

# Reliability of microwave towers against extreme winds

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**Abstract.** The reliability of antenna tower designed for a  $n$ -year design wind speed is determined by considering the variability of the strength of the component members and of the mean wind speed. For obtaining the  $n$ -year design wind speed, maximum annual wind speed is assumed to follow Gumbel Type-1 distribution. Following this distribution of the wind speed, the mean and standard deviation of stresses in each component member are worked out. The variability of the strength of members is defined by means of the nominal strength and a coefficient of variation. The probability of failure of the critical members of tower is determined by the first order second moment method (FOSM) of reliability analysis. Using the above method, the reliability against allowable stress failure of the critical members as well as the system reliabilities for a 75 m tall antenna tower, designed for  $n$ -year design wind speed, are presented.

**Key words:** microwave towers; reliability; system reliability; extreme wind; First Order Second Moment method (FOSM).

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## 1. Introduction

The design of microwave towers are governed by wind forces. Generally, they are designed based on the  $N$  year wind speed and an associated risk. Since yearly maximum wind speed is a random variable, the  $N$  year wind speed is calculated based on the distribution model of the yearly maximum wind speed (extreme wind). As a result, there is a level of uncertainty involved in the prediction of design wind speed. Also, there is a certain amount of uncertainty involved in the strength of available steel members to be used in the design. From the available data, it is generally observed that the strengths of members made of steel is a random variable having a normal distribution. Thus, a reliability analysis of micro wave steel tower becomes important in order to assess its vulnerability to failure over a given design life.

The reliability analysis of structures against wind forces have not been carried out as much as it has been done for the earthquake and wave loading. Rojiani and Wen (1981) presented the reliability of steel buildings under winds by considering various uncertainties involved in structural idealization, wind parameters, and strength variability. Wen (1983) studied the effect

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of wind direction on structural reliability. Wen (1984) verified the conclusion of his previous study by comparing the results at two sites. Lynn and Stathopoulos (1985) presented a simple approach for the evaluation of wind induced fatigue for low buildings. Wacker and Plate (1990) presented a method for assessing the susceptibility of surface elements of high rise buildings to fatigue due to wind pressure buffeting. Shinozuka (1990) discussed methods for digitally generating sample functions of a wind velocity field for implementation of the time and space domain analysis of wind-induced structural response. He also presented the reliability analysis of building structures subjected to multiple natural hazards (seismic and wind) within the framework of contemporary probabilistic risk assessment procedures. As such not much literature exists on the reliability analysis of tall lattice type structure under wind forces (both static and dynamic).

In this paper, reliability of antenna towers designed for a  $n$ -year wind speed is determined by considering the variability of the strength of the component members and of the mean wind speed. For obtaining the  $n$ -year design wind speed, maximum annual wind speed is assumed to follow the Gumbel Type-1 distribution. Following this distribution of the wind speed, the mean and standard deviation of stresses in each component member of the tower are obtained. The variability of the strength of members is defined by means of the nominal strength and a coefficient of variation. The probability of failure of the critical members of the tower is determined by the first order second moment method (FOSM) of reliability analysis. The reliability against allowable stress failure of the critical members as well as the system reliability for an antenna tower, 75m tall, designed for  $n$ -year design wind speed is determined as an illustrative example.

## 2. Probabilistic description of wind

### 2.1. Probabilistic nature of annual maximum (mean) wind speed

The study of very long term records of wind speed (over several years) at a given locality, reveals that wind velocity has spatial and temporal variation during a storm. At any instant of time, wind speed can be assumed to have a mean wind component and a fluctuating component. Mean wind speed is a stochastic variable and its effect is to produce quasi-static response of structures. For static design, annual maximum wind speed having certain return period is considered. The distribution of annual maximum wind speed, also known as extreme wind speed, and return period are closely related. For the reliability analysis, both are very important and are described below.

### 2.2. Distribution of annual maximum (extreme) wind speed

The Extreme Wind Speed is popularly modelled as Gumbel distribution or Extreme Value distribution (type-1), which has the following probability density and distribution functions.

$$f_v(V) = \alpha \exp[-\alpha(V-u)] - \exp\{-\alpha(V-u)\} \quad -\infty \leq V \leq \infty \quad (1)$$

$$F_v(V) = \exp[-\exp\{-\alpha(V-u)\}] \quad -\infty \leq V \leq \infty \quad (2)$$

The parameters  $u$  and  $\alpha$  are the location and dispersion parameters respectively and given by

$$\mu = u + \frac{0.5772}{\alpha} \quad (3)$$

$$\sigma^2 = \frac{\pi^2}{6\alpha^2} \quad (4)$$

where  $\mu$  and  $\sigma^2$  are the mean and the variance of the annual maximum wind speed  $V$ .

### 2.3. Return period ( $T_p$ ) and design wind speed ( $V_d$ )

The return period is defined as the expected time between two successive statically independent events. The return period,  $T_p$  which is called mean recurrence interval is defined as

$$T_p = \frac{1}{P} = \frac{1}{1 - F_v(V_d)} \quad (5)$$

where  $V_d$  is the specified design wind speed.  $F_v(V)$  is the cumulative distribution function of yearly maximum wind speed  $V$ , and  $P$  is the probability of wind storm of magnitude  $V$  exceeding  $V_d$  in any year.

A structure is required to be designed for its useful life against the wind load. The wind load is calculated for the given probability of exceedance (risk) during its life and the associated wind velocity is designated as design wind speed ( $V_d$ ) for that structure. The risk is decided on socio-economic parameters to some extent. Less risk provides higher design wind speed for the same life time.

If  $V_d$  is life time design wind speed,  $1 - F_v(V_d)$  is the probability of the annual extreme wind speed exceeding  $V_d$ . Hence, the probability of at least one event exceeding  $V_d$  in the first  $m$  years is

$$P_m = 1 - [F_v(V_d)]^m \quad (6)$$

and probability distribution

$$F_v(V_d) = [1 - P_m]^{1/m} \quad (7)$$

In the design procedures, wind loads are treated semi-probabilistically. The annual maximum wind speeds are recorded and an appropriate probability distribution is fitted to the data. A wind with some specified probability of exceedance in any one year is then selected for design purposes. Usually, a 0.02 exceedance probability for 50-year return period is used. In fact, the 0.02 exceedance level for a Type I extreme value distribution, normally used for wind speeds, corresponds to an exceedance level of 0.63 in a lifetime of 50 years (Holmes 1985).

### 2.4. Simulation of annual maximum wind speeds

For reliability analysis, the tower is to be analysed for a set of wind speeds that follows Gumbel distribution and that provides a specified design wind speed corresponding to a specified return period. It is possible to simulate different sets of wind speeds which provide the same design wind speed by different combinations of the mean and the covariance. The simulation procedure is illustrated below.

From Gumbel distribution, the probability of exceeding a design wind speed  $V_d$ , is given by

$$P(V > V_d) = [1 - F_v(V_d)] \quad (8)$$

Substituting for  $F_v$  from Eq. (2),

$$P = 1 - \exp[-\exp\{-\alpha(V_d - u)\}] \quad (9)$$

Rearranging the equation and taking log of both sides,

$$\exp\{-\alpha(V_d - u)\} = S_1 \quad (10)$$

where  $S_1 = -\ln(1-P)$

Again, taking log of both sides of Eq. (10),

$$V_d = \frac{S_2}{\alpha} + u; \text{ where } S_2 = -\ln S_1 \quad (11)$$

From Gumbel distribution parameters

$$\alpha = \frac{\pi}{\sqrt{6} \cdot \sigma_v} = 1.28255/\sigma_v \quad (12)$$

$$u = V_{mean} - \frac{0.5772}{\alpha} \quad (13)$$

From Eqs. (12) and (13),

$$u = V_{mean} - 0.5772 \cdot \sigma_v / 1.28255 = V_{mean} - 0.45\sigma_v \quad (14)$$

Substituting for  $u$  in Eq. (11),

$$V_d = \sigma_v * \left[ \frac{S_2}{1.28255} - 0.45 \right] + V_{mean} \quad (15)$$

Eq. (15) may also be written as

$$V_d = V_{mean} [1 + S_3 \cdot \text{COV}] \quad (16)$$

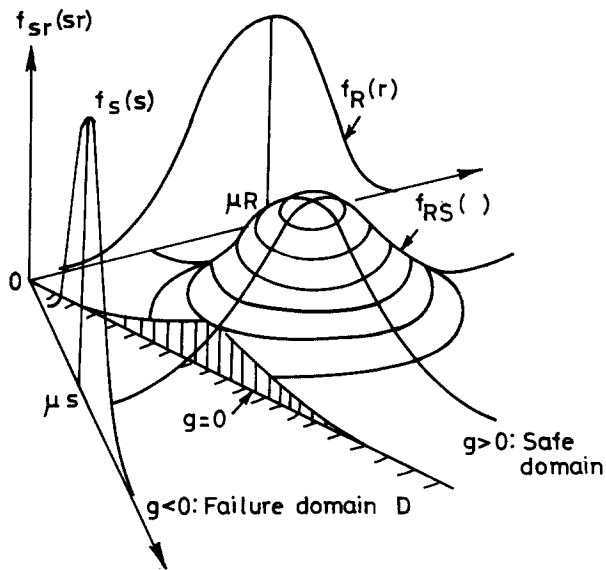
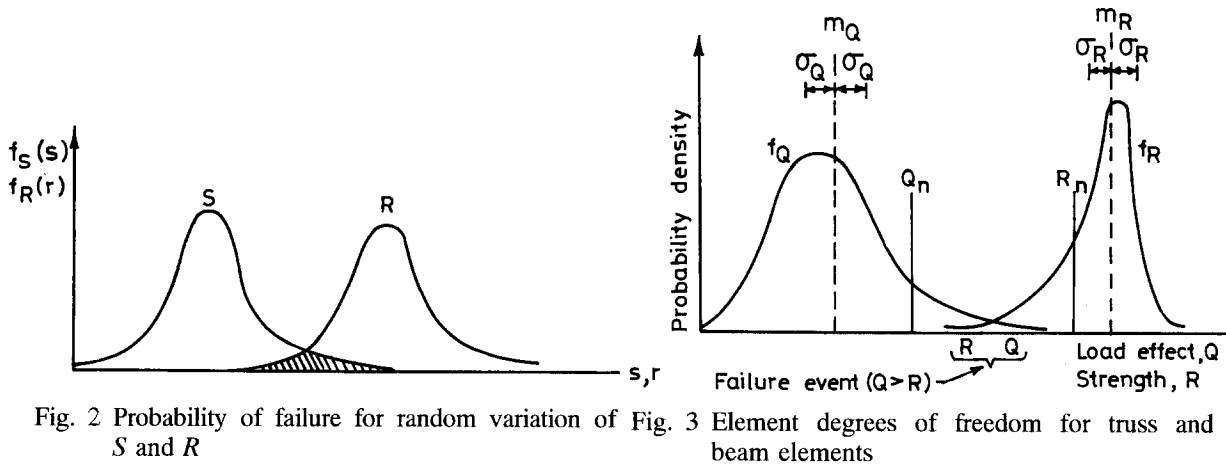
where 
$$S_3 = \left[ \frac{S_2}{1.28255} - 0.45 \right] \text{ and } \text{COV} = \frac{\sigma_v}{V_{mean}}$$

From Eqs. (10), (11) and (16), it is clear that the design wind speed, for a given value of  $P$  and Gumbel distribution parameters  $\alpha$  and  $u$ , is a function of both mean wind speed and standard deviation at a site. Therefore, different sets of wind velocities at 10 m height following Gumbel distribution may be simulated by assuming different combinations of  $V_{mean}$  and  $\sigma_v$  which provide the same design wind speed. For the simulation, the  $N$  year wind speed is specified. Using Eq. (16), COV is determined for an assumed value of  $V_{mean}$  at 10 m reference height and therefore,  $\sigma_v$  is known. Once a set of  $V_{mean}$  and  $\sigma_v$  is determined, the Gumbel distribution parameters  $u$  and  $\alpha$  are obtained using Eq. (12) and (13). With  $u$  and  $\alpha$  known, a set of wind velocities following the Gumbel distribution is computed. The set of wind velocities thus obtained, is compatible with the specified design wind speed.

### 3. Structural reliability

#### 3.1. Computation of component reliability

The structural reliability is obtained by using First Order Second Moment (FOSM) method (Appendix-I). The method requires the values of mean and standard deviation of response quantities of interest. For this purpose, the actual strength of a structural member is considered as a random variable that can be described in terms of a probability density

Fig. 1 Region  $D$  of integration for failure probability determinationFig. 2 Probability of failure for random variation of  $S$  and  $R$ 

function,  $f_R$ . This distribution is governed by variations in material properties, fabrication and erection that naturally occur from member to member. In Fig. 3,  $m_R$  and  $\sigma_R$  are the mean and standard deviation, respectively, of the member strength distribution, and  $R_n$  is the "nominal" design strength given by the design strength formula and design guide lines as given in design codes. The distribution of the strength is assumed as normal. The mean responses of all the members are determined by applying lateral wind forces corresponding to a set of simulated extreme winds generated as explained before. The mean and standard deviation of the stress in the selected members due to each set of wind velocity data are computed. The component reliability is determined using Eqs. (AI. 9) and (AI. 10).

### 3.2. System reliability

The system reliability is determined based on the failure probability of components (at the

member ends). The reliability assessment of structural system requires analysis of the various possible failure modes. Generally, two types of systems are distinguished; (1) series systems or the “weakest-link” system, and (2) parallel systems. A series system fails if any of its components fails, and a parallel system fails only if all of its components fail. In practical situations, a structural system is a combination of series and parallel systems.

$$\text{For series systems } R_{\text{sys}} = \prod_j R_j \quad (17)$$

$$\text{and for parallel systems } R_{\text{sys}} = 1 - \prod_j (1 - R_j) \quad (18)$$

where  $R_{\text{sys}}$  is the system reliability;  $R_j$  is the reliability of the  $j$ -th component and the product  $\prod$  is taken for all the components of the system. For the typical four-legged Microwave Antenna tower as shown in Fig. 4, any level failure will cause the failure of the whole system (i.e., tower). It is assumed that any level will fail if there are less than three columns (legs) remaining as supports and any column will fail if any of the member ends meeting the joint fails. It is assumed that the secondary members like, bracings etc. do not effect the reliability analysis. The computation of the system reliability involves following steps:

- (1) First the Probability of failure of a single column at  $j$ -th level ( $P_{fj}^c$ ) is determined. For the type of wind load considered, all the columns in a particular level have the same probability of failure. Since each column consists of two ends ( $i=1, 2$ ), it can be written as

$$P_{fj}^c = \prod_{i=1}^2 (1 - P_{fij}^j) \quad (19)$$

where  $P_{fij}^j$  is the failure probability of the  $i$ -th joint of any column on the  $j$ -th level.

- (2) Second, the Probability of failure of  $j$ -th level ( $P_{fj}^L$ ) is obtained. Since each level consists of four columns, level failure is the result of failure of more than one column; this probability is described by a binomial distribution;

$$P_{fj}^L = \binom{4}{4} [P_{fj}^c]^4 + \binom{4}{3} [1 - P_{fj}^c] [P_{fj}^c]^3 + \binom{4}{2} [1 - P_{fj}^c]^2 [P_{fj}^c]^2 \quad (20)$$

- (3) Finally, the probability of failure of the system ( $P_f^{\text{sys}}$ ) is determined. In the case of  $N$  levels, the probability of system failure is equal to the probability that any one level fails, which corresponds to failure of a series system.

$$P_f^{\text{sys}} = 1 - \prod_{j=1}^N (1 - P_{fj}^L) \quad (21)$$

Subsequently, the annual reliability of the system can be written in the form:

$$R_{\text{sys}} = 1 - P_f^{\text{sys}} \quad (22)$$

#### 4. Analysis of the tower due to mean wind velocity

##### 4.1. Structural model

The 3D-model of the Antenna Tower is shown in Fig. 4. For the purpose of analysis, the tower is idealised both as an assemblage of 3-D beam elements and an assemblage of 3D-

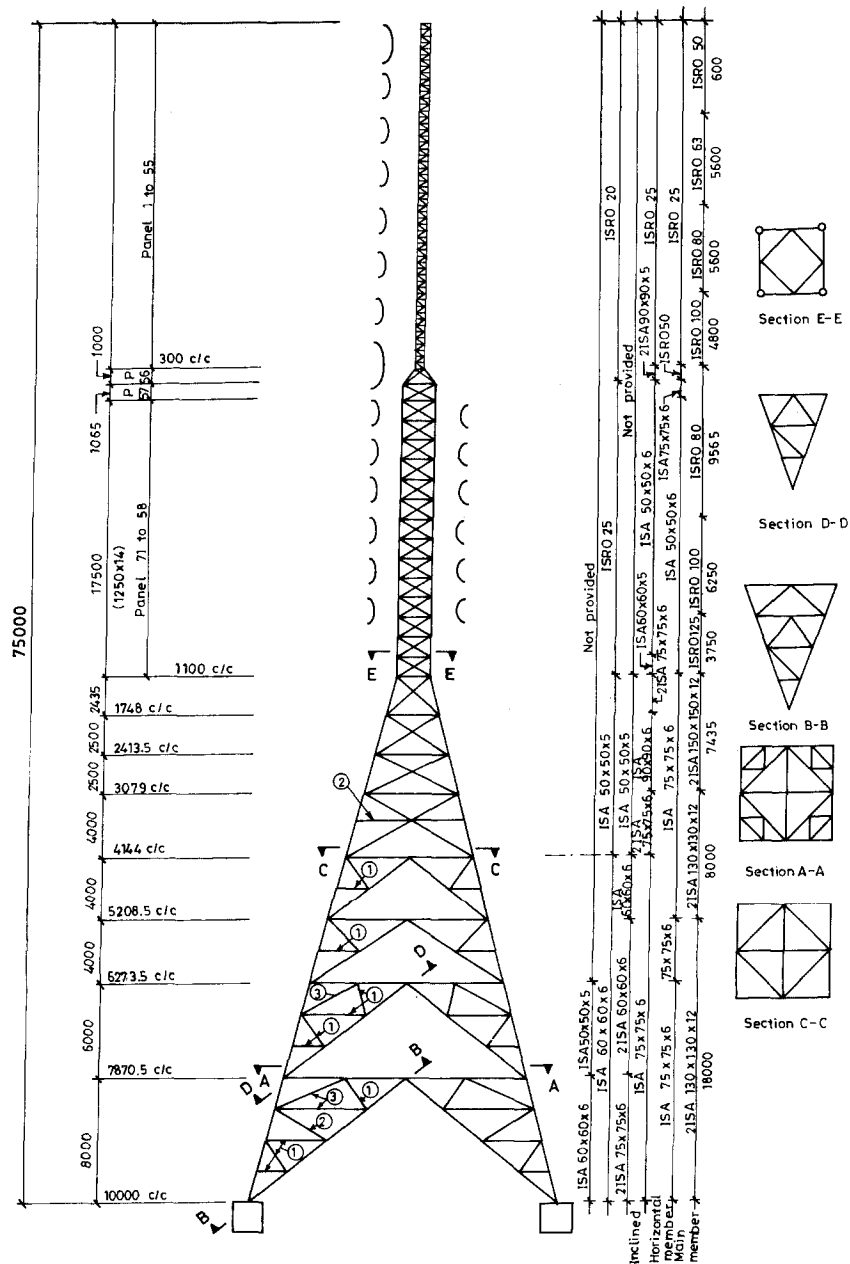


Fig. 4 Example structure

truss elements. Joints are considered as rigid joints having 6 degrees of freedom when the members are assumed as beam elements. When the joints are considered as pinned joints 3 translations are considered at each joint. Member local stiffness matrices are either the standard  $12 \times 12$  3-D beam elements stiffness matrix (without shear deformation, warping effect and beam-column effect) or the standard  $6 \times 6$  3D truss element stiffness matrix. Member stiffness matrix from local to global coordinate system is transformed with the help of usual transformation rules. The stiffness matrix corresponding to all joint degrees of

freedom are obtained by assembling the member stiffness matrices in global coordinates using the code number approach. The material behaviour is assumed to be linear elastic.

#### 4.2. Calculation of wind load vector

The wind loads are calculated using IS-codal provision [IS-875(part-3), 1987] in which overall drag coefficient depends on the solidity ratio. The solidity ratio is defined as the effective area of the tower frame normal to the wind direction divided by the area enclosed by the boundary of the frame normal to the wind direction. For the purpose of wind load calculation, the tower is divided into 'n' number of divisions, each division comprising of heights  $l$  and  $l'$  below and above a level of joints as shown in Fig. 5. For each such division, the solidity ratio is calculated and an appropriate value of the drag coefficient  $C_{di}$  is obtained.

Total wind force acting at the  $i$ -th level is given by

$$P_i = \frac{1}{2} \cdot \rho \cdot C_{di} \cdot A_{ei} \cdot V_i^2 \quad (23)$$

where  $P_i$  is the total wind forces acting at the  $i$ -th level;  $C_{di}$  is the drag coefficient which depends on the solidity ratio of the tower for the  $i$ -th division;  $V_i$  is the average mean wind velocity over the length  $l_i$  concentrated at the  $i$ -th level (Fig. 5) and  $A_{ei}$  is the effective lumped area for the  $i$ -th division given by  $A_{ei} = \sum a_{ei}^i$  where  $a_{ei}^i$  is the respective area of individual member to be lumped at the  $i$ -th level. The total wind force is equally divided to the four nodes acting along the

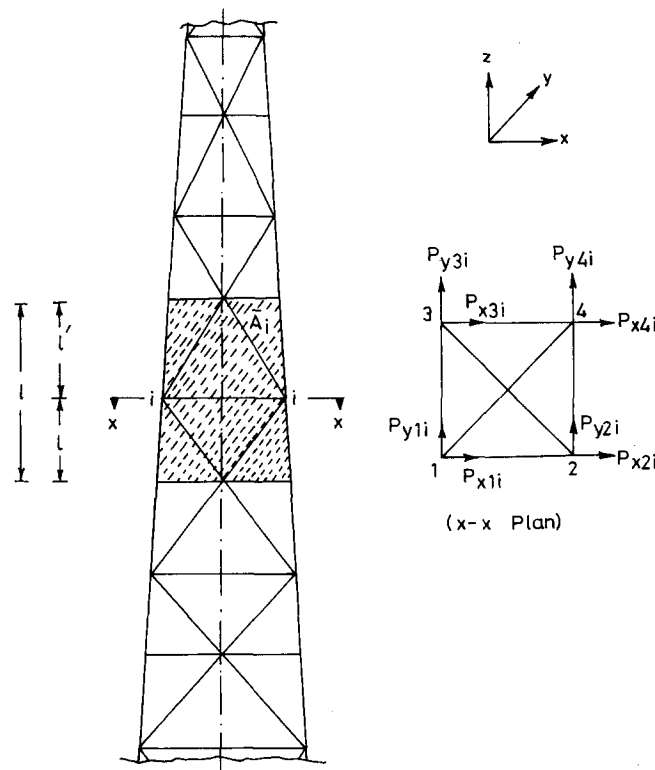


Fig. 5 Wind load computation on  $i$ -th level of tower

wind direction.

The wind loads acting on antenna disks are computed according to the Indian Railway standards for micro-wave towers for standard and shielded parabolic antennas. Total wind loads on the antennas are computed and lumped equally at the nodes to which the antennas are connected by angle members. The generated mean wind velocities are assumed to act at a reference height of 10 m. A power law variation is assumed to compute wind velocities at various levels of the tower. The power law is given by:

$$U_z = U_G (Z/Z_G)^\alpha \quad (24)$$

where  $U_z$  is the mean wind velocity at height  $Z$ ;  $U_G$  is the mean Gradient velocity;  $Z_G$  is the gradient height which is related to the surrounding structural patterns as well as terrain roughness and is the power law exponent which depends on the terrain roughness.

For the analysis, wind velocity is assumed to act along one of the principal directions of the tower. The two principal directions of the tower coincide with the  $X$  &  $Y$  axes. Thus, joint load at any joint consists of either a force in the  $X$  or in the  $Y$  direction.

#### 4.3. Determination of member end stresses

From the static analysis of the tower, the member end forces in local coordinates for the individual members are computed. The resultant stress at any end of the member is determined by

$$f = \frac{M_z}{Z_z} + \frac{M_y}{Z_y} + \frac{F_x}{A_x}; \text{ when modelled as 3-D beam element} \quad (25)$$

$$f = \frac{F_x}{A_x}; \text{ when modelled as 3-D truss element} \quad (26)$$

where  $M_z$ ,  $M_y$  are moments acting about the two principal axes of the member cross section and  $F_x$  is the axial load;  $Z_z$  and  $Z_y$  are the values of section modulus about the two member axes respectively;  $A_x$  is the cross sectional area.

The computed values of  $f$  for the member are used in the reliability analysis.

## 5. Numerical-study

A 75 m tall antenna tower is analysed. Fig. 4 shows the geometric properties of the tower. The modulus of elasticity for steel is taken as  $E=2.01 \times 10^{11}$  N/m<sup>2</sup>; the allowable mean stress used for determining the failure criterion is taken as the ratio of the yield stress ( $\sigma_y$ ) to the factor of safety ( $s$ );  $\sigma_y=2.5 \times 10^8$  N/m<sup>2</sup>. Individual member allowable stresses are determined from the  $l/r$  ratio in compression and the effective area in tension as per IS code provisions (IS: 800-1984, IS: 226-1975, IRS code for structural design of microwave antenna towers). The material density is taken as 7850.0 kg/m<sup>3</sup>.

The positioning of the antennas is indicated in Fig. 4 and the details of the antennas are given in Table 1. The solidity ratios and the  $C_d$  values are given in Table 2. The values of force coefficient used for computing the wind forces on the antenna disk are shown in Table 3.

For the computation of velocities along the height of the tower, the power law is used. The relevant information for finding the wind velocities at different heights using the power law

Table 1 Details of the antennas attached to the tower

Tower	Size ( $\phi$ )	Numbers of	Levels
75 m	2.1 m	2 nos.	(1) 72 m level: (ii) 52 m level
	1.2 m	6 nos.	One side of tower at 6 levels between 55 m and 75 m
	1.2 m	12 nos.	6 nos. of antenna on each side between 35 m level and 52 level

Table 2.1 Overall force coefficient for towers composed of flat-sided members

Solidity Ratio $\phi$	Force coefficient for	
	Square towers	Equilateral tri-angular towers
0.1	3.8	3.1
0.2	3.3	2.7
0.3	2.8	2.3
0.4	2.3	1.9
0.5	2.1	1.5

Table 2.2 Overall force coefficient for square towers composed of rounded members

Solidity ratio of front face	Force coefficient for			
	Subsritical flow		Supercritical flow	
	Onto face	Onto corner	Onto face	Onto corner
0.05	2.4	2.5	1.1	1.2
0.1	2.2	2.3	1.2	1.3
0.2	1.9	2.1	1.3	1.6
0.3	1.7	1.9	1.4	1.6
0.4	1.6	1.9	1.4	1.6
0.5	1.4	1.9	1.4	1.6

Table 3.1 Wind load on the antenna shall be calculated as follows

For standard parabolic antenna	For shielded parabolic antenna
$WL_A=400 C_{FA} P.A.$	$WL_A=400 C_{FA} P.A'.$
$WL_S=400 C_{FS} P.A.$	$WL_S=400 C_{FS} P.A'.$

$C_{FA}, C_{FS}$ =Constants

$WL_A$ =Axial force (N)

$WL_S$ =Side force (N)

$P$ =Wind pressure (N/m<sup>2</sup>)

$\theta$ =Angle between the wind direction and normal to antenna

$A$ =Projected area of standard parabolic antenna in the direction normal to antenna.

$A'$ =Projected area of shielded parabolic antenna in the direction normal to antenna.

are shown in Table 4.

For the reliability analysis, the strength of the members is assumed as a random variable with mean strength denoted by the allowable stress computed as per IS code provisions. The mean stress (strength) varies from member to member depending upon its  $l/r$  ratio and effective area. In order to determine the standard deviation of the strength of the member, a coefficient of variation (COV)=0.1 is assumed for all members. The effect of COV of strength for the members on the reliability estimate is investigated later by varying COV (which is assumed same for all members).

Table 3.2 Constants to be used in Table 3.1

$\theta^\circ$	Standard parabolic antenna		Shielded parabolic antenna	
	$C_{FA}$	$C_{FS}$	$C_{FA}$	$C_{FS}$
0	0.00400	0.00000	0.00320	0.00000
10	0.00400	0.00000	0.00320	0.00015
20	0.00400	0.00000	0.00320	0.00030
30	0.00400	0.00000	0.00320	0.00045
40	0.00414	0.00014	0.00320	0.00060
50	0.00428	0.00050	0.00320	0.00075
58.5	0.00440	-	0.00295	-
60	0.00428	0.00085	0.00285	0.00090
70	0.00350	0.00120	0.00190	0.00105
80	0.00175	0.00110	0.00095	0.00120
90	0.00000	0.00100	0.00000	0.00135

Table 4 Power law coefficient and gradient height for different terrain conditions

	Open area	Suburban area	City centre
Power law coefficient $\alpha$	0.16	0.28	0.33
Gradient height $\delta$	275.0 m	400.0 m	450.0 m

The design wind speed is obtained for a return period of 50 years (with a risk=0.63) with maximum annual wind speed to follow a Gumbel Type-I distribution.

Antenna towers are generally bolted at the joints. Depending upon the number of bolts used, the joints may behave as pin joints, semi rigid joints or rigid joints. Accordingly, the analysis of the tower varies. The 75 m tower is analyzed for a wind velocity 50 m/sec at the reference height of 10 m considering the joints as (i) Pin joints (ii) Rigid joints. The results of the analysis had shown that axial stresses in the members are nearly the same for both analyses. However, for rigid joints significant bending stresses developed at the joints, maximum to the order of 40% of the axial stress (Deoliya 1995). For the present study, stresses in the tower are obtained by considering the joints as semi rigid. 30% of the bending stresses developed for fully rigid joint condition were added to the axial stresses for determining the total stresses in the members for the reliability analysis.

### 5.1. Effect of mean and covariance of the annual maximum wind speed

As discussed before, it is possible to have different combinations of mean and standard deviation (Eq. (16)) to obtain the same design wind speed for a given return period, if the maximum annual wind speed is assumed to follow a Gumbel distribution. Four different combinations of mean and standard deviation are considered to provide the same 50 years design wind speed of 50 m/sec (Table 5). Corresponding to each combination, 66 wind speeds are simulated which follow the Gumbel Type-I distribution (Fig. 6). The tower is

Table 5 Wind statistics for different sets

	Set-1	Set-2	Set-3	Set-4
Mean	30.0 m/sec	35.0 m/sec	40.0 m/sec	45.0 m/sec
COV	$2.5857 \times 10^{-1}$	$1.6611 \times 10^{-1}$	$9.7159 \times 10^{-2}$	$4.3443 \times 10^{-2}$

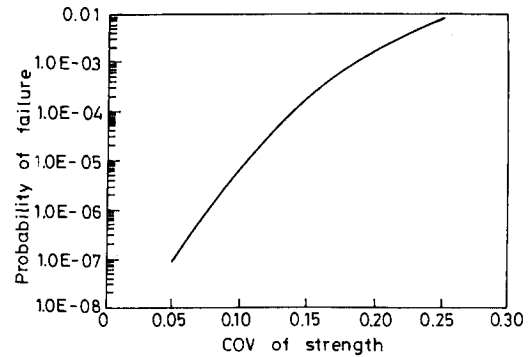
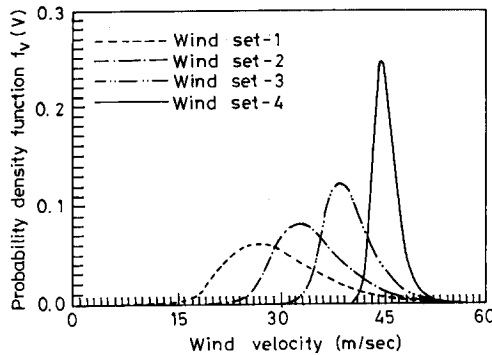


Fig. 6 Probability density functions for wind data Fig. 7 Effect of strength variability on probability of failure

Table 6 Response statistics and reliability results for 75 m tower  
( $\alpha=0.28$ ,  $\delta=400$ ,  $\mu_{10}=45.0$  m/sec,  $COV=0.0434$ )  
(Suburban Area)

Level	Mean Stress (MN/m <sup>2</sup> )	Std. Dvn. (MN/m <sup>2</sup> )	Safety Index $\beta$	$P_f$ (Leg)	$P_f$ (Level)	$R$ (Level)
1	82.99	18.86	3.287	$1.012 \times 10^{-3}$	$6.135 \times 10^{-6}$	0.9994939
5	90.50	19.13	2.849	$4.377 \times 10^{-3}$	$1.143 \times 10^{-4}$	0.9978086
10	68.83	18.42	4.135	$3.541 \times 10^{-5}$	$7.521 \times 10^{-9}$	0.9999822
50	20.94	17.48	7.096	0.000000	0.000000	0.9999999
79	14.27	16.84	7.465	0.0000000	0.000000	0.9999999

Over all system reliability=0.8308697; Minimum component reliability=0.92025632

Table 7 Response statistics and reliability results for 75 m tower  
( $\alpha=0.16$   $\delta=275.0$ ,  $COV=0.16611$ )  
(Open Terrain)

Level	Mean stress (MN/m <sup>2</sup> )	Std. Dvn. (MN/m <sup>2</sup> )	Safety index $\beta$	$P_f$ (Leg)	$P_f$ (Level)	$R$ (Level)
1	91.00	36.14	1.494	0.130612	$8.540 \times 10^{-2}$	0.9324095
5	93.06	36.77	1.412	0.151640	0.111660	0.9210641
10	70.00	29.94	2.505	$1.220 \times 10^{-2}$	$8.793 \times 10^{-4}$	0.9938785
50	17.15	18.29	6.986	0.000000	0.000000	0.9999999
79	13.89	17.40	7.243	0.000000	0.000000	0.9999999

Over all system reliability= $4.278 \times 10^{-2}$ ; Minimum component reliability=0.7677931

analysed for each simulated wind speed, considering it to be the wind speed specified at the reference height of 10 m.

The probability of failure ( $P_f$ ) of leg members of the tower and  $P_f$  obtained for a level by considering the parallel combination are determined. The typical results are shown in Tables 6 and 7. In these tables, minimum component reliability (i.e., member having maximum probability of failure) is also shown. The tables also show the mean and standard deviation of the stress, and the safety index  $\beta$  for a typical leg member at different levels. Level 1, in the tables denotes the base level and the top level corresponds to 79th level. It is seen from the tables that there is a considerable difference between the  $P_f$  values of the leg member A at different levels. However, no definite trend of variation of the  $P_f$  values with the variation of

Table 8 System reliability and minimum component reliability for different sets of wind velocities

Reliability	$\alpha=0.28, \delta=400.0 \text{ m}$			
	Set-1	Set-2	Set-3	Set-4
System reliability	0.9987605	0.9962534	0.9855359	0.8308657
Minimum component reliability	0.9942124	0.9897433	0.9791990	0.9202563

Table 9 System reliability and minimum component reliability for different terrain conditions

Reliability	Set-2 of wind		
	Open area	Suburban	City centre
System reliability	$4.728 \times 10^{-2}$	0.9962534	0.9999819
Minimum component reliability	0.7677931	0.9897433	0.9991977

the level is observed. This happens due to the fact that the cross section of the members and the stresses developed in the members vary from level to level without following any pattern. It should be noted that the cross bracings and the other secondary members are not subjected to significant stresses and the probability of failure for all these members happen to be zero.

Table 8 shows the effect of the mean and standard deviation of wind speed on the reliability estimates. It is seen that both system and component reliabilities vary with the 4 different sets of wind velocities generated to predict the same design wind speed. The reliabilities are more for less values of the mean wind speed.

### 5.2. Effect of terrain condition on reliability

The tower is designed for the terrain condition corresponding to the suburban area. The reliability estimates are made for the same tower if it were located in the open terrain and city centres. The results are obtained for the second set of wind speeds (Table 5) and are shown in Table 9. It is seen that the system reliabilities for the same tower become extremely small if they were situated in the open terrain. On the other hand, for the city centre the same tower has a system reliability close to unity. This is expected because in the open terrain the wind velocities are more along the height of the tower for the same wind speed assumed at the reference height. Thus, the power law coefficient and gradient height play an important role in the reliability estimate of the towers.

### 5.3. Effect of coefficient of variation (COV) of the strength of members

The probability of failure  $P_f$  for the component member (which has the worst stress combination) is obtained by considering the coefficient of variation of strength as 5%, 10%, 15%, 20% and 25% for the second set of wind speeds and the terrain condition taken as suburban area. The variation of  $P_f$  with COV of strength is shown in Fig. 7. It is seen from the figure that the  $P_f$  value for the component member increases with the increase in COV of strength. The  $P_f$  for the system is expected to vary in a similar manner.

## 6. Conclusions

Reliability of a steel lattice antenna tower against annual maximum wind speed over its

design life is investigated. Design wind speed of the structure is obtained by considering the return period for the wind speed to be equal to the design life of the structure. The maximum annual wind speed distribution follows a Gumbel type-I distribution. The strength of the member is assumed to be normally distributed defined by a mean strength and a coefficient of variation (COV). Probability of failure of the tower is determined using First Order Second Moment theory. Both component and system reliabilities for a 75 m tall tower are determined using the proposed method of analysis. Results of the study lead to the following conclusions:

- (i) It is possible to generate different sets of wind speeds which follow Gumbel distribution and provide the same  $N$  year return wind speed (for which the tower is designed). Each set of wind speed is characterised by a mean value and a standard deviation which are different from one set to the other.
- (ii) Reliability of the tower, which is designed for  $N$  year return wind speed may be obtained for any of these sets of wind speeds; each set providing different reliability estimate.
- (iii) Reliability estimate is more for the set of wind speeds which has less mean but more standard deviation.
- (iv) Because of the practical limitations of the design (i.e., the use of standardised sections), the probability of failure of the components (individual member of the tower) widely differ.
- (v) The terrain condition, which dictates the power law coefficient and the gradient height, significantly influences the reliability estimates. Reliability of the same tower situated in the city centre is significantly more than that if it were situated in the open terrain.
- (vi) The coefficient of variation (COV) of the strength of structural members significantly influences the probability of failure ( $P_f$ ) of the tower. Greater the value of COV more is the value of  $P_f$ .

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## Appendix-I; Basic case of structural reliability

Let  $R$  and  $S$  represent the resistance of a structural element and the dimensionally consistent actions

or the load effect on the element, respectively. The element is said to have failed if its resistance  $R$  is less than the action  $S$ . The probability that this state is reached is the probability of failure i.e.,

$$p_f = P(R \leq S) \quad (\text{AI.1})$$

$$= P(R - S \leq 0) \quad (\text{AI.2})$$

or in general,

$$P_f = P[g(R, S) \leq 0] \quad (\text{AI.3})$$

where,  $g(R, S)$  is the "limit state".

If  $f_R(r)$  and  $f_S(s)$  represent probability density functions for  $R$  and  $S$  respectively, the probability of failure is estimated as

$$p_f = P[g(R, S) \leq 0] = \int_D \int f_{RS}(r, s) dr ds \quad (\text{AI.4})$$

where  $f_{RS}(r, s)$  is the joint probability density function and the region of integration is the failure domain  $D = \{(r, s): g(r, s) \leq 0\}$ , (Fig. 1). In general,  $R$  is a function of material properties and dimensions and  $S$  is a function of applied loads, each of which may be a random variable.

Numerical computation of Eq. (AI.4) is required to compute  $P_f$  of the structural element. It can be obtained from the closed form solutions for a few cases. The closed form solution for the evaluation of  $P_f$  when both load effect  $S$  and component resistance  $R$ , are normal, is given below:

$$P_f = P[(R - S) < 0]$$

$$\text{Let } M = R - S \quad (\text{AI.5})$$

Where,  $M$  is defined as margin of safety. When  $R$  and  $S$  are independent and normally distributed, as stated before, (Fig. 2 illustrates this simplified case of structural reliability), the mean value of  $M$ ,  $\mu_M$  and the standard deviation of  $M$ ,  $\sigma_M$ , are given by

$$\mu_M = (\mu_R - \mu_S) \quad (\text{AI.6})$$

$$\sigma_M = \sqrt{(\sigma_R^2 + \sigma_S^2)} \quad (\text{AI.7})$$

Hence, the probability of failure  $P_f$  is

$$P_f = P(M < 0)$$

$$F_M(0) = \Phi\left(\frac{0 - \mu_M}{\sigma_M}\right) \quad (\text{AI.8})$$

$$P_f = \Phi\left(\frac{\mu_S - \mu_R}{\sqrt{(\sigma_R^2 + \sigma_S^2)}}\right) \quad (\text{AI.9})$$

where  $\Phi(\quad)$  is standard normal distribution function.

$$\text{Let } \beta = \frac{\mu_M}{\sigma_M}$$

Then, the value of  $P_f$  corresponding to  $\beta$  is given by

$$P_f = \Phi(-\beta) \quad (\text{AI.10})$$

It is clear from Eqn. (AI.10) that  $\beta$  is related to the probability of failure and is called the reliability index. Since mean values and standard deviations of the variables are required to compute the reliability, the method is known as the first order second moment reliability method (FOSM).