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Shear forces amplification due to torsion, explicit reliance on structural topology. Theoretical and numerical proofs using the Ratio of Torsion (*ROT*) concept

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Abstract. The recently introduced index Ratio Of Torsion (*ROT*) quantifies the base shear amplification due to torsional effects on shear cantilever types of building structures. In this work, a theoretical proof based on the theory of elasticity is provided, depicting that the ratio of torsion (*ROT*) is independent of the forces acting on the structure, although its definition stems from the shear forces. This is a particular attribute of other design and evaluation criteria against torsion such as center of rigidity and center of strength. In the case of *ROT*, this evidence could be considered as inconsistent, as *ROT* is a function solely of the forces acting on structural members, nevertheless it is proven to be independent of them. As *ROT* is the amplification of the shear forces due to in-plan irregularities, this work depicts that this increase of internal shear forces rely only on the structural topology. Moreover, a numerical verification of this theoretical finding was accomplished, using linear statistics interpretation and nonlinear neural networks simulation for an adequate database of structures.

Keywords: torsional coupling; shear center; center of rigidity; center of twist; strength center; ratio of torsion; linear statistics; neural networks; solid mechanics

1. Introduction

The coupling of floor *ROT*ational deformations with translational ones, under horizontal external loading of shear cantilever beam type of structural configurations (buildings and beams), generates internal forces to the vertical resisting elements. These shear forces and the corresponding displacements due to floor *ROT*ation, constitute the negative effects of structural forms affected by torsion. Many research works deal with the issue of torsion, nonetheless with contradictory results, as depicted in a critical review by Anagnostopoulos, Kyrkos *et al.* (2015). The confrontation of the torsional coupling remains an open issue of structural design, thus structural engineers apply building codes requirements in order to deal with torsion, in engineering practice.

This coupling occurs because of the asymmetrical arrangement of the mass and the vertical elements' stiffness of building's floor plan. Therefore, the application of an external horizontal loading, causes internal torque in the vertical resisting elements and thus, torsional oscillations. In order to define the effect of torsion in buildings, significant amount of studies have been performed around the basic concept that the structure is oscillating -in the elastic and inelastic state of response- around a specific point (centroid), while the distance of the center of mass from this point is a quantification of the torsional effect (Mylimaj and Tso 2002, Paulay 1998).

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The Ratio Of Torsion (*ROT*) was recently introduced by Stathi, Bakas *et al.* (2015) as a new index for the accurate estimation of the torsional amplification of the internal shear forces. This is achieved through the quantification of the internal forces due to torsional coupling effects in reference to the corresponding symmetrical structural design. In the next session the analytical form of *ROT* will be defined, while afterwards, an explicit form defining *ROT* as a function of shear resisting elements dimensions and topology will be extracted. This is an attribute of both Center of Rigidity (Hejal and Chopra 1987) and Center of Strength (Paulay 1998): they both are independent of loading, and therefore, an explicit function of structural dimensions and topology (geometry). It will be demonstrated theoretically and computationally, that the same stands for *ROT* criterion, its independence of the loading acting on a shear cantilever beam type of building. Thus, the novel contribution of this work consist of both theoretical and computational proofs that the shear forces amplification -defined by *ROT*- is also independent of the external loading and affected only from the structural form. Consequently, *ROT* can be utilized as a structural design and assessment criterion in conceptual design, in structural optimization as a performance objective function and in the assessment of existing buildings, by means of it quantifies the increase of the internal forces due to asymmetry, for a given external loading.

This work is derived in two parts. Part A excludes closed mathematical formulation for a generative C-shaped shear cantilever beam, deriving *ROT* from structural geometry (dimensions and topology). The same formula applies on a building structure consisting of a space frame

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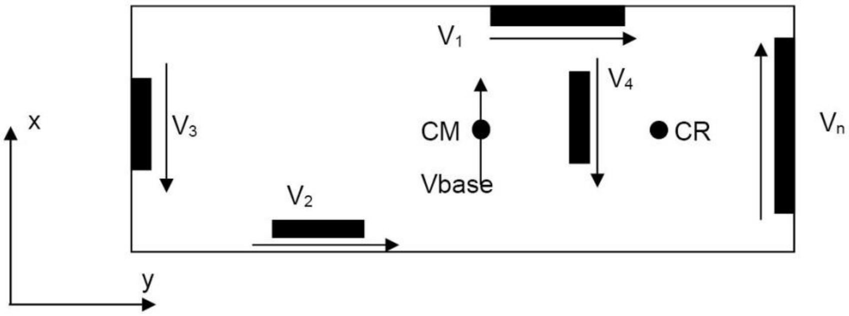


Fig. 1 Vertical resisting elements internal forces

with rigid diaphragm. Furthermore, a mathematical formulation connecting *ROT* with shear eccentricity is derived. Likewise, using equilibrium equations on the deformed shape, a general proof that *ROT* depends only on structural topology and resisting element sizes is derived. Part B uses an adequate sample of randomly generated structures and their structural analysis results, in order to exclude statistically reliable results for several generative structural configurations. The correlations of *ROT*, center of twist, shear center and omega (Ω) index, are demonstrated, using empirical results of structural analyses. Afterwards, the results are used to train neural networks of diverse architectures, in order to confirm computationally the explicit relationship between *ROT* and structural geometry. The independent variables are the sizes and positions of vertical resisting elements and structural stiffness parameters, which are independent of loading and relevant to torsion, such as omega, torsional radius and rigidity eccentricity. Using this approach, nonlinear multivariate correlations of structural topology and *ROT* index, found to be initially close and finally equal to unit (100%), validating the analytical proof, while when loading included to the independent variables, the nonlinear correlations of the raw values and the predicted ones, exhibit lower statistical correlation, giving computational evidence that *ROT* is independent of internal shear forces, confirming the theoretical proof.

2. Ratio of torsion

The Ratio Of Torsion (*ROT*) index is a quantitative valuation of the amplification of the shear forces acting on the vertical resisting elements of a building. For a generative, non-symmetric floor plan with non-coincident center of mass and rigidity and a horizontal force acting on the center of mass, the resulting base torque provokes additional shear forces on the resisting structural elements (columns). In the Fig. 1, a typical floor plan is shown where the vertical resisting elements and the corresponding shear forces acting are demonstrated. The floor plan comprises the vertical resisting elements 1,2,…,*n*. Due to torsional effects, the shear forces acting on the vertical resisting elements are not directed to the same direction (opposite to the external force to satisfy equilibrium). Therefore, the absolute summation of the shear forces satisfy the subsequent expression

$$\sum_{k=1}^{n} |V_{kij}| \neq \sum_{k=1}^{n} V_{kij} \quad (1)$$

where:

- n = the number of elements in a floor direction (x or y),
- i = the corresponding shear force of the element,
- and j = the direction of the earthquake motion

Especially for the above floor plan with a seismic action in the y direction, for the resisting elements in the x direction, Eq. (1) can be written as

$$\sum_{k=1}^{n} |V_{kxy}| \neq \sum_{k=1}^{n} V_{kxy} = 0 \quad (2)$$

and

$$\sum_{k=1}^{n} |V_{kyy}| \neq \sum_{k=1}^{n} V_{kyy} = V_{base} \quad (3)$$

V_{base} of Eq. (3) is the base shear force acting on a particular floor of a building. The floor torsion is equal to the product of the vertical resisting elements' shear forces and shear eccentricity, while the elements' torsional moments are neglected as their contribution to the total sum of toques is negligible (Kan 1977). This is a fact that many research and professional engineers have lack of care, considering that torsion on a diaphragm results torsion on the vertical structural elements (columns). Upon this is based *ROT*'s formulation as a new torsional criterion which quantifies the torsional effect on the earthquake resisting buildings, as the effect of shear forces and not torsional moments.

The definition of *ROT* for the generative floor plan of Fig. 1 is

$$ROT = \sum_{1}^{n} \sum_{i=x,j=y}^{y,x} ROT_{ij} \quad (4)$$

Where

$$ROT_{ij} = \frac{\sum_{k=1}^{n} |Vkij| - a * \sum_{k=1}^{n} Vkij}{\sum_{k=1}^{n} Vkij} \quad (5)$$

and

- n = the number of elements in a floor direction (for i,j = x or y)
- i = the corresponding shear force of the element
- j = the direction of the earthquake motion
- $\alpha = 0$ if $i \neq j$ or $\alpha = 1$ if $i = j$

Eq. (4) defines *ROT* as a summation of four particular ROT_{ij} for each combination of x and y standing for i and j. If $i=j$, then the base shear of the floor plan in not abstracted

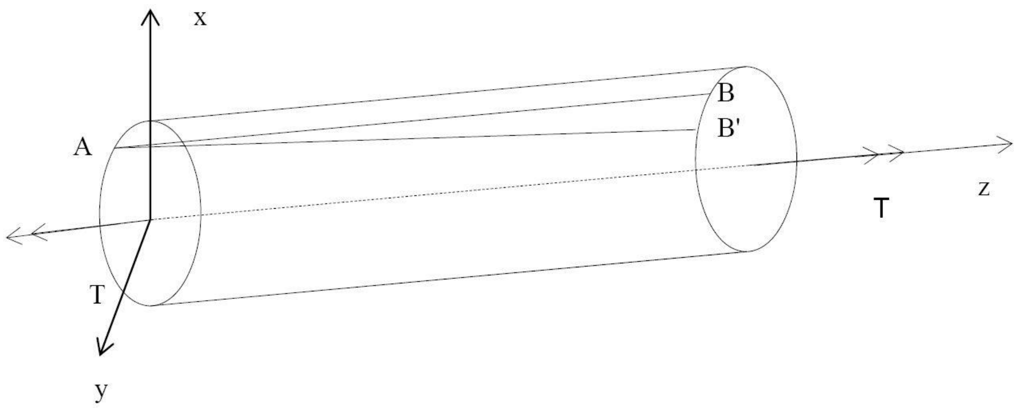


Fig. 2 Torque acting on a prismatic bar

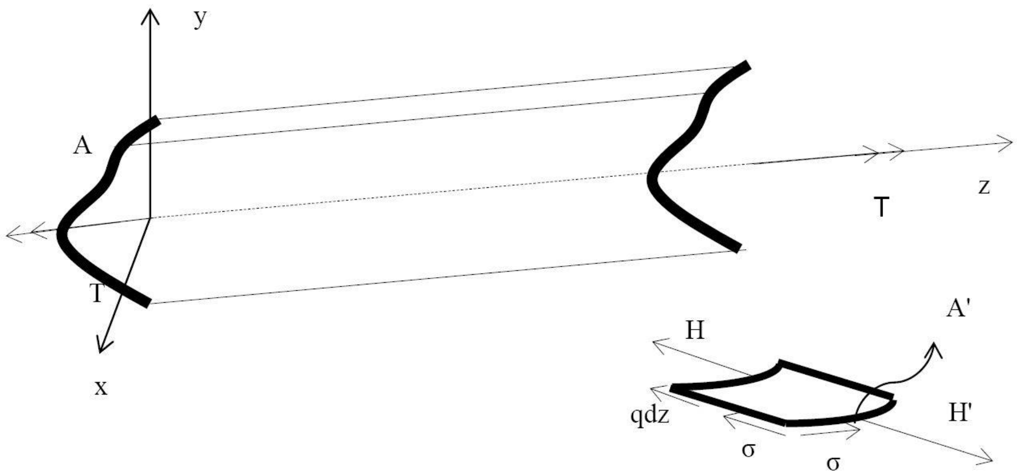


Fig. 3 Forces acting on thin walled beam

from the numerator. Thus, with the definition of *ROT* according to Eqs. (4) and (5), and the utilization of index α, a normalization of the shear forces amplification is achieved.

3. Analytical relationship between *ROT* and structural sizes and topology derived from the theory of elasticity

3.1 Definition of torsion of a prismatic bar

Torque

In this section, the constitutional equations of torsion as constituted by Boresi, Schmidt *et al.* (1993) for a deformable body, are utilized to derive the *ROT* formulation in terms of structural topology, assuming a general cross section prismatic bar, and a Torque *T*, acting on each side, as per Fig. 2. This torsion causes the cross section to *ROT*ate as a rigid body about the *z* axis (or coupling axis), also called axis of twist

The torsion acting on the cross section *A* will be equal to

$$T = \int_A \left(x\sigma_{zy} - y\sigma_{zx}\right)dA \tag{6}$$

where σ_{zy} and σ_{zx} are the in plane shear stresses acting on the cross section. From this initial formulation of Torsion, it is obvious that torsional force, is actually a result of shear stresses. This is very important for the formulation of *ROT*, as the first observation that leaded to its conceptual constitution, was that torsion in a structure is not actually torsion in the structural elements but shear.

3.2 Thin walled beam

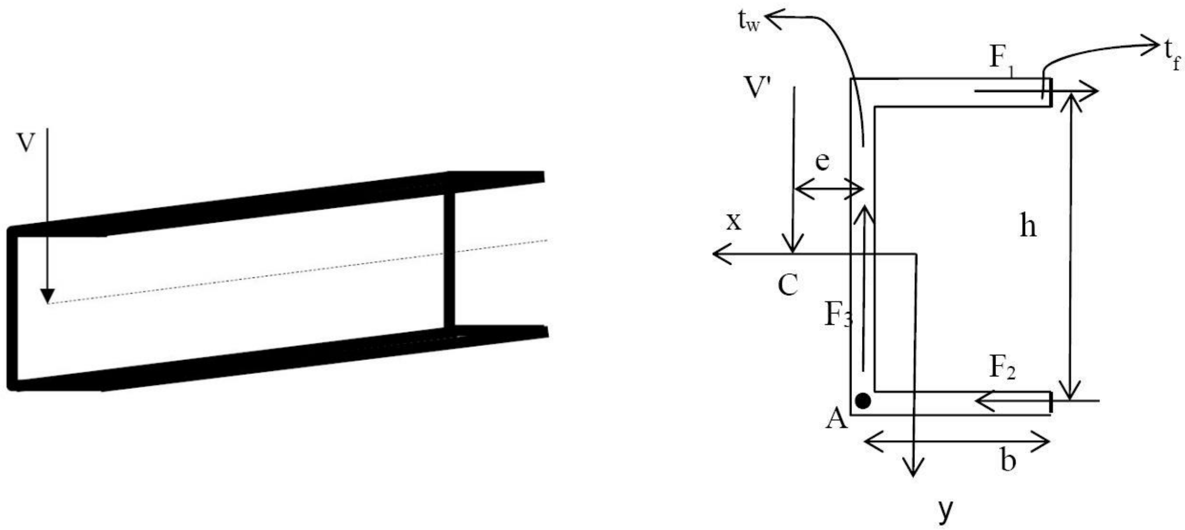
In the following, a thin walled beam cross sections will be considered. The same equations, stands for shear cantilever type of buildings, as they exhibit the same mechanical behavior. The shear flow *q* can be derived from these shears and expressed as

$$\mathrm{q} = \sigma * \mathrm{t} \tag{7}$$

where *t* is the thickness of the wall
therefore, according to the Fig. 3

$$\mathrm{qdz} = \mathrm{H}^{'} - \mathrm{H} \tag{8}$$

from the definitions of theory of elasticity, the axial force is the integral of the axial stresses



(a) Forces acting on a C-shaped thin walled beam (b) Dimensions of a C-shaped thin walled beam

Fig. 4 C-shaped thin walled beam

$$H = \int_A \sigma_{zz} dA = \int_A \frac{M_x y}{I_x} dA \tag{9}$$

while

$$H' = \int_{A'} (\sigma_{zz} + d\sigma_{zz}) dA = \int_{A'} \frac{(M_x + dM_x)y}{I_x} dA \tag{10}$$

where *A* and *A'* stands for the cross sections over which the stresses σ_z and $\sigma_z + d\sigma_z$ are integrated.

Eqs. (8),(9),(10) lead to the calculation of shear flow as

$$q = \frac{dM_x}{dz} \frac{1}{I_x} \int_{A'} y dA \tag{11}$$

Additionally, from the geometry of a cross section

$$\int_{A'} y dA = A' \overline{y'} \tag{12}$$

where $\overline{y'}$ is the distance from the *x* axis to the centroid of *A'*.

From definition of the Shear force due to bending moments

$$Vy = \frac{dM_x}{dz} \tag{13}$$

in combination with Eqs. (11) and (12), the shear flow is calculated as a function of shear forces and structural dimensions and topology as

$$q = \frac{V_y A' \overline{y'}}{I_x} \tag{14}$$

3.3 Special case: channel section

In the following, the previous defined equations, will be used to derive the relationship of *ROT* and structural dimensions and topology.

From equilibrium of forces and moments the following equations are derived

$$\sum F_x = 0 => F_2 = F_1 \tag{15}$$

$$\sum F_y = 0 => V' = F_3 \tag{16}$$

$$\sum M_A = 0 => V' e = F_1 h \tag{17}$$

While the bending moment of inertia is calculated as

$$I_x = \frac{1}{12} t_w h^3 + 2 b t_f \left(\frac{h}{2}\right)^2 + 2 \frac{1}{12} b t_f^3 \tag{18}$$

For thin walled cross sections, t_f is much small compared to *b* or *h*, thus the third term of Eq. (18) is neglected

$$I_x = \frac{1}{12} t_w h^3 + \frac{bh^2}{2} \tag{19}$$

From the definition of the axial force due to axial stresses and Eq. (14), it is derived that

$$F_1 = \int_0^b q dl = \frac{V_y}{I_x} \int_0^b A' \overline{y'} dl = \frac{V_y t_f h}{2 I_x} \int_0^b l dl = \frac{V_y t_f b^2 h}{4 I_x} \tag{20}$$

While the eccentricity where if the vertical load *V'* applies, the section bends without twist, is equal to

$$e = \frac{b}{2 + \frac{1}{3} \frac{t_w h}{t_f b}} \tag{21}$$

This is the location of the shear center of the beam.

The lamped definition of *ROT* (Eq. (5)) is written for the continuous mechanics medium as

$$ROT_{ij} = \frac{\int (|\sigma_{ij}| - a\sigma_{ij}) dA}{\int \sigma_{ij} dA} \tag{22}$$

Where:

- i = the direction of the internal shear stress
- j = the direction of the applied load
- $\alpha = 0$ if $i \neq j$ or $\alpha = 1$ if $i = j$

Thus, for a force acting on the *y* direction, the *ROT* in *x* direction is calculated as

$$ROT_{xy} = \frac{\int |\sigma_{xy}| dA}{\int \sigma_{xy} dA} \tag{23}$$

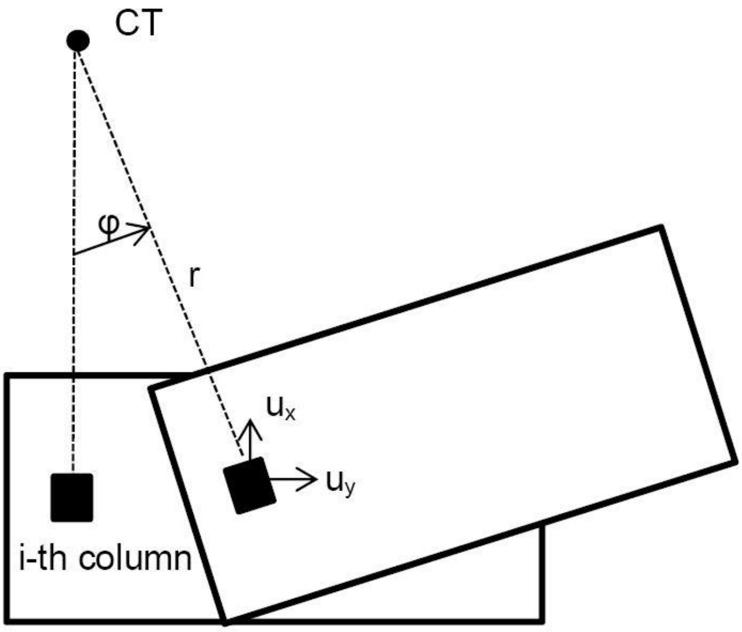


Fig. 5 Displacements of columns

$$ROT_{xy} = \frac{F_1+F_2}{V'} \tag{24}$$

From forces equilibrium F_1 equals F_2, thus using Eq. (20), the Eq. (24) is written as

$$ROT_{xy} = \frac{2V_y t_f b^2 h}{4I_x V_y} \tag{25}$$

The internal shear forces cancel each other in Eq. (25), thus *ROT* is calculated as

$$ROT_{xy} = \frac{t_f b^2 h}{2I_x} \tag{26}$$

Where I_x is calculated according to Eq. (19)

That is an explicit form of *ROT* as a product of beams dimensions. This is the first finding of this work, as *ROT* is defined by the cross section shear forces, however proved to be independent of these shear forces. According to Eq. (26), as *ROT* is a function of the cross section dimensions, the shear forces amplification due to stiffness irregularities, is also independent of the loading. In the following section 5, this is also proved for generative structural configurations, while in sections 6-8 this is confirmed computationally.

4. Analytical solution deriving *ROT* from rigidity eccentricity

Using Eqs. (16),(17) and (24), it is derived that

$$ROT = \frac{2*V'}{F_3} \tag{27}$$

and

$$\frac{ROT}{e} = \frac{2*e}{h(e+y_C)} \tag{28}$$

Eqs. (21) and (28) results

$$\frac{ROT_{xy}}{e} = \frac{2*h*(6*b*t_f + h*t_w)}{t_w*h^3 + 6*b*h^2*t_f + 2*b*t_f^3} \tag{29}$$

Therefore, *ROT* follows a relationship with center of twist eccentricity (e) independent of the acting forces. This is an explanation of the later depicted strong correlation pattern of *ROT* and center of twist. The terms *h*, *b*, t_f and t_w cause the arithmetic correlation being slightly less than 100%, however remaining high, representing almost linear patterns. Furthermore, from Eq. (29) it is derived that if eccentricity is equal to zero, *ROT* is also zero. This is conceptually correct as in this case, the structure exhibits only translational deformations.

5. General implementation of the theoretical proof, for structural systems with rigid floor diaphragms

As defined by Tso (1990), the center of twist (*CT*) is the point in a floor plan that remains stationary when the structure is subjected to torque loading. For structural systems with rigid floor diaphragms, the coupled displacements of each point of the floor diaphragm, are directly derived from the *ROT*ation regarding *CT*, as

$$u^2 = {u_x}^2 + {u_y}^2 \tag{30}$$

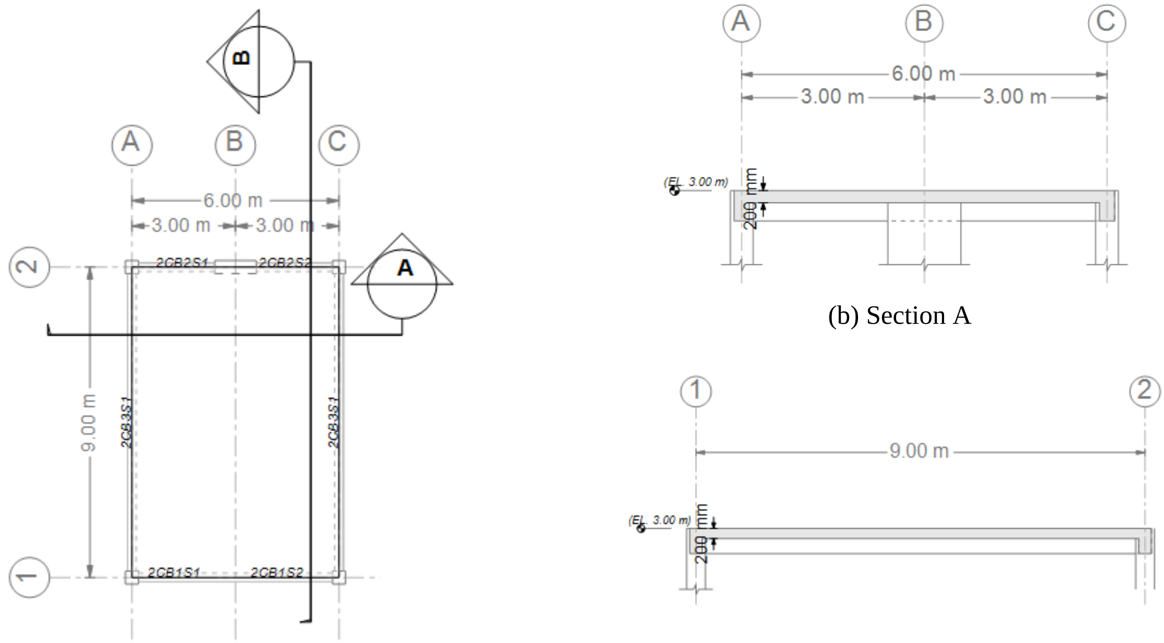
$$tan(\varphi) = \frac{u}{r} \tag{31}$$

$$\varphi \to 0 => u_{xi} = \varphi * r_i \tag{32}$$

where,

- φ is constant for all columns du to rigid floor diaphragm restraint
- r_i is distance of each column from twist center

From the assumption of linear elastic response and rigid



(a) Floor framing plan (b) Section A (c) Section B

Fig. 6 Structures family A

diaphragm constraint, the independence of twist center location from the loading is derived. Consequently the independence of r_i from the loading.

The *ROT*, as per defined in Eq. (5), can be written such as

$$ROT_{yx} = \frac{\varphi * \sum_{i=1}^{n} a_i * k_i * r_i}{\varphi * \sum_{i=1}^{n} k_i * r_i} \tag{33}$$

$$ROT_{yx} = \frac{\sum_{i=1}^{n} a_i * k_i * r_i}{\sum_{i=1}^{n} k_i * r_i} \tag{34}$$

- a_i are the signs after eliminating the absolute sign.
- k_i are the elastic stiffness's of each vertical resisting element.

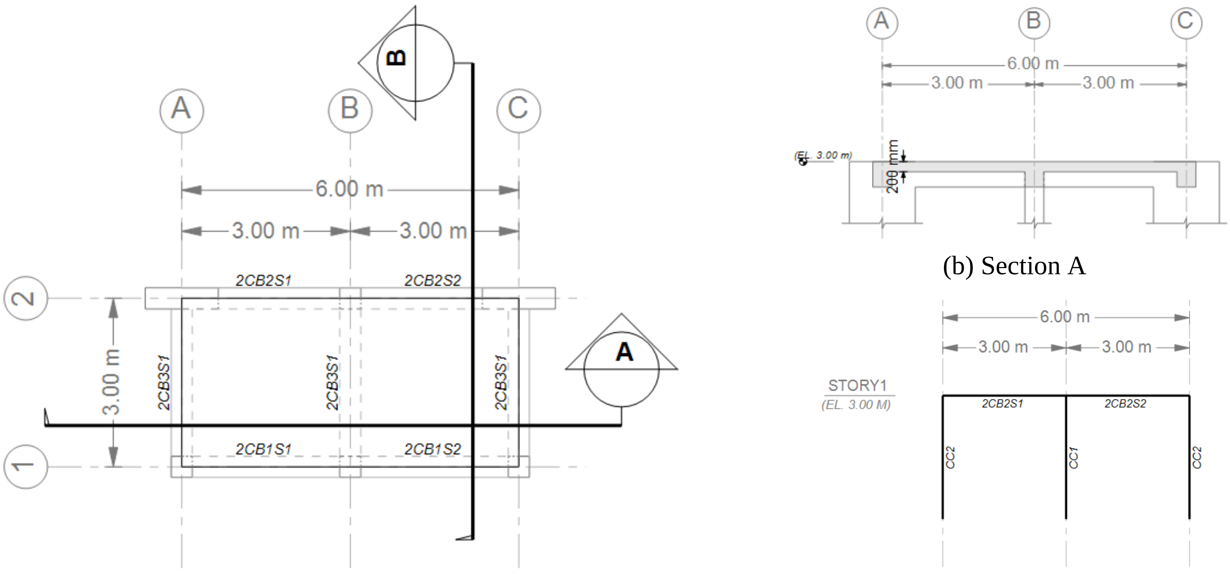
They can assumed to be independent of the loading as for the elastic state the deformed shape of the floor diaphragm is proportional to the loading without changing scheme.

6. Part B: numerical verification

6.1 Computational background-Neural networks

Neural Network (NN) analysis accomplish calculations using a circuit of interconnected computational nodes, named as neurons. NNs are based on biological neurons, which form a nervous tissue section. The aim is to achieve computational simulation of multiple variables, imitating the operation of biological neurals. A neural network is a network of computational nodes (neurons), which are interconnected between them. It is inspired by the human central nervous system, which is what it attempts to simulate. Neurons are the building blocks of the network. Each such node receives a set of numerical inputs from different sources, either from other nodes (hidden neurons) or from the environment (data values), performs a calculation based on these inputs and produces an output. The input neurons do not perform any calculations, just mediate between environmental inputs of network and computational neurons. The output neurons convey the environmental final numerical outputs of the network. The computational neurons multiply each entry with the corresponding synaptic weight and calculate the total sum of the products. This sum is supplied as an argument to the activation function, which implements inside each node. The price received by the function for this argument is the output of neuron for current inputs and weights, as described by Haykin (2004). In recent years there has been an explosion of interest in neural networks applications in an wide range of fields of science and technology, as general function approximators.

The training of Artificial Neural Networks (ANNs) finally ends to the solution of an optimization problem: to define the optimal weight that minimize the error between the predicted values and raw dependent data. Neural networks (NN) are applicable in almost any situation in which an unknown relationship between predictor variables (independent input) and predicted variables (dependent, outputs) is investigated. ANNs exhibit successful performance, even when this relationship is highly nonlinear. This is depicted in a number of research works on the prediction of mechanical behavior of structures, as demonstrated by Engin, Ozturk *et al.* (2015) for torsional strength of beams, Yavuz (2016) for shear strength estimation of RC beams, Mohammadhassani, Nezamabadi-pour *et al.* (2013) for strain prediction in tie section of concrete deep beams, Hakim and Razak (2013), for structural damage identification, Peng-hui, Hong-ping *et al.*

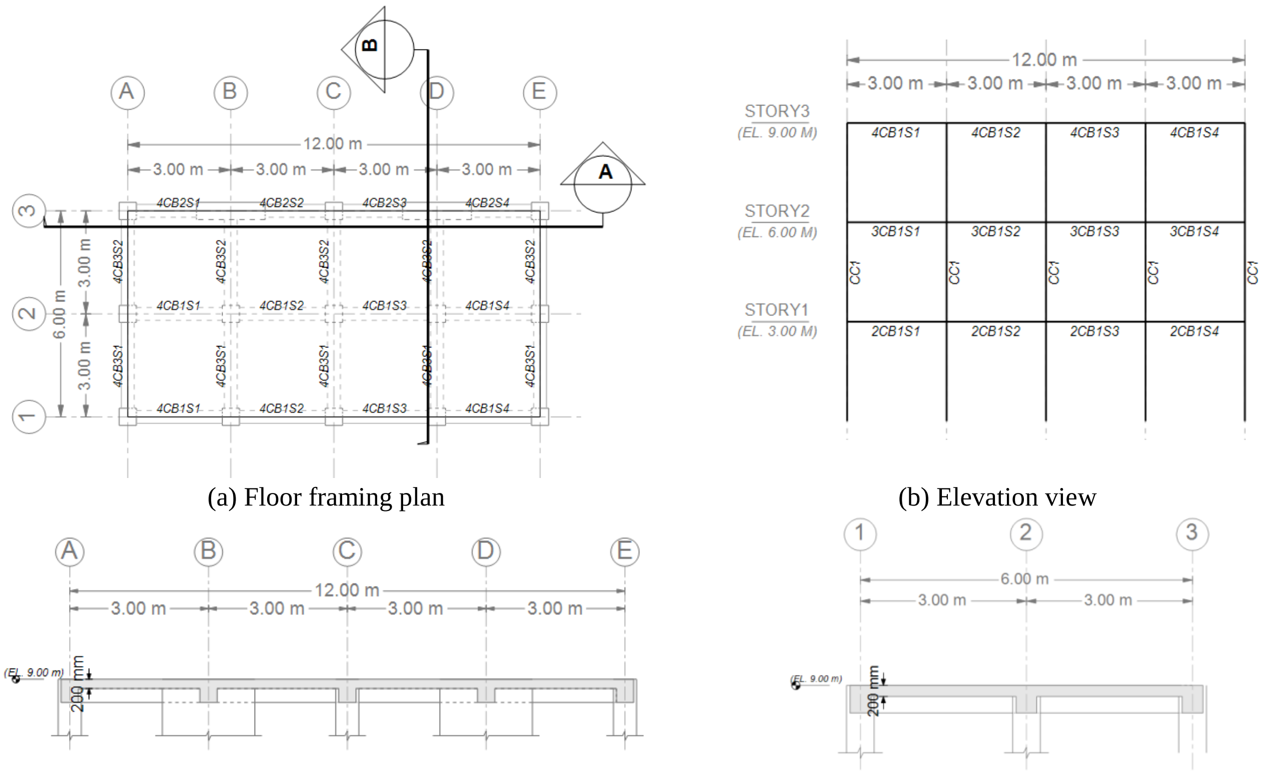


(b) Section A

(a) Floor framing plan

(c) Elevation view

Fig. 8 Structures family C



(a) Floor framing plan

(b) Elevation view

(c) Section A

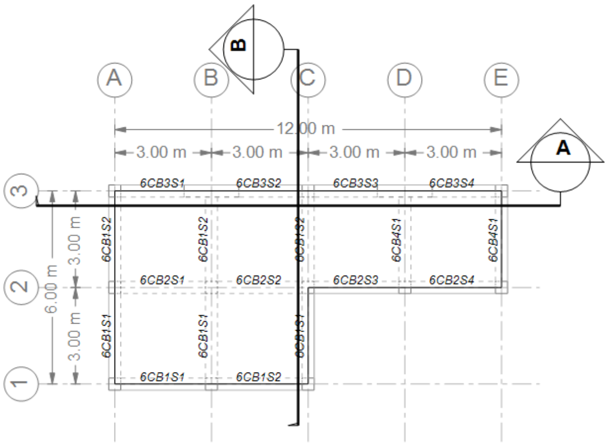
(d) Section B

Fig. 7 Structures family B

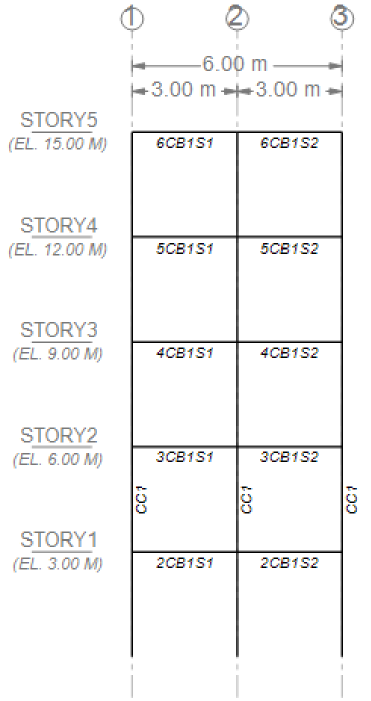
(2015) for damage identification of beams, Beycioğlu, Emiroğlu *et al.* (2015) for compressive strength of clinker mortars and Tavakkol, Alapour *et al.* (2013), for lightweight concrete strength, formulating ANNs techniques as powerful tools for the identification of structural behavior patterns and acquisition of reliable conclusions, correlating multiple independent variables with one or more dependent.

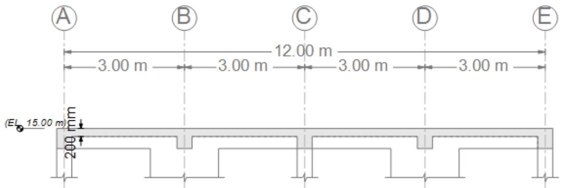
7. Definitions of examples set

In order to evaluate the theoretically derived independency of *ROT* from the loading, a population of structures was investigated. Each instantiation of the test structure was assigned initially with cross sections for the columns corresponding to the maximum dimensions for this example. Therefore, the initial cross sections represent



(a) Floor framing plan

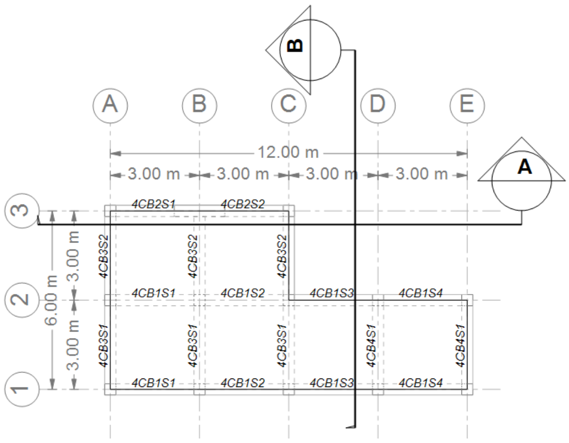




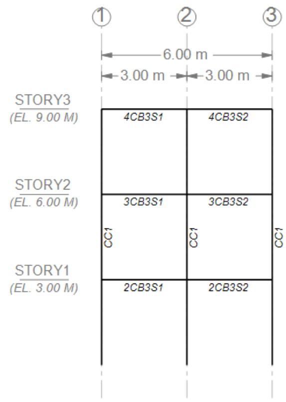
(b) Section A

(c) Elevation A

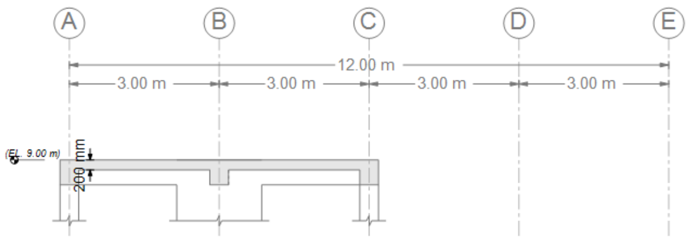
Fig. 9 Structures family D



(a) Floor framing plan



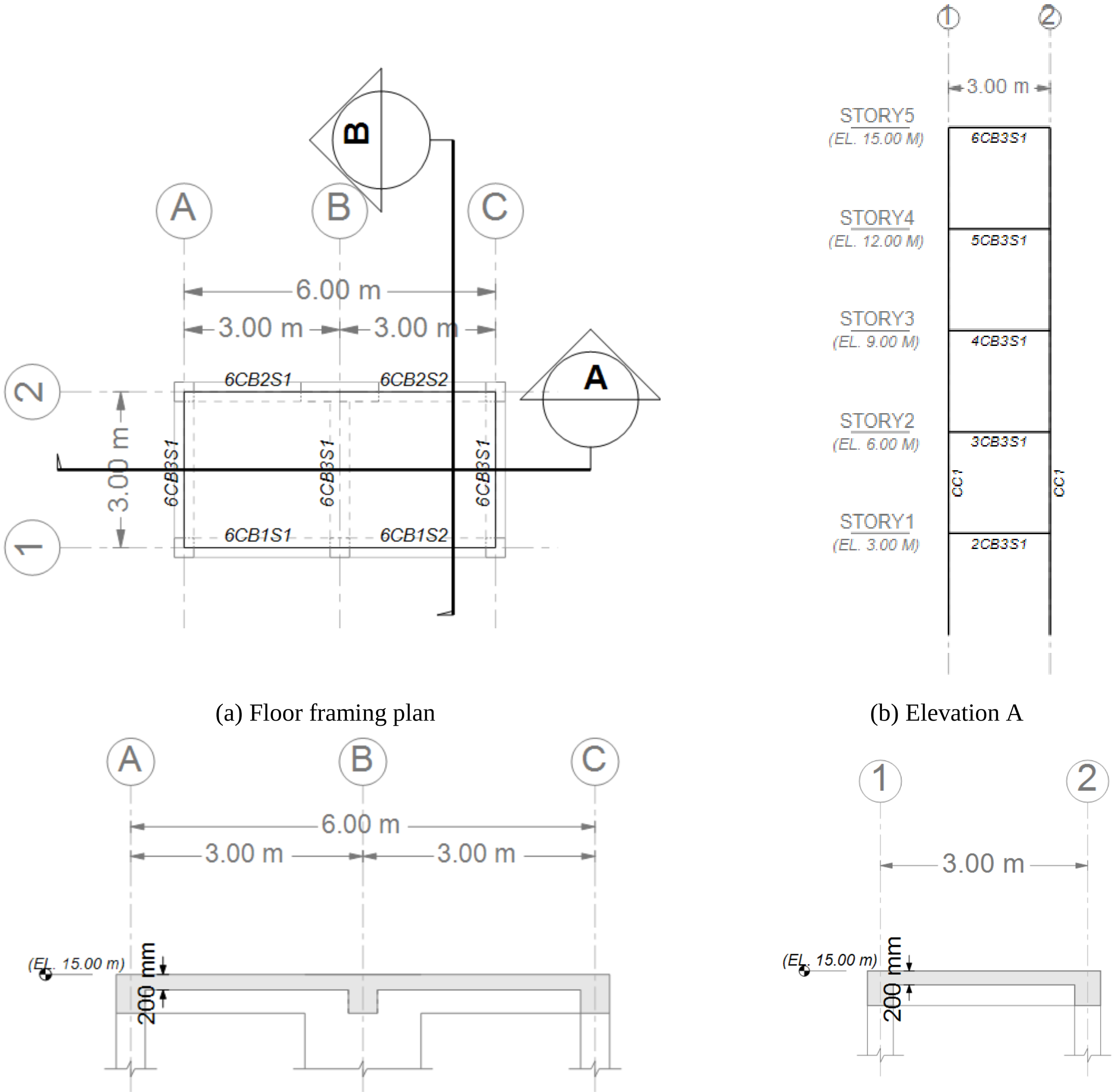
(b) Elevation A



(c) Section A

Fig. 10 Structures family E

the upper bounds of the design variables regarding the random generation algorithm. In order to derive interpretable data, the cross sections of the elements were categorized into groups. Then, a random generation algorithm was created to produce stochastically, the columns sizes. Each cross section of the corresponding group has two design variables the *x* and *y* dimensions of this cross section. The lower bounds of the design variables situated to the 0.25 cm. Subsequently, a potentially infinite number of potential configurations exist and thus eccentricities,



(a) Floor framing plan (b) Elevation A

(c) Section A (d) Section B

Fig. 11 Structures family F

response parameters and *ROT*. The algorithms implementation aimed to produce a wide variety of test structures, while the torsional attributes also varied.

As the numerical algorithm routinely produces collinear independent random variables of uniform distribution, Gaussian noise was introduced in the variables so as to have normal uncorrelated distribution, based on Box and Muller (1958) model. In order to perform the structural analysis and design for the generated population of structures (several hundreds of instances) and to derive necessary response parameters such us *ROT* and eccentricity of each one design, the random structures generator algorithm was linked to the structural analysis and design software SAP2000 (2016). For each step of the random geometry generation algorithm, the dimensions of the vertical elements were changed, and the corresponded structural response parameters were calculated and saved. Afterwards, the derived results of the structural analysis were utilized to investigate the empirical relationships between *ROT* and other torsional indices. The generative form of the structural configurations are depicted in the Figs. 6-11 for the six family of structures studied:

8. Correlations of *ROT*, Center of Twist and Shear Center

After the structural analysis and the storage of the results, the first step of the numerical interpretation was the investigation of the relationships among the torsional design parameters. The following Figs. 14-21, depict the graphical representation, the least square linear relationships and the calculated *R*-squared of the torsional indices. The high values of *R*-squared, consist numerical

Structures family A

SC = 4.251637 + 1.584118 * TwC
R-Squared = 0.97664

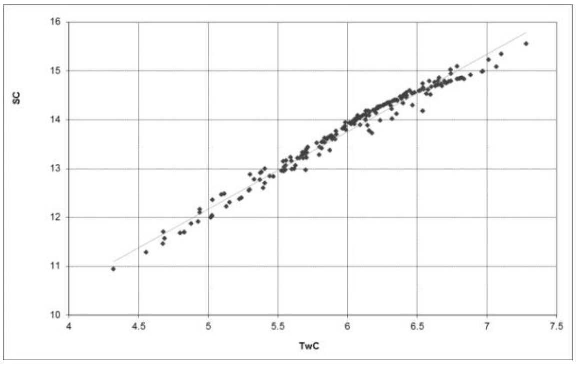


Fig. 12 Correlation of *ROT* with TwC

Structures family A

ROT = -1.063118 + 0.097991 * SC
R-Squared = 0.894526

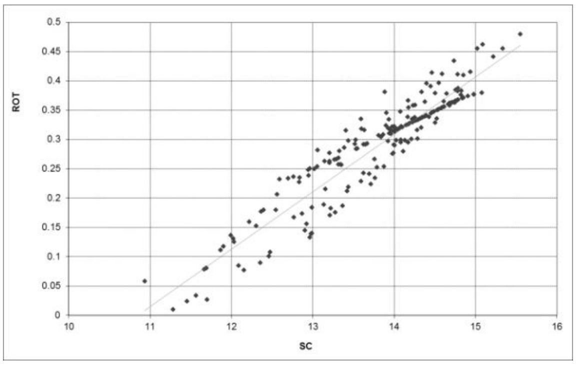


Fig. 13Correlation of *ROT* with SC

Structures family A

SC = 4.251637 + 1.584118 * TwC
R-Squared = 0.97664

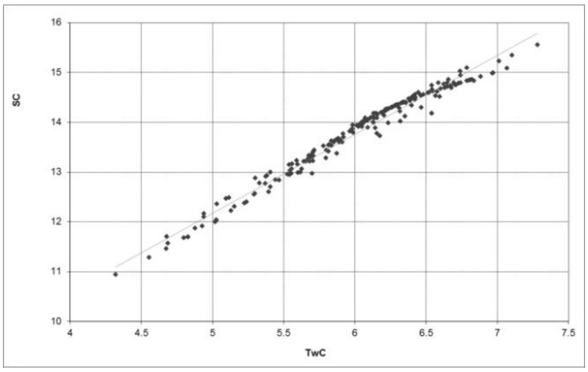


Fig. 14 Correlation of *ROT* with TwC

Structures family A

ROT = -1.063118 + 0.097991 * SC
R-Squared = 0.894526

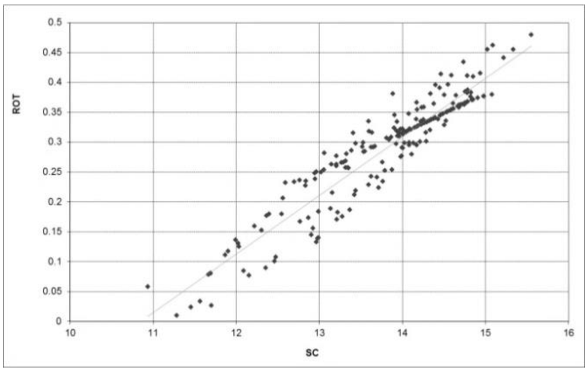


Fig. 15 Correlation of *ROT* with SC

evidence of the theoretical finding of section 4. This high correlation, is derived for a large amount of structures, of different architectural form and thus structural configurations. Hence, the empirical relationships are characterized of a high degree of statistical reliability.

9. Computational results - Ratio of torsion and structural members' dimensions explicit reliance

This section demonstrates that the vertical elements dimensions are enough to determine ratio of torsion value. In the part A of this work, an analytical solution was derived, proving a closed mathematical formula for a certain type of shear cantilever: one axis symmetric - U type. Additionally, an analytic proof for general type of buildings was demonstrated, showing that *ROT* is independent of the loading. In this section these findings will be confirmed computationally.

9.1 (Over) Fitting

Several neural networks were trained in order to examine correlations between *ROT* and structure's vertical elements dimensions. The optimum architecture of the network found to be one with a training functions using Marquardt (1963) optimization approach for the updates of the weights. It follows Bayesian regularization, and particularly minimizes a combination of squared errors and weights, and the minimum of the combination produces a network that fits best. After several tryouts, the networks hidden nodes optimum size (accurate enough and training time efficient) defined to ten (10). In order to assess the independence of the trained network from the test set of values, the total population (637 series of values) was divided to the train set and the test set, where the test set defined to the 15% of the whole sample. This procedure performed to avoid a common problems occurring during neural network training and called overfitting, when the error on the training set is very low (high *R*-squared), but when new data is presented to the network the error is large. Therefore, the test set was used to ensure generalization.

9.2 *ROT* & columns dimensions

Structures family B

ROT = -0.469843 + 0.159878 * TwC
R-Squared = 0.921969

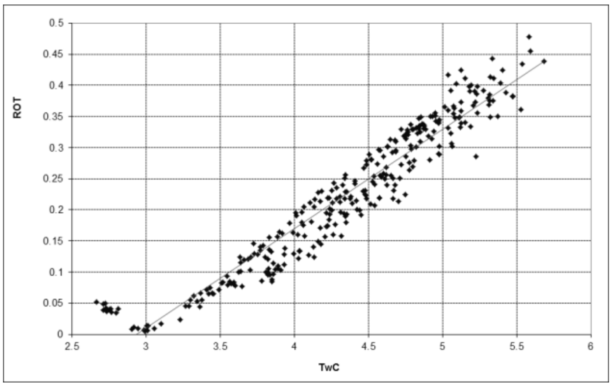


Fig. 16 Correlation of *ROT* with TwC

Structures family D

ROT = -0.488143 + 0.142622 * TwC
R-Squared = 0.888921

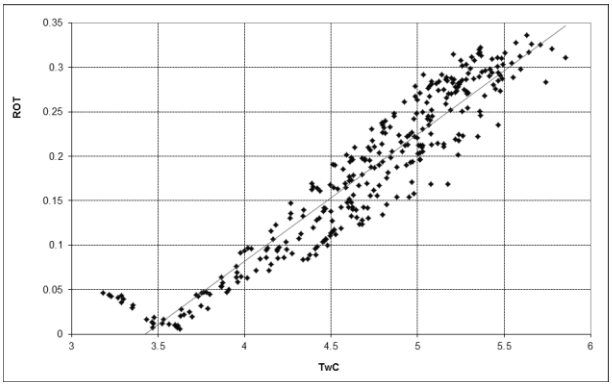


Fig. 17 Correlation of *ROT* with TwC

Structures family D

CMCV = -0.012464 + 0.020474 * Rkx
R-Squared = 0.850597

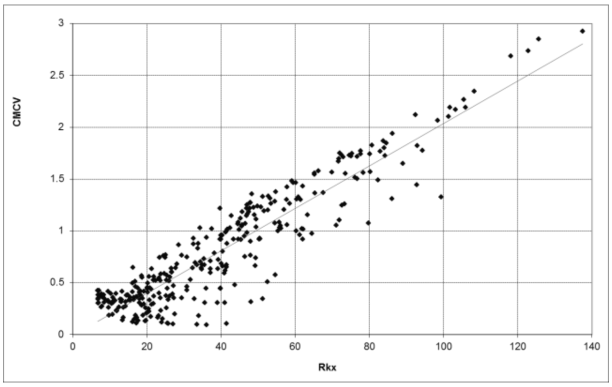


Fig. 18 Correlation of CMCV with Rkx

Structures family E

ROT = -0.391082 + 0.158161 * TwC
R-Squared = 0.887592

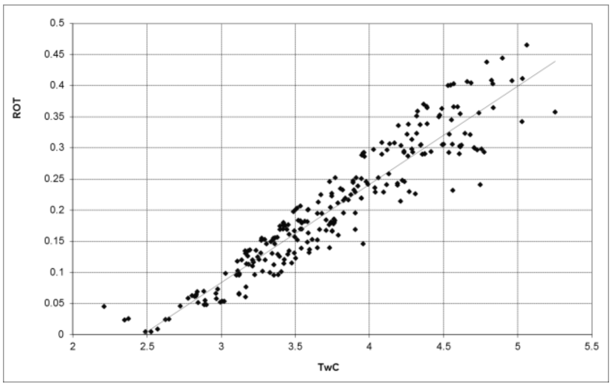


Fig. 19 Correlation of Twc with *ROT*

Structures family F

SC = -6.019494 + 5.129967 * TwC
R-Squared = 0.971503

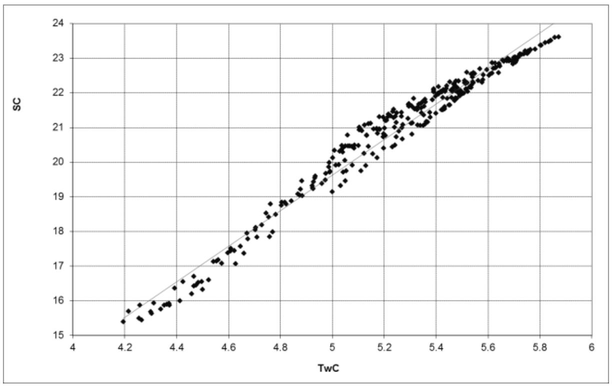


Fig. 20 Correlation of SC with TwC

Structures family F

CR = 7.017684 - 3.710073 * Omega
R-Squared = 0.460026

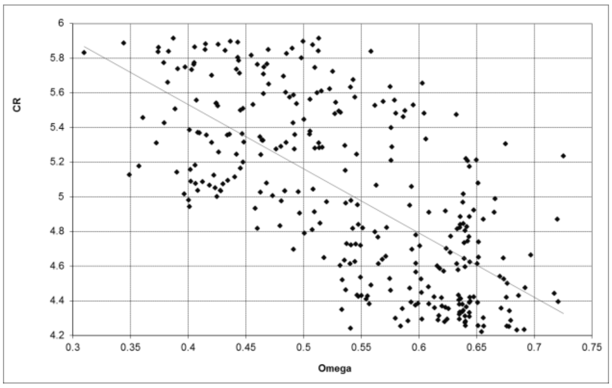


Fig. 21 Correlation of Omega with CR

As shown in the following figure, the correlation was **0.99991** for the train set. This concludes that the columns dimensions are enough to describe a nonlinear correlation with *ROT*. Furthermore, the *R* for the test set found of 0.99892 value, signing that the trained network can be generalized, and so the nonlinear correlation derived.

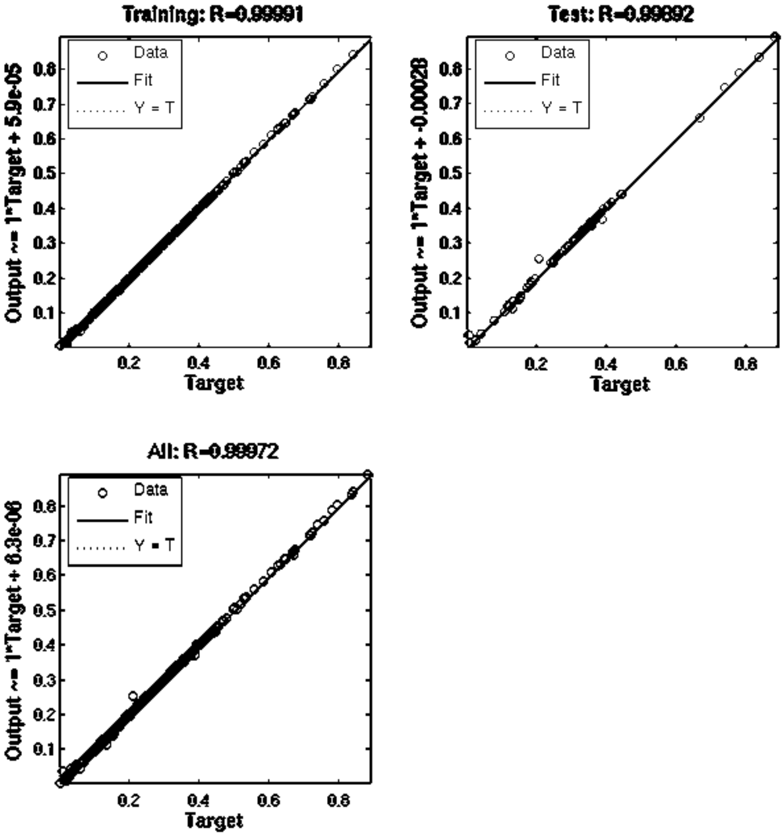


Fig. 22 correlation of *ROT* with columns dimensions

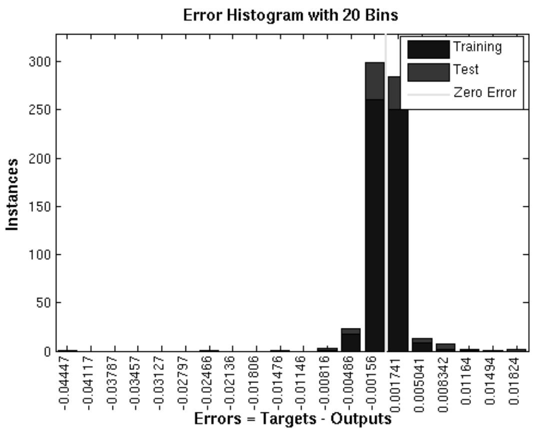


Fig. 23 error histogram

The histogram of the fitting errors, was close to normal distribution as shown in the following figure, signing that the trained network was statistically reliable.

9.3 ROT, columns dimensions & Ω

Neural networks, are not used only to find correlations between variables but also to examine whether one specific variable improves or not the correlation factor and so derive conclusions of the importance of this specific variable. Thus, the next step of this work, the network trained using columns dimensions and also omega as predictors of *ROT*. Omega as described by Hejal and Chopra (1987) is a function of eigenmodes and so independent of the load. In particular, Ω is the uncoupled torsional to lateral frequency ratio

$$\Omega = \frac{\omega_\theta}{\omega_y} \tag{35}$$

where:

- ω_θ is the uncoupled torsional frequency of the structural system
- ω_y is the lateral frequency of the structural system

As shown in the Fig. 24, a slightly increase of the *R* for the train set found, and decrease for the test set. In any case the correlation remain close to unit.

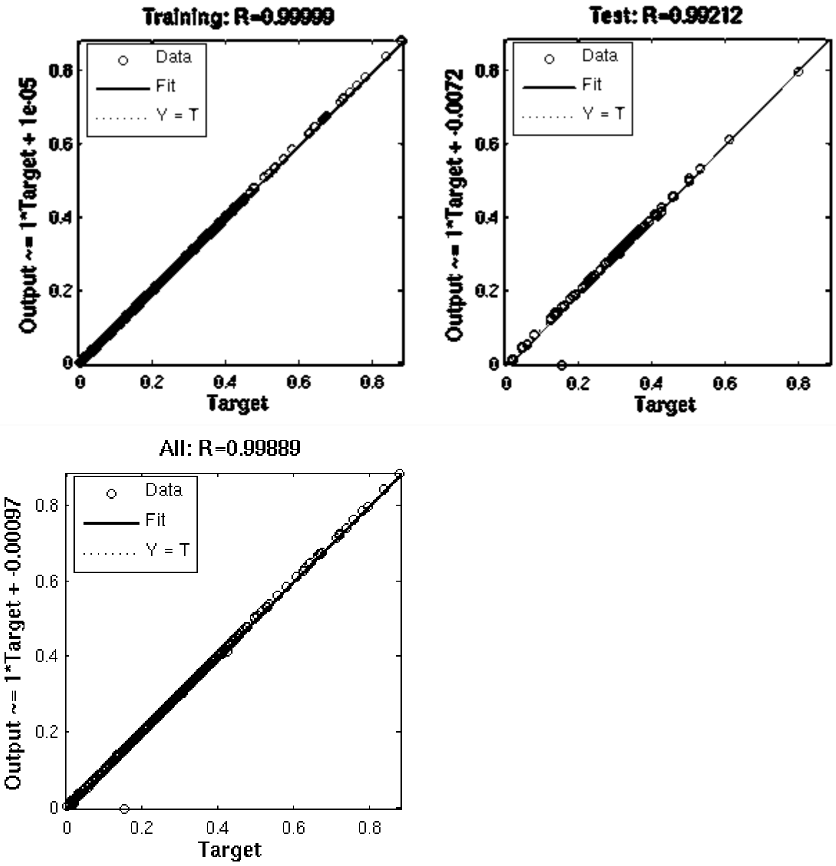


Fig. 24 correlation of *ROT* with columns dimensions and omega

9.4 ROT, columns dimensions, Ω, r_{kx} & r_{ky}

In the next trial the torsional radius r_{kx} and r_{ky} also included to the predictors. The new *R* was unit for the training and improved for the test set. This depicts that torsional radius is an important parameter for torsional phenomena interpretation and affects *ROT*. Unit signifies the computationally unconditional correlation of *ROT* and load independent predictors. Thus, the theoretically derived results confirmed by the computational ones.

9.5 ROT, columns dimensions, Ω, r_{kx}, r_{ky} & V_x

Finally, the force acting on the floor diaphragm was added to the independent variables. The new *R* was lower for the train and test set, meaning that the insertion of V_x disorientates the network and so has a negative effect on the correlation. This is in compliance with this work hypothesis, that *ROT* is independent of forces acting on structure, as it causes disorientation of the correlations.

10. Conclusions

This work is derived into two parts. Part A demonstrates an analytic for one axis symmetric C-type proof that *ROT* is independent of the loading acting onto the cross section. Additionally, an explicit form defining *ROT* as a function of shear beams dimensions was extracted, and in particular the relationship between *ROT* and eccentricity. As a building with rigid diaphragm can be simulated by a shear cantilever beam, this relationship can be generalized and is actually the relationship between *ROT* and the location of the center of twist. Furthermore, the hypothesis that *ROT* is independent of the forces acting on the floor diaphragm was confirmed analytically in section 5.

In part B, computational proofs deduced for six families of structures with one axis of symmetry. Several linear patterns of the relations depicted in the demonstrated figures showing *ROT* to be highly correlated with other criteria for the structural design against torsion. Strong correlating patterns between *ROT*, shear/twist/rigidity center and also omega index, consist numerical evidence that the numerical procedure is reliable.
Afterwards, the numerical results derived from the database of the investigated structures, were utilized to develop the nonlinear model of the correlation between *ROT*, columns dimensions, Ω, r_{kx}, r_{ky} and V_x. In agreement with the analytic proof, significant correlations found between ratio of torsion and structural elements dimensions. Furthermore, when torsional radius and omega index incorporated to the correlation model, the *R*-squared equals to the unit. Finally when the shear forces acting on members added to the predictors the correlation was decreased. This interpretation, constitutes an arithmetic verification of the analytical findings.

These two approaches, analytical and computational confirm that *ROT* is a function only of the vertical elements

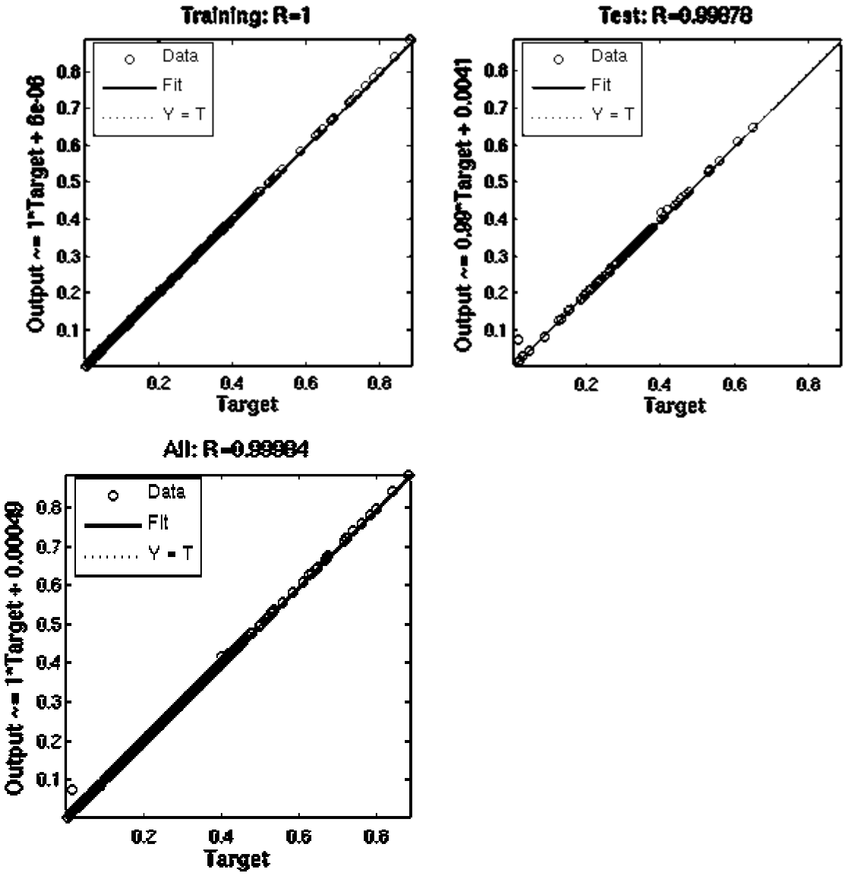


Fig. 25 correlation of *ROT* with columns dimensions and omega + rkx + rky

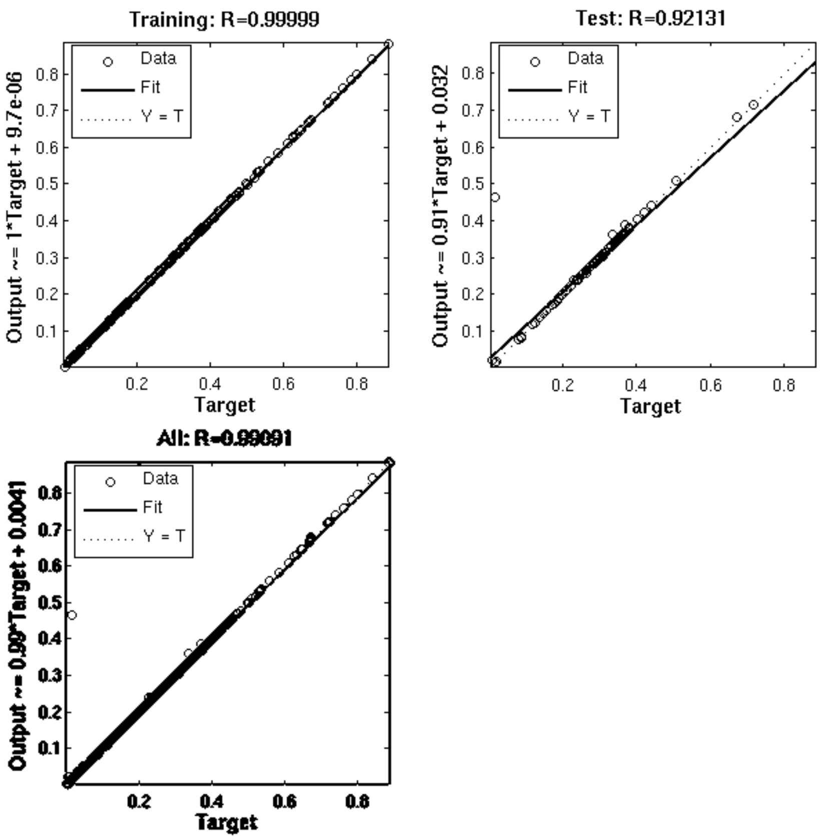


Fig. 26 Correlation of *ROT* with columns dimensions and omega + rkx + rky + Vx

dimensions and locations, likewise center of strength, rigidity and twist. However, *ROT* is defined by the internal forces, while the torsion centers are defined by structural topology and dimensions only. The importance of this finding is that the amplification of the shear forces -defined by *ROT*- due to irregularities, is also independent of the implied loading, thus is an explicit function of structural configuration only. Therefore, *ROT* can be presumed as a particular attribute of the structural system. This finding can be utilized by research and profession engineers in the design of new buildings and rehabilitation of existing ones, regarding quantification of torsional effects for a particular structural configuration.

References

Alapour, T.F., Kazemian, A., Hasaninejad, A., Ghanbari, A. and Ramezanianpour, A.A. (2013), "Prediction of lightweight concrete strength by categorized regression, MLR and ANNs" *Comput. Concrete*, **12**(2), 151-167.

Anagnostopoulos, S.A., Kyrkos, M.T. and Stathopoulos, K.G. (2015), "Earthquake induced torsion in buildings: critical review and state of the art", *Earthq. Struct.*, **8**(2), 305-377.

Beycioğlu, A., Emiroğlu, M., Kocak, Y. and Subaş, S. (2015), "Analyzing the compressive strength of clinker mortars using approximate reasoning approaches-ANN vs MLR", *Comput. Concrete*, **15**(1), 89-101.

Boresi, A.P., Schmidt, R.S. and Sidebottom, O.M. (1993), *Advanced Mechanics Of Materials*, Vol. 6, Wiley, New York, USA.

Box, G.E.P. and Muller, M.E. (1958) "A note on the generation of random normal deviates", *Ann. Math. Statist.*, **29**(2), 610-611

Computers and Structures (1997), SAP2000: Integrated Finite Element Analysis and Design of Structures, Analysis Reference, Computers and Structures, Berkeley, California, USA.

De La Llera, J.C. and Chopra, A.K. (1995), "Understanding the inelastic seismic behavior of asymmetric-plan buildings", *Earthq. Eng. Struct. Dyn.*, **24**(4), 549-572.

Engin, S., Ozturk, E.O. and Okay, F. (2015), "Estimation of ultimate torque capacity of the SFRC beams using ANN", *Struct. Eng. Mech.*, **53**(5), 939-956.

Goldberg, D.E. and Holland, J.H. (1998) "Genetic algorithms and machine learning", *Mach. Learn.*, **3**(2), 95-99.

Hakim, S.J.S. and Razak, H.A. (2013), "Adaptive Neuro Fuzzy Inference System (ANFIS) and Artificial Neural Networks (ANNs) for structural damage identification", *Struct. Eng. Mech.*, **45**(6), 779-802.

Haykin, S. (2004), *Neural Networks. A Comprehensive Foundation*, Prentice Hall, Upper Saddle River, New Jersey, USA.

Hejal, R. and Chopra, A.K. (1987). "Earthquake response of torsionally-coupled buildings", Research Report No. UCBIEERC-87/20; Earthquake Engineering Research Center, University of California, Berkeley, USA.

Kan, C.L. and Chopra A.K. (1977) "Elastic earthquake analysis of torsionally coupled multistorey buildings", *Earthq. Eng. Struct. Dyn.*, **5**(4), 395-412.

Makridakis, S., Wheelwright, S.C. and Hyndman, R.J. (2008), *Forecasting Methods And Applications*. John Wiley & Sons, Hoboken, New Jersey, USA.

Marquardt, D.W. (1963) "An algorithm for least-squares estimation of nonlinear parameters", *J. Soc. Indus. Appl. Math.*, **11**(2), 431-441.

Mohammadhassani, M., Nezamabadi-pour, M., Suhatril, M. and Shariati, M. (2013), "Identification of a suitable ANN architecture in predicting strain in tie section of concrete deep beams", *Struct. Eng. Mech.*, **46**(6), 853-868.

Mylimaj, B. and Tso, W. K. (2002), "A strength distribution criterion for minimizing torsional response of asymmetric wall-type systems", *Earthq. Eng. Struct. Dyn.*, **31**(1), 99-120.

Paulay, Th. (1998), "Torsional mechanisms in ductile building systems", *Earthq. Eng. Sstruct. Dyn.*, **27**(10), 1101-1121.

Peng-hui, L., Hong-ping, Z., Hui, L. and Shun, W. (2015), "Structural damage identification based on genetically trained ANNs in beams", *Smart Struct. Syst.*, **15**(1), 227-244.

Stathi, C.G., Bakas, N.P., Lagaros, N.D. and Papadrakakis, M. (2015), "Ratio of Torsion (*ROT*): an index for assessing the global induced torsion in plan irregular buildings", *Earthq. Struct.*, **9**(1), 145-171.

Tavakkol, S., Alapour, F., Kazemian, A., Hasaninejad, A., Ghanbari, A. and Ramezanianpour, A.A. (2013), "Prediction of lightweight concrete strength by categorized regression, MLR and ANN", *Comput. Concrete*, **12**, 151-167.

Tso, W.K. (1990), "Static eccentricity concept for torsional moment estimations", *J. Struct. Eng.*, **116**(5) 1199-1212.

Yavuz, G. (2016), "Shear strength estimation of RC deep beams using the ANN and strut-and-tie approaches", *Struct. Eng. Mech.*, **57**(4), 657-680.

CC

Nomenclature

α:	sign of internal stresses
A:	cross section
CT:	center of twist
H:	axial forces on a cross section
i:	the direction of the internal shear force (*x or y*)
I_x:	bending moment of inertia about x axis
j:	the direction of the external loading (*x or y*)
M:	bending moment
n:	the number of elements in a floor direction
q:	shear flow
r:	distance between a column and center of twist
r_{kx}:	torsional radius in x direction
r_{ky}:	torsional radius in y direction
ROT:	ratio of torsion
t:	thin walled cross section thickness
T:	torque
u:	joint displacement
V_i:	shear force acting on column i
V_x:	base shear force
σ:	stresses
φ:	diaphragm *ROT*ation
ω_θ:	uncoupled torsional frequency of a structural system
ω_y:	lateral frequency of a structural system
Ω:	uncoupled torsional to lateral frequency ratio