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Technical Note

A mathematical model to predict fatigue notch factor of butt joints

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Abstract. A mathematical model is developed to predict the fatigue notch factor of butt welds subject to number of parameters such as weld geometry, residual stresses under dynamic combined loading conditions (tensile and bending). Linear elastic fracture mechanics, finite element analysis, dimensional analysis and superposition approaches are used for the modelling. The predicted results are in good agreement with the available experimental data. As a result, scatters of the fatigue data can be significantly reduced by plotting *S-N* curve as $(S \cdot K_f)$ vs. *N*.

Key words: fatigue notch factor; stress intensity factor; crack propagation; weld geometry; residual stress and butt welded joints.

1. Introduction

In practice, in order to assess the fatigue strength of the welded structures which are prone to a large number of parameters the fatigue notch factor K_f is usually employed. The fatigue notch factor represents a reduction in the fatigue strength of welded joints as a result of stress concentrations at the weld toes. In general, a material is regarded as more sensitive to a "fatigue notch" if the value of K_f is larger for a given value of stress concentration factor K_t. Amongst the equations that available for the calculation of the fatigue notch factor (K_f), Peterson's equation (Peterson 1959) and Buch's equation (Buch 1988) are two of the most widely used and they are described respectively as follows

$$K_f = 1 + (K_t - 1)/(1 + a^*/r) \tag{1}$$

$$K_f = K_t[(1 - 2.1h/(r + \rho_o))]/B \tag{2}$$

where a^*, B, h and ρ_o are material constants and r is weld toe radius (Fig. 1).

It should be noted here that Eqs. (1) and (2) do not take into account the variations of weld geometry, residual stresses and the loading conditions but only the weld toe radius (r) and stress concentration (K_t). Hence, these equations generally are not suitable for the fatigue assessment of welded joints and the improved model for fatigue notch factor is needed. Therefore, this study aims to develop a mathematical model to predict fatigue notch factor of butt welded joints subjected to the effects of various weld geometrical parameters, residual stresses and the combined loading conditions (tensile and bending).

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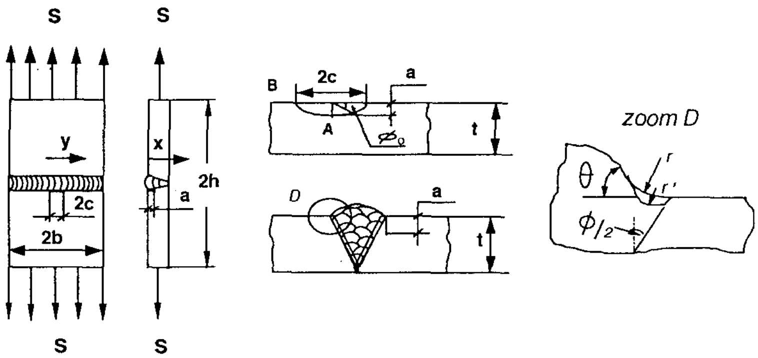

Fig. 1 Surface crack model for transverse butt joint

2. Modelling concepts

2.1. Linear elastic fracture mechanics crack propagation model

Using Fracture Mechanics, the rate of fatigue crack growth under cyclic loading can be expressed in terms of the range of the effective stress intensity factor $(\Delta K)_{eff}$ through Paris-Erdogan's power law (Paris and Erdogan 1963) as follows

$$da/dN_p = C(\Delta K)_{eff}^m \tag{3}$$

The effective stress intensity factor in model-I loading can be written in the following form to allow for the effect of weld geometry and residual stress in combined loading condition as follows:

$$K_{I,eff} = Y_{o,a} \cdot M_{k,eff} \cdot S_A \cdot \sqrt{\pi a} \tag{4}$$

where

$$M_{k,eff} = M_{k,a} + \frac{Y_{o,b}}{Y_{o,a}} \cdot M_{k,b} \cdot R_{ba} + M_{k,r} \cdot \frac{S_r}{S_A}$$

When the range of the stress intensity factor of a cracked body is known, the fatigue crack propagation life, N_p can be calculated by integrating Eq. (3) between the inital crack length (a_i) and the final crack length at failure (a_f). In this study, the range of stress intensity factor is replaced by the range of effective stress intensity factor (ΔK_{eff}) to allow for the effect of weld geometry and residual stresses in combined loading. More details about numerical procedures used in this study can be found elsewhere (Nguyen 1996).

2.2. A mathematical model for fatigue notch factor

In order to quantify the combined effect of all the weld geometry parameters, residual stresses and combined loading conditions on fatigue behaviour of butt joints, a mathematical model for *S-N* curve is proposed on the basis of the fatigue design rule BS 5400 (Maddox 1988) as follows

$$S^m \cdot N_f = A \tag{5}$$

where $$m/m_o = k_m \cdot f_m(r'/r) \cdot f_m(r/t) \cdot f_m(\theta) \cdot f_m(t/b) \cdot f_m(S_r/S_r) \cdot f_m(R_{ba}) \quad (6)$$

$$A/A_o = k_A \cdot f_A(r'/r) \cdot f_A(r/t) \cdot f_A(\theta) \cdot f_A(t/b) \cdot f_A(S_r/S_r) \cdot f_A(R_{ba}) \quad (7)$$

k_A, k_m-proportional constants related to A and m respectively

Eq. (5) is determined once Eqs. (6) and (7) were determined by using dimensional analysis technique and transformations (Nguyen 1996). Furthermore, Eq. (5) can be rewritten for flush-ground welded joints or flat plate as:

$$S_o^{m_o} \cdot N_f = A_o \quad (8)$$

from Eqs. (5) and (8) $$\frac{S_o^{m_o}/S^{m_o}}{S^{m-m_o}} = \frac{A_o}{A} \quad (9)$$

substituting $K_f = S_o/S$ into Eq. (9) and modifying using Eq. (5)

$$K_f = \left(\frac{A}{N_f}\right)^{\frac{1}{m_o} - \frac{1}{m}} \cdot \left(\frac{A}{A_o}\right)^{-\frac{1}{m_o}} \quad (10)$$

3. Results and discussion

The above mathematical model has been verified by comparing the predicted results of the fatigue notch factor (K_f) with that predicted by Peterson's and Buch's models and with the available fatigue test data. Fig. 2 shows a comparison between the predicted values of (K_f) and the available experimental data. It can be seen that the scatter band of (K_f) corresponding to the lower and upper bounds for the weld geometry parameters, welding residual stress and loading condition. The aligned, undercut-free, butt-joints in perfectly stress-relieved condition were selected for the lower boundary condition (r'=0, R_{ba}=0, S_r=0). In this case, only three weld geometry parameters namely weld toe radius (r), flank angle (θ) and plate thickness (t) were varied.

For the upper boundary condition three parameters (r', R_{ba} and S_r) were selected so as to simulate the conditions of the test specimens i.e., r'=0.05 mm, R_{ba}=0.15 and S_r/S_y=0.1 which were based on the fatigue test results (Nguyen 1996). Fig. 2 also shows that the variations of (K_f) predicted by the present model covers most of the available experimental data. The values of (K_f)

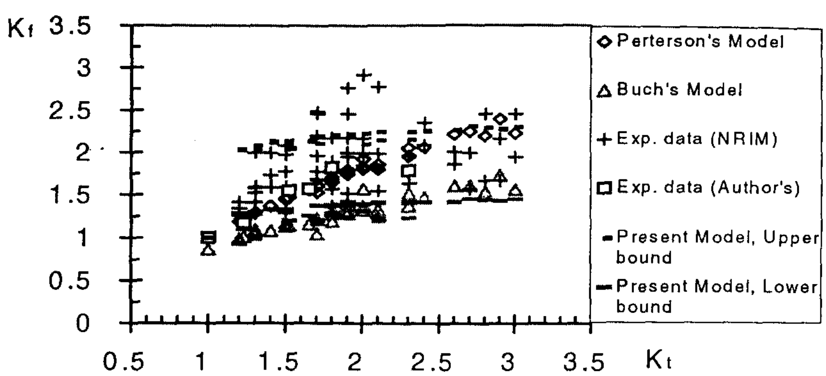

Fig. 2 A comparison between the predicted values of (K_f) and available experimental data

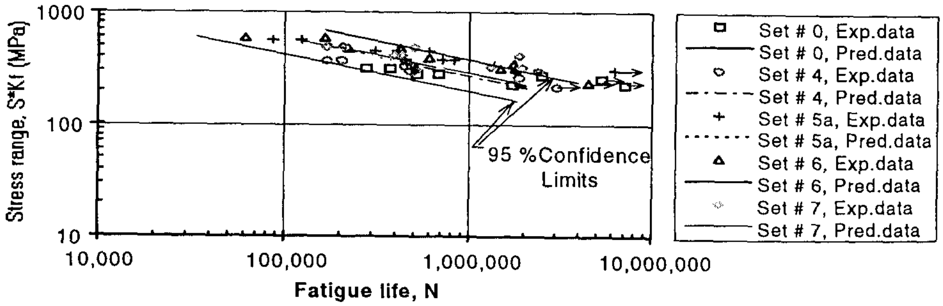

Fig. 3 A comparison between calculated $(S \cdot K_f)$-N curves and the fatigue test data

increased as the value of (K_t) increased. Furthermore, the values of (K_f) predicted by Peterson's model failed to cover a large scatter band of the experimental data and thus highlights the weakness of that nodel. On the other hand, the values of (K_f) predicted by Buch's model underestimate the experimental data due to stress concentration factor of less than 2.5 $(K_t<2.5)$ though a better agreement was found for the experimental data with higher stress concentration factor $(K_t>2.5)$.

Fig. 3 shows the fatigue data plotted in terms of $(S \cdot K_f)$ vs. N in log-log scale. The values of (K_f) for each set of fatigue data were calculated by using Eqs. (5) to (7) and (10). It can be seen from this figure that scatters of the fatigue data have become relatively small when are corrected by fatigue notch factor (K_f). The 95% confidence limits of predicted $(S \cdot K_f)$ vs. N curves are also plotted by using standard deviation value specified for S-N design class B in BS 5400 (Maddox 1988). Fig. 3 also shows that majority of the fatigue test results fall within the 95% confidence limit lines. This means that the present model can satisfactorily predict the fatigue behaviour of butt joints due to the variations of weld geometry, residual stresses and combined loading conditions (axial and bending).

4. Conclusions

Several important conclusions can be drawn from this study as follows:

(1) The available models for fatigue notch factor fail to predict the fatigue strength of welded joints subject to large number of parameters such as weld geometry, residual stresses and the combined loading conditions.

(2) The mathematical model developed in this study can be used to predict the overall effect of butt weld geometry parameters and residual stresses in the combined loading conditions satisfactorily.

(3) Using model equations developed in this study for the fatigue notch factor (K_f), the scatter band of the fatigue test data can be significantly reduced by plotting new S-N curve as $(S \cdot K_f)$ vs. N. This suggests a new initiative for the establishment of a new standard procedure for the evaluation of the fatigue test with reduced scatters.

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Notations

A, m	are constants related to weld geometry, residual stresses and loading condition
A_o, m_o	the regression constants (Eq. (8)) corresponding to fatigue curves of flush-ground butt-joints in residual stress-free condition (m_o=3, A_o=$f(t/b)$)
C, m	material constants in Paris' equation
$M_{k,a}$	stress intensity magnification factor produced by weld geometry in axial loading
$M_{k,b}$	stress intensity magnification factor produced by weld geometry in bending
$M_{k,eff}$	effective stress intensity magnification factor produced by weld geometry
$M_{k,r}$	stress intensity magnification factor produced by weld geometry & residual stress
R_{ba}	ratio between bending and axial nominal stress range (R_{ba}=S_B/S_A)
S_A, S_B	axial nominal stress range and bending nominal stress range
S_r	maximum residual stress at weld toe surface
S_y	yield strength of parent material
$Y_{o,a}$, $Y_{o,b}$	stress intensity geometry-configuration factors in axial & bending loading for flat plate