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Generalized coupled non-Fickian/non-Fourierian diffusion-thermoelasticity analysis subjected to shock loading using analytical method

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Abstract. In this article, the generalized coupled non-Fickian diffusion-thermoelasticity analysis is carried out using an analytical method. The transient behaviors of field variables, including mass concentration, temperature and displacement are studied in a strip, which is subjected to shock loading. The governing equations are derived using generalized coupled non-Fickian diffusion-thermoelasticity theory, which is based on Lord-Shulman theory of coupled thermoelasticity. The governing equations are transferred to the frequency domain using Laplace transform technique and then the field variables are obtained in analytical forms using the presented method. The field variables are eventually determined in time domain by employing the Talbot technique. The dynamic behaviors of mass concentration, temperature and displacement are studied in details. It is concluded that the presented analytical method has a high capability for simulating the wave propagation with finite speed in mass concentration field as well as for tracking thermoelastic waves. Furthermore, the obtained results are more realistic than that of others.

Keywords: non-Fickian diffusion; wave propagation; mass concentration; thermoelasticity; analytical method; coupled problems

1. Introduction

The coupled problems in engineering are very important from designing point of view. The interaction between various fields causes the coupled governing equations such as coupled thermoelasticity, thermo-electro-elasticity and etc.

Biot (1956) developed the coupled theory of thermoelasticity, considering the second law of

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thermodynamics to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. During the recent decades, many generalized thermoelastic theories were proposed to allow a finite velocity for the propagation of a thermal wave such as: Lord and Shulman (1967), Green and Lindsay (1972), Green and Naghdi (1993) and the inertial entropy by Kuang (2009).

Some research works were carried out for uncoupled (Hosseini and Akhlaghi 2009) and coupled thermoelasticity analysis based on the non-classical theories using analytical (Hosseini and Abolbashari 2012) and numerical (Hosseini 2009, Hosseini 2013, Hosseini and Shahabian 2014) methods.

Lee and Chang (2003) presented a numerical method based on Laplace transforms and finite difference methods for axisymmetric quasi-static coupled thermoelasticity analysis in multilayered spheres. Yang, Qin *et al*. (2010) presented a theoretical model and the corresponding FE formulation for thermo-electro-chemo-mechanical coupled problems, developed by redefining linearly coupled constitutive relations and extending the traditional Gibb's free energy to include chemical effects.

Sharyiat (2012) carried out nonlinear generalized (with second sound effect) and classical thermoelasticity analyses for functionally graded thick cylinders subjected to various thermomechanical shocks at their inner and outer surfaces employing Hermitian elements. Recently, Sharyiat, Lavasani *et al*. (2010a, b) investigated some aspects of the vibration and wave propagation phenomena in thick hollow functionally graded cylinders under thermomechanical loads using a numerical method.

Diffusion can be defined as the random walk, of an ensemble of particles, from regions of high concentration to regions of low concentration (Levine Ira 2009). The diffusion behavior of many materials cannot be adequately described by a concentration dependent form of Ficks law. Generally, this is the case which is said non-Fickian behavior. Against Fickian diffusion which in it the rate of diffusion is much less than relaxation, Non-Fickian diffusion occurs when the diffusion and relaxation rates are comparable. The coupling between the thermoelastic fields and concentration of diffusive gas takes place when a solid body is immersed in a gas. Using the variational principle and reciprocity theorem, Kumar and Kansal (2010) derived the governing equations for generalized thermoelastic diffusion analysis, which was based on the theory of Lord and Shulman with one relaxation time. Hosseini, Abolbashari *et al*. (2014) presented an analytical method for one dimensional analysis of coupled non-Fickian diffusion and mechanics.

Sherief, Hamza *et al*. (2004) developed a theory of generalized thermo-elastic diffusion based on Lord-Shulman theory. In other work, Sherief and Saleh (2005) studied a thermoelastic half-space with a permeating substance in contact with the bounding plane in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Liu and Reissig (2014) studied the Cauchy problem for 1D models of thermodiffusion with consideration of hyperbolic-parabolic character of the system. Recently, Sue and Shen (2012) presented several variational principles for coupled temperature-diffusion-mechanics and their corresponding governing equations and boundary conditions. In another work (Sue and Shen 2013), they solved one dimensional problem of coupled non-Fickian diffusion and mechanics without considering temperature effects using an approximate analytical method. Deswal and Choudhary (2009) investigated the distribution of temperature, displacement components, stresses, concentration and chemical potential in a semi-infinite medium having an impulsive mechanical load at the origin by using the joint Laplace and Fourier transforms. The application of meshless local Petrov-Galerkin (MLPG) method for two dimensional coupled non-Fickian diffusion-elasticity analysis was presented by Hosseini *et al*.

(2013). In their work, the profiles of molar concentration and displacements are illustrated in two orthogonal directions at various time instants. In another research (Hosseini, Abolbashari *et al*. 2014), they employed this method for 2D coupled non-Fickian diffusion-thermoelasticity based on Green-Naghdi theory of coupled thermoelasticity. They studied the dynamic behaviors of molar concentrations, temperature and displacement fields.

This work presents an effective analytical method to study the generalized coupled non-Fickian diffusion'-thermoelasticity in a domain subjected to shock loading. The molar concentration, temperature and displacements are obtained in series form using the presented analytical method. In this way, the wave fronts of mass concentration, displacement and temperature can be tracked at various time instants, which are propagated with finite speeds.

2. Governing equations

The constitutive equations for the coupled non-Fickian diffusion'-thermoelasticity can be written as follows

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \delta_{ij}\left(\lambda\varepsilon_{kk} - \beta_1\theta - \beta_2 c\right) \tag{1a}$$

$$P = -\beta_2\varepsilon_{kk} + bc - a\theta \tag{1b}$$

where σ_{ij}, ε_{ij},c, θ, P, a and b are the stress, strain, mass concentration, temperature, chemical potential, measure of thermodiffusion effect and measure of diffusive effect, respectively. Also, λ, μ are the Lames constants and β_1=(3λ+2μ)α_t, β_2=(3λ+2μ)α_c.

The generalized coupled diffusion-thermoelasticity governing equations for an isotropic homogeneous elastic strip can be presented as follows (Sherief, Hamza *et al*. 2004)

$$(2\mu+\lambda)\frac{\partial^2 u}{\partial x^2} - \beta_1\frac{\partial\theta}{\partial x} - \beta_2\frac{\partial c}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} \tag{2a}$$

$$k\frac{\partial^2\theta}{\partial x^2} = \rho c_E\frac{\partial\theta}{\partial t} + \rho c_E\tau_0\frac{\partial^2\theta}{\partial t^2} + T_0\beta_1\frac{\partial^2 u}{\partial x\partial t} + T_0\beta_1\tau_0\frac{\partial^3 u}{\partial x\partial t^2} + aT_0\frac{\partial c}{\partial t} + aT_0\tau_0\frac{\partial^2 c}{\partial t^2} \tag{2b}$$

$$D\beta_2\frac{\partial^3 u}{\partial x^3} + Da\frac{\partial^2\theta}{\partial x^2} + \frac{\partial c}{\partial t} + \tau\frac{\partial^2 c}{\partial t^2} - Db\frac{\partial^2 c}{\partial x^2} = 0 \tag{2c}$$

where u, x, ρ, t, k, c_E, τ_0, τ, T_0 and D are the displacement, position, density, time, thermal conductivity, specific heat, thermal relaxation time, diffusion relaxation time, reference temperature and thermo diffusion constant, respectively.

For convenience, the following non-dimensional variables are employed for analysis

$$\begin{gathered} X = c_0\omega x\ ,\ t^* = c_0^2\omega t,\ \tau^* = c_0^2\omega\tau,\ \tau_0^* = c_0^2\omega\tau_0,\ U = c_0\omega u,\ \sigma_{ij}^* = \frac{\sigma_{ij}}{\lambda+2\mu},\ C = \frac{\beta_2 c}{\lambda+2\mu}, \\ \theta^* = \frac{\beta_1\left(T-T_0\right)}{\lambda+2\mu},\ P^* = \frac{P}{\beta_2}, \qquad c_0^2 = \frac{\lambda+2\mu}{\rho},\quad \omega = \frac{\rho c_E}{k} \end{gathered} \tag{3}$$

Omitting the superscript "*", Eq. (2) can be expressed in the following dimensionless forms

$$\frac{\partial^2 U}{\partial X^2}-\frac{\partial \theta}{\partial X}-\frac{\partial C}{\partial X}=\frac{\partial^2 U}{\partial t^2} \tag{4a}$$

$$\frac{\partial^2 \theta}{\partial X^2}=\frac{\partial \theta}{\partial t}+\tau_0\frac{\partial^2 \theta}{\partial t^2}+b_1\frac{\partial^2 U}{\partial X\partial t}+b_1\tau_0\frac{\partial^3 U}{\partial X\partial t^2}+b_2\frac{\partial C}{\partial t}+b_2\tau_0\frac{\partial^2 C}{\partial t^2} \tag{4b}$$

$$\frac{\partial^3 U}{\partial X^3}+b_3\frac{\partial^2 \theta}{\partial X^2}+b_4\left(\frac{\partial C}{\partial t}+\tau\frac{\partial^2 C}{\partial t^2}\right)-b_5\frac{\partial^2 C}{\partial X^2}=0 \tag{4c}$$

where

$$b_1=\frac{\beta_1{}^2T_0}{\rho c_E\left(\lambda+2\mu\right)},\ b_2=\frac{a\beta_1T_0}{c_E\rho\beta_2},\quad b_3=\frac{a\left(\lambda+2\mu\right)}{\beta_1\beta_2},\ b_4=\frac{k\left(\lambda+2\mu\right)}{D\beta_2{}^2\rho c_E},\ b_5=\frac{b\left(\lambda+2\mu\right)}{\beta_2{}^2}$$

The above formulations can be used for generalized coupled non-Fickian diffusion-thermoelasticity analysis for a strip. The following homogenous initial conditions are assumed for the problem.

$$U\left(X,t\right)=0,\frac{\partial U\left(X,t\right)}{\partial t}=0 \qquad \text{at} \qquad t=0 \tag{5a}$$

$$C\left(X,t\right)=0,\frac{\partial C\left(X,t\right)}{\partial t}=0 \qquad \text{at} \qquad t=0 \tag{5b}$$

$$\theta\left(X,t\right)=0,\frac{\partial \theta\left(X,t\right)}{\partial t}=0 \qquad \text{at} \qquad t=0 \tag{5c}$$

It is assumed that the one boundary of strip is excited by suddenly increasing of mass concentration and temperature in the form of Heaviside unit step function of time. This may be written in mathematical form as

$$\sigma\left(X,t\right)=0 \qquad \text{at} \qquad X=0 \tag{6a}$$

$$C\left(X,t\right)=c_1H\left(t\right) \qquad \text{at} \qquad X=0 \tag{6b}$$

$$\theta\left(X,t\right)=\theta_1H\left(t\right) \qquad \text{at} \qquad X=0 \tag{6c}$$

$$U\left(X,t\right)=0 \qquad \text{at} \qquad X=1.5 \tag{6d}$$

$$C\left(X,t\right)=0 \qquad \text{at} \qquad X=1.5 \tag{6e}$$

$$\theta\left(X,t\right)=0 \qquad \text{at} \qquad X=1.5 \tag{6f}$$

where $H(t)$ is the Heaviside unit step function.

The application of Laplace transformation on Eqs. (4a), (4b) and (4c) with respect to time, yields

$$\frac{\partial^2 \bar{U}(X,s)}{\partial X^2} - \frac{\partial \bar{\theta}(X,s)}{\partial X} - \frac{\partial \bar{C}(X,s)}{\partial X} = s^2 \bar{U}(X,s) \tag{7}$$

$$\frac{\partial^2 \bar{\theta}(X,s)}{\partial X^2} = (s+\tau_0 s^2)\bar{\theta}(X,s) + b_1(s+\tau_0 s^2)\frac{\partial \bar{U}(X,s)}{\partial X} + b_2(s+\tau_0 s^2)\bar{C}(X,s) \tag{8}$$

$$\frac{\partial^3 \bar{U}(X,s)}{\partial X^3} + b_3 \frac{\partial^2 \bar{\theta}(X,s)}{\partial X^2} + b_4\left(s+\tau s^2\right)\bar{C}(X,s) - b_5 \frac{\partial^2 \bar{C}(X,s)}{\partial X^2} = 0 \tag{9}$$

where the terms $\bar{U}(X,s)$, $\bar{C}(X,s)$ and $\bar{\theta}(X,s)$ are displacement, molar concentration and temperature in Laplace domain.

Also, the boundary conditions may be written in Laplace domain as

$$\bar{\sigma}(X,s) = 0 \quad \text{at} \quad X = 0 \tag{10a}$$

$$\bar{C}(X,s) = \frac{c_1}{s} \quad \text{at} \quad X = 0 \tag{10b}$$

$$\bar{\theta}(X,s) = \frac{\theta_1}{s} \quad \text{at} \quad X = 0 \tag{10c}$$

$$\bar{U}(X,s) = 0 \quad \text{at} \quad X = 1.5 \tag{10d}$$

$$\bar{C}(X,s) = 0 \quad \text{at} \quad X = 1.5 \tag{10e}$$

$$\bar{\theta}(X,s) = 0 \quad \text{at} \quad X = 1.5 \tag{10f}$$

To solve the coupled system of PDEs (7)-(9), an analytical method is proposed, which is explained in the next section.

3. Analytical solution

The following series are assumed as solution of coupled PDEs (7)-(9), which are analytical at ($X-1$).

$$\bar{U}(X,s) = \sum_{n=0}^{\infty} A_n(s)(X-1)^n \tag{11}$$

$$\bar{C}(X,s) = \sum_{n=0}^{\infty} B_n(s)(X-1)^n \tag{12}$$

$$\bar{\theta}(X,s) = \sum_{n=0}^{\infty} D_n(s)(X-1)^n \tag{13}$$

Where $A_n(s)$, $B_n(s)$ and $C_n(s)$ are unknown coefficients.

By substituting Eqs. (11)-(13) into Eqs. (7)-(9), the following equations can be derived.

$$\sum_{n=2}^{\infty} A_n(s)n(n-1)(X-1)^{n-2} - \sum_{n=1}^{\infty} B_n(s)n(X-1)^{n-1} - \sum_{n=1}^{\infty} D_n(s)n(X-1)^{n-1} - s^2\sum_{n=0}^{\infty} A_n(s)(X-1)^n = 0 \tag{14}$$

$$b_1\left(s+\tau_0 s^2\right)\sum_{n=1}^{\infty} A_n(s)n(X-1)^{n-1} + b_2\left(s+\tau_0 s^2\right)\sum_{n=0}^{\infty} B_n(s)(X-1)^n + \left(s+\tau_0 s^2\right)\sum_{n=0}^{\infty} D_n(s)(X-1)^n - \sum_{n=2}^{\infty} D_n(s)n(n-1)(X-1)^{n-2} = 0 \tag{15}$$

$$\sum_{n=3}^{\infty} A_n(s)n(n-1)(n-2)(X-1)^{n-3} + b_3\sum_{n=2}^{\infty} D_n(s)n(n-1)(X-1)^{n-2} + b_4\left(s+\tau s^2\right)\sum_{n=0}^{\infty} B_n(s)(X-1)^n - b_5\sum_{n=2}^{\infty} B_n(s)n(n-1)(X-1)^{n-2} = 0 \tag{16}$$

Eqs. (14)-(16) may be rewritten in the new forms as

$$\sum_{n=0}^{\infty}[(n+1)(n+2)A_{n+2}(s) - (n+1)B_{n+1}(s) - (n+1)D_{n+1}(s) - s^2 A_n(s)](X-1)^n = 0 \tag{17}$$

$$\sum_{n=0}^{\infty}\left[b_1\left(s+\tau_0 s^2\right)(n+1)A_{n+1}(s) + b_2\left(s+\tau_0 s^2\right)B_n(s) + \left(s+\tau_0 s^2\right)D_n(s)\right](X-1)^n - \sum_{n=0}^{\infty}\left[(n+1)(n+2)D_{n+2}(s)\right](X-1)^n = 0 \tag{18}$$

$$\sum_{n=0}^{\infty}\left[(n+1)(n+2)(n+3)A_{n+3}(s) + b_4\left(s+\tau s^2\right)B_n(s) + b_3(n+1)(n+2)D_{n+2}(s)\right](X-1)^n - \sum_{n=0}^{\infty}\left[b_5(n+1)(n+2)B_{n+2}(s)\right](X-1)^n = 0 \tag{19}$$

To find the unknown coefficients, the following recurrence relations can be derived as follows

$$A_{n+2}(s) = \frac{s^2}{(n+1)(n+2)}A_n(s) + \frac{1}{(n+2)}B_{n+1}(s) + \frac{1}{(n+2)}D_{n+1}(s) \tag{20}$$

$$B_{n+2}(s) = \left[\frac{y_1 s^2 + b_1 y_2\left(s+\tau_0 s^2\right)}{n+2}\right]A_{n+1}(s) + \left[\frac{y_1 b_4\left(s+\tau s^2\right) + b_2 y_2\left(s+\tau_0 s^2\right)}{(n+1)(n+2)}\right]B_n(s) + \left[\frac{y_2\left(s+\tau_0 s^2\right)}{(n+1)(n+2)}\right]D_n(s) \tag{21}$$

$$D_{n+2}(s)=\frac{b_1\left(s+\tau_0 s^2\right)}{(n+2)}A_{n+1}(s)+\frac{b_2\left(s+\tau_0 s^2\right)}{(n+1)(n+2)}B_n(s)+\frac{s+\tau_0 s^2}{(n+1)(n+2)}D_n(s) \tag{22}$$

where

$$y_1=\frac{1}{b_5-1},\quad y_2=\frac{\left(b_3+1\right)}{b_5-1}$$

It can be seen in Eqs. (20)-(22) that all coefficients $A_n(s)$, $B_n(s)$ and $D_n(s)$ can be expressed in terms of $A_0(s)$, $A_1(s)$, $B_0(s)$, $B_1(s)$, $D_0(s)$ and $D_1(s)$ when $n>1$ .The terms $A_0(s)$, $A_1(s)$, $B_0(s)$, $B_1(s)$, $D_0(s)$ and $D_1(s)$ should be determined using boundary conditions. Therefore, the solutions (11)-(13) can be rewritten in new forms as

$$\begin{aligned}\bar{U}\left(X,s\right)=\sum_{n=0}^{\infty}\Big\{&g_1^n(s)A_0(s)+g_2^n(s)A_1(s)+g_3^n(s)B_0(s)+g_4^n(s)B_1(s)+g_5^n(s)D_0(s)+\\&+g_6^n(s)D_1(s)\Big\}\left(X-1\right)^n\end{aligned} \tag{23}$$

$$\begin{aligned}\bar{C}\left(X,s\right)=\sum_{n=0}^{\infty}\Big\{&K_1^n(s)A_0(s)+K_2^n(s)A_1(s)+K_3^n(s)B_0(s)+K_4^n(s)B_1(s)+K_5^n(s)D_0(s)+\\&+K_6^n(s)D_1(s)\Big\}\left(X-1\right)^n\end{aligned} \tag{24}$$

$$\begin{aligned}\bar{\theta}\left(X,s\right)=\sum_{n=0}^{\infty}\Big\{&L_1^n(s)A_0(s)+L_2^n(s)A_1(s)+L_3^n(s)B_0(s)+L_4^n(s)B_1(s)+L_5^n(s)D_0(s)+\\&+L_6^n(s)D_1(s)\Big\}\left(X-1\right)^n\end{aligned} \tag{25}$$

The coefficients $g_i^n(s)$, $K_i^n(s)$ and $L_i^n(s)$ may be calculated using the following equations: (i=1,2,…,6)

$$g_i^{n+2}(s)=\frac{1}{(n+2)}K_i^{n+1}(s)+\frac{s^2}{(n+1)(n+2)}g_i^n(s)+\frac{1}{(n+2)}L_i^{n+1}(s) \tag{26}$$

$$\begin{aligned}k_i^{n+2}(s)=&\left[\frac{y_1 b_4\left(s+\tau s^2\right)+b_2 y_2\left(s+\tau_0 s^2\right)}{\left(n+1\right)\left(n+2\right)}\right]K_i^n(s)+\left[\frac{y_1 s^2+b_1 y_2\left(s+\tau_0 s^2\right)}{n+2}\right]g_i^{n+1}(s)+\\&+\left[\frac{y_2\left(s+\tau_0 s^2\right)}{\left(n+1\right)\left(n+2\right)}\right]L_i^n(s)\end{aligned} \tag{27}$$

$$L_i^{n+2}(s)=\frac{s+\tau_0 s^2}{(n+1)(n+2)}L_i^n(s)+\frac{b_1\left(s+\tau_0 s^2\right)}{(n+2)}g_i^{n+1}(s)+\frac{b_2\left(s+\tau_0 s^2\right)}{(n+1)(n+2)}k_i^n(s) \tag{28}$$

and also

$$g_1^0(s)=1 \quad g_2^0(s)=0 \quad g_3^0(s)=0 \quad g_4^0(s)=0 \quad g_5^0(s)=0 \quad g_6^0(s)=0$$

$$g_1^1(s)=0 \quad g_2^1(s)=1 \quad g_3^1(s)=0 \quad g_4^1(s)=0 \quad g_5^1(s)=0 \quad g_6^1(s)=0$$

$$g_1^2(s)=\frac{s^2}{2} \quad g_2^2(s)=0 \quad g_3^2(s)=0 \quad g_4^2(s)=\frac{1}{2} \quad g_5^2(s)=0 \quad g_6^2(s)=\frac{1}{2} \tag{29}$$

$$K_1^0(s)=0 \quad K_2^0(s)=0 \quad K_3^0(s)=1 \quad K_4^0(s)=0 \quad K_5^0(s)=0 \quad K_6^0(s)=0$$

$$K_1^1(s)=0 \quad K_2^1(s)=0 \quad K_3^1(s)=0 \quad K_4^1(s)=1 \quad K_5^1(s)=0 \quad K_6^1(s)=0$$

$$K_1^2(s)=0 \quad K_2^2(s)=\frac{L_1 s^2+b_1 L_2\left(s+\tau_0 s^2\right)}{n+2} \quad K_3^2(s)=\frac{L_1 b_4\left(s+\tau s^2\right)+b_2 L_2(s+\tau_0 s^2)}{2D_0}$$

$$K_4^2(s)=0 \quad K_5^2(s)=\frac{L_2(s+\tau_0 s^2)}{2} \quad K_6^2(s)=0 \tag{30}$$

as well as

$$\begin{array}{llllll} L_1^0(s)=0 & L_2^0(s)=0 & L_3^0(s)=0 & L_4^0(s)=0 & L_5^0(s)=1 & L_6^0(s)=0 \\ L_1^1(s)=0 & L_2^1(s)=0 & L_3^1(s)=0 & L_4^1(s)=0 & L_5^1(s)=0 & L_6^1(s)=1 \\ L_1^2(s)=0 & L_2^2(s)=\frac{b_1(s+\tau_0 s^2)}{2} & L_3^2(s)=\frac{b_2(s+\tau_0 s^2)}{2} & L_4^2(s)=0 & & \\ L_5^2(s)=\frac{(s+\tau_0 s^2)}{2} & L_6^2(s)=0 & & & & \end{array} \tag{31}$$

Eqs. (23)-(25) are the analytical solutions for displacement, molar concentration and temperature fields in Laplace domain. To determine the solutions in time domain, the present work uses the Talbot algorithm (Cohen 2007), which is based on deforming the contour in the Bromwich inversion integral to reduce numerical error. This formulation yields the relations in time domain as follows

$$\begin{aligned} U(X,t)&=\frac{2}{5t}\sum_{k=0}^{M-1}\mathrm{Re}(\gamma_k \bar{U}(X,s_k)) \\ C(X,t)&=\frac{2}{5t}\sum_{k=0}^{M-1}\mathrm{Re}(\gamma_k \bar{C}(X,s_k)) \\ \theta(X,t)&=\frac{2}{5t}\sum_{k=0}^{M-1}\mathrm{Re}(\gamma_k \bar{\theta}(X,s_k)) \end{aligned} \tag{32}$$

where

Table 1 Material constants of the problem

$\lambda = 7.76\times10^{10}\ (\frac{N}{m^2})$	$\mu = 3.86\times10^{10}\ (\frac{N}{m^2})$	$\rho = 8954\ (\frac{kg}{m^3})$	$D = 0.85\times10^{-8}(\frac{kg.s}{m^3})$
$b = 0.9\times10^{6}\ (\frac{m^5}{kg.s^2})$	$\alpha_c = 1.98\times10^{-4}\ (\frac{m^3}{kg})$	$\tau_0 = 0.02$	$\alpha_t = 1.78\times10^{-5}\ (\frac{1}{^\circ K})$
$k = 386\ (\frac{W}{m.K})$	$a = 1.2\times10^{4}\ (\frac{m^2}{s^2.K})$	$\tau = 0.2$	$c_E = 383.1\ (\frac{J}{kg.^\circ K})$
$c_1 = 10$	$T_0 = 300(^\circ K)$	$\theta_1 = 1$	

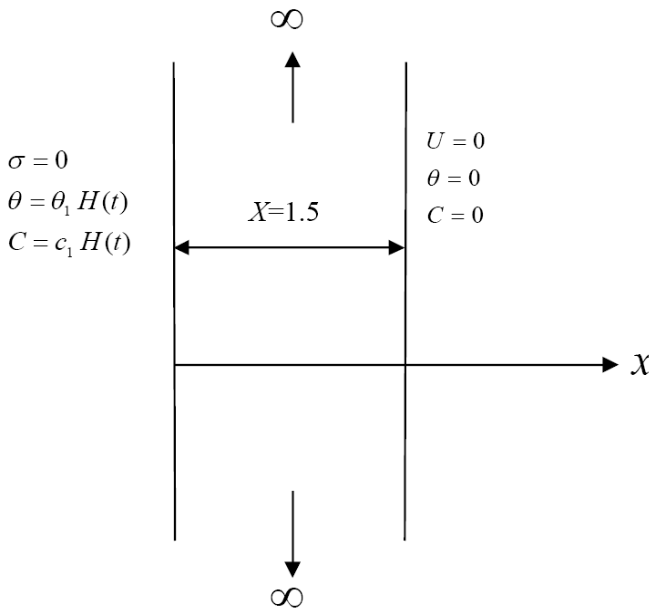

Fig. 1 Schematic of the boundary conditions for the strip

$$
\begin{aligned}
&s_k = \frac{\delta_k}{t},\ \delta_0 = \frac{2M}{5},\ \gamma_0 = 0.5e^{\delta_0},\ \delta_k = \frac{2k\pi}{5}\left(\cot\left(\frac{k\pi}{M}\right)+i\right) \\
&\gamma_k = \left[1+i\left(\frac{k\pi}{M}\right)\left(1+\left[\cot\left(\frac{k\pi}{M}\right)\right]^2\right)-i\cot\left(\frac{k\pi}{M}\right)\right]e^{\delta_k},\ 0<k<M
\end{aligned}
\tag{33}
$$

4. Numerical results and discussion

The material constants of the problem are taken as (Sherief and Saleh 2005) and are given in Table 1 in SI units. Using the presented material specifications in Table 1, the following coefficients can be calculated as:

$$b_1 = 0.0168,\ b_2 = 0.0921,\ b_3 = 5.4846,\ b_4 = 0.5439,\ b_5 = 36.9794$$

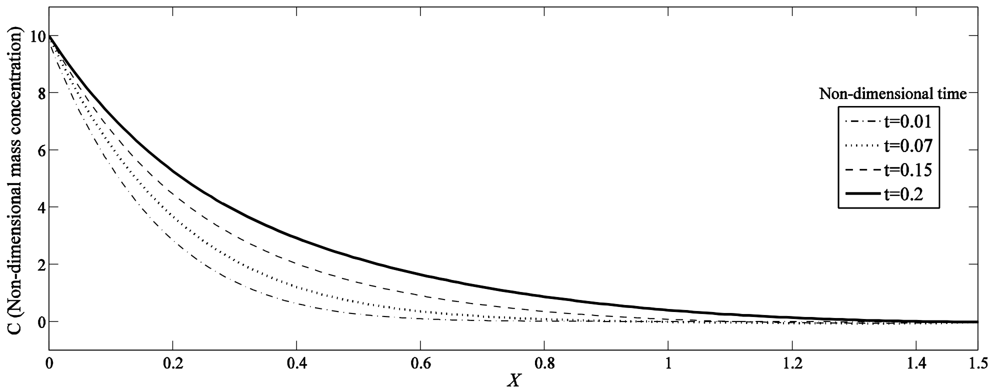

Fig. 2 Distribution of non-dimensional mass concentration along *X* direction

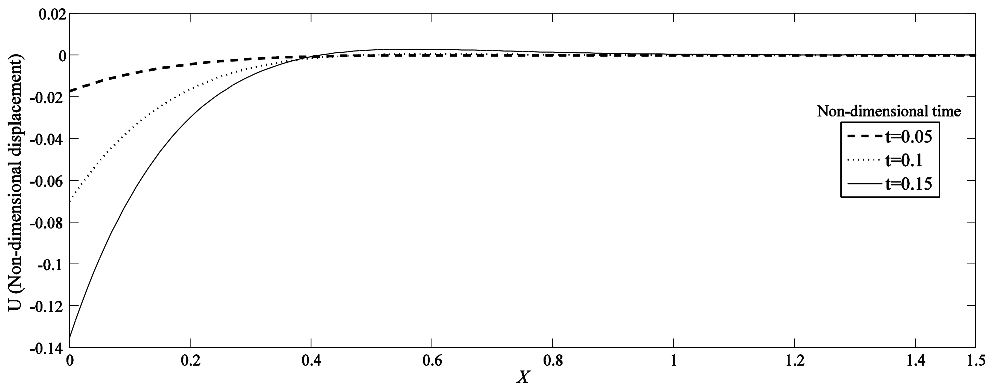

Fig. 3 Distribution of non-dimensional displacement along *X* direction

To show the wave propagation of field variables including mass concentration, temperature, displacement and stress, it is assumed that the strip is made of copper with suddenly excitations on one side (see Fig. 1). It should be noted that a unit of non-dimensional time corresponds to 6.5×10^{-12} s, while a unit of non-dimensional length corresponds to 2.7×10^{-8} m.

Using the presented analytical method, the field variables are obtained in the strip. The distributions of field variables are shown in Figs. 2 to 5 at various time instants. The distributions of mass concentration in various non-dimensional times along the X direction are shown in Fig. 2. It can be observed that the wave fronts of mass concentration are propagated with finite velocity through X direction. Also, as shown in this figure, the diffusion distance gradually increases when the diffusion time is increased. The distributions of displacement, temperature and stress along the X direction are illustrated in Figs. 3-5 at various time instants, respectively. Obviously, as the diffusion distance increases, the displacement is gradually approaching to zero. It is also seen that the assumed mechanical boundary conditions are satisfied at each side of domain. It means that the presented method has a high convergence rate for satisfying the boundary conditions. In Figs. 3-5, the wave fronts are propagated with finite velocity similar to mass concentration fields. Also, it can be concluded that the elasticity field is influenced by excitations in mass concentration and temperature fields. It is caused by considering the coupling effects in the employed generalized

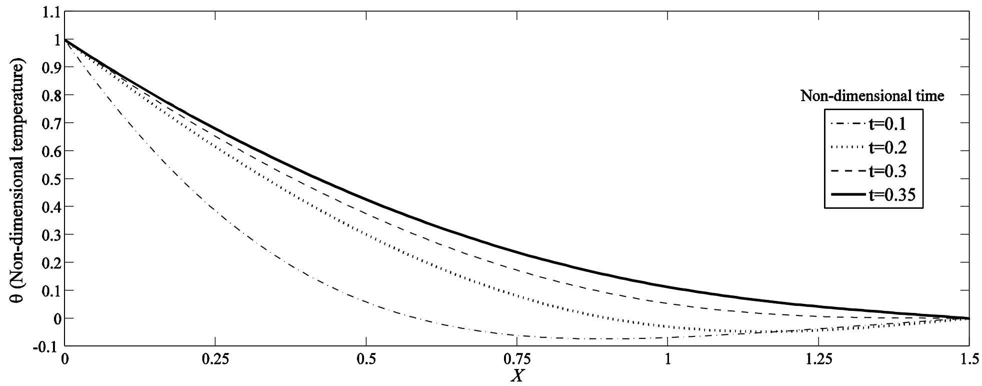

Fig. 4 Distribution of non-dimensional temperature along *X* direction

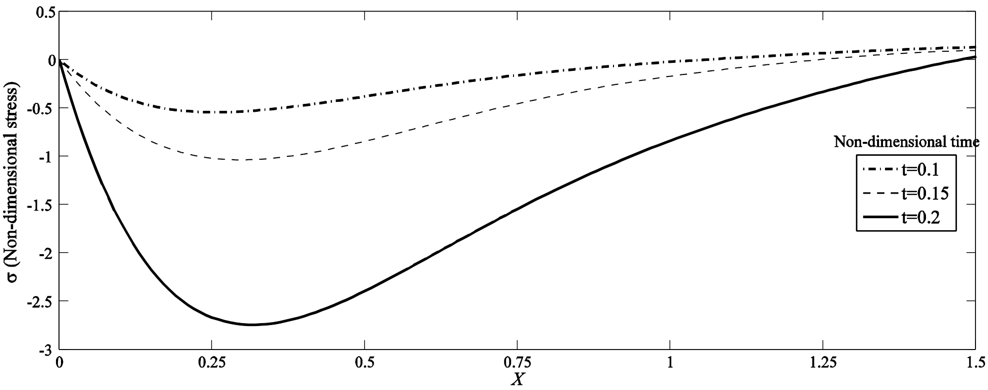

Fig. 5 Distribution of non-dimensional stress along *X* direction

coupled diffusion-thermoelasticity theory, which is based on the interaction between three fields of mass concentration, temperature and displacement fields. As it can be observed in Figs. 2-5, there are not any disturbances in the vicinity of wave fronts or other situations in diagrams. The reason may be suggested as the superiority of the convergence rate and high accuracy of the presented analytical method.

The variation of temperature, mass concentration and displacement in time domain are respectively presented in Figs. 6-8 for different positions. The transient and steady state behaviors can be seen in these figures. From time histories of displacement, temperature and mass concentration, it is clearly concluded that the presented analytical method can be successfully employed to solve the coupled system of PDEs such as the presented problem. The effect of the diffusion relaxation time τ on the velocity of wave propagation in displacement and mass concentration fields can be observed in Figs. 9 and 10, respectively. When the relaxation time is decreased, the wave propagation velocities are decreased in both displacement and concentration fields.

The results obtained from the presented method are verified using the known results as a benchmark solution. The following boundary conditions are assumed which are the same as reported in (Sherief and Saleh 2005).

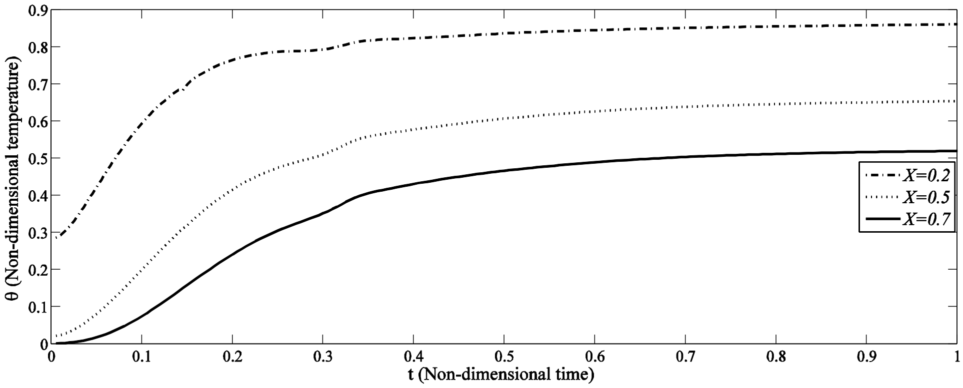

Fig. 6 Time histories of non-dimensional temperature at three different positions

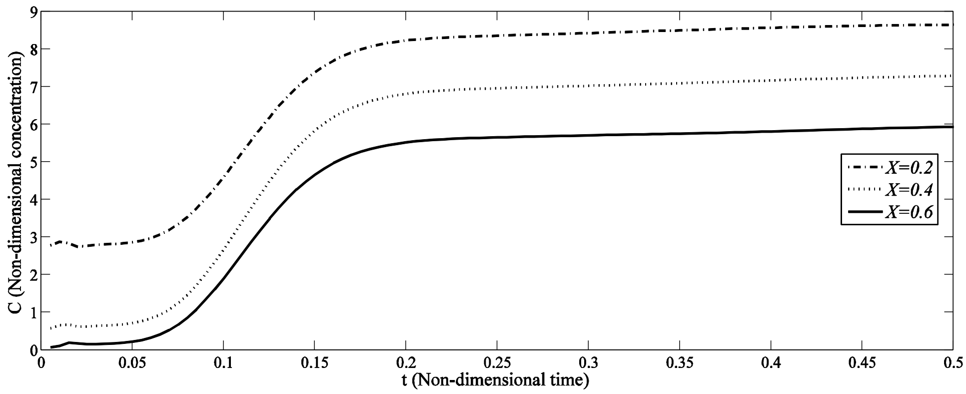

Fig. 7 The dynamic behaviors of non-dimensional mass concentration at three different positions in time domain

$$\sigma(X,t)=0 \quad \text{at} \quad X=0 \tag{34a}$$

$$P(X,t)=P_1H(t) \quad \text{at} \quad X=0 \tag{34b}$$

$$\theta(X,t)=\theta_1H(t) \quad \text{at} \quad X=0 \tag{34c}$$

$$\sigma(X,t)=0 \quad \text{at} \quad X=1.5 \tag{34d}$$

$$P(X,t)=0 \quad \text{at} \quad X=1.5 \tag{34e}$$

$$\theta(X,t)=0 \quad \text{at} \quad X=1.5 \tag{34f}$$

where the terms P_1 and θ_1 are taken as P_1=1 and θ_1=1.

The propagation of mass concentration and temperature along the X direction are compared with those obtained by (Sherief. and Saleh 2005), which was presented for this material in Figs. 11 and 12. It is evident that the presented results in this article are realistic. Also, the distributions of

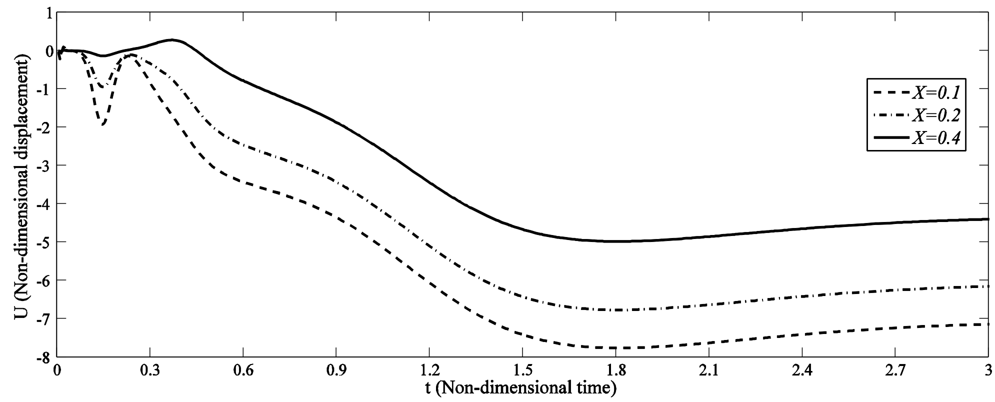

Fig. 8 Time histories of non-dimensional displacement at three positions

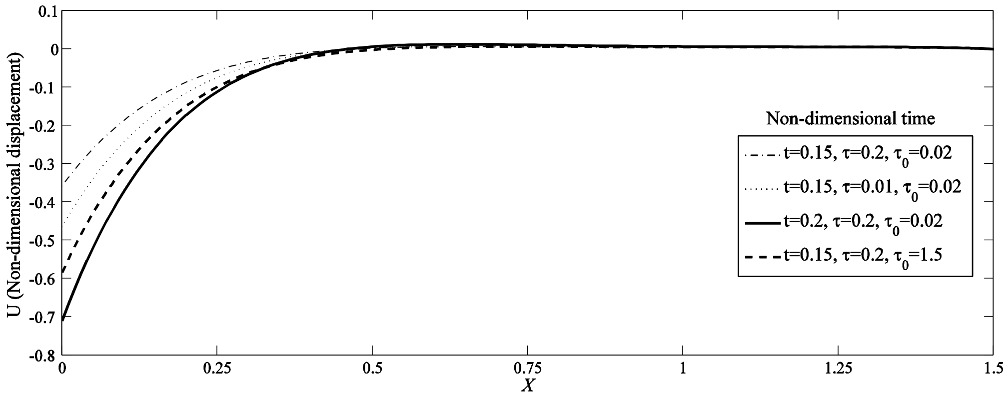

Fig. 9 The effects of relaxation times on non-dimensional displacement field

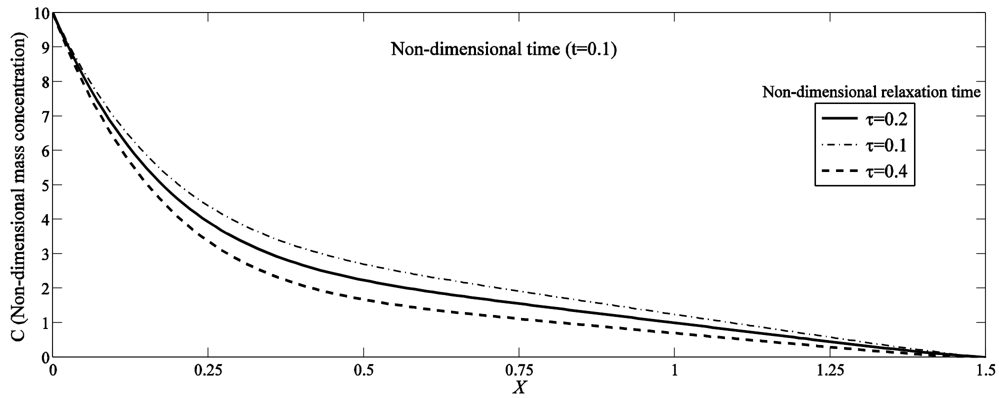

Fig. 10 The effects of relaxation times on non-dimensional mass concentration field

stresses along the X direction are illustrated in Fig. 13 at various time instants. It can be observed in this figure that the wave fronts of stresses propagate with finite speed.

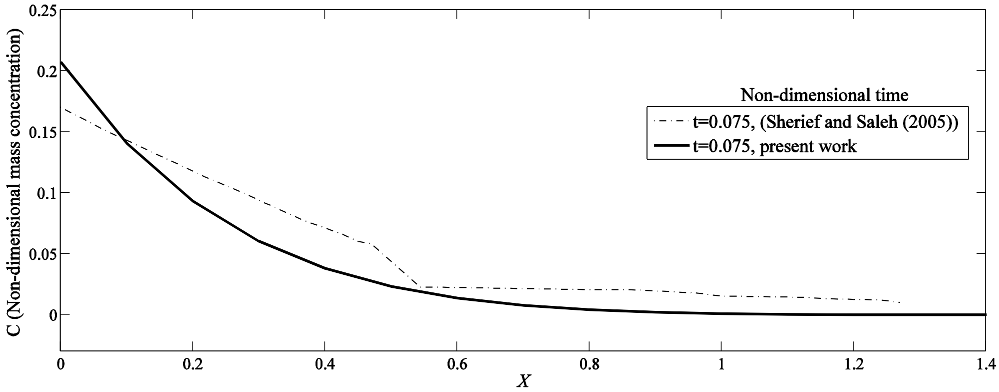

Fig. 11 The comparison of distribution of mass concentration along *X* direction between obtained results and published data

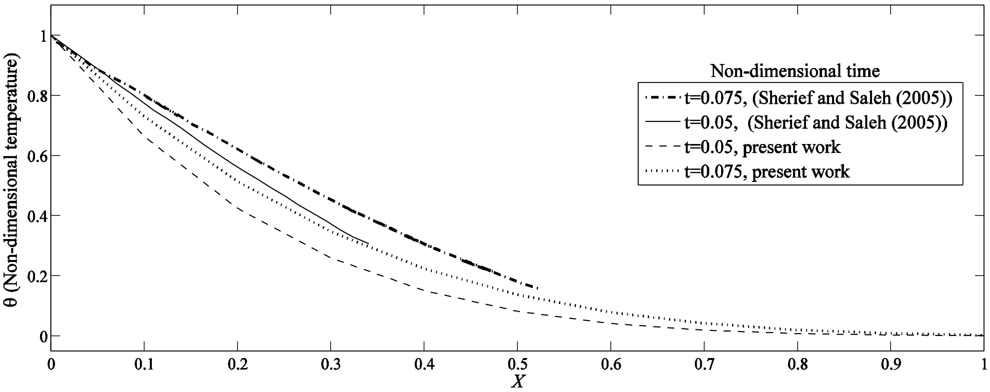

Fig. 12 The comparison of distribution of temperature along *X* direction between obtained results and published data

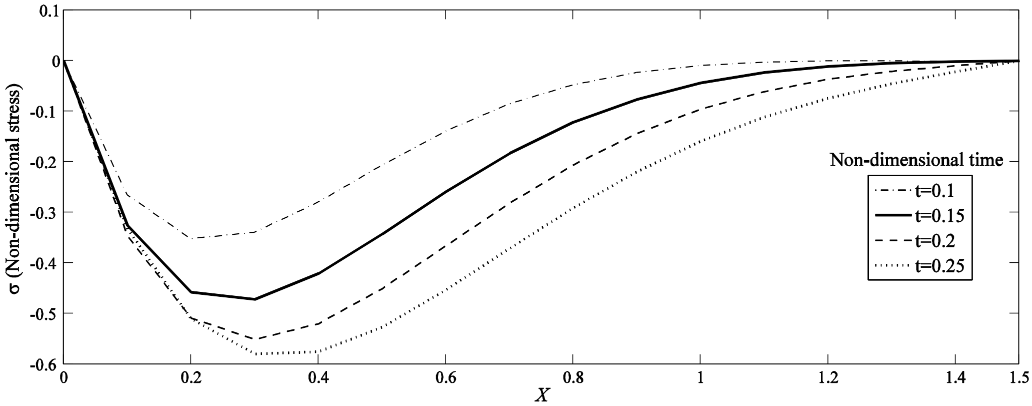

Fig. 13 Non-dimensional stress wave propagation along *X* direction

5. Conclusions

In this paper, an analytical method is proposed for generalized coupled non-Fickian diffusion-thermoelasticity analysis in a strip based on the Lord-Shulman theory of coupled thermoelasticity. The employed theory consists of both thermal and diffusion relaxation times. The derived governing equations are transferred to Laplace domain to find the solutions and then the obtained results are transferred back to time domain employing Talbot technique.

The main results of the presented research can be addressed as follows:

• The field variables including displacement, temperature and mass diffusion are obtained in series forms.

• The transient behaviors of field variables are studied in details. Also, the influences of variations in fields variables on each other are discussed.

• The wave propagation of field variables with finite speed is observed using the presented analytical method.

• The presented results can provide useful information for researchers experimentally work on wave propagation. The study of thermo-diffusion effect may be used to improve the conditions of processes like oil extractions.

This analytical method can be applied to two or three-dimensional problems of coupled diffusion thermoelasticity with various boundary conditions.

It can be concluded that the presented method has a high capability for solution of coupled system of PDEs such as coupled non-Fickian diffusion-thermoelasticity.

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CC

Nomenclature

a	Measure of thermodiffusion effect $(\frac{m^2}{s^2.K})$	X	Non-dimensional form of position
b	Measure of diffusive effect $(\frac{m^5}{kg.s^2})$	α_c	Coefficient of linear diffusion expansion $(\frac{m^3}{kg})$

c	Mass concentration $(\frac{kg}{m^3})$	α_t	Coefficient of linear thermal expansion $(\frac{1}{^\circ K})$
c_E	Specific heat $(\frac{J}{kg.^\circ K})$	β_1	$(3\lambda+2\mu)\alpha_t$
c_1	Boundary mass concentration (Non-dimension)	β_2	$(3\lambda+2\mu)\alpha_c$
C	Non-dimensional form of mass concentration	θ	$T-T_0$
D	Thermo diffusion constant $(\frac{kg.s}{m^3})$	θ_1	Boundary temperature (Non-dimension)
$A_n(s)$,$B_n(s)$, $D_n(s)$	Unknown coefficients	λ, μ	Lame's constants $(\frac{N}{m^2})$
k	Thermal conductivity $(\frac{W}{m.K})$	ρ	Density $(\frac{kg}{m^3})$
P	Chemical potential $(\frac{N.m}{kg})$	σ_{ij}	Components of stress tensor $(\frac{N}{m^2})$
P_1	Boundary chemical potential (Non-dimension)	τ_0	The thermal relaxation time (s)
t	Time (s)	τ	The diffusion relaxation time (s)
T	Absolute temperature (k)	ε_{ij}	Components of strain tensor (Non-dimension)
T_0	Reference temperature (k)		
u	Displacement (m)		
U	Non-dimensional form of displacement		
x	Position (m)		