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Dynamic sensitivity analysis and optimum design of aerospace structures

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Abstract. The research and applications of numerical methods of design optimization on structural dynamic behaviors are presented in this paper. The emphasis is focused on the dynamic design optimization of aerospace structures, particularly those composed of composite laminate and sandwich plates. The methods of design modeling, sensitivity analysis on structural dynamic responses, and the optimization solution approaches are presented. The numerical examples of sensitivity analysis and dynamic structural design optimization are given to demonstrate the effectiveness of the numerical methods.

Key words: sensitivity; dynamic optimization; aerospace structures.

1. Introduction

The structural design optimization is an important research and application branch of the computational mechanics. The research of modern theories and methods of structural design optimization is closely related with the finite element method. Although most of the research work has been concentrated on the design optimization of static behaviors of the structures, much attention has been focused on the optimum design concerning structural dynamic behaviors (Fox and Kapoor 1969). Compared with the static behavior design of structures, the dynamic behavior design is more difficult, since the structural dynamic properties are more implicitly related to the design parameters and there are fewer design criteria and experiences on them.

Aerospace structures are typically large scale flexible ones made of light-weight materials such as composite materials. Therefore the natural frequencies of the structures are relatively low. Moreover, some part of the structures may be subjected to external excitations caused by eccentricity, collision and so on. For this reason, the structural dynamic behaviors such as vibration frequencies, dynamic deformation and stress responses are usually critical design requirement for aerospace structures. For example, the vibration frequencies of flexible space structures such as satellite and its wing structures must be designed higher enough to ensure the required controlling performance. Along with the advances of scientific research and industrial technologies, in more and more structural designs the dynamic property requirement becomes dominant. Due to the complicated relationships between the structural dynamic properties and design parameters,

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and the lack of design experience on the structural dynamic properties, the traditional design methods are tedious or even impracticable in the design of the aerospace structures. Consequently, it is important to applying the optimization techniques in the design process to obtain an optimum design which satisfy both the dynamic property requirements and other ones.

The structural optimization method is an important and efficient tool for dynamic property design. The present paper addresses the research and applications of this numerical method to the dynamic design, particularly, the design of space structures. In the design of space structures, the weight minimizing is desired for economic consideration and the structural vibration frequencies are strictly limited by the design requirements on working performance and operating control. It is very difficult to make balance between these two conflicting requirements. Part of the difficulty arises from the great number of design variables and complicate relations between the structural dynamic properties and the design variables. Moreover, the space structures are usually composed of composite materials which are very flexible and possess very low vibration frequencies due to their shape and size features and the material properties. This makes the structural dynamic analysis and sensitivity analysis more difficult. On the other hand, the composite components used in space structures, e.g. laminate plates, sandwich plates, composite thin-walled beams, provide more design possibilities too. On the basis of the research work (Gu and Cheng 1990), the structural analysis and design optimization program MCADS was extended to the dynamic design optimization of structures. The design optimization is based on the versatile structural modeling and finite element analysis for general structures and composite space structures, for which the optimization facilities of laminate and sandwich plates are developed. Linear approximation of the behavior constraint is usually needed if mathematical programming methods are employed to search for optimal solutions. The sensitivity analysis for vibration frequency and dynamic responses, particularly, a direct derivation method for the latter, is studied. The optimization method with basic sequential linear programming algorithm has been improved stable and efficient for dynamic design and multi-objective optimization. These numerical methods and the program MCADS have been applied to space structures such as satellites.

2. Structural design modeling

The practical applicability of design optimization program is principally determined by the structural design and analysis modeling. The modeling of structural analysis with finite element method is normally presented by the types of elements, loading and boundary conditions, while the design modeling is described with design variables, constraint and objective functions of structural behaviors. The structural modeling of MCADS is developed to optimize general structures and, particularly, the composite space structures made of laminate plates, sandwich plates, and special thin-walled beams. The element library is composed of bar, beam, membrane, plate, shell, 3D solid brick, axisymmetric solid and shell, spring, composite honeycomb sandwich plate and laminate plate. To deal with the laminate plate and sandwich plate, the quadrilateral and triangular Mindlin-type lower-order plate elements free of shear locking have been developed.

The design variables of MCADS can be classified into three categories:

(1) Size design variables, including the cross section area of bar; thickness of membrane, plate and shell; cross section sizes of beam with various shaped cross sections.

(2) Composite design variables, including the ply orientation angles and the layer thickness

of laminate plate and faces of sandwich plate, the height of honeycomb core, and the material parameters of some kinds special composite plates.

(3) Shape design variables, including the coordinates of special nodes and geometric parameters used in the shape description of structural boundaries, e.g. interpolation parameters and interpolation point positions of curves or surfaces.

Particularly, for the beam elements extensively used in engineering structures, the design variables can be any size of cross section. Furthermore, a cross section library has been built for commonly used cross sections of beam. The design variables and stress calculation of beam cross section within this library can be dealt with uniformly.

The constraint functions of dynamic design optimization with MCADS cover the following structural behaviors: structural weight, vibration frequencies, dynamic displacement and stress responses. The objective function of design optimization can be selected from above constraint functions or be combination of several constraint functions. This way, the objective of design optimization can meet different design requirements such as reducing structural weight, increasing structural fundamental frequency, adjusting the distribution of a group of vibration frequencies, or minimizing structural dynamic responses.

3. Optimization solution algorithm

The mathematical formulation of structural design optimization problem is as following.

$$
\begin{aligned}
&\text{To find } \boldsymbol{X}=(x_1, x_2, \ldots, x_n)^T, \\
&\min. f(\boldsymbol{X}) \\
&s.t.\ g_j(\boldsymbol{X})\le 0, \qquad j=1, 2, \ldots, m \\
&\boldsymbol{X}^L \le \boldsymbol{X} \le \boldsymbol{X}^U
\end{aligned} \tag{1}
$$

where X is the vector of design variables and $f(X)$ is the objective function. $g_j(X)\le 0$ ($j=1, 2, \ldots, m$) represents the constraint conditions such as the structural weight or the dynamic behaviors constraints. X^L and X^U are lower and upper bound of the design variables, respectively. n denotes the number of the design variables and m the number of the constrains.

The basic optimization algorithms of MCADS are sequential quadratic programming (SQP) and sequential linear programming (SLP). At each design point, the constraints are approximated by linear inequalities, and the objective function $f(\boldsymbol{X})$ is replaced with linear or quadratic approximation for SLP or SQP algorithm respectively. By this means, the original optimization problem is approximated by a linear programming or quadratic programming problem and solved with the standard Lamke pivot algorithm.

In some circumstances, an infeasible design may be encountered and this is often the case in the dynamic property design. For example, the natural frequency of the structure may be lower than the lower bound of the frequency constraint. In order to overcome the difficulty caused by the infeasible design, a goal programming treatment is employed to find feasible solution. If some constraint conditions are deviated noticeably, then these constraint functions are added into objective function with weighting factors λ_j ($j\in J$) and the corresponding constraint conditions are temperately relaxed by introducing positive quantities δ_j. Thus the modified problem can be formulated as:

$$
\begin{aligned}
&\text{To find } X=(x_1, x_2, \ldots, x_n)^T \\
&\min F(X)=f(X)+\sum_{j\in J}\lambda_j g_j(X) \\
&s.t.\ g_j(X)\le 0,\ (j\notin J) \\
&g_j(X)\le \delta_j,\ (j\in J) \\
&X^L\le X\le X^U
\end{aligned} \tag{2}
$$

This way, by means of assigning suitable bounds to some constraints, a multi-objective optimization problem can be defined and solved. Particularly, this approach is effective when a group of frequencies are subjected to lower bound constraints which are not satisfied in the initial design.

The nonlinearity of dynamic properties of the structures makes it difficult to obtain a stable convergence during the optimization iteration procedure. Therefore the development of efficient and robust optimization algorithm suitable for dynamic optimum design problems continues to be a topic of much research interest. To ensure the convergence of iteration, the methods of approximate line search and adaptive move limit have been studied and implemented in the presented paper. The approximate line search is to search a better design on the line between old design and new design obtained by solving current quadratic or linear programming problem. At each iteration of optimization procedure, the design is checked according to the following Goldstein criterion.

$$
\begin{aligned}
&\beta(1-\eta)\ \nabla^T F(X_0)\boldsymbol{d}\le F(X_0+\beta\boldsymbol{d})-F(X_0)\le \beta\eta\nabla^T F(X_0)\boldsymbol{d} \\
&\boldsymbol{d}=X-X_0;\quad 0<\eta<0.5
\end{aligned} \tag{3}
$$

where X is a new design point obtained from quadratic or linear programming, $\boldsymbol{d}$ is the direction of line search, β is the step length of line search with initial value 1.0, and $F(X)$ is the original objective function or the modified multi-criterion objective function in Eq. (2). η is a prescribed factor. Starting from X and with initial $\beta=1.0$, the approximate line search is carried out within a few steps as following.

1) If the Goldstein criterion (3) is satisfied, then $X_{new}=X_0+\beta\boldsymbol{d}$ is accepted as a new design and stop the line search.
2) If $F(X_0+\beta\boldsymbol{d})-F(X_0)>\beta\eta\nabla^T F(X_0)\boldsymbol{d}$, then reduce step length β to continue line search, and reduce the move limit for the next iteration;
3) If $F(X_0+\beta\boldsymbol{d})-F(X_0)<\beta(1-\eta)\nabla^T F(X_0)\boldsymbol{d}$, then take $X_{new}=X_0+\delta\boldsymbol{d}$ as a new design and stop the line search. In this case, the move limit of the next iteration can be enlarged.

4. Dynamic property approximation

When sequential quadratic programming or sequential linear programming methods are employed to solve the optimization problem, it is desired that objective function or the constraint function be approximated with a high enough quality to facilitate efficient solutions. In the dynamic optimization problem, a first order approximation for the constraints concerning structural natural frequency or frequency field response is needed.

The structural response and sensitivity analysis are implemented using the finite element method in MCADS.

The free vibration problem of the discrete model of the structure can be represented by the following eigenvalue problem:

$$K\varphi - \lambda M\varphi = 0 \tag{4}$$

where M and K are structural mass and stiffness matrix, respectively. λ is the square of the eigen-frequency ω and φ is the mass normalized eigenvector associated with ω.

Generally, the vibration frequency constraint can be written as

$$\lambda_j \geq \underline{\lambda} \tag{5}$$

where $\underline{\lambda}$ is the lower bound of λ_j.

By differentiating Eq. (4) with respect to design variable, we have the derivative of the j-th eigenvalue

$$\frac{\partial \lambda_j}{\partial X} = \frac{\varphi_j^T \left(\dfrac{\partial K}{\partial X} - \lambda_j \dfrac{\partial M}{\partial X} \right) \varphi_j}{\varphi_j^T M \varphi_j} \tag{6}$$

By virtue of this, an approximation of the eigenvalue based upon the first-order Taylor's series expansion is straightforward. Thus the constraint condition (5) is approximated by a linear inequality at the vicinity the current design point X.

On the other hand, since the eigenvalues are highly nonlinear in design variable space, the approximation quality is not good enough. However, it has been shown (Canfield 1990) that the linear approximation quality of vibration frequency can be greatly improved by the Rayleigh Quotient approximation.

The j-th order frequency can be represented by the Rayleigh Quotient

$$\omega_j^2 = \lambda_j = \frac{E_j}{T_j}\ , \tag{7}$$

where E_j and T_j are the modal strain energy and modal kinetic energy, respectively.

$$E_j = \varphi_j^T K \varphi_j,\ T_j = \varphi_j^T M \varphi_j \tag{8}$$

At an new design point X, the j-th order frequency is approximated as

$$\tilde{\omega}_j^2 = \tilde{\lambda}_j = \frac{\tilde{E}_j}{\tilde{T}_j} \tag{9}$$

with

$$\tilde{E}_j = E_j(X_0) + \sum_{i=1}^{n} \frac{\partial E_j}{\partial x_i}(x_i - x_{0i}) \tag{10}$$

$$\tilde{T}_j = T_j(X_0) + \sum_{i=1}^{n} \frac{\partial T_j}{\partial x_i}(x_i - x_{0i}) \tag{11}$$

where X_0 is the previous design point.

By this means, the frequency constraint (5) can be represented as the following linear inequality

$$\sum_{i=1}^{n} \left(\frac{\partial E_j}{\partial x_i} - \underline{\lambda} \frac{\partial T_j}{\partial x_i} \right) x_i \geq \underline{\lambda}\left(T_j(X_0) - \sum_{i=1}^{n} \frac{\partial T_j}{\partial x_i} x_{0i}\right) - E_j(X_0) + \sum_{i=1}^{n} \frac{\partial E_j}{\partial x_i} x_{0i} \tag{12}$$

It has been shown from the numerical examples that the improvement of the quality of approximation for the frequency constraint results in a more stable convergence.

The govening equation for structural dynamic response is represented in the following form

$$M\ddot{u}+C\dot{u}+Ku=p(t) \tag{13}$$

where C is dumping matrix, and u is displacement vector. There exist two forms of harmonic exciting load vector p, $p=p_h\sin\theta t$ and $p=p_h\cos\theta t$, where p_h denotes the vector of the amplitude of the load, which is caused by external nodal force or acceleration of base movement.

The structural displacement response is solved with the modal superposition method in the following form

$$u=s\sin\theta t+c\cos\theta t \tag{14}$$

where s and c are both linear combination of a number of vibration modes of the structure.

The derivatives of u with respect to design variables can be evaluated by several approaches, e.g., the modal superposition method. However, the eigenvector derivative calculation is tedious and time-consuming. Here, a direct sensitivity analysis method by means of solving a dynamic equation in eigenvector space is employed. Taking $p=p_h\cos\theta t$ as example, the derivative of Eq. (13) with respect to design variable X yields

$$M\ddot{u}'+C\dot{u}'+Ku'=F_s\sin\theta t+F_c\cos\theta t \tag{15}$$

with

$$\begin{aligned}F_s&=M's\theta^2-K's\\F_c&=M'c\theta^2-K'c\end{aligned} \tag{16}$$

where the superscript $'$ denotes the derivative with respect to the design variable.

Here, the dumping matrix C is assumed independent of the design variables. It is worth noting that Eq. (15) is the same as Eq. (13) except for the amplitude and the phase angle of the load vector. Following the similar procedure as in the analysis of frequency response, we have two solutions corresponding to excitation $F_s\sin\theta t$ and $F_c\cos\theta t$, respectively.

$$\begin{aligned}u_s'&=s_s\sin\theta t+c_s\cos\theta t\\u_c'&=s_c\sin\theta t+c_c\cos\theta t\end{aligned} \tag{17}$$

Finally, the derivative of displacement response can be obtained as $u'=u_s'+u_c'$. This direct sensitivity analysis method are easy to implement and suitable for different types of design variables and structures without lengthy calculation of eigenvector derivatives for repeated eigenvalues.

5. Numerical examples

The numerical methods mentioned above have been implemented in MCADS system and applied to the dynamic optimization problem of structures.

Example 1. Dynamic response sensitivity analysis of cantilever plate (see Fig. 1). Material parameters: $E=3*10^7$(Pl/in^2), $\mu=0.3$, $\rho=2.82$(Pl/in^3), dumping ratio $c=0.03$. The thickness of plate $t=0.1$(in). The 55-th node of the plate is subjected to harmonic excitation with amplitude $P=1.0$ and frequencies θ varying from 20 to 1000 Hz. Two design variables are treated, i.e., the thickness of the half part of clamped end $x_1=t_1$, and the thickness of the half part of free end $x_2=t_2$.

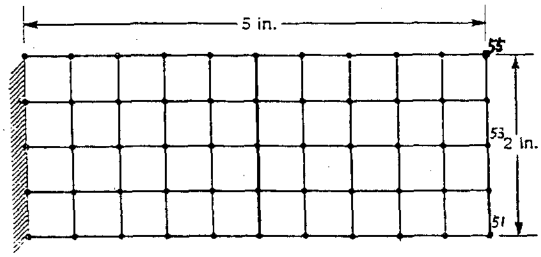

Fig. 1 The finite element model of cantilever plate.

Table 1 Eigenvalues of cantilever plate (HZ)

j	1	2	3	4	5	6
Numerical:	42.18	208.50	263.31	662.46	743.97	1218.48
Analytical:	42.69	220.20	266.30	709.72	748.26	1315.45

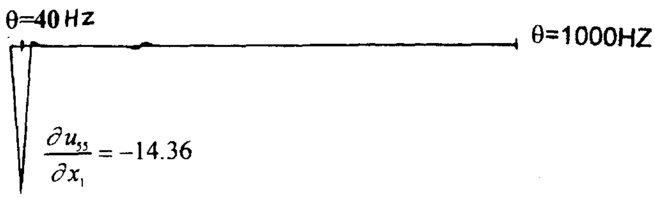

Fig. 2 $\partial u_{55}/\partial x_1$ versus θ.

Fig. 3 $\partial u_{55}/\partial x_2$ versus θ.

The computational results of the eigenfrequencies as well as the analytical solutions are listed in Table 1. Fig. 2 and Fig. 3 show the displacement derivatives of node 55 with respect to x_1 and x_2, respectively, as a function of θ. It is revealed that the numerical results obtained using the presented method agree well with that of the finite difference method.

Example 2. The solar wing of satellite in deployed status, the finite element model and first three vibration modes of which are shown in Fig. 4, is optimized with frequency constraints. The solar wing structure is composed of three pieces of honeycomb sandwich plate with ply reinforced face plate. The support frame, composite thin-walled beam, and the joints linking the three pieces of sandwich plates are modeled with beam elements. The first four vibration frequencies are considered as constraints. Two group of design variables are tested: (a) Surface plate thickness t_i and core heights h_i of the three pieces of sandwich plates. (b) t_i, h_i, and the ply orientation angles θ_i ($i=1, 2, 3$) of the face plate. The initial, optimum and constraint bounds of sandwich plate weight, frequencies and design variables are given in Table 2. The sandwich plate weight is reduced by 25.7% and 27.2% in the optimization results of the two models, respectively.

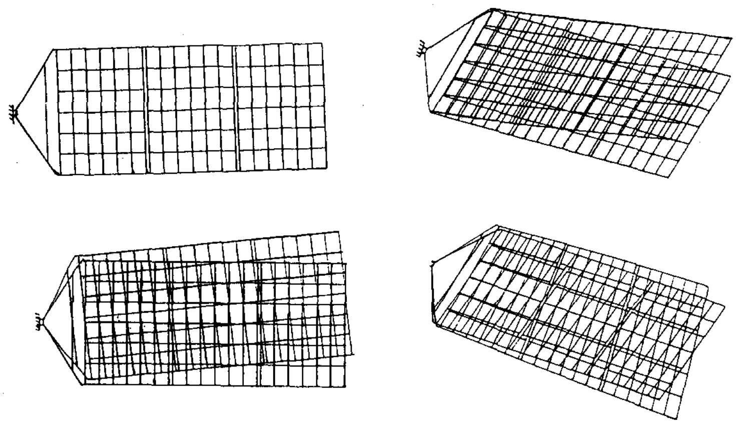

Fig. 4 The FEM model and the first three vibration modes of solar wing structure.

Table 2 Dynamic optimization results of solar wing sandwich plates

	Weight	λ_1	λ_2	λ_3	λ_4	t_1	t_2	t_3	h_1	h_2	h_3	θ_1	θ_2	θ_3
initial value	16.0	0.21	0.80	1.05	1.59	0.3	0.3	0.3	25.0	25.0	30.0	0	0	0
lower-bounds		0.22	0.80	1.10	1.50	0.2	0.2	0.2	20.0	20.0	20.0	−45	−45	−45
upper-bounds						0.4	0.4	0.4	30.0	30.0	30.0	45	45	−45
optimum(a)	12.33	0.24	0.872	1.10	1.705	0.2	0.2	0.2	30.0	30.0	30.0	0	0	0
optimum(b)	12.08	0.22	0.824	1.277	1.633	0.2	0.2	0.2	20.9	26.8	20.0	43	35	45

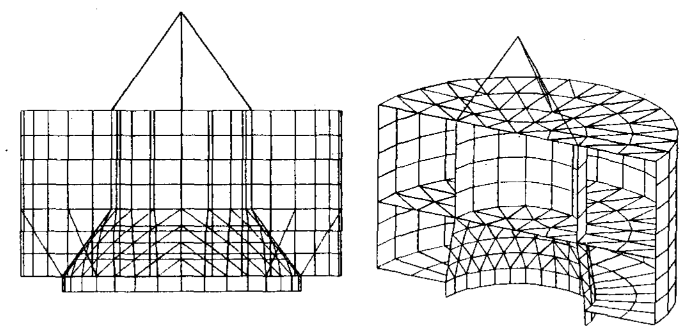

Fig. 5 The finite element model of satellite structure.

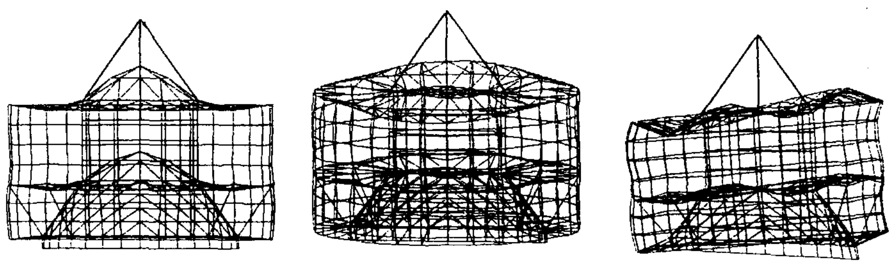

Fig. 6 The first three vibration modes of satellite structure.

Example 3. The dynamic design optimization of satellite structure. The finite element model, shown in Fig. 5, is composed of 571 nodes and 924 elements (beam, bar, shell, composite honeycomb sandwich plate). The first three vibration modes of a satellite structure are shown in Fig.

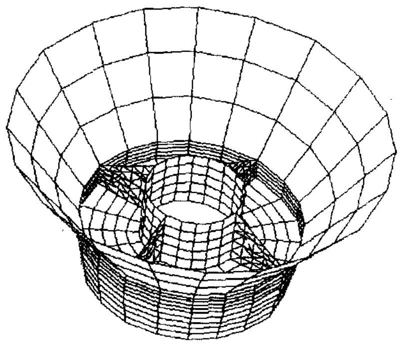

Fig. 7 The finite element model of rocket component structure.

6. The design variables include cross section areas of bars and beams, thickness of shell elements, core heights and face thickness of sandwich plates. After the variable link, twenty master variables are selected. By means of the goal programming method, a multi-objective optimization model is built to increase the first three vibration frequencies of the satellite. The structural weight is constrained not increase in design optimization. The result of design optimization is that the first vibration frequency has been increased by 9.5%, and the second and third vibration frequencies have been increased by 4.3%.

Example 4. The static and dynamic design optimization of structural component of carrier rocket CZ-2E/TS, which is designed for the future launching program of China. In this problem, the upper-stage structure of the rocket is modeled with 580 nodes and 1460 elements (see Fig. 7) and the objective is to minimizing the structural weight under the fundamental frequency constraint. There are totally 20 design variables considered, including thickness of plates, size parameters of beam cross-section, and so on.

In the optimum design obtained, the structural weight is reduced by 12% without violation of the frequency constraint, which indicates a great improvement of the initial design.

6. Conclusions

The sensitivity analysis for dynamic response and the linear approximation of the constraints concerning structural dynamic properties are addressed in this paper. The optimization algorithm is improved by several numerical methods, such as approximate line search, adaptive move limit. Furthermore, the numerical methods mentioned above are implemented in the general purposed structural analysis and optimization program package MCADS. Numerical examples show that the propose methods of dynamic optimization for space structures are effective and efficient.

Acknowledgements

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