sets of measured data under different external forces are used. Remaining the location of the static loads unchanged, two sets of measured data under different magnitude of loads, i.e., F_1=300 N, F_2=300 N and F_1=350 N, F_2=450 N are used in this study. After 33 iterations, the final result for damage extent identification is shown in Fig. 7 with the tolerance of 10^{-8}. And the optimal regularization parameter is found to be $\lambda=6.0882\times10^{-7}$.

3.2 A three-span continuous beam

A three-span continuous beam shown in Fig. 8 is considered in this section. The geometrical parameters of the beam structure are presented in Fig. 8. And the physical parameters are: Young's modulus $E=3.0\times10^{10}$ N/m^2, mass density ρ=2800 kg/m^2. Then the structure is divided into 24 Euler-Bernoulli beam elements. And the best load case for localization shown in Fig. 8 is selected previously based on the criterion described in section 2.4. Considering the locations of the external forces and the constrains of the structure the measurement points are evenly distributed in each span of the beam. Therefore static displacement data obtained from nodes 3, 5, 7, 11, 13, 15, 19, 21 and 23 is used for damage localization and quantification. Two damage cases are studied here.

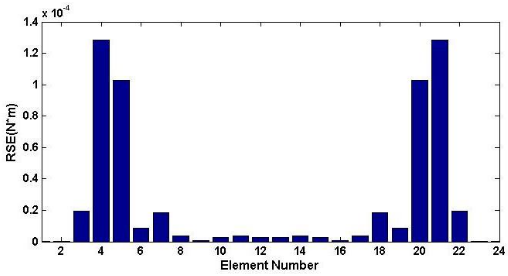

Fig. 9 Localization of two far away damages with 9 deflection data

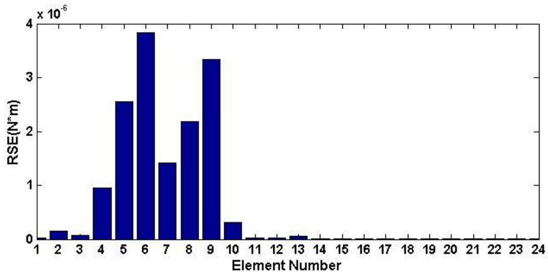

Fig. 11 Localization of two adjacent damages with 9 deflection data

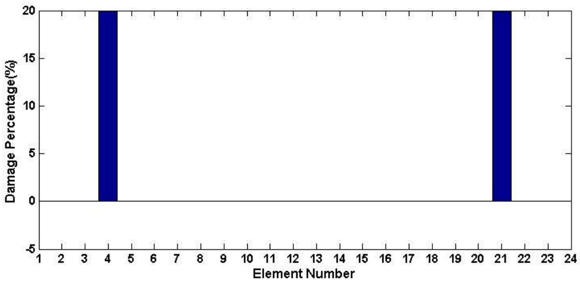

Fig. 10 Quantification of two far away damage extents with 9 deflection data

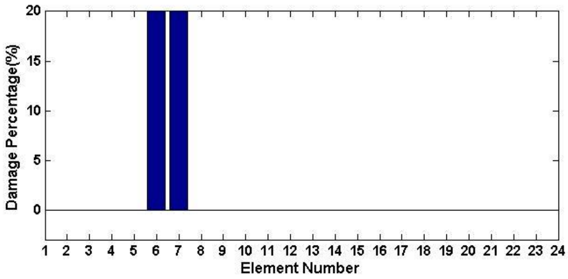

Fig. 12 Quantification of two adjacent damage extents with 9 deflection data

Case 3: Identification of two far away damages

In this case the two damaged elements are far from each other. The stiffness parameters of elements 4 and 21 are both reduced by 20%. The localization result is presented in Fig. 9. From this figure, one can find that the values of *RSE* for elements 4, 5, 20 and 21 are significant big comparing with the values of other elements. Therefore these 4 elements are regarded as the suspicious damaged elements. It is shown that the localization result is not exact to the true damage location because of the limited measurements. But it can identify the suspicious elements well and the other 20 elements are excluded as undamaged ones.

Then the model updating method based on sensitivity analysis is utilized for accurate identification. Changing the magnitude of the static loads to F_1=1.5 kN, F_2=3.0 kN, F_3=2.0 kN and one more set of static displacement measurement can be obtained. Therefore two sets of measured data can be used in the model updating. The final identification result is presented by Fig. 10 after 23 iterations. Fig. 10 shows that the local damages are successfully identified with final optimal regularization parameter $\lambda=7.2868\times10^{-7}$.

Case 4: Identification of two adjacent damages

In this case, the proposed method is used to differentiate two adjacent damaged elements. Elements 6 and 7 are assumed to be damaged elements. The stiffness parameters of these two elements are reduced by 20%. The same external forces and measurements as the last study case are used. The localization result is shown in Fig. 11. It can be found that the *RSE* values of elements 4 to 9 are relatively bigger than others. So elements 4, 5, 6, 7, 8 and 9 are identified as suspicious elements. Once again, model updating is used to obtain the accurate damage location and extent. After 24 iterations, the result of damage identification is shown in Fig. 12 with the final optimal regularization parameter $\lambda=1.9247\times10^{-7}$. It is shown that the two adjacent damages have been differentiated successfully through the proposed method.

4. Conclusions

A structural local damage localization and quantification method using incomplete static measured data is proposed in this study. Based on the residual force vector in the static system, the residual strain energy (*RSE*) is introduced. Making use of the incomplete static displacement data of both intact and damaged structure, the proposed method can indicate the suspicious elements while lots of undamaged ones will be excluded. As the suspicious elements are determined, a model updating method based sensitivity analysis is utilized to further identify local damages. Numerical simulations show that the proposed method is effective for identifying the damage location and quantity even when the measurement data are incomplete. The advantage of the proposed method is that only limited static displacement measurement is needed in the damage localization and quantification. In this study the effect of artificial noise in the static displacement measurement on damage identification results is not taken into account. It will be investigated by the authors in the future.

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