Structural Engineering and Mechanics, *Vol. 62, No. 1 (2017) 55-64*
DOI: https://doi.org/10.12989/sem.2017.62.1.055

Optimization and application of multiple tuned mass dampers in the vibration control of pedestrian bridges

Zheng Lu[1,2], Xiaoyi Chen[2], Xiaowei Li[2,3] and Peizhen Li*[1,2]

[1]*State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, Shanghai 200092, China*
[2]*Research Institute of Structural Engineering and Disaster Reduction, Tongji University, Shanghai 200092, China*
[3]*Tongji Architectural Design (Group) Co., Ltd (TJAD), Shanghai 200092, China*

(Received October 9, 2014, Revised October 1, 2016, Accepted January 5, 2017)

Abstract. An effective design approach for Multiple Tuned Mass Dampers (MTMDs) in pedestrian bridges was proposed by utilizing the transfer function to obtain each TMD's optimum stiffness and damping. A systematic simulation of pedestrian excitations was described. The motion equation of a typical MTMD system attached to a Multi-degree-of-freedom (MDOF) system was presented, and the transfer function from the input pedestrian excitations to the output acceleration responses was defined. By solving the minimum norm of the transfer function, the parameters of the MTMD which resulted in the minimum overall responses can be obtained. Two applications of lightly damped pedestrian bridges attached with MTMD showed that MTMDs designed through this method can significantly reduce the structural responses when subjected to pedestrian excitations, and the vibration control effects were better than the MTMD when it was considered as being composed of equal number and mass ratios of TMDs designed by classical Den Hartog method.

Keywords: multiple tuned mass dampers; pedestrian bridge; optimization design; vibration control; pedestrian excitation

1. Introduction

Nowadays, the concept of vibration control (Lu *et al.* 2016b) has become more and more important in structural engineering, and various strategies have been proposed to attenuate the responses (Zhou *et al.* 2016, Gong and Zhou 2016, Lu *et al.* 2017b). When Frahm invented a vibration control device called dynamic vibration absorber in 1909 (Rana and Soong 1998), people began to use mass dampers to control the vibration (Lu *et al.* 2010, Lu *et al.* 2012, Lu *et al.* 2014, Lu *et al.* 2017a). In the past decades, numerous studies have focused on the development of Tuned Mass Damper (TMD) system design and it has been widely used in actual projects (Lu *et al.* 2012, Lu *et al.* 2016c, Dai *et al.* 2016). As passive control devices, TMDs have simple characteristics, favorable controlling effects under specific tuned frequencies and low cost, thus they are widely used in wind- or earthquake-induced (Xiang and Nishitani 2015, Chakraborty and Debbarma 2016, Lu *et al.* 2016a) vibration control.

One of the classical methods for design of TMDs is the Den Hartog method (Abubakar and Farid 2009). Recently, new methods have been proposed to determine the optimum parameters of TMDs. For example, a robust optimal design criterion in case of random vibration was developed by Marano *et al.* (2008). Anh and Nguyen (2012) obtained the optimum tuning ratio of a TMD attached to a damped primary system by using equivalent linearization method. Salvi and Rizzi (2015) suggested a tuning procedure to deal with the optimum parameters of a TMD applied to frame structures, which consisted of a numerical minimax optimization algorithm within MATLAB. Mrabet *et al.* (2015) adopted the reliability based optimization strategy where the failure probability was related to the primary structure displacement to design TMD. Tubino and Piccardo (2015) proposed a numerical optimization criterion based on the maximization of an efficiency factor, defined as the ratio between the uncontrolled acceleration standard deviation and the controlled one to design TMD in footbridges.

However, in the aspect of tuning TMD to the desired structural frequency, uncertainty exists due to the difficulties in predicting the structure's natural frequencies in a practical implementation. Consequently, Multiple Tuned Mass Dampers (MTMDs) have been proposed to broaden the bandwidth of suppression frequencies. To effectively reduce the responses in different modes of the primary structure, the natural frequencies of MTMD are usually equally distributed over a range. Researchers have shown that MTMDs can not only improve the control robustness of the system and reduce the sensitivity to mis-tuning design of dampers (Singh *et al.* 2002), but efficiently control the seismic responses of structures where multiple modes are dominant. Xu and Igusa (1992) examined a main oscillator with multiple sub-oscillators and found these sub-oscillators were more effective than a single TMD at low damping values when subjected to wide-band input. The effectiveness of MTMDs has also been validated by investigating the seismic energy dissipation of inelastic

*Corresponding author, Professor
E-mail: lipeizh@tongji.edu.cn

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

structures with MTMDs under earthquakes (Wong and Johnson 2009).

MTMDs are also widely applied for pedestrian bridges. Carpineto *et al.* (2010) investigated the dynamic responses of a suspension footbridge under pedestrian-induced excitations and its passive mitigation by using MTMD. Chen *et al.* (2012) proposed a design method for long-span floors installed with a MTMD system and conducted field tests of several large scale buildings, showing that the MTMD system was capable of effectively mitigating the vertical vibration of long-span floor structures. In terms of the optimum design of MTMDs, Li *et al.* (2010) investigated the vibration characteristics of a footbridge with MTMD based on crowd-footbridge random vibration model, and an optimization procedure based on the minimization of maximum root-mean-square acceleration of footbridge was introduced to determine the optimum design parameters of MTMD system. Based on all these efforts, as for vibration control of pedestrian bridges, more effective and realistic methods for the optimum design of MTMD should be proposed with deep understanding of the physics of the MTMD.

In this paper, a new optimization design method for MTMD in pedestrian bridges was proposed by determining the minimum norm of the transfer function to obtain each TMD's optimum stiffness and damping. Different from the aforementioned method by Li *et al.* (2010) that may consider two ultimate excitation scenarios and determine the optimum parameters of the MTMD system in each case to ensure safety and comfortability, the new proposed method took both the input excitations and output responses into account. Systematic simulations of pedestrian excitations were firstly described. The motion equation of a typical MTMD system attached to a Multi-degree-of-freedom (MDOF) system was presented followed, and the transfer function from the input pedestrian excitations to the output acceleration responses was defined. By solving the minimum norm of the transfer function, the parameters of the MTMD which resulted in the minimum overall responses can be obtained. Two applications of lightly damped pedestrian bridges showed that MTMDs designed through this method can significantly reduce the structural responses when subjected to pedestrian excitations, and the vibration control effects were better than the MTMD when it was considered as being composed of equal number and mass ratios of TMDs designed by classical Den Hartog method.

2. Excitation simulation

Pedestrian walking induces excitations including vertical forces, lateral forces and torsional forces. Since torsional forces are relatively low compared to vertical and lateral forces, the simulation of excitations was focused on vertical forces resulting from people walking and jumping, and lateral forces in the following sections.

2.1 Vertical excitation of walking

The vertical excitation of people walking can be seen as a cyclic excitation. The period of this excitation, known as gait cycle, is the interval between adjacent ground touches by the same foot. By applying the Fourier transform to the time history of the walking force, the force can be expressed as a periodic function as follows (Allen and Murray 1993).

$$F(t) = p \times \left[1 + \sum \alpha_i \cos\left(2\pi i f_s t + \varphi_i\right)\right] \tag{1}$$

where p is the weight of the people walking. α_i is the dynamic factor of the i th order harmonic wave. f_s is the frequency of the gait cycle. t is the time and ψ_i is the phase angle in i th order harmonic wave. Generally, the dynamic factor α_i continuously decreases as the order of the harmonic wave increases.

Different methods including first-order and third-order harmonic excitations have been proposed by researchers to simplify the expression of the walking force. This paper only considered the first three order harmonic excitations to decide the single-person walking loads, which was suggested by the International Association for Bridge and Structure Engineering (IABSE). The continuous contact model was selected to present single-person walking excitation (Da Silva *et al.* 2003), as shown in Fig. 1(a).

However, for most of the time, excitations applied to a structure are induced by more than one people. The upper limit of people density walking in group without interfering with others is 0.3 people per square meter (Yang and Ke 2008). When the pedestrian density reaches 0.6-0.8 people per square meter, the blockage of other people will hinder the normal movement. In that situation, people are forced to adjust their step length and speed to coordinate with other people around, thus the walking group tends to walk in the same manner with the same frequency. According to Fujino *et al.* (1993), the number of people walking with the same frequency when they are on a pedestrian bridge is 0.2n, where n is the total number of people walking on a bridge and can be approximated obtained through the following equation.

$$n = \lambda T_0 \tag{2}$$

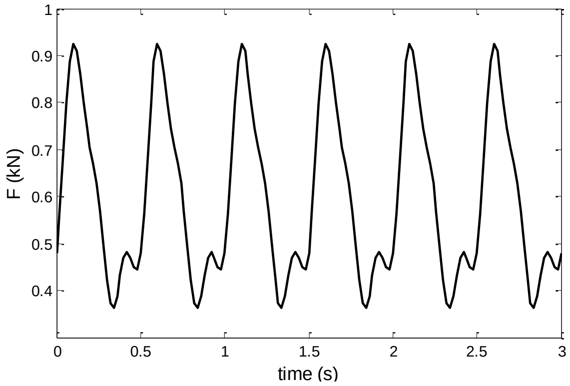
where λ is the number of people passing over the width of the bridge per second and T_0 is the time needed to walk through the bridge.

2.2 Vertical excitation of jumping (running)

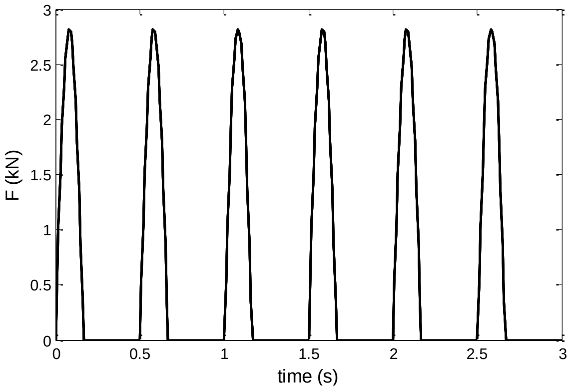
When the movement is performed by a group of people, it is necessary to consider the excitation caused by people jumping or running. The excitation of jumping or running is different from the excitation of walking as sudden changes of force are existed in jumping or running. A model for excitation of jumping or running is proposed, in which there is a force when the foot touches the ground and a zero-force when the feet are in no contact with the ground as shown in Eq. (3) (Da Silva *et al.* 2003).

$$F_p(t) = \begin{cases} K_p G \sin(\pi t / t_p) & t \in \left[0, t_p\right] \\ 0 & t \in \left[t_p, T\right] \end{cases} \tag{3}$$

where K_p is the impact factor and K_p=$F_{\max}/G$. $F_{\max}$ is the



(a) Single-person walking excitation (2Hz)



(b) Jumping/running excitation (2Hz)

Fig. 1 Pedestrian excitation time history

maximum of the force and G is the weight of people. t_p is the contacting time and T is the time period of the jumping or running excitation. With the assumption that the weight of a single people is 600 N, K_p is 4.7, and the contacting time t_p is one third of T, the time history of a 2 Hz jumping (running) excitation is shown in Fig. 1(b).

2.3 Lateral excitation of walking

The lateral force of walking pedestrian is calculated according to the British standard 5400 (2006). A simple sinusoidal function is used in the following equation.

$$F_{pv}(t)=\begin{cases}0.033\times 700\sin(2\pi f_0 t) & 0.8\text{Hz}\le f_0\le 1.2\text{Hz}\\ 0.009\times 700\sin(2\pi f_0 t) & 1.6\text{Hz}\le f_0\le 2.4\text{Hz}\end{cases} \tag{4}$$

where f_0 is the fundamental lateral frequency of the pedestrian bridge.

2.4 Force on handrail

The forces on the handrail from people's leaning are considered as static loads in current practice as described in the Chinese code "Technical Specifications of Urban Pedestrian Overcrossing and Underpass" CJJ69 (1995). However, in real situation, the handrail bears sudden forces which usually cause the lateral vibration of the structure. The vibration caused by the forces applied on the handrail is even stronger than that caused by the lateral excitations of walking pedestrian. Consequently, modelling the forces on handrail from people's leaning as static loads is not appropriate and should be considered in the lateral vibration analysis. For convenience, distributed sudden pushing forces (250 N/m) were selected to equivalently model the handrail forces.

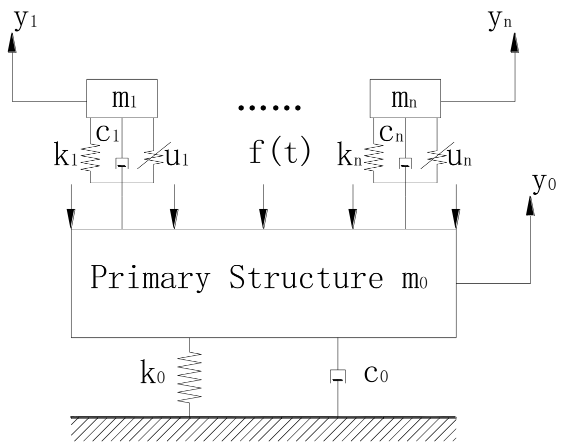


Fig. 2 Computational model of pedestrian bridge system with MTMD

3. MTMD design

The proposed design method for MTMD system was based on structures (Singh *et al.* 2002, Li 2002, Miguel *et al.* 2016). Once the frequencies of the primary structure were determined, the TMDs of MTMD system can be designed to correspond to the dominant vibration modes of the primary structure. Then the optimization design of MTMD focuses on the determination of each TMD's optimum stiffness and damping coefficient to obtain overall satisfied structural performance. This can be achieved by solving problems of transfer function concerning the input and output, given a predefined displacement and velocity (Yang *et al.* 2010). Assuming that the state of the system is known, the transfer function from the excitations to the structural responses is related with the gains of TMD's stiffness and damping coefficient. Combining the initial parameters with gains that give the minimum norm of the transfer function, the optimum stiffness and damping coefficient can be obtained.

3.1 Motion equations of MTMD system

Fig. 2 shows the analytical model of the pedestrian bridge system attached with MTMD consisting of n distributed TMDs. The primary structure that was symbolizing a typical MDOF system was characterized by the generalized mass m_0, damping coefficient c_0 and stiffness k_0. m_i, c_i and k_i are the mass, damping coefficient and stiffness of the i th TMD, respectively. $f(t)$ is pedestrian excitation, which is directly applied on the primary structure. y is the displacement response, and u_i is the passive control force between the primary structure and the i th TMD, which is relevant to the stiffness and damping coefficient of the i th TMD.

The motion equation of the MTMD system can be written as

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=[E_s]f(t)+[P_s]\{u\} \tag{5}$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping, and stiffness matrix of the whole system, respectively. $[E_s]$ and $[P_s]$ are the direction matrixes of the excitation $f(t)$ and the passive control force vector $\{u\}$, respectively. $\{q\}$, $\{\dot{q}\}$, and $\{\ddot{q}\}$ are the displacement, velocity, and acceleration vector, respectively. Full expressions of these matrixes are shown as follows.

$$[M]_{(n+1)\times(n+1)}=\begin{bmatrix} m_0 & 0 & \cdots & 0 \\ 0 & m_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_n \end{bmatrix}$$

$$[C]_{(n+1)\times(n+1)}=\begin{bmatrix} c_0+c_1+\cdots+c_n & -c_1 & \cdots & -c_n \\ -c_1 & c_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -c_n & 0 & \cdots & c_n \end{bmatrix}$$

$$[K]_{(n+1)\times(n+1)}=\begin{bmatrix} k_0+k_1+\cdots+k_n & -k_1 & \cdots & -k_n \\ -k_1 & k_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -k_n & 0 & \cdots & k_n \end{bmatrix}$$

$$[E_s]=-\begin{Bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \qquad [P_s]_{(n+1)\times n}=\begin{bmatrix} 1 & 1 & \cdots & 1 \\ -1 & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 \end{bmatrix}$$

$$\{u\}_{n\times 1}=\begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{Bmatrix} \qquad \{q\}_{(n+1)\times 1}=\begin{Bmatrix} y_0 \\ y_1 \\ \vdots \\ y_n \end{Bmatrix}$$

3.2 Transfer function of MTMD system

The passive control force vector $\{u\}$ is related with the relative movement of each TMD to the primary structure. To present the system motion equations in terms of relative response of TMD to the primary structure, the absolute displacement vector $\{q\}$ can be transformed into relative displacement vector $\{p\}$ as

$$\{q\}=\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 1 \end{bmatrix}\begin{Bmatrix} y_0 \\ y_1-y_0 \\ \vdots \\ y_n-y_0 \end{Bmatrix}=[T_s]\{p\} \tag{6}$$

where $[T_s]$ is the transfer matrix.

Substituting Eq. (6) in Eq. (5), then Eq. (5) in terms of state equation can be written as

$$\{\dot{x}\}=[A]\{x\}+[E]f(t)+[P]\{u\} \tag{7}$$

where $\{x\}$ is the state vector $\{p, \dot{p}\}^T$,

$$[A]=\begin{bmatrix} [0] & I \\ -[T_s]^{-1}[M]^{-1}[K][T_s] & -[T_s]^{-1}[M]^{-1}[C][T_s] \end{bmatrix},$$

$$[E]=\begin{bmatrix} [0] \\ [T_s]^{-1}[M]^{-1}[E_s] \end{bmatrix}, \quad [P]=\begin{bmatrix} [0] \\ [T_s]^{-1}[M]^{-1}[P_s] \end{bmatrix}$$

Considering that the absolute acceleration of the primary structure can be easily measured, the matrix of structural absolute acceleration z is set as the output matrix of the system, as shown in Eq. (8).

The passive control force vector $\{u\}$ is determined by gains of continuously varying displacement and velocity, and can be presented as Eq. (9), where Δk_i and Δc_i are gains of the i th TMD's stiffness and damping.

Substituting Eq. (9) in Eq. (7) and Eq. (5), respectively, then, Eq. (10) and Eq. (11) can be obtained.

$$\begin{aligned} z=&-[1\ \ 0\ \ \cdots\ \ 0]\Big[[T_s]^{-1}[M]^{-1} \\ &[K][T_s]\quad [T_s]^{-1}[M]^{-1}[C][T_s]\Big]\{x\} \\ &+[1\ \ 0\ \ \cdots\ \ 0]\Big[[T_s]^{-1}[M]^{-1}[P_s]\Big]\{u\} \\ &=[C_1]\{x\}+[D]\{u\} \end{aligned} \tag{8}$$

$$\{u\}=-[F]\{x\}= \\ -\begin{bmatrix} 0 & \Delta k_1 & 0 & \cdots & 0 & 0 & \Delta c_1 & 0 & \cdots & 0 \\ 0 & 0 & \Delta k_2 & \cdots & 0 & 0 & 0 & \Delta c_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \Delta k_n & 0 & 0 & \cdots & 0 & \Delta c_n \end{bmatrix}\{x\} \tag{9}$$

$$\{\dot{x}\}=([A]-[P][F])\{x\}+[E]f(t) \tag{10}$$

$$z=([C_1]-[D][F])\{x\}+[0]f(t) \tag{11}$$

Therefore, the transfer function from the input pedestrian excitations to the output acceleration responses is defined as

$$\left[G_{zf(t)}\right]=\frac{[z]}{[f(t)]}= \\ ([C_1]-[D][F])(s[I]-[A]+[P][F])^{-1}[E] \tag{12}$$

3.3 Optimization design procedure of MTMD system

The steps that one can follow to design an optimum MTMD attached to a SDOF primary structure are summarized as follows:

(1) Determine the initial values of each TMD's stiffness k_i and damping coefficient c_i according to the classical Den Hartog method. This method is usually used to optimize the stiffness and damping of a TMD attached to a SDOF system when subjected to sinusoidal excitations (Masaki and Hirata 2004; Iba *et al.* 2006).

The frequency ratio and damping ratio of the *i* th TMD to the primary structure is

$$f_{\mathrm{TMD}i} = 1/(1+\mu_i) \tag{13}$$

$$\varepsilon_{\mathrm{TMD}i} = \sqrt{3\mu_i/8(1+\mu_i)} \tag{14}$$

where μ_i is the mass ratio of the *i* th TMD to the primary structure

$$\mu_i = \frac{m_{\mathrm{TMDi}}}{m_0} \tag{15}$$

f_{TMDi} and ε_{TMDi} can also be presented as follow

$$f_{\mathrm{TMD}i} = \frac{\omega_{\mathrm{TMD}i}}{\omega_0} = \frac{\sqrt{k_{\mathrm{TMD}i} m_{\mathrm{TMD}i}}}{\sqrt{k_0 m_0}} \tag{16}$$

$$\varepsilon_{\mathrm{TMD}i} = \frac{c_{\mathrm{TMD}i}}{2\sqrt{k_{\mathrm{TMD}i} m_{\mathrm{TMD}i}}} \tag{17}$$

Thus, given the mass of each TMD m_{TMDi}, corresponding stiffness k_i and damping coefficient c_i can be obtained.

(2) Define H_2 norm of the system transfer function $[G_{zf}(t)]$ as the objective function. H_2 norm of the transfer function, as shown in Eq. (12), is

$$\|H\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} tr\left[\left[G_{zf(t)}(j\omega)\right]\left[G_{zf(t)}(j\omega)\right]\right]d\omega \tag{18}$$

where $tr[M]$ is the trace of matrix $[M]$, meaning the algebraic sum of the primary diagonal elements of $[M]$.

(3) Solve for the minimum value of Eq. (18) to obtain the gain matrix of TMD's stiffness and damping, as shown in Eq. (9). To avoid negative stiffness and damping, Δk_i and Δc_i are confined in the following ranges,

$$-k_i < \Delta k_i < k_i \qquad -c_i < \Delta c_i < c_i$$

Finally, the optimum stiffness and damping coefficient of each TMD can be obtained as

$$k_{\mathrm{T}i} = k_i + \Delta k_i \tag{19}$$

$$c_{\mathrm{T}i} = c_i + \Delta c_i \tag{20}$$

4. Application in pedestrian bridges

Two applications in pedestrian bridges enhanced by MTMD were introduced in this and next section. Since the TMDs are usually demanded and expected to achieve good vibration control effects for lightly damped structures, two applications with damping ratios of 0.02 and 0.01 were presented. In the first application, a vertical excitation of walking pedestrians was applied to the bridge and the MTMD only functioned in the vertical direction. In the second application, more complicated excitations were considered and both vertical and lateral TMDs were attached to the bridge. The excitation simulation and optimum design of MTMD utilized the methods described in this paper. The responses of the pedestrian bridge without MTMD, with MTMD that was regarded as consisting of TMDs with same number and mass ratios designed by the classical Den Hartog method (refered to as Den Hartog's MTMD) and that by the new proposed method were compared.

4.1 Primary structure and MTMD design

The primary structure was a two-span steel pedestrian bridge with a 32 m left span and a 36 m right span, and the width of the bridge is 15 m. For each span, it can be considered as a simply supported beam. The natural frequencies were 2.763 Hz, 3.737 Hz, and 3.991 Hz for the first three vibration modes. The first mode was a symmetric bending mode, and the second and the third modes were asymmetric and symmetric torsional modes, respectively. In this case, the first mode was the primary focus, and only the vibration control in the vertical direction was taken into consideration. The damping ratio of the primary structure was 0.02.

The optimization design of the MTMD followed the procedure in Section 3. Considering several engineering experiences, the total mass of the MTMD system was chosen to be 2% of the mass of the primary structure (308.56 t). 16 TMDs were equally distributed along the bridge with 6 TMDs on the left span and 10 TMDs on the right span. The mass of each TMD was thus 1000 kg. Take the optimization design for the left span as an example, and the steps were described as follows.

(1) Determine the initial values of each TMD's stiffness k_i and damping coefficient c_i by applying Eqs. (13)-(17). The parameters of the primary bridge and the initial values of the TMD on the left span based on preliminary design are listed in Table 1.

(2) Determine the gains of each TMD's stiffness and damping coefficient by solving the H_2 norm for its minimum. H_2 norm of the transfer function was defined by Eq. (18). The optimum parameters of the MTMD

Table 1 Parameters of the primary structure and preliminary design of MTMD

m_0 (kg)	k_0 (kN/m)	c_0 (kNs/m)	ξ_0	m_i (kg)	k_i (kN/m)	c_i (kNs/m)
308560	37760	136.536	0.02	1000	239.282	0.767

Table 2 Optimization process of TMDs on the left span

MTMD No.	Gain		Optimum value	
	Δk_i (kN/m)	Δc_i (kNs/m)	k_{Ti} (kN/m)	C_{Ti} (kNs/m)
1	1.6775	0.017	237.6004	0.784
2	-14.389	-0173	253.6710	0.940
3	16.6003	-0.253	222.6816	1.020
4	-34.022	-0.241	273.3039	1.007
5	31.6853	-0.185	273.3039	0.952
6	-2.7117	-0.216	241.9936	0.983

Table 3 Unified optimization results of the MTMD

Group No.	Stiffness (kN/m)	Damping (kNs/m)	Frequency (Hz)
220	1	2.67	3.34
270	1	2.36	0.290
240	1	2.47	23.63
0.021	0.161	0.161	0.042

were the sum of the initial values and the gains. The gains and optimum stiffness and damping coefficient of six TMDS on the left span are listed in Table 2. The minimum value of the H_2 norm was 7.6483.

(3) To be economical and convenient for industry construction, all the TMDs were categorized into three groups and in each group, values of TMD parameters were very close. For each group of TMDs, a set of unified stiffness and damping coefficient was decided, as listed in Table 3, as well as corresponding frequencies.

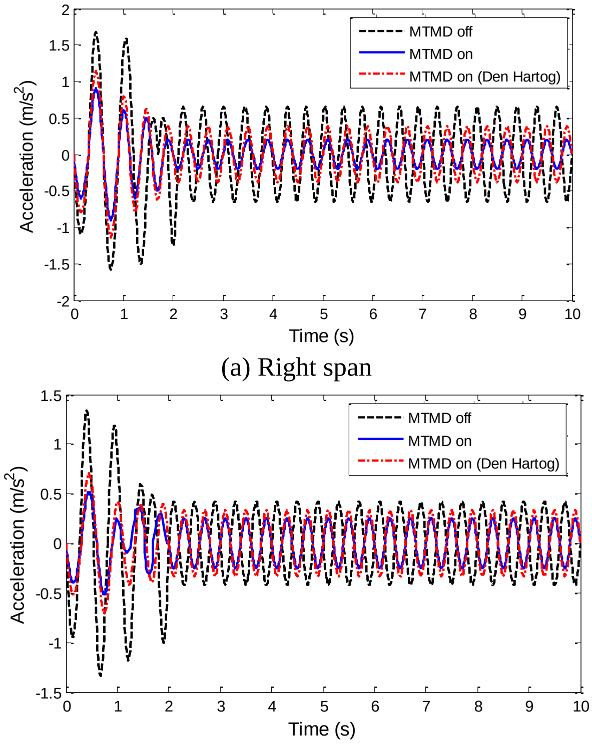
4.2 Excitation and structural response

Vertical excitation from walking pedestrians was considered in this application, as shown in Eq. (1). The walking speed was assumed to be 3.25 m/s and the pedestrian density was one person per square meter. The frequency of the excitation was set as 2.5 Hz which was in line with the walking speed in normal situation.

Fig. 3 compares the acceleration time histories at the middle position of each span of pedestrian bridge with and without MTMD. In addition, the acceleration responses of the bridge designed by the proposed optimization method and that by the classical Den Hartog method are compared. It can be observed that an overall reduction of the absolute acceleration responses in both spans can be achieved with both two kinds of MTMD. However, the vibration control effects of new MTMD were better than that of the classical Den Hartog's MTMD. By attaching with the new proposed MTMD, the peak acceleration of both spans can be significantly reduced. For example, this value of the right span was reduced from 1.68 m/s^2 to 0.89 m/s^2, whereas it was reduced from 1.68 m/s^2 to 1.14 m/s^2 by the classical Den Hartog's MTMD, suggesting an improvement of 28.5% of new method.

In the steady-state range, the performance of the left span was less satisfied compared to that of the right span. There were two possible reasons. For one thing, in this application, 6 TMDs were attached on the left span, whereas 10 TMDs were attached on the right span. Generally speaking, the vibration reduction effects increase as the auxiliary mass ratio increases for TMDs. For another thing, the responses of the right span were larger than the left span, which may make TMDs fully take effects.

Theoretically speaking, the optimization method proposed in this paper improved the vibration control effects on the basis of the classical Den Hartog method. It firstly determined the initial parameters of MTMD in the way of the classical Den Hartog method, and then optimized the transfer function, and finally obtained the optimum stiffness and damping coefficient. As a result, the comparison results showed that the new optimization method in this paper was more efficient in designing MTMD.



(a) Right span

(b) Left span

Fig. 3 Comparison of acceleration time histories without MTMD, with new MTMD and with Den Hartog MTMD

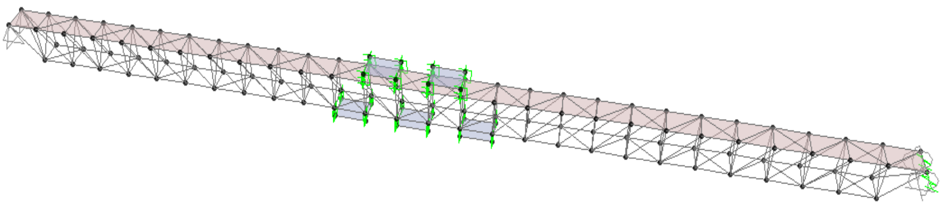
5. Application in an art museum connecting bridge

5.1 Primary structure and MTMD design

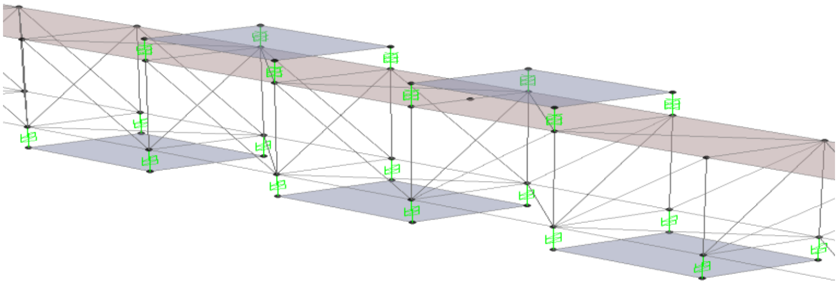
The primary structure was a connecting bridge located in an art museum. The bridge was a single-span truss structure consisting of upper and lower chords, and web members, which were made of box shape steel. The bridge was 2.2 m in height and 1.25 m in width, with a length of 56 m. A layer of 100 mm thick reinforced concrete was paved on the top of the truss with outstretched width of 575 mm on both sides in the lateral direction. The total weight of the bridge was 231.83 t. And the damping ratio of the primary structure was 0.01.

The first and third modes of the structure vibrated in the lateral direction with frequencies of 1.49 Hz and 3.88 Hz, respectively. The second and fourth modes of the structure were bending modes in the vertical direction with frequencies of 1.64 Hz and 5.58 Hz, respectively. The fifth mode was the torsional mode along the longitude direction with a frequency of 5.80 Hz. In this case, the first mode and the second mode were mainly paid attention to, and the vibration control both in the lateral and vertical directions were taken into consideration.

The optimization design of the MTMD followed the



(a) Bridge attached with MTMD



(b) Zoom in MTMD part

Fig. 4 Model of bridge in SAP2000

Table 4 Parameters of vertical TMDs

Mass (kg)	Stiffness (kN/m)	Damping ratio (%)	Damping Coefficient (kNs/m)
2000	53.04×4	10	4.12

Table 5 Parameters of suspension TMDs

Mass (kg)	Pendulum length (mm)	Damping ratio (%)	Damping Coefficient (kNs/m)
2000	112	10	3.74

procedure in Section 3. A total number of 5 TMDs were installed on the bridge, including 3 TMDs attached to the lower chords to control the vertical vibration and 2 suspension TMDs attached to the upper chords to control the lateral vibration. Both kinds of TMDs were positioned around the middle of the bridge, and Fig. 4 shows the location of MTMD. The mass ratio of each TMD to the primary structure was set as 0.86%, resulting in 2 t of each TMD. The natural frequency of the vertical TMDs was 1.64 Hz, and that of the suspension TMDs was 1.49 Hz. The detailed optimization results for vertical and suspension TMDs are listed in Tables 4-5, respectively.

5.2 Excitations

Excitations including vertical and lateral excitations from walking, vertical excitation from jumping (running), and the lateral sudden force on the handrails were considered.

Assume that the average weight of a person was 600 N and 13 people were walking in the same gait cycle. According to the International Association for Bridge and Structural Engineering (IABSE), the pedestrian walking loads were applied to the structure in the mid-span with frequencies of 1.2 Hz, 1.5 Hz, 1.7 Hz, 2.0 Hz, 2.2 Hz, and 2.4 Hz in the vertical direction, and with frequencies of 1.2 Hz, 1.5 Hz, 2.0 Hz and 2.4 Hz in the lateral direction. The same weight and number of people were used for the simulation of jumping excitation. The frequencies for the simulation of vertical jumping excitation were 1.2 Hz, 1.5 Hz, 2.0 Hz, and 3.0 Hz. The bridge was excited in the mid-span. A sudden force of 250 N/m was applied to the full length of the bridge as the simulation of lateral force on handrails.

5.3 Structural response

The dynamic characteristics of the bridge were slightly changed after attaching with the MTMD. The first mode was the in-phase lateral vibration of the bridge and the suspension TMDs with a frequency of 1.388 Hz. The second mode was the in-phase vertical vibration of the bridge and the vertical TMDs with a frequency of 1.49 Hz. At this mode, the suspension TMDs vibrated in the longitudinal direction. The fifth mode was similar to the second mode with a frequency of 1.50 Hz. The sixth mode was the out-of-phase lateral vibration of the bridge and the suspension TMDs with a frequency of 1.626 Hz. In the third, the fourth, the seventh, and the eighth modes, only the movement of the TMDs can be observed. In the ninth mode, the vertical TMDs vibrated in the opposite direction of the bridge with a frequency of 1.856 Hz.

5.3.1 Vibration control in the vertical direction

The peak responses of displacement, velocity, and acceleration of the bridge at mid span with and without MTMD when subjected to walking excitations are compared in Table 6. Also, the responses of the bridge with the new optimum MTMD and that with the classical Den Hartog's MTMD (referred to as DH in Tables 6-8) are compared in Table 6. It can be found that with the designed MTMD, the responses of the bridge were significantly reduced in a wide band of frequencies, which showed the robustness of MTMD. The reduction effects for displacement and velocity ranged from 22.38% to 66.37 %, from 33.33% to 78.26%, respectively. The acceleration response of the bridge without MTMD was above 10 cm/s^2,

Table 6 Responses at mid span under walking excitations

Frequency		1.2 Hz		1.5 Hz		1.7 Hz	
		New	DH	New	DH	New	DH
Displacement (mm, %)	MTMD off	1.43		2.13		3.42	
	MTMD on	1.11	1.27	1.14	1.33	1.15	1.29
	Reduction	22.38	11.19	46.48	37.56	66.37	62.28
Velocity (cm/s, %)	MTMD off	0.86		1.52		2.99	
	MTMD on	0.56	0.64	0.61	0.72	0.65	0.76
	Reduction	34.88	25.58	59.87	52.63	78.26	74.58
Acceleration (cm/s^2, %)	MTMD off	11.57		15.64		32.26	
	MTMD on	5.97	6.70	6.38	7.14	6.92	10.16
	Reduction	48.40	42.09	59.21	54.35	78.55	68.51
Frequency		1.2 Hz		1.5 Hz		1.7 Hz	
		New	DH	New	DH	New	DH
Displacement (mm, %)	MTMD off	1.74		1.48		1.46	
	MTMD on	1.15	1.32	1.14	1.15	1.32	1.14
	Reduction	33.9	24.14	22.97	33.9	24.14	22.97
Velocity (cm/s, %)	MTMD off	1.27		1.11		0.99	
	MTMD on	0.67	0.79	0.67	0.67	0.79	0.67
	Reduction	47.24	37.80	39.63	47.24	37.80	39.63
Acceleration (cm/s^2, %)	MTMD off	16.03		15.08		13.57	
	MTMD on	8.23	9.20	8.86	8.23	9.20	8.86
	Reduction	48.65	42.61	41.24	48.65	42.61	41.24

Table 7 Responses at mid span under jumping excitations

Frequency		1.2 Hz		1.5 Hz		2.0 Hz		3.0 Hz	
		New	DH	New	DH	New	DH	New	DH
Displacement (mm, %)	MTMD off	4.04		9.72		4.58		1.92	
	MTMD on	3.14	3.16	3.89	5.28	3.14	3.16	3.89	5.28
	Reduction	22.27	21.78	59.97	45.68	22.27	21.78	59.97	45.68
Velocity (cm/s, %)	MTMD off	4.02		9.64		4.83		1.86	
	MTMD on	2.74	2.76	3.30	5.04	2.74	2.76	3.30	5.04
	Reduction	31.84	31.34	65.76	47.72	31.84	31.34	65.76	47.72
Acceleration (cm/s^2, %)	MTMD off	44.78		90.47		67.08		39.59	
	MTMD on	23.61	36.07	43.32	56.77	23.61	36.07	43.32	56.77
	Reduction	47.27	19.45	52.11	37.25	47.27	19.45	52.11	37.25

the upper limit required by the owner for all frequencies. However, with the control by MTMD, the acceleration responses can meet the requirement and the best reduction effects for acceleration can achieve 78.55%.

In addition, the closer the frequency of excitation to the frequency of the bridge was, the better the vibration control effects were. For example, the highest response reduction occurred at 1.7 Hz walking excitation which was close to the natural frequency of the bridge. Besides, the designed MTMD can achieve better vibration control effects compared with the classical Den Hartog's MTMD, which demonstrated that the optimization method proposed in this paper was efficient in designing MTMD. For example, under 1.2 Hz walking excitations, the displacement vibration control effects of the new MTMD were almost twice of the classical Den Hartog's MTMD.

Table 7 lists the responses under jumping/running excitations. When subjected to 1.5 Hz excitations, which

Table 8 Accelerations at mid span under lateral excitations

Frequency	1.2 Hz		1.5 Hz		2.0 Hz		3.0 Hz		Lateral	
	New	DH	New	DH	New	DH	New	DH	New	DH
MTMD off (cm/s^2)	0.89		1.19		0.77		0.59		3.46	
MTMD on (cm/s^2)	0.61	0.65	0.70	0.81	0.60	0.65	0.35	0.39	2.25	2.58
Reduction (%)	31.46	26.97	41.17	31.93	22.07	15.58	40.67	33.90	34.97	25.43

were the closest frequency to the natural frequency of the bridge, the bridge vibrated the most due to the resonance. As the frequency of the excitation deviated away from the natural frequency of the bridge, the responses decreased. And the MTMD reduced the structural responses the most when the bridge resonated, where the reduction effects were 59.97%, 65.76%, and 52.11% for displacement, velocity, and acceleration responses, respectively. At other frequencies, the overall performance of the MTMD was also effective. Similarly, the designed MTMD by this new method can get better vibration control effects than the classical Den Hartog's MTMD.

5.3.2 Vibration control in the lateral direction

The acceleration responses of the controlled structure with two kinds of MTMD and the uncontrolled structures under the lateral walking excitations are compared in Table 8. The highest response reduction effect occurred under 1.5 Hz excitation, showing a similar tendency of the vibration control in the vertical direction. The reduction effects achieved by the new MTMD ranged from 22.07% to 41.2%. The acceleration responses at mid span when subjected to the lateral sudden impact resembling the force on the handrail were 2.25 cm/s^2 and 3.46 cm/s^2 when the suspension TMDs were on and off the bridge, respectively. Also, the classical Den Hartog's MTMD can reduce the vibration, but the effects were not as satisfying as new MTMD. The reduction effects reached 34.97% and 25.43% for new MTMD and classical MTMD, respectively, under the lateral force case, indicating the high efficiency of new MTMD.

6. Conclusions

An effective approach for optimization design of MTMD in pedestrian bridges was proposed by utilizing the transfer function to obtain each TMD's optimum stiffness and damping. The transfer function from the input pedestrian excitations to the output acceleration responses was defined. By solving for the minimum norm of the transfer function, the optimized parameters of MTMD can be obtained. Two applications of lightly damped pedestrian bridges attaching with MTMD designed by new proposed optimization method and that by the classical Den Hartog method were presented and their vibration reduction effects were compared. Based on above analysis, the following conclusions can be drawn:

- For lightly damped bridges, both MTMD using the optimization method proposed in this paper and MTMD using the classical Den Hartog method can significantly reduce the structural responses under a series of pedestrian excitations, including vertical excitations of walking and jumping/running, lateral excitations of walking and lateral forces on handrails.
- The vibration control effects are the best when the frequency of excitations is the closest to the frequency of the structure.
- The MTMDs are capable of reducing responses under pedestrian excitations among a reasonably wide range of frequencies, showing the MTMD's robustness.
- The new approach for optimization design of MTMD shows high efficiency of vibration control effects compared with the classical Den Hortag method. Its vibration reduction effects achieve even 78% in the case of resonance, and are twice of the classical MTMD in some cases, which illustrates the effectiveness of MTMDs in the vibration control.

In conclusion, the procedure presented and validated in the paper provides a practical and efficient way for design of MTMD, and promises an encouraging development of MTMD systems.

Acknowledgments

Financial support from the National Natural Science Foundation of China through grant 51478361 is highly appreciated. The work is also supported by the Fundamental Research Funds for the Central Government Supported Universities.

References

Abubakar, I.M. and Farid, B.J.M. (2009), "Generalized Den Hartog tuned mass damper system for control of vibrations in structures", *Seismic Control .*, **59**, 185-193.

Allen, D.E. and Murray, T.M. (1993), "Design criterion for vibrations due to walking", *AISC Eng. J.*, **30**(4), 117-129.

Anh, N.D. and Nguyen, N.X. (2012), "Extension of equivelent linearization method to design of TMD for linear damped systems", *Struct. Control. Hlth.*, **19**(6), 565-573.

British Standard 5400 (2006), Specification for Loads, Steel, concrete and composite bridges.

Carpineto, N., Lacarbonara, W. and Vestroni, F. (2010), "Mitigation of pedestrian-induced vibrations in suspension footbridges via multiple tuned mass dampers", *J. Vib. Control.*, **16**(5), 749-776.

Chakraborty, S. and Debbarma, R. (2016), "Robust optimum design of tuned liquid column damper in seismic vibration control of structures under certain bounded system parameters", *Struct. Infrastr. Eng.*, **12**(5), 592-602.

Chen, X., Ding, Y.L., Li, A.Q., Zhang, Z.Q. and Sun, P. (2012). "Investigations on serviceability control of long-span structures under human-induced excitation", *Earthq. Eng. Struct. D.*, **11**(1), 57-71.

Chinese Code CJJ69 (1995), Technical Specifications of Urban Pedestrian Overcrossing and Underpass, Beijing.

Da Silva, J.G.S., Vellasco, P.C.G., De Andrade, S.A.L., Soeiro, F.J.D.C.P. and Werneck, R.N. (2003). "An evaluation of the dynamical performance of composite slabs", *Comput. Struct.*, **81**(1), 1905-1913.

Dai, K.S., Wang, J.Z., Mao, R.F., Lu, Z. and Chen, S.E. (2016), "Experimental investigation on dynamic characterization and seismic control performance of a TLPD system", *Struct. Des. Tall Spec. Build.*, DOI: 10.1002/tal.1350.

Fujino, Y., Pacheco, B.M., Nakamura, S. and Warnitchai, P. (1993), "Synchronization of human walking observed during lateral vibration of a congested pedestrian bridge", *Earthq. Eng. Struct. D.*, **22**(9), 741–785.

Gong, S.M. and Zhou, Y. (2016), "Experimental study and numerical simulation on a new type of viscoelastic damper with strong nonlinear characteristics", *Struct. Control. Hlth*, DOI: 10.1002/stc.1897.

Iba, D., Masuda, A. and Sone, A. (2006), "Robust design method of multi-degree-of-freedom passive tuned mass damper by control theory", *ASME 2006 Pressure Vessels and Piping/ICPVT-11 Conference*, Vancouver, BC, Canada, July .

Li, C.X. (2002), "Optimum multiple tuned mass dampers for structures under the ground acceleration based on DDMF and ADMF", *Earthq. Eng. Struct. D.*, **31**, 897-919.

Li, Q., Fan, J.S., Nie, J.G., Li, Q.W. and Chen, Y. (2010), "Crowdinduced random vibration of footbridge and vibration control using multiple tuned mass dampers", *J. Sound. Vib.*, **329**(19), 4068-4092.

Lu, X.L., Ding, K., Shi, W.X. and Weng, D.G. (2012), "Tuned mass dampers for human-induced vibration control of the Expo Culture Centre at the World Expo 2010 in Shanghai, China", *Struct. Eng. Mech.*, **43**(5), 607-621.

Lu, Z., Chen, X.Y., Lu, X.L. and Yang, Z. (2016b), "Shaking table test and numerical simulation of an RC frame-core tube structure for earthquake-induced collapse", *Earthq. Eng. Struct. D.*, **45**(9), 1537-1556.

Lu, Z., Chen, X.Y., Zhang, D.C. and Dai, K.S. (2016a), "Experimental and analytical study on the performance of particle tuned mass dampers under seismic excitation", *Earthq. Eng. Struct. D.*, DOI: 10.1002/eqe.2826.

Lu, Z., Lu, X.L, Jiang H.J. and Masri, S.F. (2014), "Discrete element method simulation and experimental validation of particle damper system", *Eng. Comput*, **31**(4):810-823.

Lu, Z., Lu, X.L, Lu, W.S. and Masri, S.F. (2012), "Shaking table test of the effects of multi-unit particle dampers attached to an MDOF system under earthquake excitation", *Earthq. Eng. Struct. D.*, **41**(5), 987-1000.

Lu, Z., Lu, X.L. and Masri, S.F. (2010), "Studies of the performance of particle dampers under dynamic loads", *J. Sound Vib.*, **329**(26):5415-5433.

Lu, Z., Wang, D.C. and Zhou, Y. (2017a), "Experimental parametric study on wind-induced vibration control of particle tuned mass damper on a benchmark high-rise building", *Struct. Des. Tall Spec.*, DOI: 10.1002/tal.1359

Lu, Z., Wang, D.C., Masri, S.F. and Lu, X.L. (2016c), "An experimental study of vibration control of wind-excited high-rise buildings using particle tuned mass dampers", *Smart. Struct. Syst.*, **18**(1), 93-115.

Lu, Z., Yang, Y.L., Lu, X.L. and Liu, C.Q. (2017b), "Preliminary study on the damping effect of a lateral damping buffer under a debris flow load", *Appl. Sci.*, **7**(2), 201.

Marano, G.C., Sgobba, S., Greco, R. and Mezzina, M. (2008), "Robust optimum design of tuned mass dampers devices in random vibrations mitigation", *J. Sound. Vib*., **313**(3), 472-492.

Masaki, N. and Hirata, H. (2004),"Vibration control performance of damping coupled tuned mass dampers", *ASME/JSME 2004 Pressure Vessels and Piping Conference*, San Diego, California, July.

Miguel, L.F.F, Lopez, R.H., Miguel, L.F.F. and Torri, A.J. (2016), "A novel apporach to the optimum design of MTMDs under seismic excitations", *Earthq. Eng. Struct. D.*, DOI: 10.1002/stc.1845.

Mrabet, E., Guedri, M., Ichchou, M. and Ghanmi, S. (2015), "New approaches in reliability based optimization of tuned mass damper in presence of uncertain bounded parameters", *J. Sound Vib.*, **355**, 93-116.

Rana, R. and Soong, T.T. (1998), "Parametric study and simplified design of tuned mass dampers", *Eng. Struct.*, **20**(3), 193-204.

Salvi, J. and Rizzi, E. (2015), "Optimum tuning of tuned mass dampers for frame structures under earthquake excitation", *Struct. Control. Hlth.*, **22**(4), 707-725.

Singh, M.P., Singh, S. and Moreschi, L.M. (2002), "Tuned mass dampers for response control of torsional buildings", *Earthq. Eng. Struct. D.*, **31**, 749-769.

Tubino, F. and Piccardo, G. (2015), "Tuned mass damper optimization for the mitigation of human-induced vibrations of pedestrian bridges", *Meccanica*, **50**(3), 809-824.

Wong, K.K.F. and Johnson, J. (2009), "Seismic energy dissipation of inelastic structures with multiple tuned mass dampers", *J. Eng. Mech.*, **135**(4), 265-275.

Xiang, P. and Nishitani, A. (2015), "Optimum design and application of non-traditional tuned mass damper toward seismic response control with experimental verification", *Earthq. Eng. Struct. D.*, **44**(13), 2199-2220.

Xu, K. and Igusa, T. (1992). "Dynamic characteristics of multiple substructures with closely spaced frequencies", *Earthq. Eng. Struct. D.*, **21**(12), 1059-1070.

Yang, F., Sedaghati, R. and Esmailzadeh, E. (2010), "Vibration suppression using distributed tuned mass damper", *Proceedings of the ASME 2010 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference*, Montreal, Quebec, Canada, August.

Yang, Y.Q. and Ke, Z.T. (2008), "Vibration control of footbridge", *Constr. Des. Project*, **2**, 93-97.

Zhou, Y., Zhang, C.Q. and Lu, X.L. (2016), "Seismic performance of a damping outrigger system for tall buildings", *Struct. Control. Hlth.*, DOI: 10.1002/stc.1864.

CC