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Predicting shear strength of SFRC slender beams without stirrups using an ANN model

Riza S.O. Keskin*

Department of Civil Engineering, Yildiz Technical University, Istanbul, Turkey

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Abstract. Shear failure of reinforced concrete (RC) beams is a major concern for structural engineers. It has been shown through various studies that the shear strength and ductility of RC beams can be improved by adding steel fibers to the concrete. An accurate model predicting the shear strength of steel fiber reinforced concrete (SFRC) beams will help SFRC to become widely used. An artificial neural network (ANN) model consisting of an input layer, a hidden layer of six neurons and an output layer was developed to predict the shear strength of SFRC slender beams without stirrups, where the input parameters are concrete compressive strength, tensile reinforcement ratio, shear span-to-depth ratio, effective depth, volume fraction of fibers, aspect ratio of fibers and fiber bond factor, and the output is an estimate of shear strength. It is shown that the model is superior to fourteen equations proposed by various researchers in predicting the shear strength of SFRC beams considered in this study and it is verified through a parametric study that the model has a good generalization capability.

Keywords: artificial neural network; steel fiber reinforced concrete; slender beam; shear strength

1. Introduction

Steel reinforcing bars have been traditionally used with concrete to enhance its mechanical properties since it has a relatively low tensile strength and a brittle nature. In recent decades, the use of steel fibers has been investigated in the context of improving the mechanical properties of concrete. One of the major concerns of structural engineers is shear failure of reinforced concrete (RC) beams. It has been shown that the shear strength and ductility of RC beams can be increased significantly by adding steel fibers to the concrete (Batson *et al*. 1972, Kadir and Saeed 1986, Mansur *et al*. 1986, Uomoto *et al*. 1986, Lim *et al*. 1987, Narayanan and Darwish 1987, Li *et al*. 1992, Swamy *et al*. 1993, Khuntia *et al*. 1999, Lim and Oh 1999, Noghabai 2000, Kwak *et al*. 2002, Rosenbusch and Teutsch 2002, Dupont and Vandewalle 2003, Cucchiara *et al*. 2004, Parra-Montesinos 2006, Parra-Montesinos *et al*. 2006, Choi *et al*. 2007, Dinh *et al*. 2010, Ding *et al*. 2011, Aoude *et al*. 2012, Minelli and Plizzari 2013). Steel fibers randomly dispersed through the concrete matrix provide a resistance against the formation and growth of cracks, thus they are able to increase the cracking strength, improve the post-cracking behavior and change the failure mode from a brittle shear failure to a flexural ductile failure. Moreover, it has been suggested that steel fibers can be used as a replacement for stirrups (Lim *et al*. 1987, Dinh *et al*. 2010), offering a reduction in reinforcement congestion at critical sections such as beam-column joints. A comprehensive review of improvements that can be provided by steel fibers is available in the report by ACI Committee 544 (1996).

Accurate prediction tools for steel fiber reinforced concrete (SFRC) structural members will help SFRC to become widely used. There exist several equations proposed by various researchers for predicting the shear strength of SFRC beams (Sharma 1986, Narayanan and Darwish 1987, Ashour *et al*. 1992, Swamy *et al*. 1993, Imam *et al*. 1994, Khuntia *et al*. 1999, Kwak *et al*. 2002, RILEM 2003, Dinh *et al*. 2011, Yakoub 2011). Alternative prediction methods based on empirical modelling have been developed as computational power has increased. An effective numerical method is to develop an artificial neural network (ANN) which is a powerful tool to extract the relationships between the parameters involved and deliver predictions without requiring any functional form assumed a priori. ANNs have been widely used for seeking solutions to various structural engineering problems (Bagdatli *et al*. 2009, Pendharkar *et al*. 2010, Keskin and Arslan 2013, Njomo and Ozay 2014). The shortcoming of an ANN is that it cannot deliver a solution in a functional form.

Adhikary and Mutsuyoshi (2006) developed ANN models to predict the shear strength of SFRC beams and compared the models with the equations proposed by Swamy *et al*. (1993) and Khuntia *et al*. (1999). Ahn *et al*. (2007) developed five ANN models predicting the shear strength of SFRC beams, conducted experiments to verify the selected ANN model and compared the fittest model with the equation proposed by Zsutty (1971). Naik and Kute (2013) used ANN models to study the shear strength of steel fiber reinforced high-strength concrete deep beams. With increasing computational power, various approaches have been developed. Gandomi *et al*. (2011) and Kara (2013) used an approach known as genetic programming to develop models for predicting the shear strength of SFRC

∗Corresponding author, Ph.D.
E-mail: okeskin@yildiz.edu.tr

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beams.

The paper presents an ANN model developed for predicting the shear strength of SFRC beams without stirrups. The model is limited to the beams with the ratio of shear span to depth greater than 2.5, which are referred to as slender beams. It is shown that the performance of the developed ANN model is better than those of fourteen equations proposed by various researchers and it is verified that the model is able to generalize to new data. The database used in developing the ANN model is larger than the ones used by both Adhikary and Mutsuyoshi (2006) and Ahn *et al*. (2007). Besides, the number of existing models considered for comparison is also greater than those considered in the previous studies.

2. Existing shear strength models

It has been observed through various experimental studies (Batson *et al*. 1972, Kadir and Saeed 1986, Mansur *et al*. 1986, Uomoto *et al*. 1986, Lim *et al*. 1987, Narayanan and Darwish 1987, Li *et al*. 1992, Swamy *et al*. 1993, Noghabai 2000, Kwak *et al*. 2002, Rosenbusch and Teutsch 2002, Dupont and Vandewalle 2003, Cucchiara *et al*. 2004, Parra-Montesinos 2006, Parra-Montesinos *et al*. 2006, Dinh *et al*. 2010, Ding *et al*. 2011, Aoude *et al*. 2012, Minelli and Plizzari 2013, Minelli *et al*. 2014, Shoaib *et al*. 2014) that the shear strength of an SFRC beam is significantly higher than the shear strength of its companion RC beam due to the increased post-cracking tensile strength. Fiber length L_f, fiber diameter D_f, fiber bond factor d_f and volume fraction of fibers V_f have been identified as the parameters affecting the shear strength of SFRC beams in addition to the ones affecting the shear strength of RC beams, that is, concrete compressive strength f_c, tensile reinforcement ratio ρ, shear span-to-depth ratio a/d and effective depth d.

There exist several equations developed for predicting the shear strength of SFRC beams. Minelli (2005) classified the existing models broadly into two groups: (i) the ones considering the contributions of concrete and steel fibers to the shear strength separately, and (ii) the ones assuming a direct improvement of shear strength by steel fibers due to the improved post-cracking tensile strength. Some of the existing equations are considered here. The empirical equation developed by Sharma (1986) is recommended for shear design of SFRC beams by ACI Committee 544 (1988) and is given as

$$v_u = k f_{ct} \left(\frac{d}{a} \right)^{0.25} \quad \text{(MPa)}, \tag{1}$$

where f_{ct} is the concrete tensile strength, k=1 if f_{ct} is obtained by direct tension test, k=2/3 if f_{ct} is obtained by indirect tension test and k=4/9 if f_{ct} is obtained by using modulus of rupture or $f_{ct}=0.79 f_c^{0.5}$. Narayanan and Darwish (1987) proposed an empirical equation based on a superposition of the contributions of concrete and steel fibers to the shear strength under the assumption of a 45° crack angle as

$$v_u = e\left(0.24 f_{sp} + 80\rho \frac{d}{a} \right) + v_b \quad \text{(MPa)}, \tag{2}$$

$$f_{sp} = \frac{f_{cuf}}{20 - \sqrt{F}} + 0.7 + \sqrt{F} \quad \text{(MPa)}, \tag{3}$$

$$v_b = 0.41 \tau F \quad \text{(MPa)}, \tag{4}$$

$$F = \frac{L_f}{D_f} V_f d_f , \tag{5}$$

where e=1.0 for a/d>2.8 and e=2.8(d/a) for $a/d\leq 2.8$, f_{sp} is the splitting tensile strength of fiber reinforced concrete, f_{cuf} is the cube strength of fiber reinforced concrete, v_b is the pull-out strength of fibers along the inclined crack, τ is the average fiber matrix interfacial bond stress equal to 4.15 MPa, F is a factor considering the effect of geometry and volume fraction of fibers on the shear strength, d_f is 0.5 for round, 0.75 for crimped and 1.0 for indented fibers. Ashour *et al.* (1992) proposed empirical equations by revising the empirical shear strength equations for RC beams given by ACI 318-11 (2011) and Zsutty (1971) as

$$v_u = \left(0.7\sqrt{f_c} + 7F \right)\frac{d}{a} + 17.2\rho \frac{d}{a} \quad \text{(MPa)}, \tag{6}$$

$$v_u = \left(2.11\sqrt[3]{f_c} + 7F \right)\left(\rho \frac{d}{a} \right)^{1/3} \quad \text{(MPa) for } a/d \geq 2.5, \tag{7}$$

respectively. The version of Eq. (7) for a/d<2.5 is not considered here. Swamy *et al*. (1993) developed a simple method based on a truss model resulting in an equation as

$$v_u = 0.37 \tau V_f \frac{L_f}{D_f} + 0.167\sqrt{f_c} \quad \text{(MPa)}. \tag{8}$$

Imam *et al.* (1994) modified the equation of Bazant and Sun (1987), which is based on non-linear fracture mechanics, to propose an equation as

$$v_u = 0.6 \frac{1 + \sqrt{\dfrac{5.08}{d_a}}}{\sqrt{1 + \dfrac{d}{25 d_a}}} \sqrt[3]{\rho(1+4F)} \times \left[f_c^{\,0.44} + 275 \sqrt{\frac{\rho(1+4F)}{\left(\dfrac{a}{d} \right)^5}} \right] \quad \text{(MPa)}, \tag{9}$$

where d_a is the maximum aggregate size and d_f is 0.5 for smooth, 0.9 for deformed and 1.0 for hooked fibers. Khuntia *et al.* (1999) derived a simple equation based on the basic shear transfer mechanisms and experimental data as

$$v_u = (0.167 + 0.25F)\sqrt{f_c} \quad \text{(MPa)}, \tag{10}$$

where d_f is 2/3 for plain or round and 1.0 for hooked or crimped fibers. Kwak *et al*. (2002) developed an equation by introducing additional terms to the equation derived through a multiple regression analysis by Zsutty (1971) as

$$v_u = 3.7 e f_{sp}^{\,2/3} \left(\rho \frac{d}{a} \right)^{1/3} + 0.8 v_b \quad \text{(MPa)}, \tag{11}$$

where e=1.0 for a/d>3.4 and e=3.4(d/a) for a/d≤3.4. Yakoub (2011) developed an expression for predicting the contribution of steel fibers to the shear strength of SFRC beams and used this to modify the equations of Bazant and Kim (1984), which is based on non-linear fracture mechanics, and CSA A23.3-04 (2004), which is based on the modified compression field theory, as

$$v_u = 0.83\xi\sqrt[3]{\rho}\left(\sqrt{f_c} + 249.28\sqrt{\frac{\rho}{\left(\frac{a}{d}\right)^5}}\right) \tag{12}$$

$$+0.162F\sqrt{f_c} \quad \text{(MPa)},$$

$$v_u = \beta\sqrt{f_c}\left(1+0.70F\right) \quad \text{(MPa)}, \tag{13}$$

respectively, where d_f is 0.79 for sheared, 0.83 for crimped, 0.89 for duoform, 0.91 for rounded, 0.92 for indented cut wire and 1.00 for hooked fibers,

$$\xi = \frac{1}{\sqrt{1+\frac{d}{25d_a}}}, \tag{14}$$

is the aggregate size effect factor,

$$\beta = \frac{0.4}{\left(1+1500\varepsilon_x\right)}\frac{1300}{\left(1000+s_{xe}\right)}, \tag{15}$$

$$\varepsilon_x = \frac{M/d_v + V}{2E_s A_s} \tag{16}$$

is the longitudinal strain at the mid-depth of the beam web, M and V are the external failure moment and shear acting on the section, respectively, d_v is the flexural lever arm equal to 0.9d or 0.72h (h is the beam height), whichever is greater,

$$s_{xe} = \frac{35s_x}{16+d_a} \geq 0.85s_x \tag{17}$$

is the equivalent crack spacing factor that accounts for the maximum aggregate size effects on the shear strength and s_x is the crack spacing parameter that accounts for the crack spacing at the mid-depth of the beam. Eqs. (12)-(13) are for a/d≥2.5. The versions for a/d<2.5 are not considered here. Dinh *et al.* (2011) proposed a model where the shear strength of SFRC beams are calculated as the summation of the shear stress carried across the compression zone and the vertical component of the diagonal tension resistance provided by steel fibers, and the resulting equation is

$$v_u = 0.13\rho f_y + 1.2\left(\frac{V_f}{0.0075}\right)^{1/4}\left(1-\frac{c}{d}\right) \quad \text{(MPa)}, \tag{18}$$

where f_y is the yield strength of flexural reinforcement and c is the depth of the compression zone, which can simply be taken as 0.1h. Arslan (2014) proposed an equation by modifying his equation which is based on the principal shear strength carried in the compression zone and predicts the shear strength of RC beams without stirrups (Arslan 2008, Arslan 2012) as

$$v_u = \left(0.2f_c^{\,2/3}\left(\frac{c}{d}\right) + \sqrt{\rho\left(1+4F\right)f_c}\right)\left(\frac{3}{a/d}\right)^{1/3} \quad \text{(MPa)}, \tag{19}$$

where

$$\left(\frac{c}{d}\right)^2 + 600\frac{\rho}{f_c}\frac{c}{d} - 600\frac{\rho}{f_c} = 0\cdot \tag{20}$$

Gandomi *et al.* (2011) developed an equation as

$$v_u = 2\frac{d}{a}\left(\rho f_c + v_b\right) + 2\frac{d}{a}\frac{\rho}{\left(288\rho-11\right)^4} + 2 \quad \text{(MPa)}, \tag{21}$$

using linear genetic programming. Similarly, Kara (2013) used genetic programming to develop an equation as

$$v_u = \left(\frac{\rho d}{c_0c_1\left(a/d\right)}\right)^3 + \frac{Fd^{1/4}}{c_2} + \sqrt{\frac{c_3f_c}{d}} \quad \text{(MPa)}, \tag{22}$$

where c_0=3.324, c_1=0.909, c_2=2.289 and c_3=9.436.

3. ANN model

The basic unit of an ANN is referred to as neuron. It receives data from one or more neighbouring neuron(s), processes the data and transmits the processed data to another neuron. A typical neuron is shown in Fig. 1, where S is the number of input elements, p_i, i=1,…,S, is the i-th input element, w_i, i=1,…,S, is the weight of i-th input element, b is the bias that can be viewed as a weight of a constant input of 1, n is the net input which is the summation of weighted inputs with the bias, f(.) is the transfer function, which must be differentiable, and q is the output.

A common type of ANNs used for solving engineering problems is multi-layer feed-forward network, which consists of an input layer, one or more hidden layers and an output layer. A layer can be made of either a single neuron or a group of neurons. The input layer transmits input elements to a hidden layer, which processes the supplied data and sends the output to either another hidden layer or the output layer, which produces the final output. An ANN is developed in two stages: training and testing. In the training stage, weights and biases are tuned by using input data with known output. A common learning algorithm used for training ANNs is error back-propagation algorithm. In the testing stage, the performance of the network over the data that is never presented to the network is evaluated.

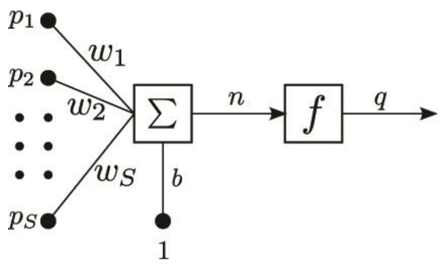


Fig. 1 A typical neuron

Table 1 Properties of database

Parameter	f_c (MPa)	ρ (%)	a/d	d (mm)	L_f/D_f	V_f (%)
Min.	20.6	0.99	2.5	126	45	0.22
Max.	68.4	5.72	5.0	910	133	2.00

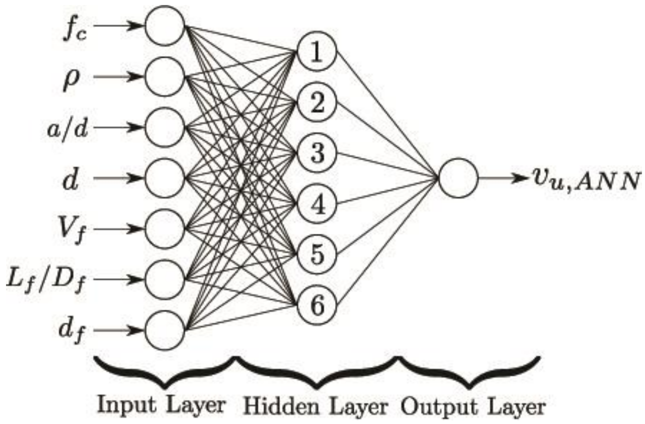


Fig. 2 The architecture of the ANN model

3.1 Experimental data

A database that will be used for developing an ANN model needs to be sufficiently large, accurate and evenly distributed so that the network can establish the relationships between the parameters involved and acquire the ability to deliver reliable predictions. A database was compiled by scanning experimental studies on the shear strength of SFRC slender beams without stirrups (Batson *et al*. 1972, Kadir and Saeed 1986, Mansur *et al*. 1986, Uomoto *et al*. 1986, Lim *et al*. 1987, Narayanan and Darwish 1987, Li *et al*. 1992, Swamy *et al*. 1993, Noghabai 2000, Kwak *et al*. 2002, Rosenbusch and Teutsch 2002, Dupont and Vandewalle 2003, Cucchiara *et al*. 2004, Parra-Montesinos 2006, Parra-Montesinos *et al*. 2006, Dinh *et al*. 2010, Ding *et al*. 2011, Aoude *et al*. 2012, Minelli and Plizzari 2013,). The database includes 129 rectangular beams with the ranges of parameters shown in Table 1, where f_c denotes the mean concrete compressive cylinder strength. d_f is taken as 0.79 for sheared, 0.83 for crimped, 0.89 for duoform, 0.91 for rounded, 0.92 for indented cut wire and 1.00 for hooked fibers (Yakoub 2011).

3.2 The model

A multi-layer feed-forward network consisting of an input layer, a hidden layer and an output layer was developed for predicting the shear strength of SFRC beams without stirrups by using MATLAB Neural Network Toolbox. The input layer consists of seven neurons receiving input parameters: f_c, ρ, a/d, d, V_f, L_f/D_f, d_f. The output layer is a single neuron delivering an estimate of shear strength $v_{u,ANN}$. The number of neurons in the hidden layer was determined to be six after carrying out simulations with models having a hidden layer of four to ten neurons. Also, simulations with models consisting of more than one hidden layer were conducted, but no improvement in the performance was observed. The network topology is shown in Fig. 2 schematically.

The net input function is the summation of weighted inputs with the bias, so that the output of a neuron is calculated as

$$q = f(n) = f\left(\sum_{i=1}^{S} w_i p_i + b\right) \cdot \tag{23}$$

The transfer functions of hidden and output layers are log-sigmoid and linear transfer functions given as $q=1/(1+e^{-n})$ and $q=n$, respectively.

It is possible for an ANN to memorize the data used for training the network but fail to generalize to new data, known as overfitting. In such a case, the error on the training set is very small, but it is likely to observe a large error on a set of data never presented to the network. MATLAB Neural Network Toolbox offers two methods - Bayesian regularization and early stopping technique- for improving generalization. The details of Bayesian regularization can be found elsewhere (Foresee and Hagan 1997). Early stopping technique was used for training the ANN model. For this purpose, the database was divided into two subsets as training and validation sets having 110 and 19 beams, respectively. Weights and biases were tuned using the training set according to the Levenberg-Marquardt back-propagation algorithm (Hagan *et al.* 1996*).* Early stopping technique consists of monitoring the errors on the training and validation sets simultaneously and stopping the training process when the validation set error starts to increase for a prescribed number of successive epochs. The performance function was determined to be the mean squared error (MSE) between the network outputs $\{v_{u,ANN}\}_i$ and the corresponding experimental values $\{v_{u,exp}\}_i$, $i=1,\ldots,N$, that is

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left(\{v_{u,\exp}\}_i - \{v_{u,ANN}\}_i\right)^2, \tag{24}$$

where N is the number of beams in the set. The errors on the training and validation sets monitored during the training process are plotted in Fig. 3. They are equal to 0.130 and 0.239, respectively, for the optimized network. ANN models resulting in much smaller errors were developed, but they were rejected due to overfitting issues. Once the network was trained, its performance on a test set consisting of 17 beams tested by Minelli *et al*. (2014) and Shoaib *et al*. (2014) was examined. It is to be noted that two of these beams have an effective depth of 1440 mm, which is far above the range used in training the network, and three of them have a concrete compressive strength of 80 MPa, which is outside the range used in training the network. Yet, the performance is satisfactory, where the error on the testing set is 0.533. The ANN model outputs against the corresponding experimental values for the training, validation and test sets are plotted in Fig. 4. The correlation coefficients (R) for the training, validation and test sets are 0.882, 0.848 and 0.812, respectively. The statistics of the $v_{u,ANN}/v_{u,exp}$ are given in Table 2. The mean, standard deviation (SD) and coefficient of variation (COV) of $v_{u,ANN}/v_{u,exp}$ for the whole database are 1.039, 0.177 and 0.171, respectively. A good agreement between the

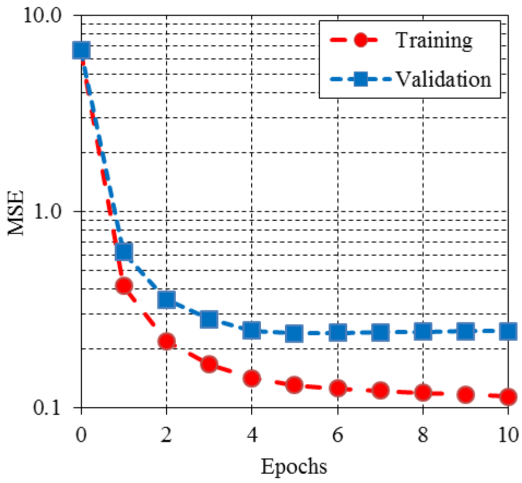
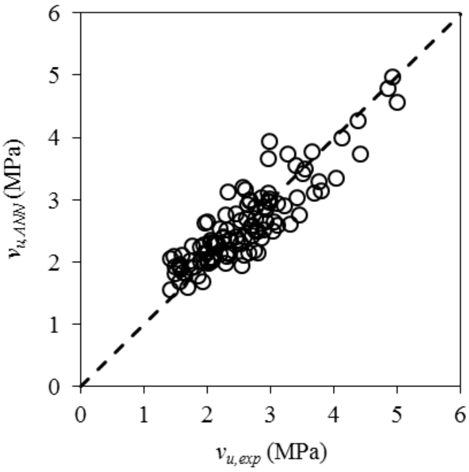


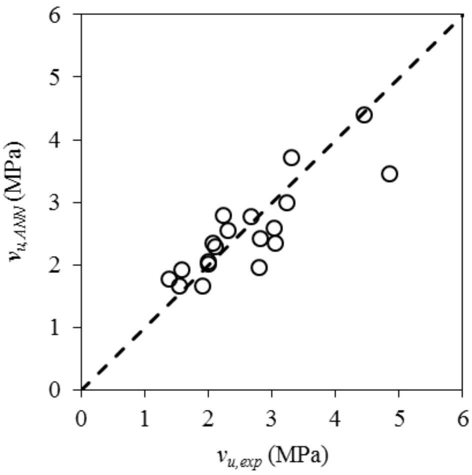
Fig. 3 Performance of the developed ANN model

Table 2 Statistics of $v_{u,ANN}/v_{u,exp}$

Set	Min.	Max.	Mean	SD	COV
Training	0.761	1.448	1.025	0.150	0.146
Validation	0.702	1.281	1.001	0.172	0.172
Test	0.762	1.701	1.171	0.276	0.236
All	0.702	1.701	1.039	0.177	0.171

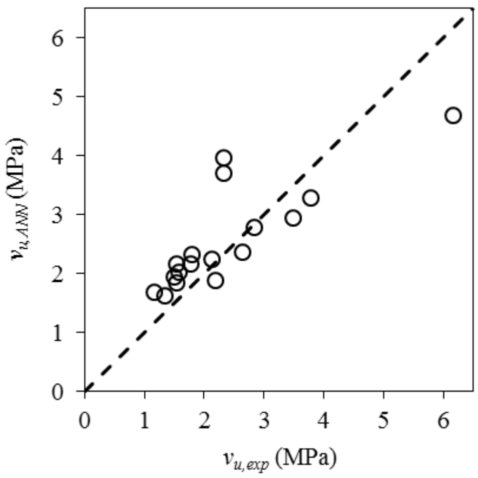


(a) Training set



(b) Validation set

Fig. 4 The ANN outputs vs. the experimental values



(c) Test set

Fig. 4 Continued

numerical and experimental results is observed through Fig. 4 and Table 2. The parameters of the ANN model are given in Appendix A.

4. Results and discussion

4.1 Comparison with the existing models

The ANN model was compared with fourteen equations developed for predicting the shear strength of SFRC beams by various researchers (Sharma 1986, Narayanan and Darwish 1987, Ashour *et al*. 1992, Swamy *et al*. 1993, Imam *et al*. 1994, Khuntia *et al*. 1999, Kwak *et al*. 2002, Dinh *et al*. 2011, Yakoub 2011, Gandomi *et al*. 2011, Kara 2013, Arslan 2014). The shear strength of each beam in the considered database, consisting of training, validation and test sets, was estimated through the equations given in Section 2. The predictions by the equations against the corresponding experimental values are plotted in Figs. 5-6. The statistics of the ratio of predictions to experimental values, correlation coefficients and errors for each model are given in Table 3, where MSE is calculated from Eq. (24) and relative absolute error (RAE) is calculated as

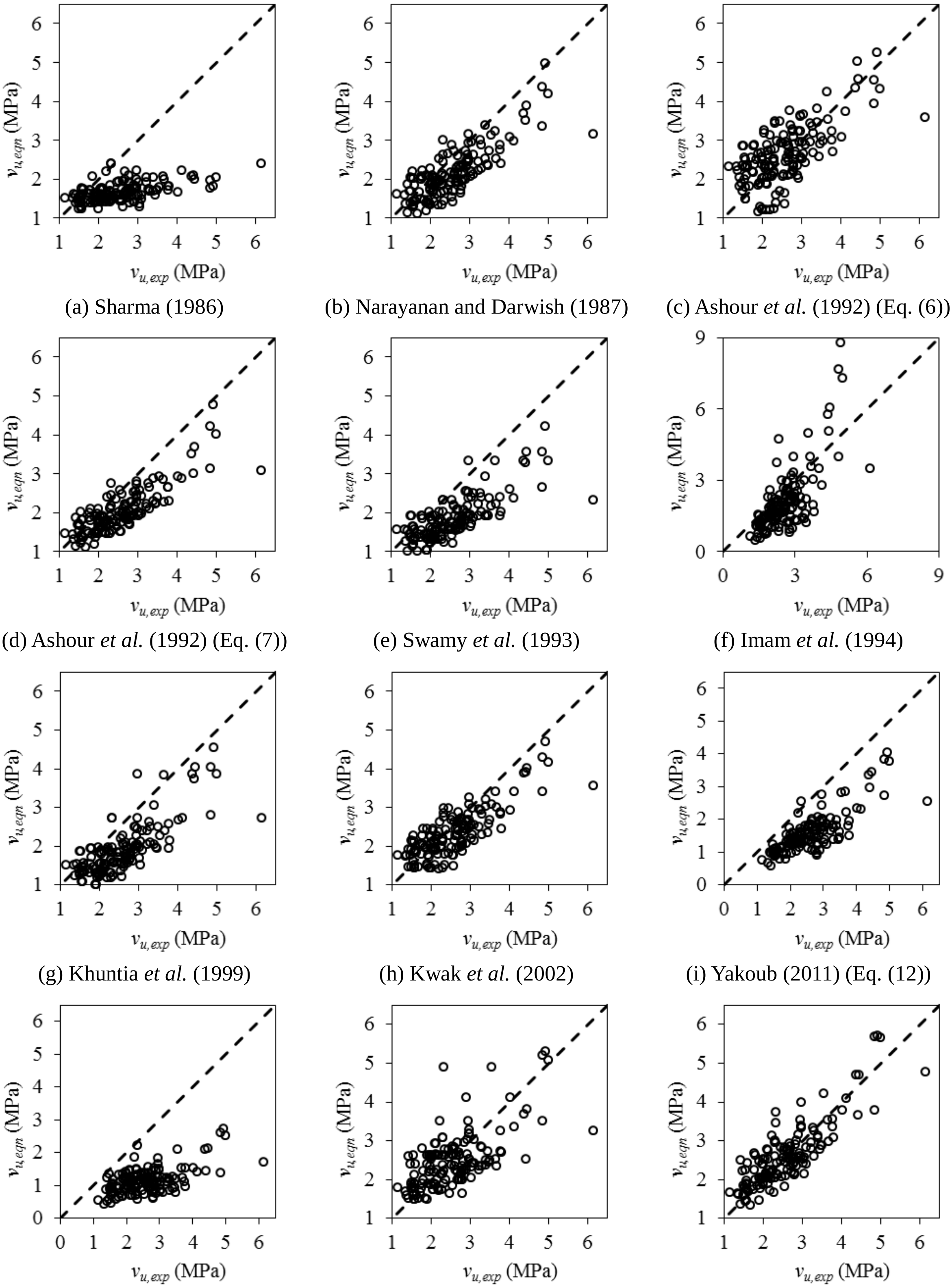
$$RAE = \frac{\sum_{i=1}^{N}\left|\left\{v_{u,\exp}\right\}_i - \left\{v_{u,eqn}\right\}_i\right|}{\sum_{i=1}^{N}\left|\left\{v_{u,\exp}\right\}_i - \bar{v}_u\right|}, \tag{25}$$

where

$$\bar{v}_u = \frac{1}{N}\sum_{i=1}^{N}\left\{v_{u,\exp}\right\}_i . \tag{26}$$

$v_{u,eqn}$ instead of $v_{u,ANN}$, and vice versa, are used in Eqs. (24) and 25, respectively, when necessary.

The equation of Sharma (1986), which is recommended by ACI Committee 544 (1988), delivers highly conservative predictions that are poorly correlated with the experimental values. The predictions obtained from the equations of



(a) Sharma (1986) (b) Narayanan and Darwish (1987) (c) Ashour *et al.* (1992) (Eq. (6))

(d) Ashour *et al.* (1992) (Eq. (7)) (e) Swamy *et al.* (1993) (f) Imam *et al.* (1994)

(g) Khuntia *et al.* (1999) (h) Kwak *et al.* (2002) (i) Yakoub (2011) (Eq. (12))

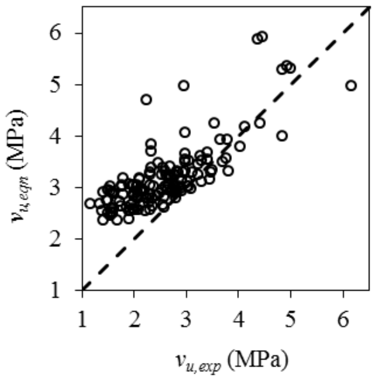
(j) Yakoub (2011) (Eq. (13)) (k) Dinh *et al*. (2011) (l) Arslan (2014)

Fig. 5 The predictions by various equations vs. the experimental values

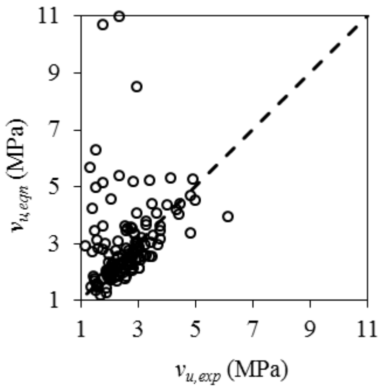
Narayanan and Darwish (1987), Ashour *et al*. (1992) (the revised version of Zsutty's equation (Zsutty 1971), Swamy *et al*. (1993) and Khuntia *et al*. (1999) are also conservative but the correlation between the predictions and the experimental values are better. Similarly, the equations of Yakoub (2011) deliver conservative predictions, where the ones obtained from the modified version of Bazant and Kim's equation (Bazant and Kim 1984) have a better

Table 3 Statistics of $v_{u,ANN}/v_{u,exp}$ and $v_{u,eqn}/v_{u,exp}$, correlation coefficients and errors

Model	Mean	SD	COV	R	MSE	RAE
The ANN model	1.039	0.177	0.171	0.857	0.191	0.515
Sharma (1986) (Eq. (1))	0.700	0.191	0.273	0.523	1.374	1.439
Narayanan and Darwish (1987) (Eq. (2))	0.860	0.183	0.212	0.810	0.425	0.821
Ashour *et al.* (1992) (Eq. (6))	1.068	0.295	0.276	0.684	0.420	0.823
Ashour *et al.* (1992) (Eq. (7))	0.813	0.156	0.192	0.834	0.528	0.924
Swamy *et al.* (1993) (Eq. (8))	0.742	0.168	0.226	0.763	0.833	1.177
Imam *et al.* (1994) (Eq. (9))	0.783	0.292	0.373	0.722	1.063	1.256
Khuntia *et al.* (1999) (Eq. (10))	0.747	0.171	0.229	0.779	0.759	1.147
Kwak *et al.* (2002) (Eq. (11))	0.940	0.197	0.210	0.807	0.314	0.682
Yakoub (2011) (Eq. (12))	0.611	0.125	0.205	0.804	1.276	1.573
Yakoub (2011) (Eq. (13))	0.469	0.144	0.307	0.544	2.524	2.206
Dinh *et al*. (2011) (Eq. (18))	1.008	0.264	0.261	0.638	0.469	0.815
Arslan (2014) (Eq. (19))	1.030	0.198	0.192	0.838	0.224	0.576
Gandomi *et al.* (2011) (Eq. (21))	1.307	0.290	0.222	0.785	0.644	1.036
Kara (2013) (Eq. (22))	1.198	0.760	0.635	0.286	2.263	1.172

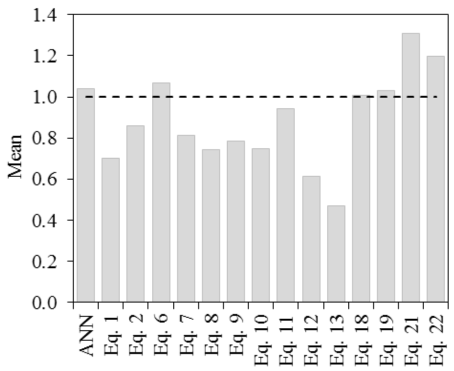


(a) Gandomi *et al.* (2011)

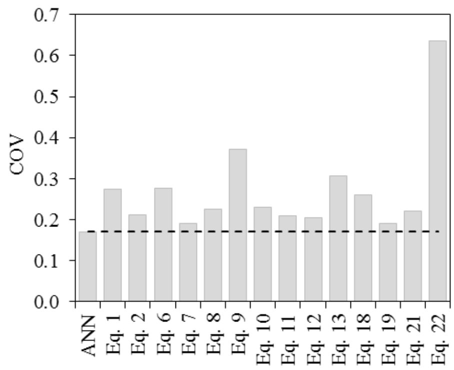


(b) Kara (2013)

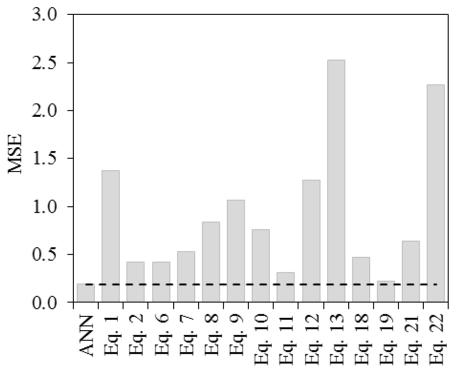
Fig. 6 The predictions by various equations vs. the experimental values



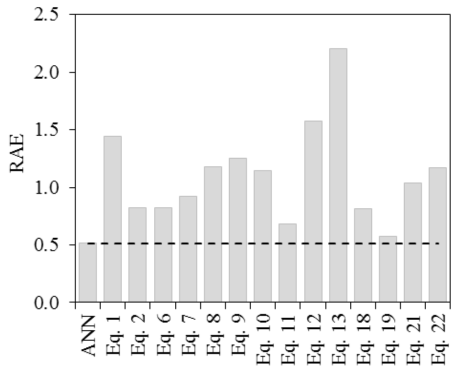
(a) Mean



(b) COV



(c) MSE



(d) RAE

Fig. 7 Mean and COV of $v_{u,ANN}/v_{u,exp}$ and $v_{u,eqn}/v_{u,exp}$, MSE and RAE for each model correlation with the experimental values. On the other hand, the predictions delivered by the equation of Ashour *et al*. (1992) (the revised version of ACI 318-11's equation (ACI Committee 318 (2011)) are scattered above and below the experimental values. The equation of Imam *et al*. (1994) delivers predictions far above the experimental values for some beams. The predictions from the equation proposed by Dinh *et al*. (2011) are largely scattered. Two equations derived by using genetic programming are also considered. The equation of Gandomi *et al*. (2011) delivers predictions greater than the experimental values. The equation of Kara (2013) delivers better predictions compared to the equation of Gandomi *et al*. (2011) except for few beams.

The mean value of the ratio of the predictions obtained from the equation of Dinh *et al*. (2011) to the experimental values is almost equal to 1.0, however the predictions are so scattered that the COV of the ratio is 53% greater than that for the ANN model. Among the considered equations, it can be observed from Table 3 and Fig. 7 that the ones proposed by Kwak *et al*. (2002) and Arslan (2014) have better performances than the others do, where the equation of Kwak *et al*. (2002) delivers more conservative predictions. The mean value of the ratio of the predictions by the equation of Arslan (2014) to the experimental values is almost the same as that for the ANN model, but the COV of the ratio is 13% greater than that for the ANN model. The MSE and RAE for the predictions obtained from the equation of Arslan (2014) are 17% and 12% greater than those for the predictions delivered by the ANN model, respectively.

It can be seen from Table 3 and Fig. 7 that the ANN model has the smallest COV of the prediction to experimental value ratio, the largest R, and the smallest MSE and RAE, so it is deduced that it is superior to the considered equations over the considered database.

4.2 Parametric study

It is mentioned in Section 3.2 that early stopping technique was used to develop an ANN model with a good generalization capability. A parametric study was conducted to verify the generalization capability of the developed ANN model. It also enables to observe the effect of considered parameters on the shear strength of SFRC beams. The ranges of parameters used in the parametric study are consistent with those of the experimental database (Table 1).

The change in the shear strength against the volume fraction of fibers for various values of aspect ratio of fibers is plotted in Fig. 8, where concrete compressive strength is 40 MPa, shear span-to-depth ratio is 3, tensile reinforcement ratio is 1.5%, effective depth is 400 mm and hooked fibers are used. It is observed that the shear strength increases with the volume fraction of fibers. For all values of aspect ratio of fibers considered, increasing the volume fraction of fibers from 0.25% to 2.00% almost doubles the shear strength. The shear strength for the volume fraction of fibers equal to 2.00% is 2.1, 2.1, 1.98 and 1.86 times the shear strength for the volume fraction of fibers equal to 0.25% for the aspect ratio of fibers equal to 50, 75, 100 and 133, respectively.

The change in the shear strength against the aspect ratio of fibers for various values of volume fraction of fibers is plotted in Fig. 9, where concrete compressive strength is 40 MPa, shear span-to-depth ratio is 3, tensile reinforcement ratio is 1.5%, effective depth is 400 mm and hooked fibers are used. An increasing relationship between the shear strength and the aspect ratio of fibers is observed. Increasing the aspect ratio of fibers from 45 to 133 increases the shear strength 1.40, 1.35, 1.31, 1.29 and 1.23 times for the volume fraction of fibers equal to 0.25%, 0.50%, 1.00%, 1.50 % and 2.00%, respectively.

The effect of concrete compressive strength on the shear strength for various values of volume fraction of fibers is depicted in Fig. 10, where shear span-to-depth ratio is 3, tensile reinforcement ratio is 1.5%, effective depth is 400 mm, aspect ratio of fibers is 100 and hooked fibers are used. The shear strength is observed to increase with the concrete compressive strength. The increase in the shear strength resulting from the change in the concrete compressive strength from 25 MPa to 65 MPa is 74%, 72%, 63%, 51% and 37% for the volume fraction of fibers equal to 0.25%, 0.50%, 1.00%, 1.50 % and 2.00%, respectively.

Fig. 11 shows the size effect on the shear strength by plotting the change in the shear strength against the

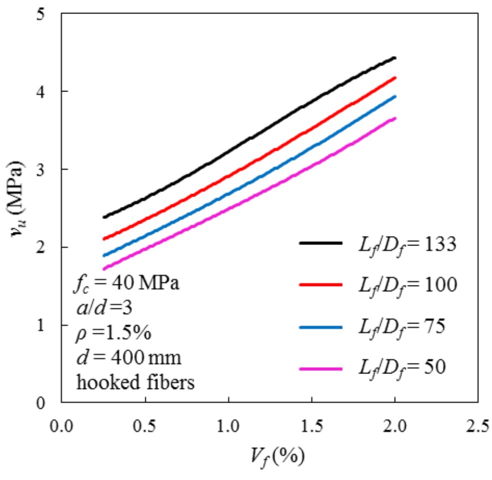


Fig. 8 Shear strength vs. volume fraction of fibers

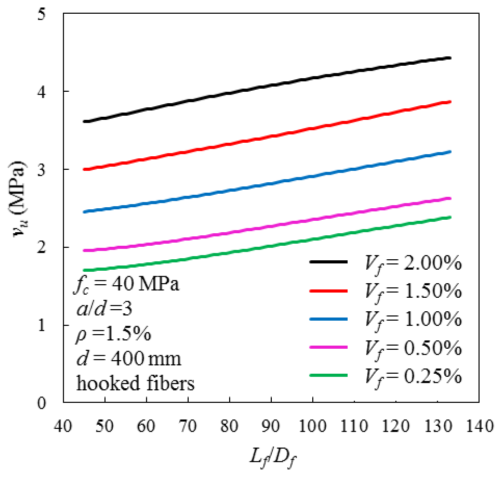


Fig. 9 Shear strength vs. aspect ratio of fibers

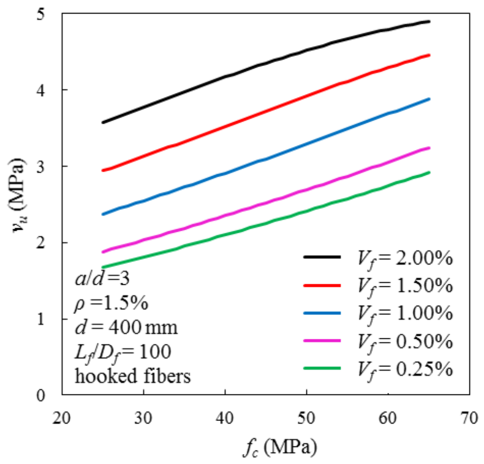


Fig. 10 Shear strength vs. concrete compressive strength

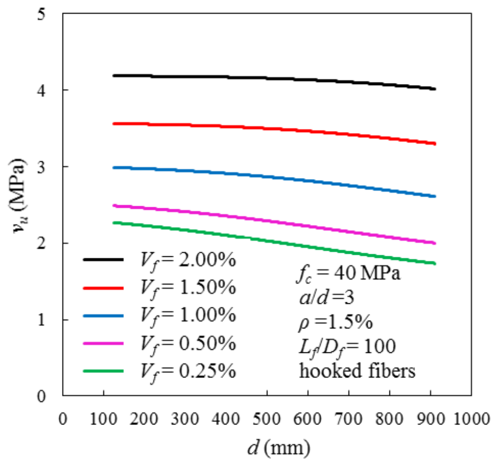


Fig. 11 Shear strength vs. effective depth

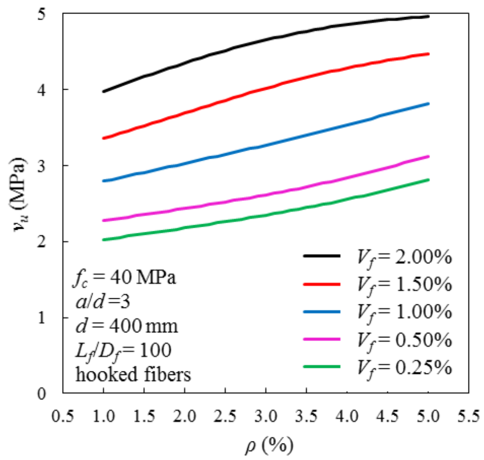


Fig. 12 Shear strength vs. tensile reinforcement ratio

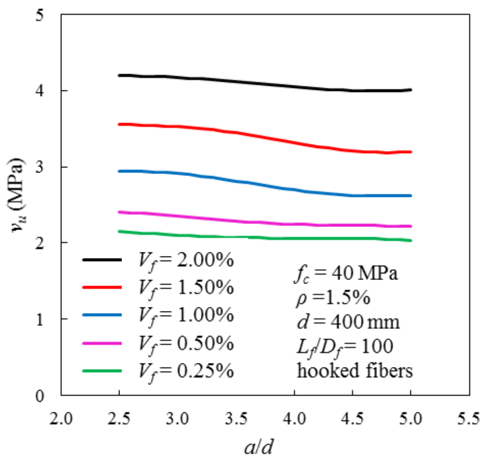


Fig. 13 Shear strength vs. shear span-to-depth ratio

effective depth for various values of volume fraction of fibers, where concrete compressive strength is 40 MPa, shear span-to-depth is 3, tensile reinforcement ratio is 1.5%, aspect ratio of fibers is 100 and hooked fibers are used. It is observed that the reduction in the shear strength due to the increase in the effective depth decreases with the increasing volume fraction of fibers. In other words, the size effect gets less prominent as the volume fraction of fibers increases. The decrease in the shear strength due to the increase in the effective depth from 126 mm to 910 mm is 23%, 20%, 13%, 7% and 4% for the volume fraction of fibers equal to 0.25%, 0.50%, 1.00%, 1.50 % and 2.00%, respectively.

The change in the shear strength against the tensile reinforcement ratio is plotted in Fig. 12 for various values of volume fraction of fibers, where concrete compressive strength is 40 MPa, shear span-to-depth is 3, effective depth is 400 mm, aspect ratio of fibers is 100 and hooked fibers are used. The shear strength is observed to increase with the tensile reinforcement ratio. The increase in the shear strength with an increase in the tensile reinforcement ratio from 1.0% to 5.0% is 39%, 37%, 37%, 33% and 25% for the volume fraction of fibers equal to 0.25%, 0.50%, 1.00%, 1.50 % and 2.00%, respectively.

The effect of shear span-to-depth ratio on the shear strength is shown in Fig. 13, where concrete compressive strength is 40 MPa, tensile reinforcement ratio is 1.5%, effective depth is 400 mm, aspect ratio of fibers is 100 and hooked fibers are used. There is no significant effect of shear span-to-depth ratio observed within the considered range. The decrease in the shear strength with the increase in the shear span-to-depth ratio from 2.5 to 5.0 is less than 11% for the considered volume fractions of fibers.

The effects of volume fraction of fibers, aspect ratio of fibers, concrete compressive strength, effective depth, tensile reinforcement ratio and shear span-to-depth ratio are depicted in Figs. 8-13, respectively. The trends observed through the parametric study are consistent with the behavior of SFRC beams, so that it can be inferred that the developed ANN model is capable of generalizing to data never presented to the network. On the other hand, it should be noted that the data used for developing the ANN model is limited even though all data available to the author is used.

5. Conclusions

An ANN model was developed for predicting the shear strength of SFRC slender beams without stirrups. A database of 146 rectangular beams, which is larger than the ones used in similar studies conducted previously, was used for this purpose. The network consists of an input layer of seven neurons receiving concrete compressive strength, tensile reinforcement ratio, shear span-to-depth ratio, effective depth, volume fraction of fibers, aspect ratio of fibers and fiber bond factor as input parameters, a hidden layer of six neurons and an output layer of a single neuron delivering an estimate of shear strength. It is shown that the developed ANN model is superior to fourteen equations proposed by various researchers in predicting the shear strength of SFRC slender beams without stirrups included in the database. Among the considered equations, the one proposed by Arslan (2014) exhibit the best performance in predicting the shear strength of SFRC beams in the database. The mean value of the ratios of the predictions by the equation of Arslan (2014) and those delivered by the ANN model to the experimental values is almost the same. However, the COV of the ratio for the equation of Arslan (2014) is 13% greater than that for the ANN model, and the MSE and RAE for the predictions by the equation of Arslan (2014) are 17% and 12% greater than those for the predictions of the ANN model, respectively.

It is verified through a parametric study that the ANN model has a good generalization capability, that is, it can generalize to data never presented to the network previously. It is to be noted that the size of the database used to develop the ANN model is limited even though all data available to the author is used. Besides, it is observed through the parametric study that the shear strength of SFRC beams without stirrups increases with concrete compressive strength, tensile reinforcement ratio, volume fraction of fibers and aspect ratio of fibers, and decreases with effective depth within the considered ranges. Moreover, it is also observed that the effects of aspect ratio of fibers, concrete compressive strength, effective depth and tensile reinforcement ratio on the shear strength get less pronounced as the volume fraction of fibers increases.

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CC

Appendix: Parameters of the ANN model

The weights and biases of the developed ANN model are given in Tables A1, A2 and A3 in order to make it possible for anyone to use the developed ANN model. It is to be noted that the input parameters should be mapped to the interval [-1,1] with respect to the ranges of input parameters given in Table 1 and the result delivered by the ANN model should be back-mapped with respect to the output interval [1.38,5.00] (MPa).

Table A1 Weights between neurons of input and hidden layers

Hidden Layer	Input Layer						
Neuron #	f_c	ρ	a/d	d	V_f	L_f/D_f	d_f
1	-1.0578	-0.8675	-1.6440	-0.5999	0.3517	2.4332	1.4182
2	1.3684	1.1581	-0.0847	0.1158	0.6547	1.4481	0.8978
3	0.1271	2.0005	-1.1431	-0.3284	-1.6736	1.8750	-1.8363
4	-0.9364	1.4208	2.2779	-0.1749	0.9154	-1.8558	0.7072
5	1.4214	1.5609	1.1416	-1.1337	0.3897	1.0727	1.3446
6	-0.3214	0.1469	-2.4667	0.0289	0.2326	-1.5449	1.9248

Table A2 Weights between neurons of hidden and output layers

Hidden Layer Neuron #	Output Neuron
1	0.4662
2	1.2384
3	0.1950
4	-0.4004
5	0.6467
6	-0.5714

Table A3 Biases of neurons of hidden and output layers

Layer	Neuron #	Bias
Hidden	1	3.6023
Hidden	2	-0.9493
Hidden	3	0.8790
Hidden	4	-0.0739
Hidden	5	2.6455
Hidden	6	3.7652
Output	1	-0.7239