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Expected extreme value of pounding force between two adjacent buildings

Sepideh Rahimi[a] and Masoud Soltani*

Faculty of Civil and Environmental Engineering, Tarbiat Modares University, Jalaale-al Ahmad Ave., Tehran, Iran

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Abstract. Seismic pounding between adjacent buildings with inadequate separation and different dynamic characteristics can cause severe damage to the colliding buildings. Efficient estimation of the maximum pounding force is required to control the extent of damage in adjacent structures or develop an appropriate mitigation method. In this paper, an analytical approach on the basis of statistical relations is presented for approximate computation of extreme value of pounding force between two adjacent structures with equal or unequal heights subjected to stationary and non-stationary excitations. The nonlinearity of adjacent structures is considered using Bouc-Wen model of hysteresis and the pounding effect is simulated by applying the nonlinear viscoelastic model. It is shown that the proposed approach can significantly save computational costs by obviating the need for performing dynamic analysis. To assess the reliability and accuracy of the proposed approach, the results are compared with those obtained from nonlinear dynamic analysis.

Keywords: pounding force; probability distribution; random vibration; Bouc-Wen; adjacent building

1. Introduction

Adjacent buildings with different dynamic properties and inadequate gap distance are highly susceptible for experiencing non-phase vibration during the earthquake. This is why the pounding is too probable in such cases. Previous investigations on seismic responses of adjacent buildings have clearly shown the vast damages resulted from the pounding effects (Jankowski. 2008, Muthukumar and DesRoches 2006, Favvata *et al*. 2009, Cole *et al*. 2010, Efraimiadou *et al*. 2012, Jankowski and Mahmoud 2015). Although the newly built structures are indispensable to meet the code-based requirements for developing the gap distance, existing adjacent buildings may disregard these requirements. However, construction process of newly built structures and common difficulties may cause unreliable and inexact developed distance between adjacent buildings. Therefore, different construction details may be considered for preventing the occurrence of contact of adjacent buildings and/or reducing its intensity. Connecting the adjacent buildings with a beam (Westermo 1989), adopting energy-absorbing materials for filling the gap distance (Anagnostopoulos 1996), application of MR dampers in order to reduce the seismic pounding effect of base-isolated multi-span RC highway bridges (Sheikh 2012), using the optimally tuned mass dampers to decrease the displacement vibrations of adjacent structures (Negdeli and Bekdas 2014) and linking the two structures with fluid-viscous dissipaters (Licari *et al*. 2015) are from these details.

Since the seismic pounding generally causes the increase in the developed story shear force (Naserkhaki 2012), adopting a numerical approach for estimating the probability of pounding and its maximum induced force and intensity, is of interest. Having such an approach, probable damages due to impact could be controlled and effective construction methods for reducing the pounding force would be investigated. In this regard, Jankowski *et al.* proposed the concept of force response spectrum in order to evaluate the pounding force between two adjacent Single-degree-of-freedom (SDOF) systems (Jankowski 2006). The research which can be regarded as a development of the approach proposed by Ruangrassamee and Kawashima (2001) in determining the relative displacement response spectrum, and utilized the linear visco-elastic model for simulating the seismic induced impact. Jalili and Yaghmaei (2012) investigated the pounding force response spectra for elastic SDOF systems due to near-field and far-field ground motions. They evaluated the influence of effective structural parameters on pounding force, including mass, stiffness and damping ratio of neighboring structures and gap distance between them. Regarding the results, evaluation of the pounding force is highly related to the earthquake characteristics as well.

Despite of valuable researches in this field, development of elastic response spectrum for the impact force is the only reliable outcomes of existing approaches which has been fundamentally developed by adopting linear SDOF models for adjacent buildings under specific numbers of earthquake records. As the amount of the seismic pounding force highly depends on dynamic properties, nonlinearity of structures, gap distance, and specially the uncertainties of earthquake records, it is importance to consider these features in evaluating the pounding effect and its intensity. Most accurate approach for investigating all these features is

*Corresponding author, Associate Professor
E-mail: msoltani@modares.ac.ir
[a]Ph.D., E-mail: se.rahimi@modares.ac.ir

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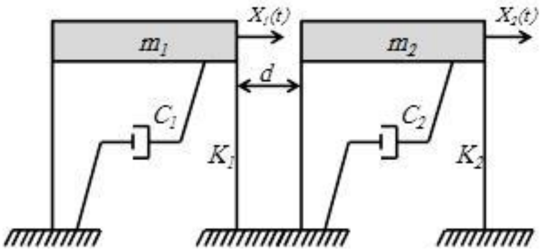


Fig. 1 Model of interacting structure with equal heights

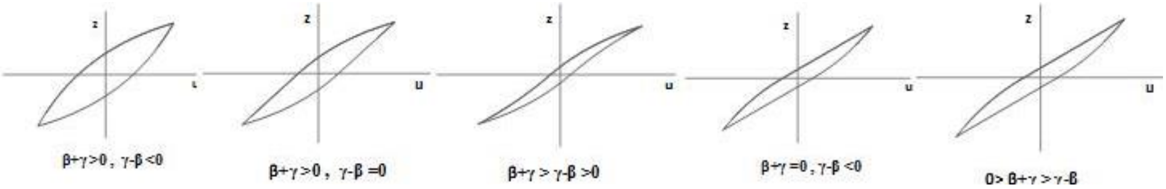


Fig. 2 Classic Bouc-Wen models for different values of β, γ

nonlinear time history analysis which must be conducted for several earthquake seismic records with different characteristics. Of course this is an expensive and time consuming approach. Therefore, it is appreciated if - instead- a reliable numerical approach could be introduced in a closed form expression and efficient algorithm.

In this paper, a closed-form numerical solution method is proposed in a step-by-step algorithm to determine the maximum pounding force developed between two adjacent buildings. A probability based approach is utilized to consider the uncertainties of the earthquake features, and the hysteresis Bouc-Wen model is adopted for simulating the nonlinear behavior of buildings considering most of the effective properties.

The proposed approach is presented in a step-by-step calculation process and the details of each step are specifically discussed in reference to the basics and assumptions. Development of a closed-form expression for impact force between two nonlinear structural systems, on the basis of linearization of complex pounding phenomenon, is the outstanding innovation of this research which is not previously reported. The proposed approach is extended and validated for stationary and non-stationary excitations with specific spectral densities. The results show reliable agreements with the expected values.

2. Basic assumptions of the proposed method

Adjacent structures “1” and “2” are modeled as classic Bouc-Wen SDOF systems (Fig. 1). The Bouc-Wen (BW) model, is a phenomenological based model used to describe different hysteretic characteristics of nonlinear systems (Wen 1976). With this mode it is possible to capture a wide range of different hysteretic cycle shapes. Therefore, using this model, variety of behavioral models can be investigated for the nonlinear behavior of adjacent systems. The differential equation of the model is as follows

$$m\ddot{x}(t) + c\dot{x}(t) + \alpha k x(t) + (1-\alpha)kz(t) = P(t) \tag{1}$$

$$\dot{z} = A\dot{x} - \beta|\dot{x}||z|^{n-1}z - \gamma\,\dot{x}|z|^{n} \tag{2}$$

Eq. (1) is a differential equation of motion for a SDOF system including: stiffness (k), mass (m), damping ratio (c), and ratio of the stiffness after yielding to the stiffness before yielding (α). The hysteretic variable z is a fictitious displacement related to the actual displacement, x. Changes in parameters such as n, β, γ and A will tend to produce different models of hysteresis (Fig. 2).

Herein, a nonlinear viscoelastic model is used for simulating the interacting structures (Jankowski 2005). If the displacement response processes of the SDOF systems “1” and “2” are denoted by $x_1(t)$ and $x_2(t)$ respectively and the separation distance between them by d, the value of pounding force from nonlinear viscoelastic model is determined by

$$\begin{cases} F(t) = 0 & for \quad \delta(t) \le 0 \\ \quad (no\,contact) & \\ F(t) = \bar{\beta}\,\delta^{\frac{3}{2}}(t) + \bar{c}\,\dot{\delta}(t) & for \quad \delta(t) > 0 \quad and \quad \dot{\delta}(t) > 0 \\ \quad (contact - approach\ period) & \\ F(t) = \bar{\beta}\,\delta^{\frac{3}{2}}(t) & for \quad \delta(t) > 0 \quad and \quad \dot{\delta}(t) \le 0 \\ \quad (contact - restitution\ period) & \end{cases} \tag{3}$$
$$\begin{cases} \delta(t) = x_1(t) - x_2(t) - d \\ \dot{\delta}(t) = \dot{x}_1(t) - \dot{x}_2(t) \end{cases}$$

where $\bar{\beta}$ is the impact stiffness parameter that depends on the material properties of the pounding structures and the geometry of the contact area, and $\bar{c}(t)$ is the impact element’s damping, which can be obtained at any instant of time from following equation (Jankowski 2005)

$$\bar{c}(t) = 2\bar{\xi}\sqrt{\bar{\beta}\sqrt{\delta(t)}\left(\frac{m_1 m_2}{m_1 + m_2}\right)} \tag{4}$$

where, m_1, m_2 are masses of systems “1” and “2” respectively, $\bar{\xi}$ denotes an impact damping ratio correlated with a coefficient of restitution, and e, accounts for the energy dissipation during pounding. The approximate relation between $\bar{\xi}$ and e in the non-linear viscoelastic model is expressed by the following equation (Jankowski 2005)

$$\bar{\xi} = \frac{9\sqrt{5}}{2}\frac{(1-e^2)}{e(e(9\pi-16)+16)} \tag{5}$$

where the coefficient of restitution, e, is defined as the ratio of separation relative velocity of the bodies after impact, $\dot{\delta}_f$, to their approaching relative velocity before impact ($\dot{\delta}_0$) (Jankowski 2005)

$$e = \frac{\dot{\delta}_f}{\dot{\delta}_0} \tag{6}$$

The artificial seismic excitation is considered as a Gaussian, zero mean stationary random process, $\ddot{u}_g(t)$. These earthquake motions are generated from Kanai-Tajimi power spectral density function, PSDF, of alluvial terrain. The power spectrum of Kanai-Tajimi is expressed as

$$S_g(\omega) = \frac{\omega_g^4 + 4\xi_g^2\,\omega_g^2\,\omega^2}{(\omega_g^2-\omega^2)^2 + 4\xi_g^2\,\omega_g^2\,\omega^2} G_0 \tag{7}$$

where ω_g and ξ_g are the ground damping ratio and frequency respectively and G_0 is the constant power spectral intensity of the bed rock excitation. 0.34, 65.03, 27.02 are respectively considered for ξ_g, G_0 (cm^2/sec^3), and ω_g (rad/sec).

The Kanai-Tajimi power spectral density, adopted in this research, is shown in Fig. 3. It should be noted that the approach presented in this paper is independent of the records and their compatible spectrum. The spectral density model of Kanai-Tajimi -used for generating the required accelerograms- was selected just because it is widely used to express PSDF of earthquake ground acceleration, by the way any other functions can be adopted for spectral density stimulation.

To simulate the non-stationary character of real ground motions, the excitation is modified by

$$\ddot{U}_g(t) = A(t)\;\; \ddot{u}_g(t) \tag{8}$$

where $A(t)$ is a deterministic, modulating function and $\ddot{u}_g(t)$ is a stationary random process. The modulating function employed in this study is that of Shinozuka and Sato, given by

$$A(t) = \frac{1}{C}(e^{-B_1 t} - e^{-B_2 t}) \;\;,\;\; C = \max(e^{-B_1 t} - e^{-B_2 t}) \tag{9}$$

where B_1 and B_2 are constants equal to 0.085 and 0.17, respectively for a long-duration earthquake.

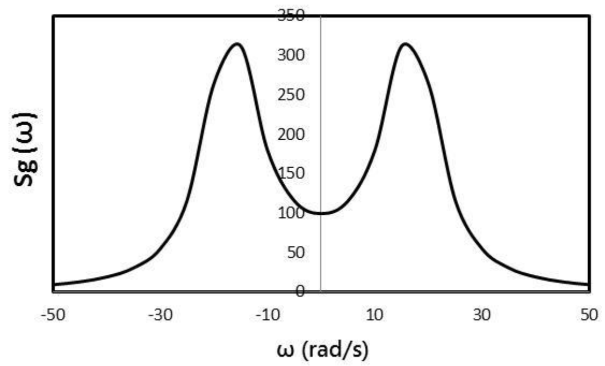


Fig. 3 Kanai-Tajimi power spectral density

3. Calculation of pounding force

The dynamic equations of motion including the pounding force during impact between two adjacent SDOF systems (Fig. 1), can be written as follows

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{Bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{Bmatrix}$$
$$\begin{bmatrix} \alpha_1 k_1 & 0 \\ 0 & \alpha_2 k_2 \end{bmatrix}\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} +$$
$$\begin{bmatrix} (1-\alpha_1)k_1 & 0 \\ 0 & (1-\alpha_2)k_2 \end{bmatrix}\begin{Bmatrix} z_1(t) \\ z_2(t) \end{Bmatrix} + \begin{Bmatrix} F(t) \\ -F(t) \end{Bmatrix} \tag{10}$$
$$= -\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_{g1}(t) \\ \ddot{x}_{g2}(t) \end{Bmatrix}$$

In this system of equations, $x_i(t), \dot{x}_i(t), \ddot{x}_i(t), C_i$ and K_i are the values of horizontal displacement, velocity, acceleration, damping coefficient and stiffness for structures 1, 2 respectively (i=1, 2). Moreover, $\ddot{x}_{gi}(t)$ denotes the input ground acceleration and $F(t)$ is the pounding force. The time history of pounding force can be obtained by numerical solving of the above equations of motion and its extreme value, $F_{\max}$, is obtained by

$$F_{\max} = \max_t |F(t)| \tag{11}$$

The amount of seismic pounding force highly depends on characteristics of earthquake records. To consider record to record uncertainties, these nonlinear time history analyses must be conducted for several earthquake seismic records which are, of course, time consuming. Therefore, it is appreciated if -instead- a reliable numerical approach could be introduced in a closed form expression and efficient algorithm. This is the main aim of this research which is presented in following sections.

4. Description of the proposed method

The maximum pounding force between two dynamic systems based on the nonlinear viscoelastic model, is equal to

$$F_{\max} = \bar{\beta}\delta_{\max}^{\frac{3}{2}} \tag{12}$$

where, $\delta_{\max}$ is the maximum deformation, which is calculated by equating the loss in the kinetic energy with the loss of energy through the work done by the damping force and is equal to

$$\delta_{\max} = \left(\frac{5}{4} \frac{m_1 m_2 (\dot{\delta}_f)^2}{(m_1 + m_2)\overline{\beta}} \right)^{\frac{2}{5}} \tag{13}$$

The maximum deformation ($\delta_{\max}$) depends on the relative velocity after pounding ($\dot{\delta}_f$) and according to Eq. (6), the velocity after pounding is a coefficient of relative velocity before pounding. Regarding the constancy of all parameters in Eq. (13) except relative velocity, it is proposed to determine the expected extreme value of relative velocity before the pounding to calculate the expected extreme values of maximum deformation and then, the pounding force based on Eq. (12). Thus, the expected extreme value of pounding force is equal to

$$F_e = \overline{\beta} \delta_e^{\frac{3}{2}} = \overline{\beta} \left(\frac{5}{4} \frac{m_1 m_2 (e\dot{\delta}_e)^2}{(m_1 + m_2)\overline{\beta}} \right)^{\frac{3}{5}} \tag{14}$$

where $\dot{\delta}_e$ is the expected extreme value of the relative velocity before pounding. To calculate the expected extreme value of the relative velocity before pounding, adjacent structural systems "1" and "2" (Fig. 1) are considered. The relative velocity response process $V_{rel}(t)$ is given by

$$V_{rel}(t) = V_1(t) - V_2(t) \tag{15}$$

and the expected value of the square of relative velocity is

$$\begin{aligned} \sigma^2_{V_{rel}} &= E\left\{V^2_{rel}(t)\right\} = E\left\{[V_1(t) - V_2(t)]^2\right\} \\ &= E\left\{V_1^2(t)\right\} + E\left\{V_2^2(t)\right\} - 2E\left\{V_1(t) V_2(t)\right\} \\ &= \sigma^2_{V_1} + \sigma^2_{V_2} - 2E\left\{V_1(t) V_2(t)\right\} \end{aligned} \tag{16}$$

where $E\{\}$ is the expected value and $\sigma^2_{v_1}$ and $\sigma^2_{v_2}$ are the standard deviations of velocity of systems "1" and "2", The standard deviation of the velocity of the system "i" is equal to

$$\begin{aligned} {\sigma_{V_i}}^2 &= E[{V_i}^2] = \int_{-\infty}^{+\infty} \omega^2 S_{xx}(\omega)\, d\omega \\ &= \int_{-\infty}^{+\infty} \omega^2 \,|H_i(\omega)|^2\; S_g(\omega)\, d\omega \end{aligned} \tag{17}$$

and the covariance of (V_1, V_2) is defined as

$$E\left\{V_1(t) V_2(t)\right\} = \int_{-\infty}^{+\infty} \omega^2 H_1(\omega) S_g(\omega) H_2(\omega)^*\, d\omega \tag{18}$$

In this equation, $H(\omega)$ is the frequency response function and $S_g(\omega)$ is input excitation density function (Eq. (7)). Frequency response function of the system has to be detected using the system's equation of motions (Eq. (1)-(2)), which are initially partial derivative equations. These equations are mainly nonlinear with no exact closed form solution. In some cases, approximate solutions are derived by simple methods such as linearization method. For this purpose, the governing differential equations of the BW model have to be converted into linear form. The parameters for the equivalent linear classic BW are introduced by minimizing least square error as shown below. An equivalent linear equation will be looked for in the form of

$$\dot{z} = c_e\, \dot{x} + k_e\, z \tag{19}$$

Corresponding values for participating coefficient are

$$c_e = A - \beta F_1 - \gamma F_2 \;,\; k_e = -\beta F_3 - \gamma F_4 \tag{20}$$

where functions F_i, i=1, 2, 3, 4 are given by

$$\begin{aligned} F_1 &= \frac{\sigma_z^n}{\pi} \Gamma\left(\frac{n+2}{2}\right) 2^{n/2} I_s \;,\; F_2 = \frac{\sigma_z^n}{\sqrt{\pi}} \Gamma\left(\frac{n+1}{2}\right) 2^{n/2}, \\ F_3 &= \frac{n\,\sigma_{\dot{x}}\, \sigma_z^{n-1}}{\pi} \Gamma\left(\frac{n+2}{2}\right) 2^{n/2} \\ &\quad \times \left\{ \frac{2(1-\rho^2_{\dot{x}z})^{(n+1)/2}}{n} + \rho_{\dot{x}z} I_s \right\} \\ F_4 &= \frac{n}{\sqrt{\pi}} \rho_{\dot{x}z} \sigma_{\dot{x}}\, \sigma_z^{n-1} \Gamma\left(\frac{n+1}{2}\right) 2^{n/2}, \\ I_s &= 2\int_l^{\pi/2} \sin^n\theta\, d\theta \;,\; l = \tan^{-1}\left(\frac{\sqrt{1-\rho^2_{\dot{x}z}}}{\rho^2_{\dot{x}z}} \right) \\ \sigma_z &= \sqrt{E[z^2]},\; \sigma_{\dot{x}} = \sqrt{E[\dot{x}^2]},\; \rho_{\dot{u}z} = \frac{E[\dot{x}z]}{\sqrt{E[\dot{x}^2]\; E[z^2]}} \end{aligned} \tag{21}$$

Based on the linear coefficients, the frequency response function for the linear BW is

$$|H(\omega)|^2 = \frac{1}{\left(-\omega^2 + \alpha{\omega_0}^2 + \frac{(1-\alpha)\omega_0^2 \omega^2 c_e}{\omega^2 + k_e^2} \right)^2 + \left(2\xi\omega_0\omega - \frac{(1-\alpha)\omega_0^2 \omega c_e k_e}{\omega^2 + k_e^2} \right)^2} \tag{22}$$

In Eq. (22), ω_0 and ξ are natural frequency and damping coefficients of the Bouc-Wen model, respectively. With using the frequency response function, the variance of relative velocity is computable (Eq. (16)). For a zero-mean stationary Gaussian process $V_{rel}(t)$, Davenport has shown that the expected extreme-values of relative velocity, $\dot{\delta}_e$, is given by the following approximate relation

$$\dot{\delta}_e = \left((2\ln(\upsilon T))^{0.5} + \frac{\gamma}{(2\ln(\upsilon T))^{0.5}}\right) \sigma_{V\,rel} \tag{23}$$

$$\upsilon = \frac{\sigma_{V_{rel}}}{\pi\, \sigma_{X_{rel}}} \;,\; \gamma = 0.5772 \;,\; T = time\;\; duration \tag{24}$$

where, $\sigma_{X_{rel}}$ is equal to

$$\begin{aligned} \sigma^2_{X_{rel}} &= \sigma^2_{X_1} + \sigma^2_{X_2} - 2E\left\{X_1(t)\, X_2(t)\right\} \\ {\sigma_X}^2 &= \int_{-\infty}^{+\infty} |H_i(\omega)|^2\; S_g(\omega)\, d\omega \\ E\left\{X_1(t)\, X_2(t)\right\} &= \int_{-\infty}^{+\infty} H_1(\omega) S_g(\omega) H_2(\omega)^*\, d\omega \end{aligned} \tag{25}$$

When a modulating function is introduced in connection with the Kanai-Tajimi filter, the non-stationary model of seismic excitation is produced. The variance of non-stationary excitations can be found by following numerical approximations developed by Michaelov *et al.* (2001) for the simple oscillator with the natural frequency of ω_0 and

Table 1 Expected extreme value of the pounding force between equal height systems under stationary records

No.	T_1, T_2 (sec)	300 nonlinear dynamic analyses (exact method)-(MN)	Frequency domain (proposed method)-(MN)
1	1.2, 0.4	4.8	4.9
2	1.2, 0.6	4.4	4.3
3	1.2, 0.8	3.7	3.6

Table 2 Expected extreme value of the pounding force between equal height systems under non-stationary records

No.	T_1, T_2 (sec)	300 nonlinear dynamic analyses (exact method)-(MN)	Frequency domain (proposed method)-(MN)
1	1.2, 0.4	3.4	3.6
2	1.2, 0.6	3.1	3.1
3	1.2, 0.8	2.7	2.7

damping ratio of ξ and the excitation modulation given in (Eq. (9))

$$\begin{aligned}\sigma_X^2(t) \approx \sigma_{X\infty}^2\, C^2\, \{ & \frac{\xi}{\xi_1}\left[\exp(-2B_1 t) - \exp(-2\xi\omega_0 t)\right] \\ & + \frac{\xi}{\xi_2}\left[\exp(-2B_2 t) - \exp(-2\xi\omega_0 t)\right] \\ & - 2\frac{\xi}{\xi_m}\left[\exp(-2B_m t) - \exp(-2\xi\omega_0 t)\right]\}\end{aligned} \tag{26}$$

$$\xi_1 = \xi - \frac{B_1}{\omega_0}\ ,\quad \xi_2 = \xi - \frac{B_2}{\omega_0}\ ,\qquad \xi_m = \frac{\xi_1 + \xi_2}{2}\ ,\ B_m = \frac{B_1 + B_2}{2} \tag{27}$$

and $\sigma_{X\infty}^2$ denotes the variance of the stationary response of the oscillator to the stationary filtered white noise. To avoid this computational hurdle, the approximation of the above function is proposed herein

$$\sigma_X^2(t) \approx A^2(t)\ \sigma_{X\infty}^2 \tag{28}$$

The approximate procedure can significantly facilitate the utilization of non-stationary models in engineering practice since it prevents computational difficulties. The method is based on the approximation of a non-stationary process by an "equivalent" stationary process. The variance of this "equivalent" stationary process is σ_{eq}^2 , and the process must be considered for the time interval [0, *Teq*] in order to create the same probability of occurrence as the original non-stationary process $X(t)$. The formulas of the equivalent variance and the equivalent duration of the non-stationary process developed by Michaelov *et al*. (2001) are

$$\sigma_{eq}^2(n_1) = \frac{I(n_1+1)}{I(n_1)}\ ,\quad T_{eq}(n_1) = \frac{I(n_1)}{\sigma_{eq}^{2n}(n_1)}\ ,\qquad I(n_1) = \int_0^T \sigma_X^{2n_1}(t)\ dt \tag{29}$$

This equation with n_1=3 or n_1=4 usually results in approximations with maximum error about 10%. For a zero-mean non-stationary process, the mean of extreme values of velocity is equal to

$$\dot{\delta}_e = ((2\ln(\upsilon T_{eq}))^{0.5} + \frac{\gamma}{(2\ln(\upsilon T_{eq}))^{0.5}})\,\sigma_{eq_{V\,rel}} \tag{30}$$

5. Verifications

In order to verify the reliability of the proposed method, two adjacent SDOF systems with the BW model and different periods (T_1=1.2 sec, T_2=0.4, 0.6, 0.8 sec) were considered. The Bouc-Wen model applied in the SDOF systems with the period of 1.2 seconds is defined by the following parameters:

k=1097 kg/cm, ξ=0.05, α=0.05, β=2(1/cm),

γ=−1 (1/cm), n=1, A=1

For the other SDOF systems with the periods of 0.4, 0.6 and 0.8, stiffness values are 9870, 4386 and 2467 kg/cm, respectively, where the other parameters are as mentioned above. The gap distance between two adjacent SDOF systems has been set to 1.0cm, the coefficient of restitution is 0.65 and stiffness of the contact element is 2×10^5 (kg/cm$^{1.5}$). These models were analysed under 300 stationary and non-stationary records compatible with the Kanai-Tajimi power spectral density, and then, the average value of the maximum pounding forces was obtained. The expected extreme value of pounding force was also investigated on the basis of statistical relations and the proposed closed-form method-without any need to perform exact dynamic analysis. Fig. 4 shows the different steps taken to find out the value of maximum pounding force from the exact solution and the proposed method. Comparisons between expected extreme values of pounding forces obtained from the performed analyses with 300 stationary and non-stationary records and from the proposed algorithm are presented in Tables 1 and 2.

According to Tables 1 and 2, the proposed algorithm has the capability of estimating the expected extreme value of the pounding force of two adjacent systems with equal heights under stationary and non-stationary excitations.

6. Extension of the proposed method for two SDOF systems with unequal heights

Until recently the research community has almost exclusively focused on modeling floor-to-floor poundings, primarily due to its simpler geometry. However, floor-to-column pounding (Fig. 4) is recognized to have more serious consequences (Karayannis and Favvata 2005a, b, Favvata and Karayannis 2013, Favvata 2015).

To model the inter-story pounding, it is proposed to use the virtual mass at the lower level of the system, thus the taller SDOF system is converted into a system with two degrees of freedom (Fig. 6). Using the dynamic equations

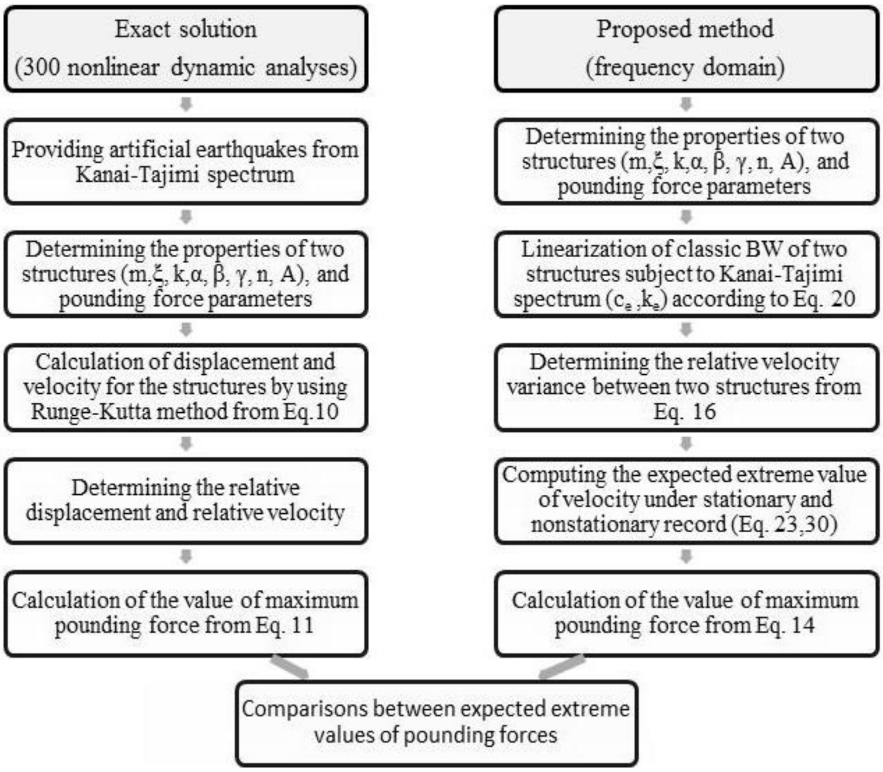


Fig. 4 Different steps taken to find out the value of maximum pounding force from exact and proposed method

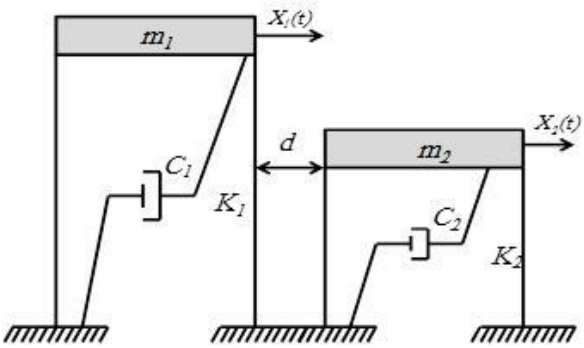


Fig. 5 Model of interacting structures with unequal heights

of motion for the shear system, the stiffness matrix is determined (k_{11}, k_{12}).

The dynamic equations of motion for systems "1", "2" which include pounding force during impact are written as

$$
\begin{aligned}
& m_{11}\ddot{x}_{11} + c_{11}\dot{x}_{11} + \alpha_{11}k_{11}x_{11} + \\
& -(1-\alpha_{12})k_{12}z_{12} + F(t) = -m_{11}\ddot{x}_g \\
& m_{12}(\ddot{x}_{12} + \ddot{x}_{11}) + c_{12}\dot{x}_{12} + \alpha_{12}k_{12}x_{12} \\
& +(1-\alpha_{12})k_{12}z_{12} = -m_{12}\ddot{x}_g \\
& m_2\ddot{x}_2 + c_2\dot{x}_2 + \alpha_2k_2x_2 + \\
& (1-\alpha_2)k_2z_2 - F(t) = -m_2\ddot{x}_g
\end{aligned}
\tag{31}
$$

where x_{11}, x_{12} are the story drifts of system "1", $\ddot{x}_g$ is the ground acceleration and $F(t)$ is the pounding force. If the relations are arranged in terms of acceleration, Eq. (31) can be written as follows

$$
\begin{aligned}
& \ddot{x}_{11} = -\ddot{x}_g - \frac{c_{11}}{m_{11}}\dot{x}_{11} - \frac{\alpha_{11}k_{11}}{m_{11}}x_{11} - \\
& \frac{(1-\alpha_{11})k_{11}}{m_{11}}z_{11} + \frac{c_{12}}{m_{11}}\dot{x}_{12} + \frac{\alpha_{12}k_{12}}{m_{11}}x_{12} \\
& + \frac{(1-\alpha_{12})k_{12}}{m_{11}}z_{12} - \frac{F(t)}{m_{11}} \\
& \ddot{x}_{12} = \frac{c_{11}}{m_{11}}\dot{x}_{11} + \frac{\alpha_{11}k_{11}}{m_{11}}x_{11} + \\
& \frac{(1-\alpha_{11})k_{11}}{m_{11}}z_{11} - c_{12}\left(\frac{1}{m_{11}} + \frac{1}{m_{12}}\right)\dot{x}_{12} \\
& -\alpha_{12}k_{12}\left(\frac{1}{m_{11}} + \frac{1}{m_{12}}\right)x_{12} - (1-\alpha_{12})k_{12} \\
& \left(\frac{1}{m_{11}} + \frac{1}{m_{12}}\right)z_{12} + \frac{F(t)}{m_{11}} \\
& \ddot{x}_2 = -a_g - \frac{c_2}{m_2}\dot{x}_2 - \frac{\alpha_2k_2}{m_2}x_2 \\
& -\frac{(1-\alpha_2)k_2}{m_2}z_2 + \frac{F(t)}{m_2}
\end{aligned}
\tag{32}
$$

The above equations can be numerically solved. In this study, COM3 finite element software (Maekawa 2003) was used to validate the above equations for two adjacent reinforced concrete (RC) frames with unequal heights (Fig. 6). Two adjacent RC frames were modeled in COM3 and Bouc-Wen model parameters of them were determined. The characteristics of these RC frames are presented in Table 3.

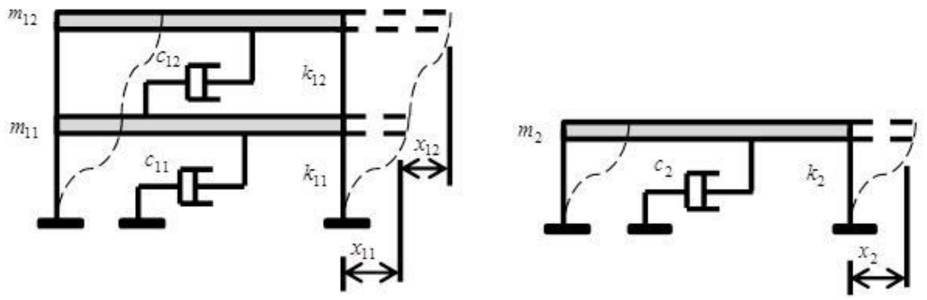


Fig. 6 Model idealization of interacting structures with unequal heights

Table 3 Characteristics of RC frames

Compressive strength of concrete	250 (kg/cm^2)	Beam Section	30×30 (cm)
Yield strength of steel	4000 (kg/cm^2)	Column Section	30×30 (cm)
Elasticity Modulus for steel	2100000 (kg/cm^2)	Mass for Frame (1)	97000 (kg)
Gap distance	2 (cm)	Mass for Frame (2)	36000 (kg)
Stiffness of the contact element	40000 (kg/cm $^{1.5}$)	Height of Frame (1)	320 (cm)
coefficient of restitution	0.65	Height of Frame (1)	160 (cm)

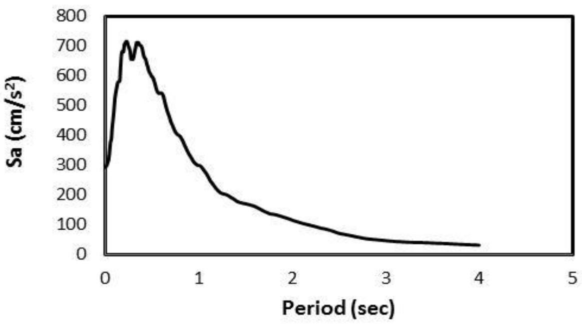
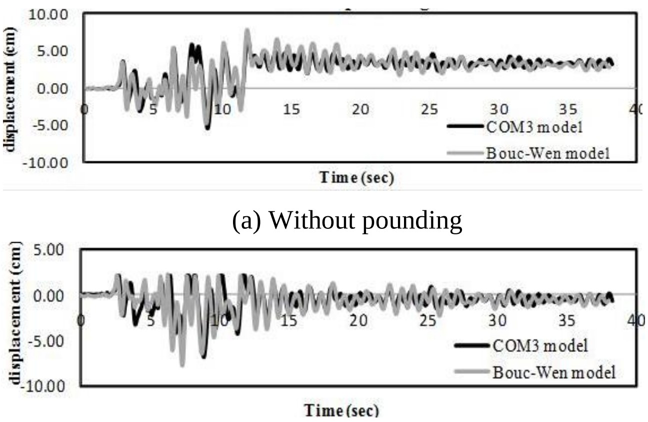


Fig. 7 The mean response spectrum of selected records



(a) Without pounding

(b) With pounding

Fig. 8 Displacement of contact point in taller frame

Table 4 Comparison between average values of maximum pounding forces computed by COM3 and equivalent Bouc-Wen model

Pounding case	Average of maximum pounding forces under 20 real records		
	COM3 software	Bouc-Wen model	Error (%)
Floor-to-floor	7.8	7.69	1.43
Floor-to-column	1.89	1.84	2.64

Parameters A, n nd α of Eq. (1) are set to 1, 2 and 0.02 respectively. Amounts of γ and β are determined with respect to the characteristics of yield state. Yield displacement and initial stiffness are estimated for each system by bilinear push curve of the force-displacement response. Adopting the following relation and accepting the applicable facts of β=−3γ, γ≤0, values of γ and β are determined.

$$x_y = \left(\frac{A}{\beta+\gamma}\right)^{\frac{1}{n}} \tag{33}$$

In order to evaluate the efficiency and accuracy of the Bouc-Wen model, a set of far-field ground motion records on the soil category C were considered. These records were selected from the records classified as LMSR (Large-Magnitude Small-Distance) by Shome and Cornell (1998). The mean response spectrum of selected records is presented in Fig. 7.

As an example, the comparison of results in terms of displacement of the contact point (roof level of the lower frame) under Loma Prieta earthquake ground motion is shown in Fig 8. Also the average values of maximum pounding forces under selected real records are presented in Table 4. As it shown, there is a good agreement between COM3 software and equivalent Bouc-Wen model (Eq. (32)).

In order to determine the maximum pounding force, two adjacent systems with the BW model and different periods (T_1=1.2 sec, T_2=0.4, 0.6, 0.8 sec) were analysed under 300 stationary and non-stationary records with Kanai-Tajimi spectral density, and then, the average value of maximum pounding force of these records were obtained. In addition, the maximum pounding force was investigated on the basis of statistical relations and the proposed closed-form method. The expected extreme value of the pounding force is equal to

$$F_e = \bar{\beta}\delta_e^{\frac{3}{2}} = \bar{\beta}\left(\frac{5}{4}\frac{m_{11}m_2(e\dot{\delta}_e)^2}{(m_{11}+m_2)\bar{\beta}}\right)^{\frac{3}{5}} \tag{34}$$

Table 5 Expected extreme value of the pounding force between unequal height systems under stationary records

No.	T_1, T_2 (sec)	300 nonlinear dynamic analyses (exact method)-(MN)	Frequency domain (proposed method)-(MN)
1	1.2, 0.4	0.45	0.45
2	1.2, 0.6	0.44	0.42
3	1.2, 0.8	0.40	0.37

Table 6 Expected extreme value of the pounding force between unequal height systems under non-stationary records

No.	T_1, T_2 (sec)	300 nonlinear dynamic analyses (exact method)-(MN)	Frequency domain (proposed method)-(MN)
1	1.2, 0.4	0.35	0.35
2	1.2, 0.6	0.31	0.31
3	1.2, 0.8	0.24	0.28

In Eq. (34), the expected extreme value of velocity is equal to

$$\dot{\delta}_e = ((2\ln(\upsilon T))^{0.5} + \frac{\gamma}{(2\ln(\upsilon T))^{0.5}})\sigma_{V\,rel}$$
$$\sigma^2_{V_{rel}} = \sigma^2_{V_{11}} + \sigma^2_{V_2} - 2E\{V_{11}(t)V_2(t)\} \tag{35}$$

where $\sigma^2_{V_{11}}$ is the standard deviation of velocity for virtual mass calculated by linearization of 2DOF system "1," Comparisons between the expected extreme values of pounding forces obtained from the performed analyses using 300 stationary and non-stationary records and the proposed algorithm are presented in Tables 5 and 6.

According to Tables 5 and 6, the proposed algorithm has the capability of estimating the expected extreme value of the pounding force of two adjacent systems with unequal heights under stationary and non-stationary excitations. It is noteworthy that the local failures of the elements and removing the failed or damaged members are not considered in the proposed approach.

The amount of pounding force created between two adjacent systems generally depends on the masses of the colliding systems (Jankowski 2005). The participating mass in floor-to-floor pounding case is larger than the participating mass of the floor-to-column pounding case. But it should be noted that although the larger impact force is detected for floor-to-floor pounding, the floor-to-column pounding is more critical. Failure in the floor-to-floor pounding occurs when the entire story reaches to the mechanism state; whereas the failure in floor-to-column pounding is a result of an imposed shear force on the columns.

7. Extension of the proposed method to MDOF systems

In the present study, the shear building model for multi-degree-of-freedom system (Fig. 9) is considered. The building is subjected to horizontal ground acceleration a_g and BW hysteresis model is used for each story. Regarding the equations represented in Fig. 2, i-th restoring force on mass i can be written as:

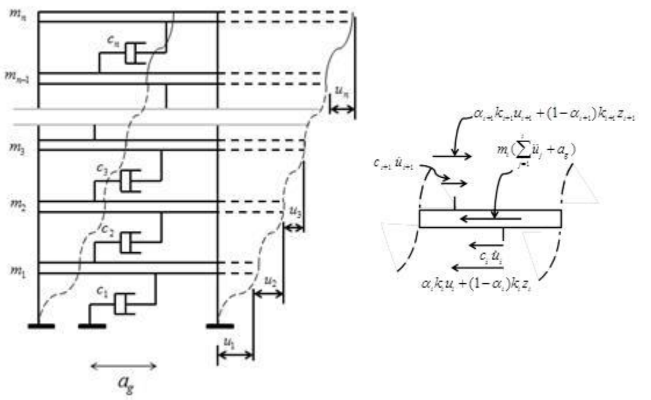


Fig. 9 Shear building structure and forces acting on i-th mass

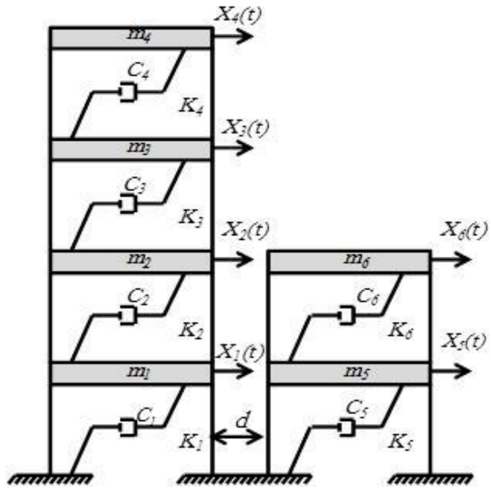


Fig. 10 Model of adjacent structures

$$q_i = \alpha_i\, k_i\, u_i(t) + (1-\alpha_i)\, k_i\, z_i(t) \tag{36}$$

where u_i is i-th story's displacement and d_i represents the displacement of i-th mass relative to the ground displacement

$$u_i = d_i - d_{i-1} \tag{37}$$

Therefore, the equation of motion for i-th mass can be written as

$$m_i\left(\sum_{j=1}^{i}\ddot{u}_j + a_g\right) + c_i\,\dot{u}_i - c_{i+1}\,\dot{u}_{i+1} + q_i - q_{i+1} = 0 \tag{38}$$

where i=1, …. , ND with ND=total number of discredited masses. Eq. (38) can be written in matrix form as follows

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} + [G]\{Z\} = -[M_0]\{I\}a_g \tag{39}$$

where $\{I\}$, $[M]$, $[C]$, $[K]$ are the influence vector, mass, damping and stiffness matrix, respectively

$$M=\begin{bmatrix} m_1 & & & 0 \\ m_2 & m_2 & & \\ \vdots & & \ddots & \\ m_{ND} & m_{ND} & \dots & m_{ND} \end{bmatrix},\quad M_0=\begin{bmatrix} m_1 & & & 0 \\ & m_2 & & \\ & & \ddots & \\ 0 & & & m_{ND} \end{bmatrix},$$

$$\{U\}=\begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{ND} \end{Bmatrix},\ \{Z\}=\begin{Bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{ND} \end{Bmatrix},\ \{\tilde{I}\}=\begin{Bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{Bmatrix},$$

$$G=\begin{bmatrix} (1-\alpha_1)k_1 & -(1-\alpha_2)k_2 & & & 0 \\ & (1-\alpha_2)k_2 & -(1-\alpha_3)k_3 & & \\ & & \ddots & & \\ & & & (1-\alpha_{ND-1})k_{ND-1} & -(1-\alpha_{ND})k_{ND} \\ 0 & & & & (1-\alpha_{ND})k_{ND} \end{bmatrix}$$

As an example, a 4-degree-of-freedom system adjacent to a 2-degree-of-freedom system is considered to determine the expected extreme value of the pounding force in MDOF systems (Fig. 10). The dynamic equations of the motion including pounding at two floor levels, are written as

$$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ m_1 & m_2 & 0 & 0 & 0 & 0 \\ m_1 & m_2 & m_3 & 0 & 0 & 0 \\ m_1 & m_2 & m_3 & m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_5 & 0 \\ 0 & 0 & 0 & 0 & m_5 & m_6 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1(t) \\ \ddot{u}_2(t) \\ \ddot{u}_3(t) \\ \ddot{u}_4(t) \\ \ddot{u}_5(t) \\ \ddot{u}_6(t) \end{Bmatrix}+$$

$$\begin{bmatrix} c_1 & -c_2 & 0 & 0 & 0 & 0 \\ 0 & c_2 & -c_3 & 0 & 0 & 0 \\ 0 & 0 & c_3 & -c_4 & 0 & 0 \\ 0 & 0 & 0 & c_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_5 & -c_6 \\ 0 & 0 & 0 & 0 & 0 & c_6 \end{bmatrix}\begin{Bmatrix} \dot{u}_1(t) \\ \dot{u}_2(t) \\ \dot{u}_3(t) \\ \dot{u}_4(t) \\ \dot{u}_5(t) \\ \dot{u}_6(t) \end{Bmatrix}+$$

$$\begin{bmatrix} \alpha_1k_1 & -\alpha_2k_2 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2k_2 & -\alpha_3k_3 & 0 & 0 & 0 \\ 0 & 0 & \alpha_3k_3 & -\alpha_4k_4 & 0 & 0 \\ 0 & 0 & 0 & \alpha_4k_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_5k_5 & -\alpha_6k_6 \\ 0 & 0 & 0 & 0 & 0 & \alpha_6k_6 \end{bmatrix}\begin{Bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \\ u_5(t) \\ u_6(t) \end{Bmatrix}+$$

$$\begin{bmatrix} (1-\alpha_1)k_1 & -(1-\alpha_2)k_2 & 0 & 0 & 0 & 0 \\ 0 & (1-\alpha_2)k_2 & -(1-\alpha_3)k_3 & 0 & 0 & 0 \\ 0 & 0 & (1-\alpha_3)k_3 & -(1-\alpha_4)k_4 & 0 & 0 \\ 0 & 0 & 0 & (1-\alpha_4)k_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-\alpha_5)k_5 & -(1-\alpha_6)k_6 \\ 0 & 0 & 0 & 0 & 0 & (1-\alpha_6)k_6 \end{bmatrix}\dots$$

$$\dots\begin{Bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \\ z_5(t) \\ z_6(t) \end{Bmatrix}+\begin{Bmatrix} F_{15}(t) \\ F_{26}(t) \\ 0 \\ 0 \\ -F_{15}(t) \\ -F_{26}(t) \end{Bmatrix}=$$

$$-\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_6 \end{bmatrix}\begin{Bmatrix} \ddot{u}_g(t) \\ \ddot{u}_g(t) \\ \ddot{u}_g(t) \\ \ddot{u}_g(t) \\ \ddot{u}_g(t) \\ \ddot{u}_g(t) \end{Bmatrix} \quad (40)$$

where u_i are

$$u_i(t)=x_i(t)-x_{i-1}(t) \quad (41)$$

and $\ddot{u}_i(t), \dot{u}_i(t), u_i(t)$, ($i$=1,...,6) are the acceleration, velocity and relative displacement of i^{th} story, respectively; $\ddot{u}_g(t)$ is the input ground motion and $F_{ij}(t)$ (i=1,2; j=5,6) denotes the pounding force between stories with masses m_i, m_j.

Two adjacent systems with different initial periods (T_1=1.2 sec, T_2=0.4, 0.6, 0.8 sec) were considered. Characteristics of the Bouc-Wen model for the 4DOF system -with the main period of 1.2 seconds- under stationary and non-stationary records are: ξ=0.05, α=0.05, β=2, γ=−1, n=1, A=1 in all stories. In the 2DOF systems, with the periods of 0.4, 0.6 and 0.8, the stiffness in all stories are 18000, 10000, and 6000 kg/cm, respectively. The other parameters are set equal to the above mentioned values. The gap distance between 2&4DOF systems and the stiffness of the contact element are the same as those considered for the SDOF systems.

This model was analysed under 300 stationary records with the Kanai-Tajimi spectral density, and then, the average value of the maximum pounding force of these records was obtained. The maximum pounding force was additionally investigated on the basis of the proposed method. According to Table 7, the proposed algorithm could accurately estimate the expected extreme value of the pounding force.

Table 7 Expected extreme value of the pounding force between 4 and 2 degree of freedom adjacent systems

No.	T_1, T_2 (sec)	300 nonlinear dynamic analyses (exact method)-(MN)	Frequency domain (proposed method)-(MN)
1	1.2, 0.4	2.0	1.9
2	1.2, 0.6	1.6	1.8
3	1.2, 0.8	1.2	1.4

8. Conclusions

The expected extreme value of the pounding force between two adjacent nonlinear systems with equal and unequal heights, subject to stationary and non-stationary excitation, was investigated on the basis of statistical relations, and a closed-form method was presented to approximately detect the expected extreme value of the pounding force without any need for performing exact dynamic analysis. Adjacent buildings with similar or different BW hysteretic behaviors can be easily modeled using the proposed approach. Compared to the exact dynamic analysis procedure, the proposed approach had acceptable results for the expected extreme value of the pounding force of SDOF and MDOF systems under stationary and non-stationary Gaussian excitations. It is noteworthy that the local failure of the elements and removing the failed or damaged members are not considered in the proposed approach, which can be the topic of future study in this area.

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