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Stress wave propagation in clearance joints based on characteristics method

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Abstract. In this paper, a stress wave model is established to describe the three states (separate, contact and impact) of clearance joints. Based on this stress wave model, the propagation characteristics of stress wave generated in clearance joints is revealed. First, the stress wave model of clearance joints is established based on the viscoelastic theory. Then, the reflection and transmission characteristics of stress wave with different boundaries are studied, and the propagation of stress wave in viscoelastic rods is described by the characteristics method. Finally, the stress wave propagation in clearance joints with three states is analyzed to validate the proposed model and method. The results show the clearance sizes, initial axial speeds and material parameters have important influences on the stress wave propagation, and the new stress waves will generate when the clearance joint in contact and impact states, and there exist some high stress region near contact area of clearance joints when the incident waves are superposed with reflection waves, which may speed up the damage of joints.

Keywords: clearance joint; stress wave; characteristics method; collision; viscoelastic; propagation

1. Introduction

Articulation linkages have been widely used in the aviation and aerospace fields, such as deployable solar panels, extendable space masts, deployable antennas, and support structures of various mechanical arms. Due to the existence of clearance joints in articulation linkages, the dynamic response of space structures has strong nonlinearity, which is mainly caused by the contact and collision in clearance joints. Such nonlinearity has brought us a great difficult to analyze the dynamic characteristics of space structures and eventually limits our ability to improve the precision of space structures. Thus, as the increasing requirements of higher reliability of space structures, the lucubration on contact impact and propagation characteristics of stress wave in clearance joints are urgent to be addressed.

In the past several years, some investigators have focused on the dynamic analysis of joint clearances. Dubowsky and Freudenatein (1971) established the one-dimensional impact pair model and studied the impact phenomenon occurring in clearance joints. Subsequently, Dubowsky and Gardner (1977), Dubowsky *et al.* (1984), Deck and Dubowsky (1994) analyzed the motion performance of flexible mechanisms comprising a plurality of clearance joints, and established the two-dimensional planar impact pair model. Muvengei *et al.* (2013) used the direct integration method to analyze the dynamic characteristics of planar slider-crank mechanism with two-clearance revolute joints. Koshy *et al.* (2013) established a multibody model of the slider-crank mechanism and studied the contact forces in revolute clearance joints by computation and experiment. Mahfouz and Badrakhan (1990) investigated the possibility of chaos in three different viscous damping systems with clearances and provided some guidance on the design of such a system. Bauchau and Rodriguez (2002) concerned with the modeling of clearance joints within the framework of finite element based dynamic analysis of nonlinear and flexible multibody system, and revealed the effects of clearance and lubrication on revolute and spherical joints.

The above methods aim at the dynamic response of mechanisms and few studies concern to the transfer and exchange of energy in clearance joints. Thus, this paper intends to discuss the wave propagation in clearance joints based on the stress wave theory.

In the past 50 years, the stress wave theory developed rapidly in the high-technology fields such as earthquake detection, engineering blasting, explosive processing, impact detection and weapons effects etc. Wang *et al.* (2013) dealt with a problem of propagation of longitudinal shock waves in two idealized material models, and given the relation of wave speed and stress with the rod material. Butt *et al.* (2014) tested the parameters of viscoelastic materials by studies the three-dimensional stress wave propagation in the split Hopkinson pressure bar. Chiffoleau *et al.* (2003) studied the reflection characteristics of structural or guided waves in rods at a solid/liquid interface, and the result shows that major reflection occurs from the solid/liquid interface not the liquid/gas one. Keskinen *et al.* (2007) studied the propagation and reflection of elastic wave in a rod chain by using finite element method and experiment method, and the result shows that different material will generate different reflection wave.

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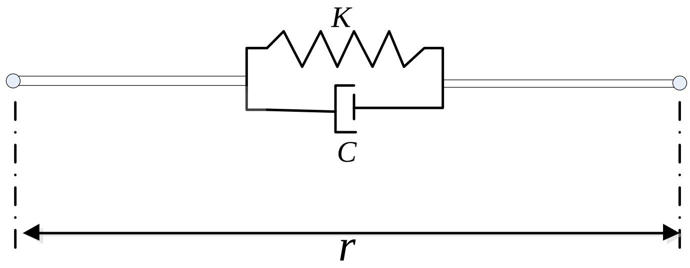


Fig. 1 MLSD model

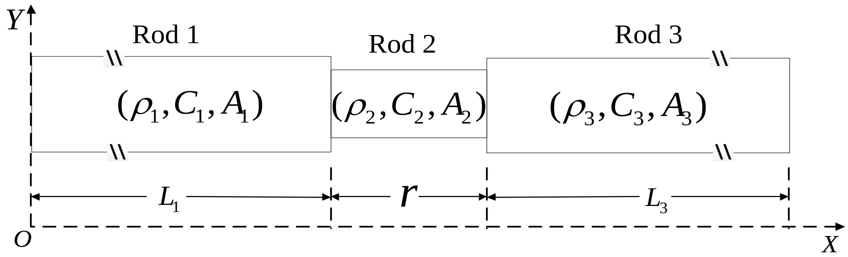


Fig. 2 Stress wave model of clearance joint

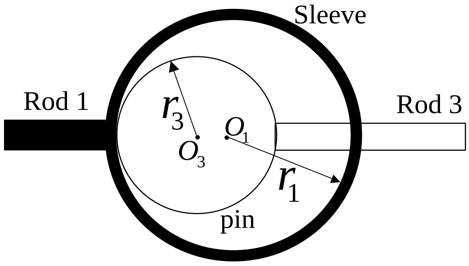


Fig. 3 Clearance joint

Wesolowski (1994) analyzed the one-dimensional wave propagating from one homogeneous material to another one. Wang (2003, 2004) did a lot of researches on the plastic stress wave and the elastic-plastic dynamic buckling stress wave. Yakupov (2007) used the Timoshenko beam theory and Laplace transform to analyze the stress wave propagate in a semi-infinite rod surrounding by elastic medium. These works focused on the characteristics of stress wave propagation, and did not involve the stress wave in clearance joints. Thus, the stress wave and clearance joint model will be combined in this work to analyze the response of clearance joints with impact loads, which may reveal propagation characteristics of stress wave in clearance joints and the damage mechanism of joints.

This paper is organized as follows. In Section 2, the stress wave model of clearance joints is proposed base on viscoelastic theory. In Section 3, the stress waves for different boundary conditions are analyzed. Then, the reflection and transmission characteristics of stress wave in viscoelastic rod are studied based on characteristics method. In Section 4, the characteristics of stress waves propagating in clearance joints with three states (separate, contact and impact) are illustrated, and the effects of different parameters on the propagation of stress wave are revealed. Finally, Section 5 presents some conclusions and discussions.

2. Stress wave model of clearance joint

The massless-link and spring-damper (MLSD) model was established by Senevirantne *et al*. (1996). In MLSD model, both the size and stiffness of clearance are considered. The joint clearance is equivalent to a bar composed of a parallel spring-damper system, as shown in Fig. 1. The bar length r is equal to clearance size. The dynamic equation can be described as

$$F = Kx + C\dot{x} \tag{1}$$

where, K and C are the contact stiffness and damping of clearance joint, respectively. x is the relative displacement of clearance joint, F is the external force.

Inspired by MLSD model, we establish a stress wave model of clearance joint which is shown in Fig. 2, and where A_i (i=1, 2, 3) is the sectional area of rods. Rod 1 and 3 are connected by clearance joint constituted of sleeve and pin shown in Fig. 3. The viscoelastic rod 2 whose length r is equal to the clearance size (r=r_1-r_3) is used to simulate

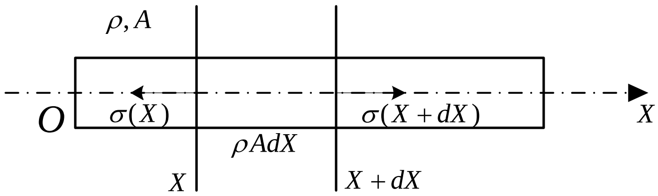


Fig. 4 The microelement of elastic rod

clearance joint.

The three states (separate, contact and impact) of clearance joint can be simulated by changing the material parameters of rod 2 in the stress wave model.

1) For separate state, the section area A_2=0 and the stress at the right boundary of rod 1 is $\sigma(L_1, t) = 0$. In this case, the stress wave cannot pass through the clearance joint, so we can set the material parameters of rod 2 as follows.

$$E = 0,\ \eta = 0 \tag{2}$$

where E is the elastic module, η is the viscosity coefficient.

2) For contact and impact states, the transmission and reflection phenomenon generated by the change of wave impedance will appear at the interface of rod 1 and rod 2. The new stress wave will generate at the interface for impact state while the contact state will not. We can get the parameters of rod 2 from Wang and Liu (2015) as follows

$$\begin{gathered} E = \frac{L}{A_2} \times \frac{4}{3\pi(\gamma_1 + \gamma_3)} \sqrt{\frac{r_1 r_3}{r_1 - r_3}} \\ \eta = \frac{L}{A_2} \times \frac{3u^{1.5}(1-c_r^2)}{4v} \times \frac{4}{3\pi(\gamma_1+\gamma_3)} \sqrt{\frac{r_1 r_3}{r_1 - r_3}} \\ \gamma_i = \frac{1-\tau_i^2}{\pi E_i} \\ (i = 1,\ 3) \end{gathered} \tag{3}$$

where u and v are the axial displacement and axial speed respectively. c_r is the restitution coefficient. τ_i and E_i are the Poisson's ratio and the elastic modulus of rod i respectively.

3. Propagation characteristics of stress wave

The stress wave model of clearance joint is comprised of three rods and two interfaces, so in this section, the propagation of stress wave in rod is discussed, and the reflection and transmission characteristics of stress wave propagation at interfaces are investigated.

3.1 Stress wave propagation in elastic rods

Fig. 4 shows a microelement of slender elastic rod, where ρ is the density, A is the sectional area, and $\sigma(X)$ is the stress at location X.

We suppose that the section plane perpendicular to axis still keeps perpendicular after the deformation, and the shear wave is ignored. By the conservation law of momentum, the equilibrium equation of axial force of microelement can be established as

$$A\rho dX \frac{\partial v}{\partial t} = A\sigma(X+dX) - A\sigma(X) = A\frac{\partial \sigma}{\partial X}dX \tag{4}$$

According to the definition of strain and velocity, we obtain

$$\varepsilon = \frac{\partial u}{\partial X},\ v = \frac{\partial u}{\partial t} \tag{5}$$

Supposing that u is continuous and second order differentiable, we have

$$\frac{\partial^2 u}{\partial t \partial X} = \frac{\partial^2 u}{\partial X \partial t} \tag{6}$$

Combining Eq. (5) and Eq. (6), the following equation can be obtained

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial v}{\partial X} = 0 \tag{7}$$

Eq. (7) is the continuous equation of axial motion of slender rod of which the wave velocity is defined as

$$C = \sqrt{\frac{1}{\rho}\frac{d\sigma}{d\varepsilon}} \tag{8}$$

where $C = C(\sigma)$ or $C = C(\varepsilon)$ can be determined completely by the constitutive relation of materials. For elastic materials, we have $d\sigma / d\varepsilon = E$, then the strain rate can be obtained as

$$\frac{\partial \varepsilon}{\partial t} = \frac{d\varepsilon}{d\sigma}\frac{\partial \sigma}{\partial t} = \frac{1}{\rho C^2(\sigma)}\frac{\partial \sigma}{\partial t} \tag{9}$$

Substituting Eq. (9) into Eq. (7), yields

$$\frac{\partial \sigma}{\partial t} - \rho C^2(\sigma)\frac{\partial v}{\partial X} = 0 \tag{10}$$

Combining Eq. (4) and Eq. (10), the simultaneous differential equations can be expressed by

$$\begin{cases} \dfrac{\partial v}{\partial t} - \dfrac{1}{\rho}\dfrac{\partial \sigma}{\partial X} = 0 \\ \dfrac{\partial \sigma}{\partial t} - \rho C^2(\sigma)\dfrac{\partial v}{\partial X} = 0 \end{cases} \tag{11}$$

The quasilinear system of partial differential equations is as follows

$$A \cdot W_t + B \cdot W_X = 0 \tag{12}$$

where

$$W = \begin{bmatrix} v \\ \sigma \end{bmatrix},\ A = I,\ B = \begin{bmatrix} 0 & -\dfrac{1}{\rho} \\ -\rho C^2 & 0 \end{bmatrix} \tag{13}$$

where W_X and W_t can be derived by taking the derivative of W with respect to X and t, respectively.

According to the method of characteristics, the points satisfied $\dfrac{dX}{dt} = \lambda$ can be solved on the X-t plane. Let $l \cdot B = \dfrac{dX}{dt} l = \lambda l$, the following characteristics relations can be obtained

$$l \cdot W_t + l \cdot B \cdot W_X = l \cdot (W_t + \lambda W_X) = l \cdot \frac{dW}{dt} = l \cdot b \tag{14}$$

where, λ is the characteristic value of the two-order tensor B, l is the left eigenvector of B corresponding to λ, and then Eq. (14) can be transformed as

$$l \cdot \frac{dW}{dt} = l \cdot b \ (\text{along } \frac{dX}{dt} = \lambda) \tag{15}$$

The characteristic values of B are $\lambda_1 = C$ and $\lambda_2 = -C$. The two left eigenvectors are $l_1 = \begin{bmatrix} -\rho C \\ 1 \end{bmatrix}$ and $l_2 = \begin{bmatrix} \rho C \\ 1 \end{bmatrix}$. Substituting λ and l into Eq. (15), we can get the following compatibility equation

$$\begin{cases} \dfrac{d\sigma}{dt} - \rho C \dfrac{dv}{dt} = 0, \ (\text{along } \dfrac{dX}{dt} = C) \\ \dfrac{d\sigma}{dt} + \rho C \dfrac{dv}{dt} = 0, \ (\text{along } \dfrac{dX}{dt} = -C) \end{cases} \tag{16}$$

Eq. (16) is the basic equations of characteristics method, which can be rewritten as follows

$$\begin{cases} \dfrac{dX}{dt} = \pm C \quad \text{characteristics equations} \\ \dfrac{d\sigma}{dv} = \mp \rho C \ \text{characteristic relation equations} \end{cases} \tag{17}$$

Transforming the four differential equations in Eq. (17) into four difference equations, and if the boundary conditions are given, we can numerically solve σ and v by partitioning t or X into calculation points.

3.2 Stress wave propagation at interfaces

The reflection and transmission phenomena will occur when the stress wave propagates between two kinds of medium with different wave impedance (Wesolowski 1994). According to the continuity conditions and Newton's third law, the velocity and stress on both sides of interface are equal to each other when the interface is in contact, and the following relationships can be obtained

$$\Delta v_i + \Delta v_r = \Delta v_t \tag{18}$$

$$(\Delta\sigma_i + \Delta\sigma_r) A_1 = \Delta\sigma_t A_2 \tag{19}$$

where, Δv is the difference of speeds between wave front and wave back, $\Delta\sigma$ is the difference of stresses between wave front and wave back, the subscript i, r, t represent the incidence, reflection and transmission waves, respectively. According to the momentum conservation equation of wave front, we have

$$\begin{cases} \Delta\sigma = -\rho C \Delta v & (\text{left traveling wave}) \\ \Delta\sigma = +\rho C \Delta v & (\text{right traveling wave}) \end{cases} \tag{20}$$

Substituting Eq. (20) into Eq. (18), yields

$$\frac{\Delta\sigma_i}{\rho_1 C_1} - \frac{\Delta\sigma_r}{\rho_1 C_1} = \frac{\Delta\sigma_t}{\rho_2 C_2} \tag{21}$$

Combining Eq. (19) and Eq. (21), the transmission wave is described as

$$\Delta\sigma_t = T(\Delta\sigma_i) A_1 / A_2 \tag{22}$$

$$\Delta v_t = nT(\Delta v_i) \tag{23}$$

And the reflection wave can be obtained as

$$\Delta\sigma_r = F(\Delta\sigma_i) \tag{24}$$

$$\Delta v_r = F(\Delta v_i) \tag{25}$$

where, $n = (\rho_1 C_1 A_1)/(\rho_2 C_2 A_2)$ is the generalized wave impedance, $F = (1-n)/(1+n)$ is the transmission coefficient, and $T = 2/(1+n)$ is the reflection coefficient. The stress wave propagation through interface can be determined from Eq. (18) to Eq. (25).

3.3 Stress wave propagation in viscoelastic rods

The velocity of viscoelastic wave, C_v (C_2) can be obtained as

$$C_v = \sqrt{4\rho_v E_v - \eta^2} / 2\rho_v,\ (4\rho_v E_v > \eta^2) \tag{26}$$

where E_v and ρ_v are the elastic modulus and density of viscoelastic material respectively.

The stress wave propagation in viscoelastic rods is described by the attenuation factor of viscoelastic material as follows (Yuan and Peng 2006)

$$v(X,\ t) = v_0(X,\ t) e^{-\frac{\eta}{\sqrt{4\rho_v E_v - \eta^2}} X} \tag{27}$$

where $v_0(X, t)$ is the initial axial speed, $e^{-\frac{\eta}{\sqrt{4\rho_v E_v - \eta^2}} X}$ is the attenuation factor of viscoelastic materials.

Substituting Eqs. (26) and (27) into Eq. (17) yields the

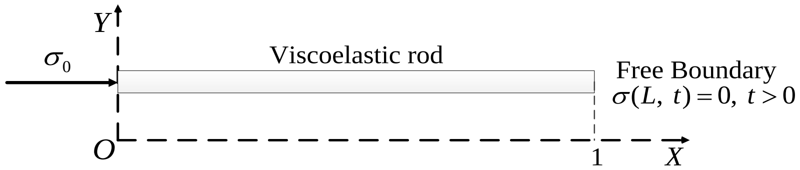


Fig. 5 A viscoelastic rod

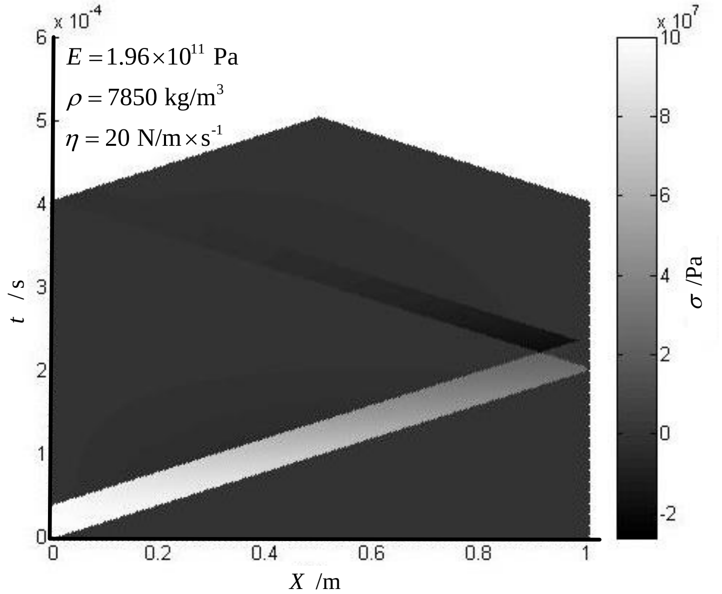
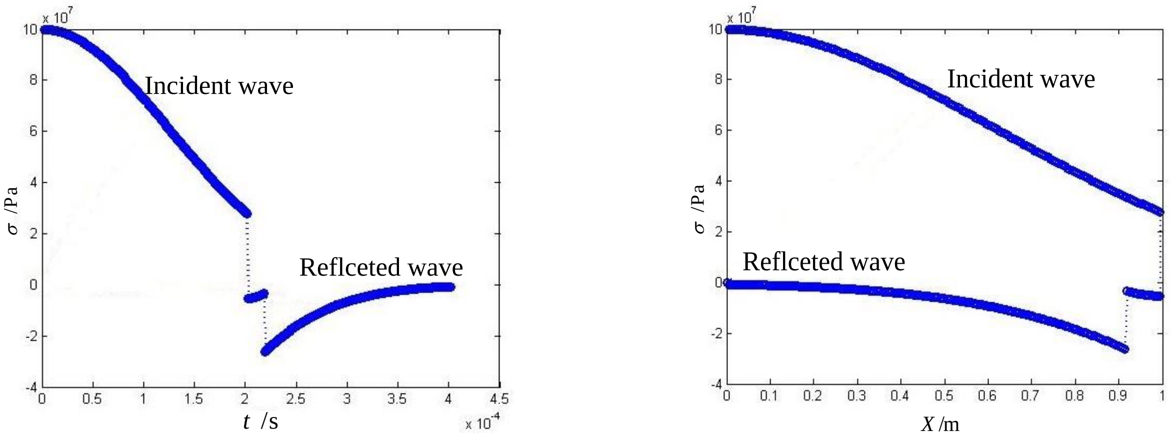


Fig. 6 The propagation of stress wave in viscoelastic rod



(a) Attenuation of stress wave with time (b) Attenuation of stress wave with location

Fig. 7 Attenuation of stress wave

basic equations of characteristics method for viscoelastic materials as follows

$$\begin{cases} \dfrac{dX}{dt}=\pm\dfrac{\sqrt{4\rho_v E_v-\eta^2}}{2\rho_v} & \text{characteristics equations} \\ \dfrac{d\sigma}{dv_0}=\mp\dfrac{\sqrt{4\rho_v E_v-\eta^2}}{2}e^{-\frac{\eta}{\sqrt{4\rho_v E_v-\eta^2}}X} & \text{characteristics relation equations} \end{cases} \tag{28}$$

In the same way, we can reveal the stress wave propagation in viscoelastic rod by numerically solving Eq. (28).

As an example, Fig. 5 shows a viscoelastic rod. The initial load stress is $\sigma_0=1\times10^8$ Pa, and the loading time is 4×10^{-5} s. The numerical results of stress wave propagation in viscoelastic rod are shown in Figs. 6 and 7.

Fig. 7 shows the attenuation of stress wave with *t* and *X*. As shown in Fig. 7(a), the stress wave keep reducing at $t<2\times10^{-4}$ s. Afterwards, the reflection occurs at the right boundary of the rod and the amplitude of stress wave jumps to zero. This is because the reflected waves unload the

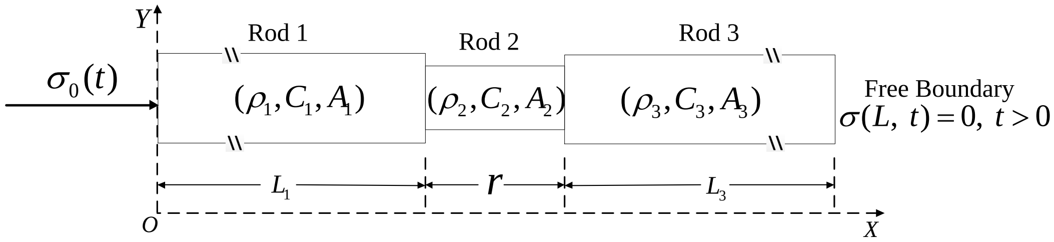


Fig. 8 Stress wave model of a clearance joint

incident wave. At t>2.2×10^{-4} s, the reflected wave proceeds to reduce because of the energy consumption of viscoelastic material. Fig. 7(b) shows the amplitude of stress wave reduces along axial direction. The incident wave reflects at X=1 m and the reflected wave reduce to zero at X=0 m.

4. Numerical examples

In this section, the stress wave model of a clearance joint, as shown in Fig. 8, is as a numerical example. The three states of clearance joint are analyzed and the influences of different parameters of rod 2 on stress wave propagation are revealed. The section area ratio of rods is $A_1/A_2=A_3/A_2=1.5$, $L_1=L_3=1$ m, the joint clearance r=0.1 mm. The initial stress is $\sigma_0=5\times10^8$ Pa, and the load time is $t=4\times10^{-5}$ s. $\rho_1=\rho_3=7850$ kg/m^3, $E_1=E_3=1.96\times10^{11}$ Pa, the initial axial speed $v_0=0.1\,\mathrm{m/s}$. E_2 and η of rod 2 can be determined from Eqs. (2) and (3) for different states. Figs. 9, 10 and 11 show the results of stress wave propagation in clearance joint for three different states.

Fig. 9 illustrates the stress wave propagation for separation state, which shows the stress wave cannot pass through the clearance joint. Fig. 10 illustrates the stress wave propagation for contact sate, which shows the stress wave propagating in rod 1 (X<1 m) generate the first reflection and transmission waves at the clearance joint (X=1 m). The wave strength weakens after passing through the clearance joint because of energy lost in clearance joint. Fig. 11 illustrates the stress wave propagation for impact state, which shows the stress wave generated in clearance joint propagates to rods 1 and 3 at the same time. With time increasing, the stress waves in rod 3 appear multiple reflections and because the reflected waves propagate in a direction contrary to the incident waves, which leads to a spallation at the region near clearance joint (X=1 m). The initial stress wave $\sigma_0(t)$ adds with the impact stress wave at the middle of rod 1, which generates a high stress region in rod 1. The results of contact and impact states show the stress wave strength reduces with the time increasing and the decaying law of stress wave coincides with the result from Tomihiko (2006).

The curves in Fig. 12 illustrate the stress on the left boundary of rod 3 with different parameters for contact sate. Fig. 12(a) shows the effect of clearance sizes on stress wave propagation in clearance joint. With the increase of clearance size, the number of wave crests and the amplitude of stress waves reduce. Fig. 12(b) shows the influence of viscoelastic coefficients on the stress wave propagation. It can be seen that the stress decay rate increases with the viscoelastic coefficient. In addition, because the clearance size r is very small, the influence of viscoelastic coefficient on the time of stress wave arriving at rod 3 is little. Fig. 13(c) shows the effect of initial axial speeds on the stress wave propagation. As the initial speed increases, the stress in rod 3 becomes larger, but the increase rate becomes smaller. And the viscoelastic coefficient η increases slower with the increase of axial speeds.

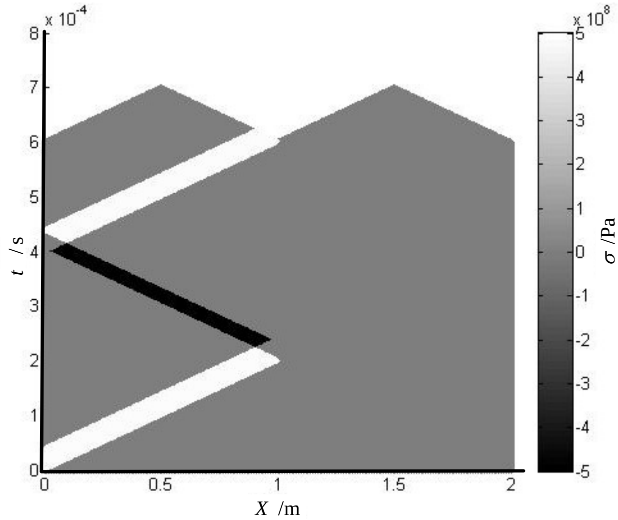


Fig. 9 Stress wave propagation of separation state

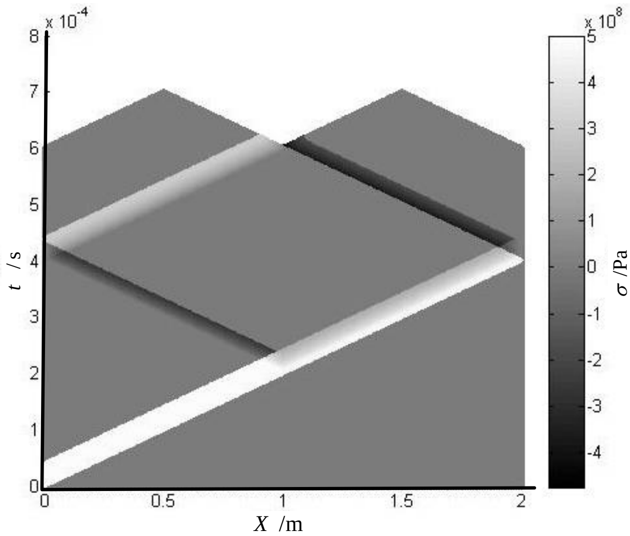


Fig. 10 Stress wave propagation of contact state

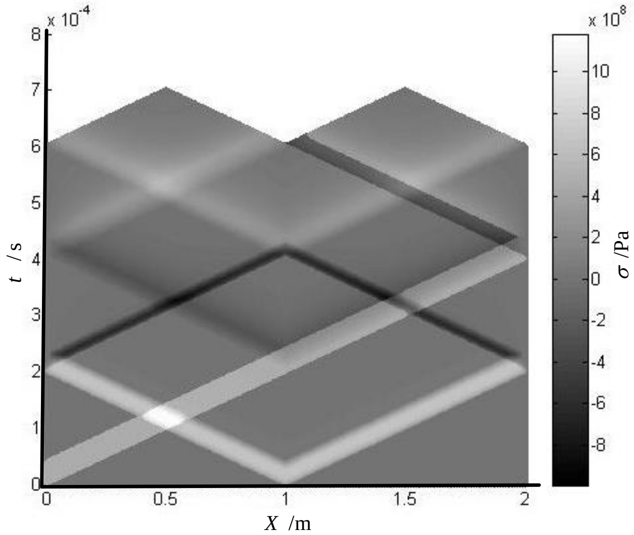


Fig. 11 Stress wave propagation of impact state

5. Conclusions

In this paper, a stress wave model is established to describe the clearance joint with three states (separate, contact and impact). And based on this stress wave model, the propagation characteristics of stress wave generated in clearance joints is revealed. The main contributions of this paper are concluded as follows.

1) The stress-wave model is first established to describe the three states of clearance joints inspired by MLSD model.

2) The stress transmission and reflection waves in different boundary conditions and the propagations of stress wave in elastic and viscoelastic rods are investigated.

3) The stress wave propagation in clearance joints with three states is analyzed, and the influences of clearance sizes, initial axial speeds and material parameters on the stress wave propagation are studied. The results show that new stress waves will generate when the clearance joint in contact and impact states, and there exist some high stress region near the contact area of clearance joints when the incidence waves are superposed with reflection waves, which may speed up the damage of joints.

Acknowledgments

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