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A new 3-unknowns non-polynomial plate theory for buckling and vibration of functionally graded sandwich plate

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Abstract. In this work a new 3-unknown non-polynomial shear deformation theory for the buckling and vibration analyses of functionally graded material (FGM) sandwich plates is presented. The present theory accounts for non-linear in plane displacement and constant transverse displacement through the plate thickness, complies with plate surface boundary conditions, and in this manner a shear correction factor is not required. The main advantage of this theory is that, in addition to including the shear deformation effect, the displacement field is modelled with only 3 unknowns as the case of the classical plate theory (CPT) and which is even less than the first order shear deformation theory (FSDT). The plate properties are assumed to vary according to a power law distribution of the volume fraction of the constituents. Equations of motion are derived from the Hamilton's principle. Analytical solutions of natural frequency and critical buckling load for functionally graded sandwich plates are obtained using the Navier solution. The results obtained for plate with various thickness ratios using the present non-polynomial plate theory are not only substantially more accurate than those obtained using the classical plate theory, but are almost comparable to those obtained using higher order theories with more number of unknown functions.

Keywords: sandwich plate; functionally graded material; vibration; buckling; a non- polynomial 3-unknown theory

1. Introduction

The conventional sandwich structures are generally fabricated from three homogeneous layers, two face sheets adhesively bonded to the core. However, the sudden change in material properties across the interface between different materials can result in large interlaminar stresses. To overcome these adverse effects, a new class of advanced inhomogeneous composite materials, that composes of two or more phases with different material properties and continuously varying composition distribution, has been developed which is referred to as functionally graded materials (FGMs). These materials are made up of mixture of ceramics and metals that are characterized by the smooth and continuous variation in the properties from one surface to another (Koizumi 1993,

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Reddy 2000, Ould Larbi, Kaci *et al*. 2013, Ait Atmane, Tounsi *et al*. 2015, Kar and Panda 2015, Pradhan and Chakraverty 2015, Sallai, Hadji *et al*. 2015, Bennai, Ait Atmane *et al*. 2015, Ebrahimi and Dashti 2015, Bounouara, Benrahou *et al*. 2016). Such materials were introduced as to take advantage of the desired material properties of each constituent material without interface problems. The sandwich plate faces are typically made from a mixture of two materials. While the core of this sandwich plate is fully homogeneous material.

In recent years, there is a rapid increase in the use of functionally graded sandwich structures in aerospace, marine and civil engineering due to high strength-to-weight ratio. With the wide application of these structures, more accurate theories are required to predict their bending, vibration and buckling response. Li, Iu *et al*. (2008) presented a three-dimensional solution for free vibration of multi-layer FGM system-symmetric and unsymmetric FGM sandwich plates using the Ritz method. Three-dimensional finite element simulations for analyzing low velocity impact behavior of sandwich panels with a functionally graded core were conducted by Etemadi, Khatibi *et al*. (2009). Anderson (2003) presented an analytical three-dimensional elasticity solution method for a sandwich composite with a functionally graded core subjected to transverse loading by a rigid spherical indentor. Natarajan and Ganapathi (2012) investigated the bending and the free flexural vibration behavior of sandwich FGM plates using QUAD-8 shear flexible element developed based on higher order structural theory. The governing equations obtained are solved for static analysis considering two types of sandwich FGM plates. Xiang, Kang *et al*. (2013) analyzed the free vibration of FG sandwich plates using a *n*th-order shear deformation theory and a meshless method, while Sobhy (2013) investigated the buckling and free vibration of FG sandwich plates using various HSDTs. Yaghoobi and Yaghoobi (2013) presented analytical solutions for the buckling of symmetric sandwich plates with FGM face sheets resting on an elastic foundation based on the first-order shear deformation plate theory and subjected to mechanical, thermal and also thermo-mechanical loads. The third order shear deformation theory (TSDT) by Reddy (1984) and Baseri, Jafari *et al*. (2016) provided better results compared to CPT and FSDT but researchers have obtained more accurate results by adopting various non-polynomial shear deformation theories. In non-polynomial shear deformation theories, the in-plane displacements are the function of thickness coordinate. The function may be trigonometric, exponential or hyperbolic. Touratier (1991) recommended sinusoidal function, Bouderba, Houari *et al*. (2013), Tounsi, Houari *et al*. (2013), Bachir Bouiadjra, Adda Bedia *et al*. (2013), Meradjah, Kaci *et al*. (2015), Al-Basyouni, Tounsi *et al*. (2015) and Ahouel, Houari *et al*. (2016) suggested four variable theory with same function. Larbi Chaht, Kaci *et al*. (2015) and Hamidi, Houari *et al*. (2015) also recommended sinusoidal function but they considered transverse deflection due to bending as well as due to shear and thickness stretching. Soldatos (1992), Akvaci (2014) and Mahi, Adda Bedia *et al*. (2015) have suggested hyperbolic shear strain function for the analysis. Saldatos (1992) employed sine hyperbolic function; whereas, Akvaci (2014) and Mahi, Adda Bedia *et al*. (2015) suggested tangential hyperbolic shear strain function. A new hyperbolic functions are also suggested by Belabed, Houari *et al*. (2014), Hebali, Tounsi *et al*. (2014), Bousahla, Houari *et al*. (2014), Bourada, Kaci *et al*. (2015), Belkorissat, Houari *et al*. (2015) and Bennoun, Houari *et al*. (2016). Recently, Akavci (2016) presented a new hyperbolic shear and normal deformation plate theory for the static, free vibration and buckling analysis of the simply supported functionally graded sandwich plates on elastic foundation. Chikh, Bakora *et al*. (2016) presented an analytical formulation based on both hyperbolic shear deformation theory and stress function, to study the nonlinear post-buckling response of symmetric FG plates supported by elastic foundations and subjected to in-plane compressive, thermal and thermo-mechanical loads. Abdelbari, Fekrar *et al*.

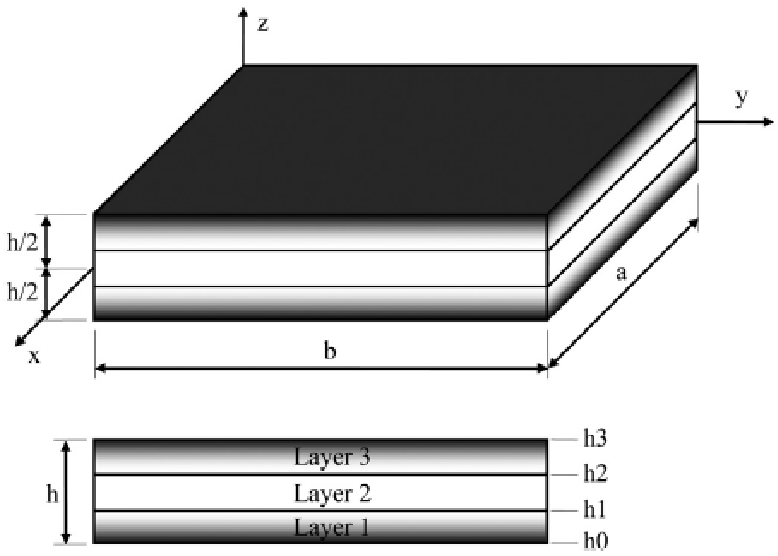

Fig. 1 Geometry and coordinates of rectangular FG sandwich plate

(2016) proposed a simple hyperbolic shear deformation theory for analysis of FG plates resting on elastic foundation. El-Hassar, Benyoucef *et al*. (2016) presented an exact analytical solution based on hyperbolic shear deformation theory for thermal stability of solar FG rectangular plates subjected to uniform, linear and non-linear temperature rises across the thickness direction is developed. To reduce computational cost, higher-order shear deformation theories (HSDTs) with four unknowns were recently developed for FG plates (Abdelaziz, Ait Atmane *et al*. 2011, Houari, Benyoucef *et al*. 2011, El Meiche, Tounsi *et al*. 2011, Benachour, Daouadji *et al*. 2011, Hadji, Ait Atmane *et al*. 2011, Bourada, Tounsi *et al*. 2012, Kettaf, Houari *et al*. 2013, Zidi, Tounsi *et al*. 2014, Nedri, El Meiche *et al*. 2014, Draiche, Tounsi *et al*. 2014, Ait Amar Meziane, Abdelaziz *et al*. 2014, Attia, Tounsi *et al*. 2015, Bouchafa, Bachir Bouiadjra *et al*. 2015, Ait Yahia, Ait Atmane *et al*. 2015, Nguyen, Thai *et al*. 2015, Tebboune, Benrahou *et al*. 2015, Benselama, El Meiche *et al*. 2015, Hadji, Hassaine Daouadji *et al*. 2015, Bellifa, Benrahou *et al*. 2016, Bouderba, Houari *et al*. 2016, Boukhari, Ait Atmane *et al*. 2016).

In this work, a new 3-unknown non-polynomial shear deformation theory is presented for buckling and vibration responses of FG sandwich plates. The principal feature of this theory is that, in addition to including the shear deformation effect, the displacement field is modeled with only 3 unknowns as the case of the CPT, which is even less than the FSDT and do not need shear correction factor. Numerical examples are presented to verify the accuracy of the present theory in predicting the buckling and free vibration responses of FG sandwich plates.

2. Theoretical formulation

A sandwich plate composed of three layers is considered in this work as shown in Fig. 1. Two FG face sheets are made from a mixture of a metal and a ceramic, while a core is made of an isotropic homogeneous material. The material properties of FG face sheets are assumed to vary continuously through the plate thickness by a power law distribution as (Majumdar and Das 2016)

$$P^{(n)}(z) = P_2 + (P_1 - P_2)V^{(n)} \tag{1}$$

where $P^{(n)}$ is the effective material property of FGM of layer n like Young's modulus E, Poisson's ratio v, and mass density ρ. P_1 and P_2 are the properties of the top and bottom faces of layer 1, respectively, and vice versa for layer 3 depending on the volume fraction $V^{(n)}$, (n=1,2,3) defined by

$$\begin{cases} V^{(1)} = \left(\dfrac{z-h_0}{h_1-h_0}\right)^p & \text{for } z\in[h_0,h_1] \\ V^{(2)} = 1 & \text{for } z\in[h_1,h_2] \\ V^{(3)} = \left(\dfrac{z-h_3}{h_2-h_3}\right)^p & \text{for } z\in[h_2,h_3] \end{cases} \tag{2}$$

where p is the power law index ($0\le p\le+\infty$), which dictates the material variation profile through the thickness.

2.1 Three-unknown non-polynomial shear deformations theory

The objective of this work is to develop a simple 3-unknown non-polynomial shear deformation theory in which in-plane displacement is expanded as a non-polynomial variation through the thickness. The advantages of the present theory is that the number of variables involved in this theory is same as that in the classical plate theory (CPT), and the stress-free boundary conditions on the top and bottom surfaces of the plate can be guaranteed without use of shear correction factors .

2.1.1 Kinematics

The displacement field satisfying the conditions of transverse shear stresses (and hence strains) vanishing at a point (x, y, $\pm h/2$) on the outer (top) and inner (bottom) surfaces of the plate, is given as follows

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} - f(z)\frac{\partial^3 w_0}{\partial x^3} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} - f(z)\frac{\partial^3 w_0}{\partial y^3} \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{3}$$

where u_0, v_0, and w_0 are three unknown displacement functions of midplane of the plate. $f(z)$ is a shape function representing the distribution of the transverse shear strains and shear stresses through the thickness of the plate and is given as

$$f(z) = -2\frac{\left(\sinh\left(\dfrac{z}{h}\right)\cosh\left(\dfrac{1}{2}\right)^2 - z\cosh\left(\dfrac{z}{h}\right)\right)h^2}{\cosh\left(\dfrac{z}{h}\right)\cosh\left(\dfrac{1}{2}\right)^2}, \tag{4}$$

The shape function in Eq. (4) assures an accurate distribution of shear deformation within the plate thickness and allows to transverse shear stresses vary as parabolic across the thickness as satisfying shear stress free surface conditions without using shear correction factors. In addition, this model is suitable for different FGM sandwich plates and easy to implement. The numerical examples show that the present theory shows good agreement with that of the results of other shear deformation theories and the 3D linear theory of elasticity.

The nonzero strains associated with the displacement field in Eq. (3) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} + f(z) \begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \tag{5}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^4 w_0}{\partial x^2} \\ -\dfrac{\partial^4 w_0}{\partial y^2} \\ -\dfrac{\partial^2 (\nabla^2 w_0)}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^3 w_0}{\partial y^3} \\ -\dfrac{\partial^3 w_0}{\partial x^3} \end{Bmatrix}, \tag{6}$$

and

$$g(z) = f'(z), \; \nabla^2 w_0 = \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \tag{7}$$

2.1.2 Constitutive relations

The linear constitutive relations of a FG sandwich plate can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}^{(n)} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}^{(n)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}^{(n)} \tag{8}$$

where (σ_x, σ_y, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively. The stiffness coefficients, C_{ij}, can be expressed as

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}, \quad C_{12} = \nu C_{11} \tag{9a}$$

$$C_{44} = C_{55} = C_{66} = G(z) = \frac{E(z)}{2(1+\nu)}, \tag{9b}$$

2.1.3 Governing equations

Using Hamilton's energy principle derives the equation of motion of the FG sandwich plate

$$\int_0^T (\delta U - \delta V - \delta T)dt = 0 \tag{10}$$

where δU is the variation of strain energy, δT is the variation of kinetic energy of the FG sandwich plate, and δV is the variation of work of external forces. The variation of strain energy of the plate is done by

$$\begin{aligned}\delta U &= \int_V \left[\sigma_x \delta\, \varepsilon_x + \sigma_y \delta\, \varepsilon_y + \tau_{xy} \delta\, \gamma_{xy} + \tau_{yz} \delta\, \gamma_{yz} + \tau_{xz} \delta\, \gamma_{xz}\right] dV \\ &= \int_A \left[N_x \delta\, \varepsilon_x^0 + N_y \delta\, \varepsilon_y^0 + N_{xy} \delta\, \gamma_{xy}^0 + M_x \delta\, k_x + M_y \delta\, k_y + M_{xy} \delta\, k_{xy}\right. \\ &\quad \left. + S_x \delta\, \eta_x + S_y \delta\, \eta_y + S_{xy} \delta\, \eta_{xy} + Q_{yz} \delta\, \gamma_{yz}^0 + Q_{xz} \delta\, \gamma_{xz}^0 \right] dA = 0\end{aligned} \tag{11}$$

where A is the top surface and the stress resultants N, M, S and Q are defined by

$$(N_i, M_i, S_i) = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} (1, z, f)(\sigma_i)^{(n)} dz, \quad (i = x, y, xy) \text{ and } Q_i = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} (\tau_i)^{(n)} g(z) dz., \quad (i = xz, yz) \tag{12}$$

where h_n and h_{n-1} are the top and bottom z-coordinates of the nth layer.

The variation of kinetic energy of the plate can be expressed in the form

$$\begin{aligned}\delta K &= \int_V \left[\dot{u}\delta\, \dot{u} + \dot{v}\delta\, \dot{v} + \dot{w}\delta\, \dot{w}\right]\rho(z) dV \\ &= \int_A \Big\{ I_0 \left[\dot{u}_0 \delta \dot{u}_0 + \dot{v}_0 \delta \dot{v}_0 + \dot{w}_0 \delta \dot{w}_0\right] \\ &\quad - I_1 \left(\dot{u}_0 \frac{\partial \delta \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial x} \delta\, \dot{u}_0 + \dot{v}_0 \frac{\partial \delta \dot{w}_0}{\partial y} + \frac{\partial \dot{w}_0}{\partial y} \delta\, \dot{v}_0 \right) \\ &\quad - J_1 \left(\dot{u}_0 \frac{\partial^3 \delta \dot{w}_0}{\partial x^3} + \frac{\partial^3 \dot{w}_0}{\partial x^3} \delta\, \dot{u}_0 + \dot{v}_0 \frac{\partial^3 \delta \dot{w}_0}{\partial y^3} + \frac{\partial^3 \dot{w}_0}{\partial y^3} \delta\, \dot{v}_0 \right) \\ &\quad + I_2 \left(\frac{\partial \dot{w}_0}{\partial x} \frac{\partial \delta\, \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial y} \frac{\partial \delta\, \dot{w}_0}{\partial y} \right) + K_2 \left(\frac{\partial^3 \dot{w}_0}{\partial x^3} \frac{\partial^3 \delta\, \dot{w}_0}{\partial x^3} + \frac{\partial^3 \dot{w}_0}{\partial y^3} \frac{\partial^3 \delta\, \dot{w}_0}{\partial y^3} \right) \\ &\quad + J_2 \left(\frac{\partial \dot{w}_0}{\partial x} \frac{\partial^3 \delta\, \dot{w}_0}{\partial x^3} + \frac{\partial^3 \dot{w}_0}{\partial x^3} \frac{\partial \delta\, \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial y} \frac{\partial^3 \delta\, \dot{w}_0}{\partial y^3} + \frac{\partial^3 \dot{w}_0}{\partial y^3} \frac{\partial \delta\, \dot{w}_0}{\partial y} \right) \Big\} dA\end{aligned} \tag{13}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t;

$\rho(z)$ is the mass density; and (I_0, I_1, J_1, I_2, J_2, K_2) are mass inertias defined as

$$\left(I_0, I_1, J_1, I_2, J_2, K_2\right) = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \left(1, z, f, z^2, z\, f, f^2\right) \rho^{(n)}(z) dz \tag{14}$$

The variation of work done by in-plane load (N_x^0 , N_y^0) can be expressed as

$$\delta V = \int_A \left(N_x^0 \frac{\partial w}{\partial x} \frac{\partial \delta\, w}{\partial x} + N_y^0 \frac{\partial w}{\partial y} \frac{\partial \delta\, w}{\partial y} \right) dA \tag{15}$$

Substituting the expressions for δU, δK, and δV from Eqs. (11), (13) and (15) into Eq. (10) and integrating by parts, and collecting the coefficients of δu_0, δv_0 and δw_0, the following equations of motion are obtained

$$\begin{aligned}
&\delta\, u_0 : \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} - J_1 \frac{\partial^3 \ddot{w}_0}{\partial x^3} \\
&\delta\, v_0 : \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial y} - J_1 \frac{\partial^3 \ddot{w}_0}{\partial y^3} \\
&\delta\, w_0 : \ \frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial^4 S_x}{\partial x^4} + \frac{\partial^4 S_{xy}}{\partial x^3 \partial y} + \frac{\partial^4 S_{xy}}{\partial y^3 \partial x} + \frac{\partial^4 S_y}{\partial y^4} \\
&\qquad - \frac{\partial^3 Q_{xz}}{\partial x^3} - \frac{\partial^3 Q_{yz}}{\partial y^3} + N_x^0 \frac{\partial^2 w}{\partial x^2} + N_y^0 \frac{\partial^2 w}{\partial y^2} = I_0 \ddot{w}_0 + I_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) \\
&\qquad + J_1 \left(\frac{\partial^3 \ddot{u}_0}{\partial x^3} + \frac{\partial^3 \ddot{v}_0}{\partial y^3} \right) - I_2 \left(\frac{\partial^2 \ddot{w}_0}{\partial x^2} + \frac{\partial^2 \ddot{w}_0}{\partial y^2} \right) - 2 J_2 \left(\frac{\partial^4 \ddot{w}_0}{\partial x^4} + \frac{\partial^4 \ddot{w}_0}{\partial y^4} \right) - K_2 \left(\frac{\partial^6 \ddot{w}_0}{\partial x^6} + \frac{\partial^6 \ddot{w}_0}{\partial y^6} \right)
\end{aligned} \tag{16}$$

By substituting Eq. (5) into Eq. (8) and the subsequent results into Eq. (12), the stress resultants are obtained as

$$\begin{Bmatrix} N \\ M \\ S \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k \\ \eta \end{Bmatrix}, \quad Q = A^s \gamma \, , \tag{17}$$

in which

$$N = \left\{ N_x, N_y, N_{xy} \right\}^t, \quad M = \left\{ M_x, M_y, M_{xy} \right\}^t, \quad S = \left\{ S_x, S_y, S_{xy} \right\}^t, \tag{18a}$$

$$\varepsilon = \left\{ \varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0 \right\}^t, \quad k = \left\{ k_x, k_y, k_{xy} \right\}^t, \quad \eta = \left\{ \eta_x, \eta_y, \eta_{xy} \right\}^t, \tag{18b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{18c}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}, \tag{18d}$$

$$Q = \{Q_{xz}, Q_{yz}\}^t, \quad \gamma = \{\gamma_{xz}^0, \gamma_{yz}^0\}^t, \quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \tag{18d}$$

and stiffness components are given as

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} C_{11}^{(n)} \left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu^{(n)} \\ \dfrac{1-\nu^{(n)}}{2} \end{Bmatrix} dz, \tag{19a}$$

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right), \tag{19b}$$

$$A_{44}^s = A_{55}^s = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} C_{44}^{(n)} [g(z)]^2 dz, \tag{19c}$$

By substituting Eq. (17) into Eq. (16), the equations of motion can be expressed in terms of displacements (u_0, v_0 and w_0) as

$$\begin{aligned} & A_{11} \frac{\partial^2 u_0}{\partial x^2} + A_{66} \frac{\partial^2 u_0}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 v_0}{\partial x \partial y} - B_{11} \frac{\partial^3 w_0}{\partial x^3} - (B_{12} + 2B_{66}) \frac{\partial^3 w_0}{\partial x \partial y^2} \\ & - B_{66}^s \frac{\partial^5 w_0}{\partial x^3 \partial y^2} - \left(B_{12}^s + B_{66}^s\right) \frac{\partial^5 w_0}{\partial x \partial y^4} - B_{11}^s \frac{\partial^5 w_0}{\partial x^5} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} - J_1 \frac{\partial^3 \ddot{w}_0}{\partial x^3}, \end{aligned} \tag{20a}$$

$$\begin{aligned} & A_{22} \frac{\partial^2 v_0}{\partial y^2} + A_{66} \frac{\partial^2 v_0}{\partial x^2} + (A_{12} + A_{66}) \frac{\partial^2 u_0}{\partial x \partial y} - B_{22} \frac{\partial^3 w_0}{\partial y^3} - (B_{12} + 2B_{66}) \frac{\partial^3 w_0}{\partial x^2 \partial y} \\ & - B_{66}^s \frac{\partial^5 w_0}{\partial x^2 \partial y^3} - \left(B_{12}^s + B_{66}^s\right) \frac{\partial^5 w_0}{\partial x^4 \partial y} - B_{22}^s \frac{\partial^5 w_0}{\partial y^5} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial y} - J_1 \frac{\partial^3 \ddot{w}_0}{\partial y^3}, \end{aligned} \tag{20b}$$

$$\begin{aligned} & B_{11} \frac{\partial^3 u_0}{\partial x^3} + (B_{12} + 2B_{66}) \frac{\partial^3 u_0}{\partial x \partial y^2} + (B_{12} + 2B_{66}) \frac{\partial^3 v_0}{\partial x^2 \partial y} + B_{22} \frac{\partial^3 v_0}{\partial y^3} - D_{11} \frac{\partial^4 w_0}{\partial x^4} \\ & - 2(D_{12} + 2D_{66}) \frac{\partial^4 w_0}{\partial x^2 \partial y^2} - D_{22} \frac{\partial^4 w_0}{\partial y^4} + B_{11}^s \frac{\partial^5 u_0}{\partial x^5} + \left(B_{12}^s + B_{66}^s\right) \frac{\partial^5 u_0}{\partial x \partial y^4} + \left(B_{12}^s + B_{66}^s\right) \frac{\partial^5 v_0}{\partial x^4 \partial y} \\ & + B_{22}^s \frac{\partial^5 v_0}{\partial y^5} + B_{66}^s \frac{\partial^5 v_0}{\partial x^3 \partial y^2} + B_{66}^s \frac{\partial^5 v_0}{\partial x^2 \partial y^3} - 2D_{11}^s \frac{\partial^6 w_0}{\partial x^6} - 2\left(D_{12}^s + 2D_{66}^s\right) \frac{\partial^6 w_0}{\partial x^2 \partial y^4} \\ & - 2\left(D_{12}^s + 2D_{66}^s\right) \frac{\partial^6 w_0}{\partial x^4 \partial y^2} - 2D_{22}^s \frac{\partial^6 w_0}{\partial y^6} - H_{11}^s \frac{\partial^8 w_0}{\partial x^8} - 2\left(H_{12}^s + H_{66}^s\right) \frac{\partial^8 w_0}{\partial x^4 \partial y^4} \end{aligned}$$

$$
\begin{aligned}
&- H_{66}^{s}\frac{\partial^8 w_0}{\partial x^6 \partial y^2} - H_{66}^{s}\frac{\partial^8 w_0}{\partial x^2 \partial y^6} - H_{22}^{s}\frac{\partial^8 w_0}{\partial y^8} + A_{44}^{s}\frac{\partial^6 w_0}{\partial x^6} + A_{55}^{s}\frac{\partial^6 w_0}{\partial y^6} + N_x^0\frac{\partial^2 w}{\partial x^2} + N_y^0\frac{\partial^2 w}{\partial y^2} = I_0\ddot{w} \\
&+ I_1\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) + J_1\left(\frac{\partial^3 \ddot{u}_0}{\partial x^3} + \frac{\partial^3 \ddot{v}_0}{\partial y^3}\right) - I_2\left(\frac{\partial^2 \ddot{w}_0}{\partial x^2} + \frac{\partial^2 \ddot{w}_0}{\partial y^2}\right) \\
&\qquad - 2J_2\left(\frac{\partial^4 \ddot{w}_0}{\partial x^4} + \frac{\partial^4 \ddot{w}_0}{\partial y^4}\right) - K_2\left(\frac{\partial^6 \ddot{w}_0}{\partial x^6} + \frac{\partial^6 \ddot{w}_0}{\partial y^6}\right)
\end{aligned}
\tag{20c}
$$

3. Analytical solutions

The above equations of motion are analytically solved for bending and free vibration problems of a simply supported rectangular plate. Based on Navier solution procedure, the displacements are assumed as follows

$$
\begin{Bmatrix} u_0(x,y,t) \\ v_0(x,y,t) \\ w_0(x,y,t) \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\lambda\, x)\sin(\mu\, y)\, e^{i\omega t} \\ V_{mn}\sin(\lambda\, x)\cos(\mu\, y)\, e^{i\omega t} \\ W_{mn}\sin(\lambda\, x)\sin(\mu\, y)\, e^{i\omega t} \end{Bmatrix}
\tag{21}
$$

where $i = \sqrt{-1}$, $\lambda = m\pi / a$, $\mu = n\pi / b$, (U_{mn}, V_{mn}, W_{mn}) are the unknown maximum displacement coefficients, and ω is the angular frequency.

Assuming that there is a given ratio between the in-plane compressive loads (N_x^0, N_y^0) such that $N_x^0 = -N_0$ and $N_y^0 = -\gamma\, N_0$; $\gamma = N_y^0 / N_x^0$

Substituting Eq. (21) into Eq. (20), the analytical solutions can be obtained from

$$
\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} \\ 0 & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}\right) = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}
\tag{22}
$$

where

$$
\begin{aligned}
a_{11} &= -\left(A_{11}\lambda^2 + A_{66}\mu^2\right) \\
a_{12} &= -\lambda\, \mu\left(A_{12} + A_{66}\right) \\
a_{13} &= \lambda\,[B_{11}\lambda^2 + (B_{12} + 2B_{66})\,\mu^2 - B_{11}^{s}\lambda^4 - B_{12}^{s}\mu^4 - B_{66}^{s}\lambda^2\mu^2 - B_{66}^{s}\mu^4] \\
a_{22} &= -\left(A_{66}\lambda^2 + A_{22}\mu^2\right) \\
a_{23} &= \mu\,[B_{22}\mu^2 + (B_{12} + 2B_{66})\,\lambda^2 - B_{22}^{s}\mu^4 - B_{12}^{s}\lambda^4 - B_{66}^{s}\lambda^2\mu^2 - B_{66}^{s}\lambda^4] \\
a_{33} &= -D_{11}\lambda^4 - 2(D_{12} + 2D_{66})\lambda^2\mu^2 - D_{22}\mu^4 + 2(D_{11}^{s}\lambda^6 + D_{22}^{s}\mu^6) \\
&\quad + 2\left(\lambda^4\mu^2 + \lambda^2\mu^4\right)(D_{12}^{s} + 2D_{66}^{s}) - H_{11}^{s}\lambda^8 - H_{22}^{s}\mu^8 - 2\lambda^4\mu^4(H_{12}^{s} + H_{66}^{s}) \\
&\quad - \left(\lambda^6\mu^2 + \lambda^2\mu^6\right)H_{66}^{s} - A_{44}^{s}\lambda^6 - A_{55}^{s}\mu^6 + N_0\left(\lambda^2 + \gamma\,\mu^2\right)
\end{aligned}
\tag{23}
$$

$$m_{11} = m_{22} = -I_0$$
$$m_{13} = \lambda\left(I_1 + J_1\lambda^2\right)$$
$$m_{23} = \mu\left(I_1 + J_1\mu^2\right),$$
$$m_{33} = -\left(I_0 + I_2\left(\lambda^2 + \mu^2\right) + 2J_2\left(\lambda^4 + \mu^4\right) + K_2\left(\lambda^6 + \mu^6\right)\right).$$

4. Numerical results and discussion

In this section, the free vibration and the buckling analysis of simply supported FG sandwich plates using the new formulation is presented. The main goal of the present work is to prove that it is possible to gain accuracy similar to other shear deformation theories by choosing the displacement field modelled with only 3 unknowns as the case of the classical plate theory (CPT). The FG plate is taken to be made of aluminum and alumina with the following material properties:

Ceramic (P_1: Alumina, Al_2O_3): E_c=380 GPa; v=0.3; ρ_c=3800 kg/m^3.

Metal (P_2: Aluminium, Al): E_m=70 GPa; v=0.3; ρ_m=2707 kg/m^3.

For simplicity, the non-dimensional natural frequency and critical buckling parameters are defined as

$$\bar{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho_0}{E_0}}, \quad \bar{N} = \frac{Na^2}{100h^2E_0} \tag{24}$$

where ρ_0=1 kg/m^3, E_0=1 GPa.

The following four layer configurations are used for multilayered FG sandwich plates:

1. 1-0-1 configuration in which is made of two layers of equal thickness without a core.
2. 1-1-1 configuration in which thickness of the core is same as the thickness of face sheets.
3. 1-2-1 configuration in which thickness of the core is twice the thickness of face sheets.
4. 2-1-2 configuration in which the core of the plate is half the face thickness.
5. 2-2-1 configuration in which the core thickness equals the lower face thickness while it is twice the upper face thickness.
6. 2-1-1 configuration in which the core thickness equals the upper face thickness while it is twice the lower face thickness.

4.1 Results for free vibration analysis

The natural frequencies of the systems are computed using Eq. (22) after setting N_x^0 and N_y^0 equal to zero. The non-dimensonalized fundamental frequencies of FG plates are presented here to estimate the accuracy of the presented new 3-unknown non-polynomial shear deformation theory.

First, for the verification purpose, the results computed using the new 3-unknown non-polynomial shear deformation theory are compared with other theories existing in the literature such as the three-dimensional linear theory of elasticity by Li, Iu *et al*. (2008) as well as the different theories presented by El Meiche, Tounsi *et al*. (2011) mainly the higher-order shear deformation theories (HSDTs) such as: parabolic shear deformation plate theory (PSDPT), sinusoidal shear deformation plate theory (SSDPT) and hyperbolic shear deformation plate theory

Table 1 Comparisons of natural fundamental frequency parameters ϖ of simply supported square power-law FG plates with other theories (h/b=0.1)

p	Theory	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
0	SSDPT [(a)]	1.82452	1.82452	1.82452	1.82452	1.82452	1.82452
	PSDPT [(a)]	1.82445	1.82445	1.82445	1.82445	1.82445	1.82445
	HSDPT [(a)]	1.82449	1.82449	1.82449	1.82449	1.82449	1.82449
	Elasticity [(b)]	1.82682	1.82682	1.82682	1.82682	1.82682	1.82682
	Present	1.83141	1.83141	1.83141	1.83141	1.83141	1.83141
0.5	SSDPT [(a)]	1.44436	1.48418	1.51258	1.51927	1.55202	1.57450
	PSDPT [(a)]	1.44424	1.48408	1.51253	1.51922	1.55199	1.57451
	HSDPT [(a)]	1.44419	1.48405	1.50636	1.51922	1.54714	1.57458
	Elasticity [(b)]	1.44614	1.48608	1.50841	1.52131	1.54926	1.57668
	Present	1.44487	1.48468	1.50716	1.51990	1.54809	1.57560
1	SSDPT [(a)]	1.24335	1.30023	1.34894	1.35339	1.40792	1.43931
	PSDPT [(a)]	1.24320	1.30011	1.34888	1.35333	1.40789	1.43934
	HSDPT [(a)]	1.24310	1.30004	1.33328	1.35331	1.39559	1.43940
	Elasticity [(b)]	1.24470	1.30181	1.33511	1.35523	1.39763	1.44137
	Present	1.24413	1.30095	1.33392	1.35387	1.39599	1.43957
5	SSDPT [(a)]	0.94630	0.98207	1.07445	1.04481	1.14741	1.17399
	PSDPT [(a)]	0.94598	0.98184	1.07432	1.04466	1.14731	1.17397
	HSDPT [(a)]	0.94574	0.98166	1.03033	1.04455	1.10875	1.17397
	Elasticity [(b)]	0.94476	0.98103	1.02942	1.04532	1.10983	1.17567
	Present	0.94801	0.98741	1.03442	1.05031	1.11271	1.17710
10	SSDPT [(a)]	0.92875	0.94332	1.04558	0.99519	1.04154	1.13460
	PSDPT [(a)]	0.92839	0.94297	1.03862	0.99551	1.10533	1.12314
	HSDPT [(a)]	0.92811	0.94275	0.99184	0.99536	1.06081	1.12311
	Elasticity [(b)]	0.92727	0.94078	0.98929	0.99523	1.06104	1.12466
	Present	0.92946	0.94851	0.99592	1.00222	1.06566	1.12746

[(a)] El Meiche, Tounsi *et al*. (2011)
[(b)] Li, Iu *et al*. (2008)

(HSDPT).

Table 1 shows a good agreement by comparisons of FG plates of five different volume fraction indices p=0,0.5,1,5,10 with other theories. Hence, the present 3-unknown model (with only three unknown variables) provides comparable results to those obtained with higher order models with more unknowns (SSDPT, PSDPT and HSDPT). Compared to the three-dimensional linear theory of elasticity (Li, Iu *et al*. 2008), the present theory gives also accurate results.

The variation of the non-dimensional fundamental frequencies of a FG sandwich plates versus the side-to-thickness ratio is illustrated in Fig. 2 for different power law index p using the present theory. It can be observed from Fig. 2 that the fundamental frequencies are reduced with increasing the power law index p. The results are the maximum for the ceramic plates and the minimum for the metal plates.

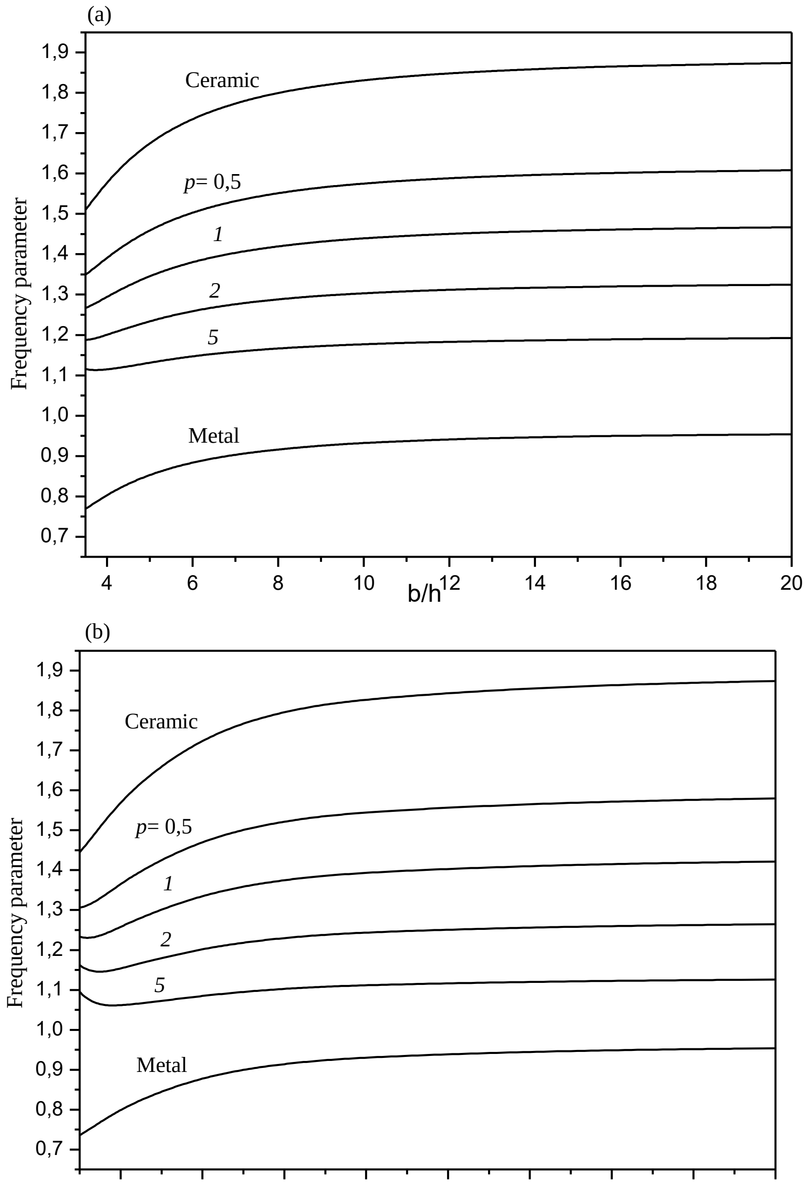

Fig. 2 Fundamental frequency ($\bar{\omega}$) as a function of side-to-thickness ratio (*b/h*) of symmetric and non-symmetric square FG sandwich plates for various values of *p*. (a) The (1-2-1) FG sandwich plate and (b) the (2-2-1) FG sandwich plate

4.2 Results for buckling analysis

The critical buckling loads of the system are predicted using Eq. (22) after setting ω equal to zero. This section aims to verify the accuracy of the present 3-unknown non-polynomial shear deformation theory in predicting the critical buckling loads of FG sandwich plates.

Tables 2 and 3 show the critical buckling loads of different types of FG sandwich plates by employing various plate theories. In these two examples, the values of the power law index are taken equal to p=0,0.5,1,5,10. It can be observed from Tables 2 and 3 that the results of the present theory are in good agreement with those of other shear deformation theories. Hence, the present model (with only three unknown functions) provides comparable results to those computed using higher order models with more unknowns. In general, the fully ceramic plates give the largest critical buckling loads. The uniaxial buckling load may be twice the biaxial one and this irrespective of the considered value of p and the type of the FG plate.

Figs. 3 and 4 present the critical buckling loads of the symmetric (1-2-1) and non-symmetric (2-2-1) types of square FG sandwich plates versus the side-to-thickness ratio using the present new 3-unknown non-polynomial shear deformation theory. The results are the maximum for the ceramic plates and the minimum for the metal plates.

5. Conclusions

A new 3-unknown non-polynomial shear deformation theory has been proposed for the

Table 2 Comparison of nondimensional critical buckling load of square FG sandwich plates subjected to uniaxial compressive load (γ=0, h/b=0.1)

p	Theory	$\bar{N}$					
		1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
0	PSDPT [(a)]	13.00495	13.00495	13.00495	13.00495	13.00495	13.00495
	SSDPT [(a)]	13.00606	13.00606	13.00606	13.00606	13.00606	13.00606
	HSDPT [(a)]	13.00552	13.00552	13.00552	13.00552	13.00552	13.00552
	Present	13.11249	13.11249	13.11249	13.11249	13.11249	13.11249
0.5	PSDPT [(a)]	7.36437	7.94084	8.22470	8.43645	8.80997	9.21681
	SSDPT [(a)]	7.36568	7.94195	8.22538	8.43712	8.81037	9.21670
	HSDPT [(a)]	7.36380	7.94046	8.22471	8.43647	8.81029	9.21757
	Present	7.36793	7.94497	8.23121	8.44250	8.81972	9.22921
1	PSDPT [(a)]	5.16713	5.84006	6.19394	6.46474	6.94944	7.50656
	SSDPT [(a)]	5.16846	5.84119	6.19461	6.46539	6.94980	7.50629
	HSDPT [(a)]	5.16629	5.83941	6.19371	6.46450	6.94952	7.50719
	Present	5.17036	5.84351	6.19506	6.46630	6.94966	7.50650
5	PSDPT [(a)]	2.65821	3.04257	3.40351	3.57956	4.11209	4.73469
	SSDPT [(a)]	2.66006	3.04406	3.40449	3.58063	4.11288	4.73488
	HSDPT [(a)]	2.65679	3.04141	3.40280	3.57874	4.11157	4.73463
	Present	2.66517	3.07268	3.42428	3.61386	4.13535	4.75567
10	PSDPT [(a)]	2.48727	2.74632	3.09190	3.19471	3.70752	4.27991
	SSDPT [(a)]	2.48928	2.74844	3.13443	3.19456	3.14574	4.38175
	HSDPT [(a)]	2.48574	2.74498	3.09111	3.19373	3.70686	4.27964
	Present	2.48889	2.77423	3.11115	3.23351	3.73515	4.30855

[(a)] El Meiche, Tounsi *et al*. (2011)

Table 3 Comparison of nondimensional critical buckling load of square FG sandwich plates subjected to biaxial compressive load (γ=1, h/b=0.1)

p	Theory	$\overline{N}$					
		1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
0	PSDPT [(a)]	6.50248	6.50248	6.50248	6.50248	6.50248	6.50248
	SSDPT [(a)]	6.50303	6.50303	6.50303	6.50303	6.50303	6.50303
	HSDPT [(a)]	6.50276	6.50276	6.50276	6.50276	6.50276	6.50276
	Present	6,55624	6,55624	6,55624	6,55624	6,55624	6,55624
0.5	PSDPT [(a)]	3.68219	3.97042	4.11235	4.21823	4.40499	4.60841
	SSDPT [(a)]	3.68284	3.97097	4.11269	4.21856	4.40519	4.60835
	HSDPT [(a)]	3.68190	3.97023	4.11236	4.21823	4.40514	4.60878
	Present	3.68397	3.97248	4.11560	4.22125	4.40986	4.61461
1	PSDPT [(a)]	2.58357	2.92003	3.09697	3.23237	3.47472	3.75328
	SSDPT [(a)]	2.58423	2.92060	3.09731	3.23270	3.47490	3.75314
	HSDPT [(a)]	2.58315	2.91970	3.09686	3.23225	3.47476	3.75359
	Present	2.58518	2.92175	3.09753	3.23315	3.47483	3.75325
5	PSDPT [(a)]	1.32910	1.52129	1.70176	1.78978	2.05605	2.36734
	SSDPT [(a)]	1.33003	1.52203	1.70224	1.79032	2.05644	2.36744
	HSDPT [(a)]	1.32839	1.52071	1.70140	1.78937	2.05578	2.36731
	Present	1.33258	1.53634	1.71214	1.80693	2.06768	2.37784
10	PSDPT [(a)]	1.24363	1.37316	1.54595	1.59736	1.85376	2.13995
	SSDPT [(a)]	1.24475	1.37422	1.56721	1.59728	1.57287	2.19087
	HSDPT [(a)]	1.24287	1.37249	1.54556	1.59687	1.85343	2.13982
	Present	1.24445	1.38712	1.55557	1.61675	1.86757	2.15428

[(a)] El Meiche, Tounsi *et al*. (2011)

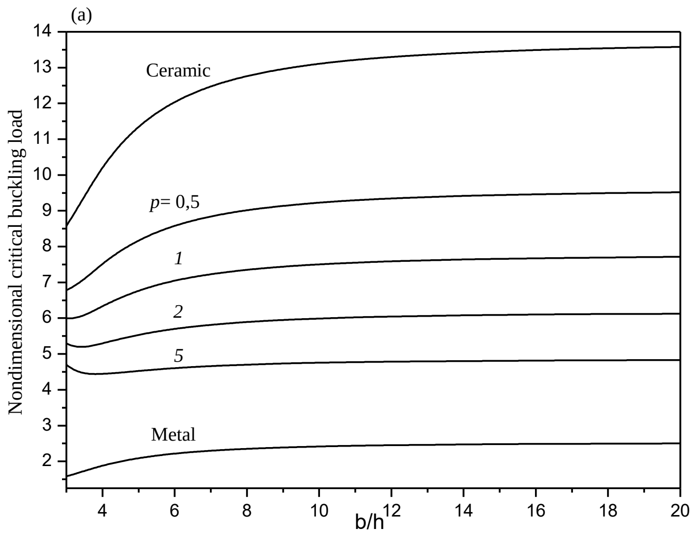

Fig. 3 Nondimensional critical buckling load ($\overline{N}$) as a function of side-to-thickness ratio (b/h) of (1-2-1) FG sandwich plates for various values of p. (a) Plate subjected to uniaxial compressive load (γ=0) and (b) Plate subjected to biaxial compressive load (γ=1)

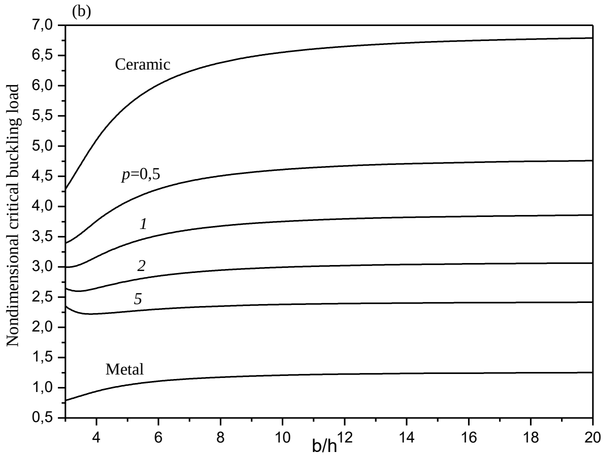

Fig. 3 Continued

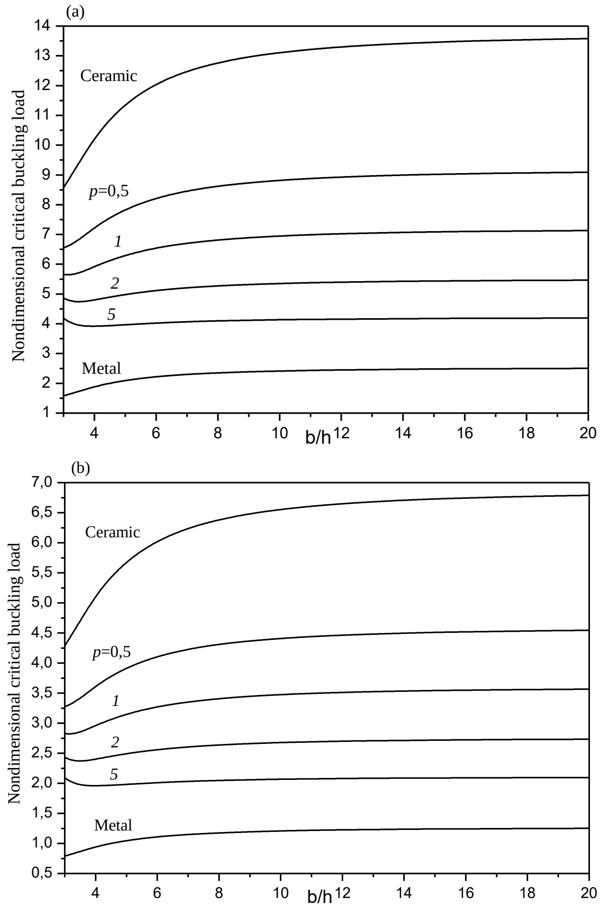

Fig. 4 Nondimensional critical buckling load ($\overline{N}$) as a function of side-to-thickness ratio (*b/h*) of (2-2-1) FG sandwich plates for various values of *p*. (a) Plate subjected to uniaxial compressive load (γ=0) and (b) Plate subjected to biaxial compressive load (γ=1)

buckling and vibration response of FG sandwich plates. The innovation of this present theory is that, in addition to considering the shear deformation effect, the displacement field is expressed with only three unknowns as the case of the CPT and which is even less than the FSDT. Verification studies demonstrate that the developed theory is not only accurate but also simple in predicting the buckling and free vibration responses of FG sandwich plates.

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