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Behaviour of lead-rubber bearings

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Abstract. Experimental work undertaken to investigate the behaviour of lead-rubber bearings under compression and a combination of compression and shear or rotation has been reported on elsewhere. However, it is difficult to determine the state of stress within the bearings in terms of the applied forces and the interaction between the lead plug and the steel shims and elastomeric layers. In order to supply some of the missing information about the stress-strain state within the bearings, an analytical study using the finite element method was carried out. The available experimental results were used to validate the model and although agreement was not as good as expected (on account of difficulties in modelling the lead plug), the analyses did provide some information about the state of the stress within the bearing.

Key words: bridge bearings; lead-rubber bearings; seismic isolation; finite element analysis; compression; shear; rotation.

1. Introduction

A lead-rubber bearing is essentially a laminated elastomeric bearing with a cylindrical lead plug inserted into a preformed hole in the bearing, the hole being of the same diameter. The lead plug provides a high stiffness up to the point at which it yields with the post-yield stiffness of the bearing being that of the rubber in the bearing. On unloading, the lead regains its elastic properties through processes of recovery, recrystallisation and grain growth.

Experimental work to investigate the behaviour of lead-rubber bridge bearings under compression, and a combination of compression and shear or rotation has been reported elsewhere (Mori, *et al.* 1996, in review 1997a,b). However, this experimental work could not provide information about the interaction between the lead plug and the surrounding laminated elastomeric bearing. As well, there has been considerable debate about the yield surface and the confinement conditions for the lead plug. A numerical analysis using the finite element model was carried out to investigate the behaviour of a lead-rubber bearing with particular reference to the interaction between the lead plug and the surrounding rubber and steel layers of the elastomeric bearing.

The bearing modelled comprised nine 10 mm thick rubber layers interspersed with eight 3

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mm thick steel shim reinforcements. The outer steel plates were 10 mm thick with 5 mm thick top and bottom cover rubber layers. Of the total bearing height of 144 mm, the total rubber thickness was 100 mm; the side rubber cover was 10 mm thick. A lead plug of 75 mm diameter was pressed into an identical diameter hole in the centre of the bearing.

2. Physical properties of lead

The purity of the lead used in a lead-rubber bearing is normally required to be over 99.99% (a pure lead). However, because lead is normally used as a component of an alloy, not many tests have been reported for the material properties of pure lead. Nevertheless, it was possible to find a few references in the literature that gave values for the elastic modulus, Poisson's ratio, and the yield point of the lead.

Hofmann (1970) collected a number of test reports and on the basis of these took the elastic modulus for lead to be 17.0 GPa (but with some scatter) at room temperature, with the Poisson's ratio lying between 0.434 and 0.44. Lead is a temperature and rate dependent material and most of the tests reported by Hofmann were carried out under relatively low strain rates with the yield point of the lead not being clearly stated. Hofmann discusses the stress-strain relationships for lead and lead alloys and from the figures for the stress-strain relationship for pure lead, the yield point was estimated to lie between 1.0 MPa and 1.6 MPa in both tension and compression. It should be noted however, that these references refer to tests on lead in an unconfined state, unlike the state in a lead-rubber bearing where the lead is confined by the steel and rubber layers when subjected to compression and shear loads.

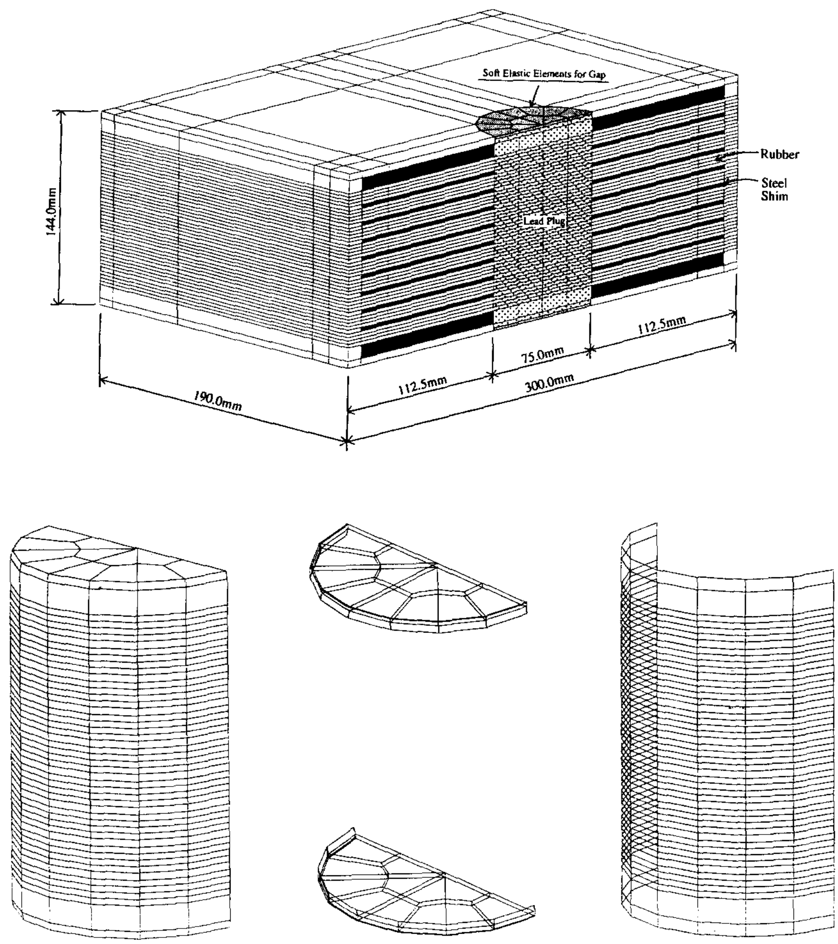
The physical properties that were assumed for pure lead for the finite element analyses that were carried out are listed in Table 1. For the numerical analysis, the lead was assumed to be a perfectly plastic material that was incompressible when behaving plastically. In the ABAQUS (1992) computer programme used to carry out the finite element analyses the Von Mises yield criterion was used and this assumes that the material is isotropic and rate independent. The yield stress given in the table is about twice the upper value derived from Hofmann (1970) as the strain rate used in the experimental programme (Mori, 1993, Mori, *et al.* 1996, 1997b,c) was much higher than those used in the literature. According to Robinson (1993), the lead yield strength when tested at a high strain rate is some two to three times higher than at a low strain rate. While Robinson's findings related to shearing behaviour, the lead stiffness should increase under higher strain rates regardless of the nature of the loading since it is a rate dependent material. Since no data could be found in the literature regarding the yield stress of lead when in a confined state, a value approximately twice that given by Hofmann (1970) was taken as reasonable for obtaining an initial value for the yield stress of the lead when carrying out a trial and error procedure for the numerical analysis.

Table 1 Assumed physical properties of pure lead

Elastic modulus E	Poisson's Ratio v	Yield Stress σ
17.0 GPA	0.44	3.0 MPa

3. Three dimensional finite element model

The finite element model used for half of the lead-rubber bearing, making use of the symmetry of the bearing and loading, is shown in Fig. 1. While 20-node isoparametric brick elements were used to model the rubber and the steel shims in an identical elastomeric bearing (Mori, *et al.* 1997a) (except for the lead plug), an 8-node element was used for this problem in order to simplify the analysis. Since the lead plug has a very low yield point compared to that of the steel plate, the incremental iteration used in the analysis requires very small steps. The computational effort will be expensive if a finite element model with a large number of degrees of freedom, such as the 20-node element, is used for this problem. The total number of degrees of freedom for the model in Fig. 1 is 14720 which is almost the same as for the elastomeric bearing model in Mori, *et al.* (1997a).



Lead Plug　　Soft Elastic Elements for Gap　　Interface Elements

Fig. 1 Three-dimensional FEM model for half of the LRB (Bearing 8).

It was observed from the experimental testing described in Mori, *et al.* (1996, 1997b, c) that the lead plug became a little short after being subjected to the initial compression load. This shortening of the lead plug was modelled as a small clearance between the level of the rubber top and bottom faces and the level of the lead top and bottom faces, using soft elastic material elements to fill the space above and below the top and bottom faces as shown in Fig. 1. An elastic modulus of 3 MPa and a Poisson's ratio of 0.4999 was assumed for these soft elements. These values roughly correspond to the material properties of the rubber in the bearing. The constitutive behaviour of the rubber was modelled using the Mooney-Rivlin form of the strain energy function. Details of the modelling have been given by Mori (1993) and Mori, *et al.* (1997a).

3.1. Boundary conditions for the interface elements

A lead-rubber bearing consists of two parts, namely a vulcanised elastomeric bearing with a hole(s) and a lead plug(s). The elastomeric bearing with its hole is moulded and the lead plug which is slightly longer and of a smaller diameter than the hole, is pressed into the hole to obtain a suitable contact condition between the lead and the surrounding rubber and steel shim face sunder the bearing design load. Because of the structure of the lead-rubber bearing, it is expected that the interaction between the lead plug and the surrounding rubber and steel layers affects the total bearing behaviour. For example, when the lead-rubber bearing is compressed, a layer which consists of the rubber, steel and lead is compressed as illustrated in Fig. 2 and the faces between them should have some frictional interaction because the lead plug and the surrounding rubber and steel layers simply touch each other without any bonding.

The interface element was used in the numerical study because it was the most appropriate element available in ABAQUS to model a contact problem between two deformable bodies with a frictional interface. The interface element can deal with problems involving contact over parts of interacting surfaces where only small relative sliding of the surfaces may occur. If the two bodies are touching, i.e., there is no clearance between them, a surface interaction theory is employed to calculate the shear and pressure stresses between the surfaces as well as the relative displacements of corresponding points along the two surfaces. In ABAQUS, the surface

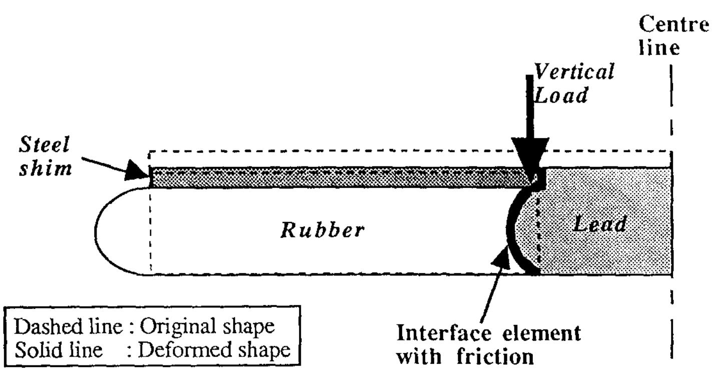


Fig. 2 Expected deformation of a layer in the bearing (LRB) under compression.

interaction theory is based on the standard Coulomb friction model. If the two bodies are not touching, no force is transmitted across the interface.

It should be noted that the relative motion between the surfaces of deformable bodies should not occur until slip occurs in a real contact problem, however, in ABAQUS a stiffness (referred to as the "sticking stiffness") is used to control this relative sliding displacement, or penalised elastic deformation at the interface, and this is a factor as to whether the program can obtain an accurate solution or not. If a larger relative sliding displacement is allowed by choosing a smaller sticking stiffness, the convergence to a solution will be very quick but the solution will be inaccurate. If a smaller relative displacement is specified, then the solution will be more accurate but will be obtained with greater computational effort. Engineering judgement therefore needs to be used to obtain balanced solutions using a computer programme such as ABAQUS. An investigation to determine the appropriate sticking stiffness at the interfaces between the lead and the surrounding rubber and steel is described in a later section.

Realistic friction coefficients between the lead and the surrounding rubber and steel layers need to be assumed to give a reasonable interaction between them. In order to determine realistic values for the friction coefficients, a lead weight was allowed to slide down an inclined sheet of vulcanised natural rubber or a steel plate having a machined surface. The inclined rubber steel or steel plate was tilted until the lead weight just began to slide. The tests were repeated several times and the average value of the inclination was determined. The sliding friction coefficient, μ and the inclination, θ, at which the lead weight begins to slide can be related by:

$$\mu = \tan\theta \tag{1}$$

From Eq. (1) and the test results, the sliding friction coefficients for lead-steel and lead-rubber contacts were determined as shown in Table 2. While it is possible that the actual sliding friction coefficients between the lead and the rubber or steel may be contact pressure dependent, this was not taken into account for the numerical analyses and was not investigated.

4. VERIFICATION OF THE MODEL

4.1. General

In metal forging, a theoretical equation is used to evaluate the stress distribution between a harder plate and a forged metal (Dieter 1976). The equation is derived from the equilibrium of forces acting on the forged metal. For a plane-strain situation, the stress, p, acting on the metal face is given by:

$$p = \frac{2}{\sqrt{3}}\sigma_y\left\{1 + \frac{2\mu(a-x)}{h}\right\} \tag{2}$$

where σ_y is the yield stress of the forged material,
μ is the friction coefficient of the forged metal surface,
$2a$ is the width of the forged metal,
x is the distance from the centre of the metal plate, and
h is the thickness of the forged metal plate

Eq. (2) is based on the Von Mises yield criterion and the assumption that the sliding friction follows Coulomb's law. The thickness of the forged plate is assumed to be small enough that the axial compressive stress is constant through the thickness.

4.2. Comparisons of pressure distributions at the contact face

The dimensions of the model used both for Eq. (2) and a two dimensional finite element analysis were a width of 75 mm and thickness of 20 mm, thus giving an aspect ratio, a/h, of 1.875. The yield stress of the lead was taken as 3.0 MPa and the friction coefficient between the lead and steel as given in Table 2. By way of comparison, analyses were also carried out using a friction coefficient of 0.10.

The normal stress distributions along the top face of the lead plate from both Eq. (2) and the finite element analyses are compared in Fig. 3. It can be seen that the finite element analysis tends to overestimate the stress though the relative error (8%) is comparatively small. The error tends to increase as the friction coefficient increases.

A similar investigation was carried out for the case of a small aspect ratio, a/h, of 0.26 (i.e., a width of 75 mm and a thickness of 144 mm). This aspect ratio was the same as that of the actual lead plug in the lead-rubber bearing modelled in Fig. 1. The results of the analyses are compared in Fig. 4. The values from the finite element analysis do not show a clear trend

Table 2 Friction coefficients

Contact Materials	Friction Coefficients, μ
Lead-steel	0.25
Lead-rubber	0.50

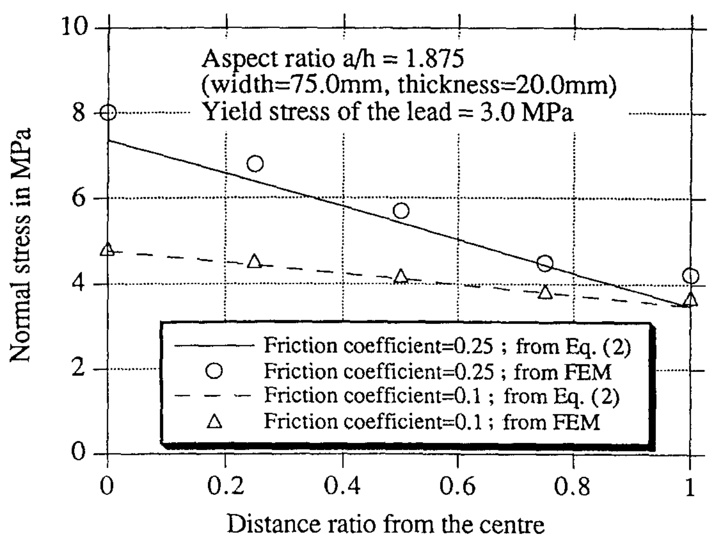


Fig. 3 Normal stresses on the lead of high aspect ratio from the theory and FEM (friction coefficients of 0.25 and 0.10).

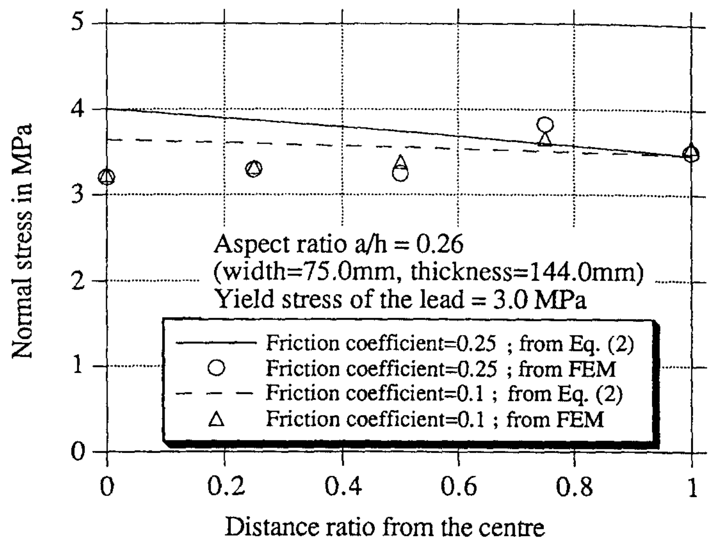


Fig. 4 Normal stresses on the lead of low aspect ratio from the theory and FEM (friction coefficients of 0.25 and 0.10).

while Eq. (2) shows a monotonic increase as the centre is approached but the difference between the centre and the edge for the theory becomes small compared with the case of high aspect ratio.

4.3. Discussion

It is difficult to determine which of the above results is more reliable. The theory assumes that the normal stress distribution is constant through the thickness of the solid metal and is more appropriate for relatively higher aspect ratios (smaller material thickness). In the case of the high aspect ratio example, the finite element results and Eq. (2) give similar results while there is some discrepancy for the case of the low aspect example. In this latter case, the finite element analysis showed a barrelling of the sides of the lead plug due to plastic stress as can be observed experimentally.

From the above, it seems that the finite element analysis is reasonably accurate when compared with the generally accepted forged metal theory for a high aspect ratio example. The finite element analysis clearly shows the limitation of the theory in terms of variations in the aspect ratio. For cases beyond the limits of the theory, the finite element analysis can give reasonable solutions by taking into account the plastic stress flow in the metal which is not considered in Eq. (2) yet is a very important factor when the total behaviour of the lead-rubber bearing is simulated.

5. Analytical approach for the bearing behaviour

Before carrying out the finite element investigation using the bearing model shown in Fig. 1, several analyses were carried out using a simpler model to investigate the effect of the penalised elastic deformation at the interface between the lead, the rubber and the steel shims. This simpler model is illustrated in Fig. 5.

As explained earlier, the analysis required the sticking condition at the interface between the lead and the rubber and steel shim faces to be controlled by an elastic stiffness ("sticking stiffness"). Several stiffness values for the frictional interface were examined. The frictional coefficients between the lead and the rubber, and the lead and steel were taken to have the values listed in Table 2. Four different sticking stiffness combinations were used as listed in Table 3.

The penalised elastic deformation, d_e, at the interface between the lead and rubber and steel shim before slipping is dependent on the interactive force normal to the interface, f_{int}, the friction coefficient, μ and the sticking stiffness, k_{stick}, as

$$d_e = \frac{\mu f_{int}}{k_{stick}} \tag{3}$$

The interactive force was maintained for all cases so that the bearing response was a function of only the different sticking stiffnesses. The most severe condition was for case *B* which had the smallest elastic deformation. Case *B* was also the upper limit for obtaining accurate solutions in terms of reasonable computational effort.

Fig. 6 shows the compression and shear forces carried by the lead plug under a constant compressive bearing displacement plotted against the bearing shear displacement. In case *B* it was possible to reach only a shear displacement of 0.86 mm because of excessive computational effort. It can be seen that there is a large difference between case *A* and the other cases even though the sticking stiffness between the lead and rubber of case *A* is the same as that of

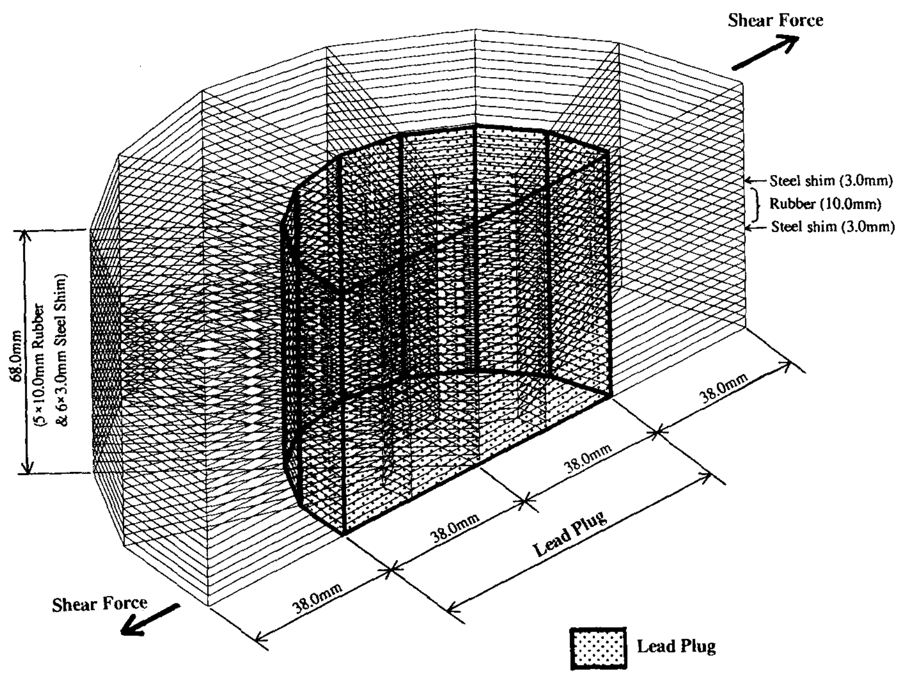


Fig. 5 Simpler bearing model.

Table 3 Sticking stiffness (kN/mm) used for comparison

Contact Materials	Case A	Case B	Case C	Case D
Lead-rubber	1,000	1,000	500	250
Lead-steel	500	10,000	5,000	2,500

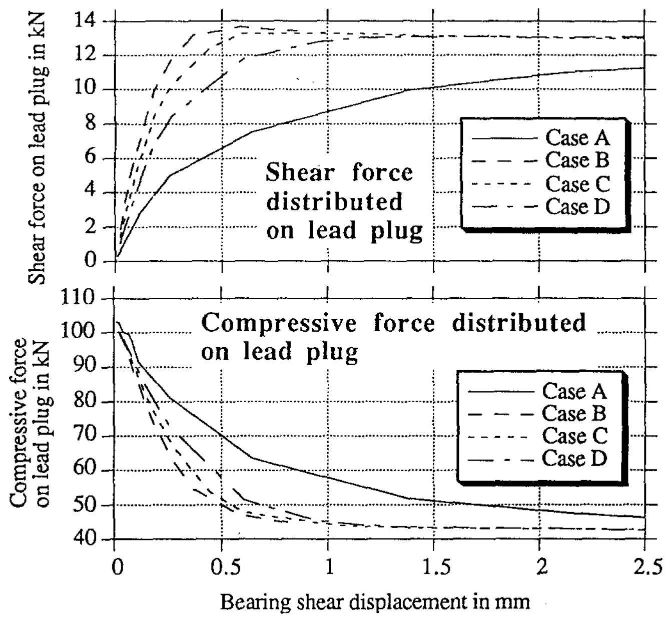


Fig. 6 Shear and compressive forces versus bearing shear displacement of lead plug.

case *B*. From this it appears that the interaction between the lead and steel shim affects the total behaviour of the lead plug. The compressive force on the lead rapidly decreases as the shear displacement increases. This is not only because the bearing compressive resistant force is maintained as constant but also that the increase of the shear force contributes to the compressive resistance force. In terms of the computational time, case *C* took twice as long as case *D* but the difference in the results was small.

The shear and compressive forces distributed to the surrounding rubber of the elastomeric bearing are shown in Fig. 7 as a function of the shear displacement for cases *A*, *C* and *D*. It can be seen from the figure that the contribution of the lead plug to the total bearing behaviour is significant due to the dominant plan area of the lead plug in this simpler bearing model. This large contribution of the lead plug does not occur in the model of Fig. 1 because of the large dimensions of the bearing. These analytical investigations illustrate the need for a careful choice of the sticking stiffness and hence the acceptable penalised elastic deformation at the interface.

6. Comparison between analysis and experiment

The investigation outlined in the previous section enabled the final model for the lead-rubber

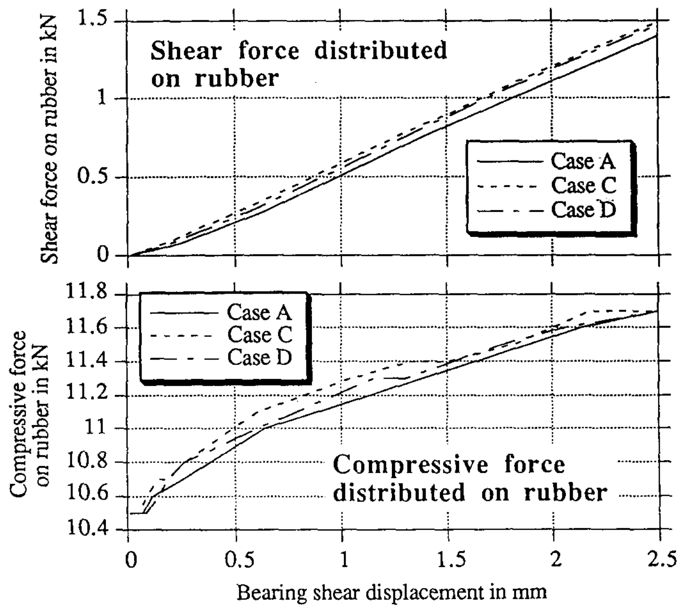


Fig. 7 Shear and compressive forces versus bearing shear displacement of rubber.

bearing to be determined as shown in Fig. 1. The bearing model was analysed for compression and shear. Figs. 8 and 9 show the compressive and shear force-displacement compared with the experimental results for monotonic force-displacement responses. The physical properties used for the lead were those listed in Table 1 and shown in Table 4 as variation *A*. As the results of these analyses did not show a satisfactory comparison with the experimental results, the lead properties were varied as given by variations *B* and *C* in Table 4. Variations in the lead stiffness (as given by the elastic modulus) while keeping the yield strength constant, do not affect the response) because the lead reaches its yield strength just after the beginning of the analysis on account of its very low strength. The response can be seen to be sensitive to the value of the lead yield strength. In compression, the high yield strength of the lead causes a greater deviation from the experimental values than does a lower yield strength. The slopes of the analytical compressive force-displacement response are similar to the experimental ones, except for the initial slope, but the analytical results show a slightly greater stiffness, even though the analytical compressive force is much larger than that of the experiment at the same compressive displacement. This discrepancy is most probably the result of there being an initial small gap between the lead and the rubber faces. This difference between the analytical and experimental responses of the lead-rubber bearing was similar to that for the laminated elastomeric bearing described in Mori, *et al.* (1997a).

In the case of shear behaviour, the analytical results for the high and low yield strengths of the lead and the experimental results indicate that a suitable value for the lead yield strength may be 7 to 8 MPa. Unfortunately, the analytical model of the lead-rubber bearing could not fully verify the experimental results as excessive computational effort was required.

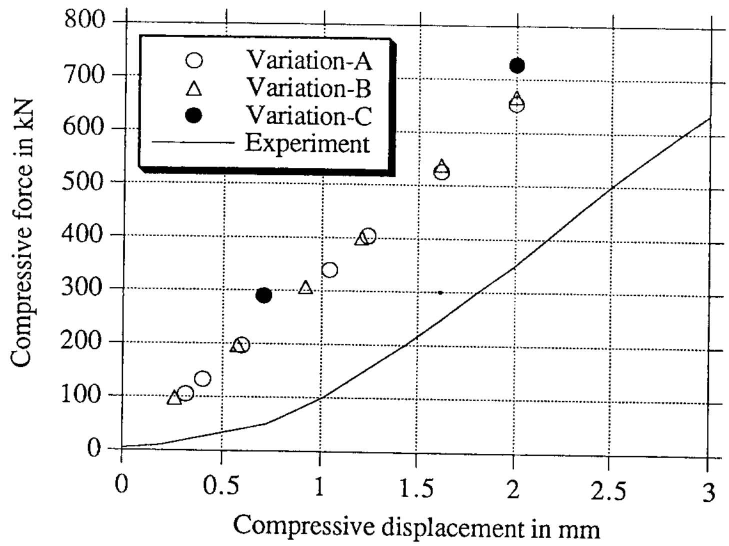


Fig. 8 Force-displacement responses in compression of the analyses and the experiment.

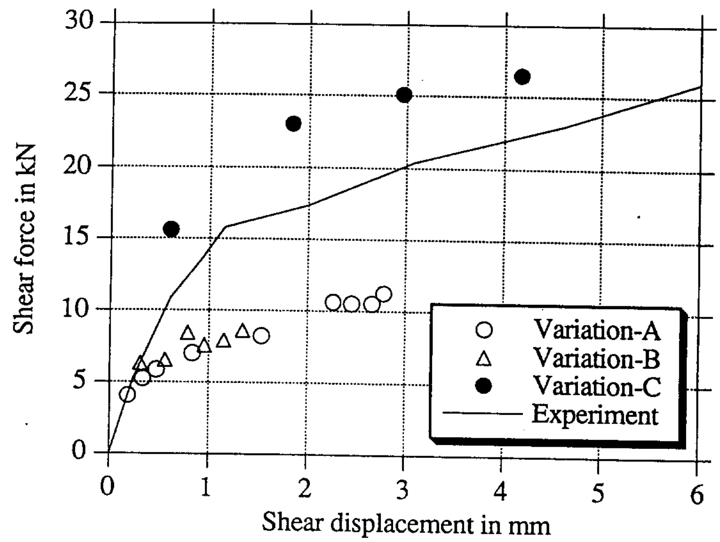


Fig. 9 Force-displacement responses in shear of the analyses and the experiment.

Table 4 Lead property variations

Variation	Elastic Modulus (GPa)	Yield Strength (MPa)
A	17.0	3.0
B	35.0	3.0
C	17.0	10.0

6.1. Deformation of the bearing and the interactive force of the interface

As described in the previous section, the model used in this study did not show good agreement with the experimental results, nevertheless the analytical results for the deformation of the LRB are useful in showing trends. The compressive deformation of the bearing is shown in Fig. 10. The bulging of the LRB at the edges of the rubber layers can be seen to be similar to that of elastomeric bearings (Mori, *et al.* 1997a), but differences appear at the cover rubber layers. These differences are that the deformation of the cover layer of the LRB seems simpler than that of the EB since the linear elements used in the model for the LRB do not have an adequate number of degrees of freedom to follow the complex deformation of the cover rubber.

The deformation of the lead plug is such that the lead plug flows outwards only at the contact faces with the rubber layers because the steel plate is much stiffer in plane than the rubber and therefore it was expected that there would be greater confinement forces at the steel plates. Fig. 11 shows the radial stress at the interface elements between the lead plug and the rubber and steel layers when under compression and also when under bearing shear displacement of 4.2 mm. The figure shows only half of the total bearing height and the stress values shown are the averaged values for the nodes in each layer.

It can be seen that the radial stress at the steel shim layers is about five times that at the rubber layers under compression. However, the radial stress at the steel shim layers for the case of shear loading has values about one third of those under compression and the radial stress tends to be more uniform through the entire height of the lead plug. This is because the interactive normal stress transfers the interactive shear stress and the axial load on the lead plug decreases as the bearing shear deformation increases. The actual values shown in the figure do not have much significance in themselves as the model is not fully satisfactory when compared with the experimental values.

6.2. Discussion

The numerical modelling of the LRB was not entirely satisfactory when compared with the

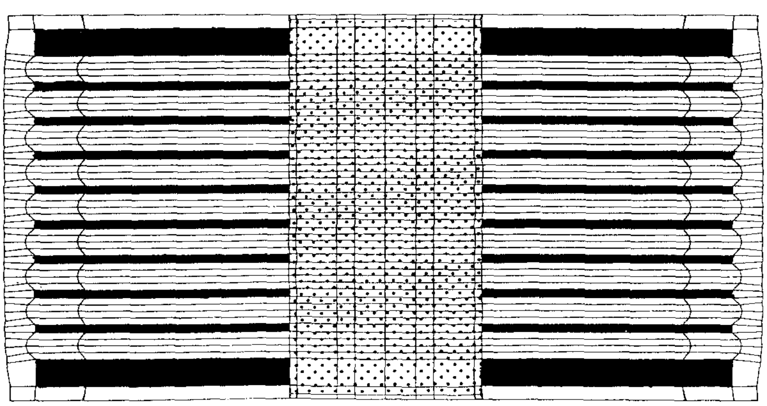


Fig. 10 Deformation of the LRB under compression.

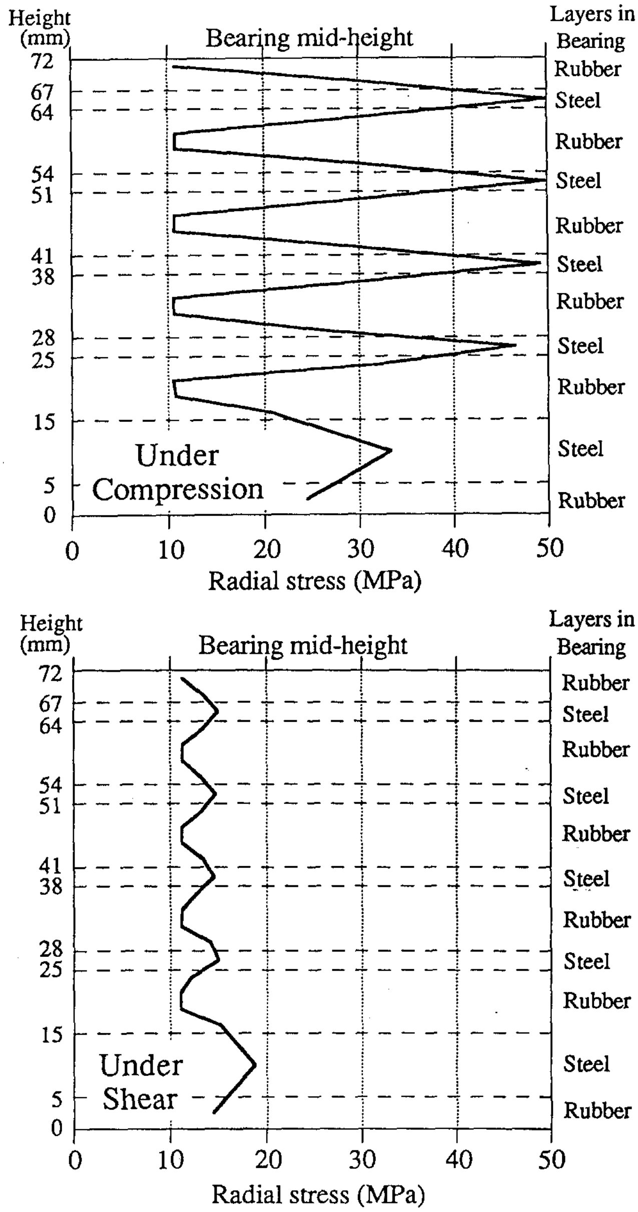


Fig. 11 Radial stress obtained at the interface elements between the lead plug, rubber and steel layers, under compression (top) and under shear (bottom).

experimental results-and the modelling of laminated elastomeric bridge bearing (Mori, *et al.* 1997a). In compression, the experimental and analytical slopes of the force-displacement responses are close if the initial slackness is ignored. It can be seen in Fig. 12 that the experimental slopes of the compressive force-displacement response for the LRB and EB are almost the same except for the behaviour under small compressive forces where the tests showed low initial stiffness. The figure shows clearly that the experimental compressive stiffness of the LRB and EB are very similar. The analytical stiffnesses are similar to the experimental ones but the LRB

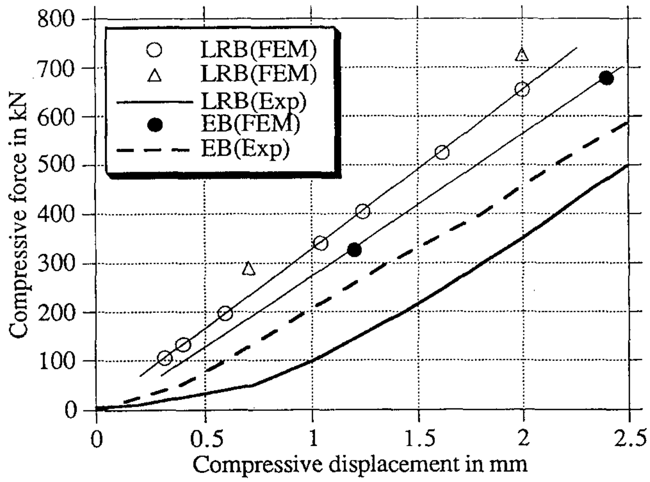


Fig. 12 Comparison of experimental and analytical compressive force-displacement responses for the LRB and EB.

appears to be slightly stiffer than the EB.

The difference between the analytical and experimental force-displacement responses has a number of possible explanations. One, is that it is difficult to model the initial slackness caused by the clearance between the face of the lead plug and the face of the rubber (this arises from the manufacture where a clearance is needed to enable the lead plug to be pressed into the hole in the elastomeric bearing). Another possible reason is that the physical properties of the lead assumed for analytical purposes were derived from a literature study and may well have been different from the actual properties of the lead used for the LRB tested. Then again, it is difficult to model an interaction (stress flow of the lead) between the lead plug and rubber and steel layers using a simple interface element. Nevertheless, if the initial slackness in the compressive force-displacement response is ignored and the analytical compressive stiffness is regarded as being reasonably similar to the experimental compressive stiffness, then a lead yield strength of 7 to 8 MPa will give a shear force-displacement response which is in reasonable agreement with the experimental response.

7. Conclusions

The analytical investigation of LRB behaviour under compression and shear shows that the finite element method can give reasonably accurate values in a contact problem when compared with classical theory. In the particular computer programme used, the value of the sticking stiffness used to model frictional action at the interface element has an important influence on obtaining accurate solutions. The analytical results obtained using the simpler bearing model showed that the bearing behaviour is sensitive to the sticking stiffness but the degree of this sensitivity depends on the dimensions of the bearing and the lead plug.

The analytical model for the LRB was not completely satisfactory when compared with the experimental results due to limitations needed to avoid excessive computational effort. However, a lead yield strength of 7 to 8 MPa does give a reasonable shear force-displacement response, although this yield strength is much higher than the values of 1.0 to 1.6 MPa for unconfined lead reported in the literature. This large difference in lead yield strength may be caused by the confinement of the lead but further research is needed to verify this. The finite element model had some difficulty in the modelling of the clearance between the lead plug and the rubber face, and the interaction between the lead plug and the rubber and steel layers using the simple stress flow rule followed by the Von Mises yield criterion used in the ABAQUS computer programme.

References

ABAQUS (1992), Hibbit, Karlsson and Sorensen Inc., Rhode Island, USA.

Dieter, G.E. (1976), *Mechanical Metallurgy*, 2nd Ed, McGraw-Hill, NY.

Hofmann, W. (1970), *Lead and Lead Alloys, Properties and Technology*, English translation of the 2nd revised German edition by Lead Development Association, London, Springer-Verlag, Berlin.

Mori, A. (1993), "Investigation of the Behaviour of Seismic Isolation Systems for Bridges", PhD Thesis, University of Canterbury, Christchurch, New Zealand.

Mori, A., Carr, A.J., Cooke, N. and Moss, P.J. (1996), "The compression behaviour of bearings used for seismic isolation", *Engineering Structures*, **18**(5), 351-362.

Mori, A., Moss, P.J., Carr, A.J. and Cooke, N. (1997a), "Behaviour of laminated elastomeric bearings", *Structural Engineering and Mechanics*. **5**(4), 451-469.

Mori, A., Carr, A.J., Moss, P.J. and Cooke, N. (1997b), "The behaviour of bearings used for seismic isolation under shear and axial load", *Earthquake Spectra*.

Mori, A., Carr, A.J., Moss, P.J. and Cooke, N. (1997c), "The behaviour of bearings used for seismic isolation under rotation and axial load", *Earthquake Spectra*.

Robinson, W.H. (1993), "Lead-rubber hysteretic bearings suitable for protecting structures during earthquakes", *Earthquake Engineering and Structural Dynamics*, **10**(3), 187-191.