Structural Engineering and Mechanics, *Vol. 58, No. 4 (2016) 719-729*
DOI: http://dx.doi.org/10.12989/sem.2016.58.4.719

Effect of the yield criterion on the strain rate and plastic work rate intensity factors in axisymmetric flow

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(Received October 18, 2015, Revised January 25, 2016, Accepted January 29, 2016)

Abstract. The main objective of the present paper is to study the effect of the yield criterion on the magnitude of the strain rate and plastic work rate intensity factors in axisymmetric flow of isotropic incompressible rigid perfectly plastic material by means of a problem permitting a closed-form solution. The boundary value problem consisting of the axisymmetric deformation of a plastic tube is solved. The outer surface of the tube contracts. The radius of the inner surface does not change. The material of the tube obeys quite a general yield criterion and its associated flow rule. The maximum friction law is assumed at the inner surface of the tube. Therefore, the velocity field is singular near this surface. In particular, the strain rate and plastic work rate intensity factors are derived from the solution. It is shown that the strain rate intensity factor does not depend on the yield criterion but the plastic work rate intensity factor does.

Keywords: strain rate intensity factor; plastic work rate intensity factor; generalised yield criterion; axisymmetric flow

1. Introduction

The strain rate intensity factor has been introduced for isotropic incompressible rigid perfectly plastic material in Alexandrov and Richmond (2001). This factor is the coefficient of the leading singular term in a series expansion of the equivalent strain rate (quadratic invariant of the strain rate tensor) in the vicinity of maximum friction surfaces. In the case of isotropic incompressible rigid perfectly plastic materials the definition for maximum friction surfaces is that the friction stress at sliding is equal to the shear yield stress. The aforementioned series is singular. In particular, the equivalent strain rate approaches infinity near the maximum friction surface and the strain rate intensity factor controls the magnitude of the equivalent strain rate in a narrow region near the surface. On the other hand, it is known that plastic deformation is one of the main contributory mechanisms responsible for hard layer generation in the vicinity of frictional interfaces (Griffiths 1987). This is in qualitative agreement with the asymptotic behaviour of the equivalent strain rate found in Alexandrov and Richmond (2001). Hard layers are often generated

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

in the vicinity of frictional interfaces in metal forming processes (Murai *et al.* 2003, Kajino and Asakawa 2006, Trunina and Kokovkhin 2008, Alexandrov *et al.* 2015). Such layers affect the performance of structures and machine parts under service conditions (Griffiths and Furze 1987, Warren and Guo 2005, Kajino and Asakawa 2006, Choi 2010). Several approaches to relate the strain rate intensity factor and properties of hard layers in the vicinity of frictional interfaces have been proposed in the literature (see, for example, Alexandrov and Lyamina 2006, Alexandrov and Goldstein 2015). In order to further develop these approaches, it is necessary to reveal the effect of constitutive equations on the magnitude of the strain rate intensity factor. In particular, the concept of the strain rate intensity factor has been extended to plastically anisotropic materials in Alexandrov and Jeng (2013). Then, it has been shown in Alexandrov and Mustafa (2014) that plastic anisotropy has a significant effect on the magnitude of the strain rate intensity factor in plane strain compression of a strip between parallel platens. In Alexandrov and Mustafa (2014), the plastic work rate intensity factor has been introduced.

In the case of isotropic incompressible rigid perfectly plastic materials, available solutions for the strain rate intensity factor are restricted to plane strain and axisymmetric problems (Alexandrov 2009). The yield criterion is immaterial in the case of plane strain problems (all possible yield criteria reduce to the same plane strain yield criterion). The available expressions for the strain rate intensity factor in axisymmetric flow are for Tresca's and Mises' yield criteria. These expressions have been derived from the solutions proposed in Shield (1955) and Spencer (1965). Note that Shield (1955) has adopted quite a general yield criterion. However, this general solution has not been used for calculating the strain rate intensity factor. In the present paper, the solution given in Spencer (1965) is extended to the yield criterion proposed in Hosford (1972). This criterion includes both the von Mises and Tresca yield criteria as particular cases. It is shown that the strain rate intensity factor does not depend on the yield criterion but the plastic work rate intensity factor does.

2. Statement of the problem

The boundary value problem considered here consists of an axisymmetric deformation of a

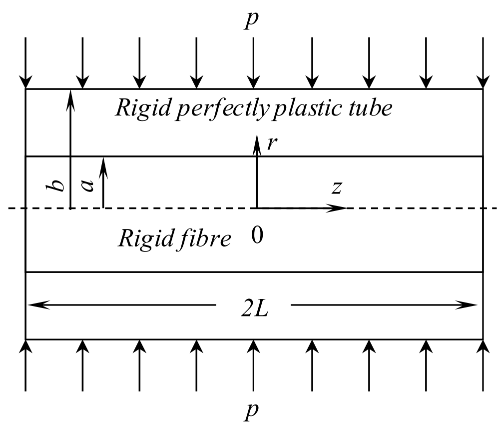

Fig. 1 Configuration and coordinate system

rigid perfectly plastic tube on a rigid fibre (Fig. 1). The length of both the fibre and tube is $2L$.

It is convenient to introduce a cylindrical coordinate system (r,θ,z) such that the z-axis coincides with the axis of symmetry of the flow and the plane z=0 is the plane of symmetry. Then, it is sufficient to consider the region $0\le z\le L$. The outer radius of the tube is defined by the equation $r=b$, and its inner radius as well as the radius of the fibre by the equation $r=a$. The constitutive equations for the tube are the yield criterion proposed in Hosford (1972) and its associated flow rule. The yield criterion reads

$$\left[\frac{(\sigma_1-\sigma_2)^n+(\sigma_2-\sigma_3)^n+(\sigma_1-\sigma_3)^n}{2}\right]^{1/n}=\sigma_0 \tag{1}$$

where σ_1, σ_2 and σ_3 are the principal stresses, n and σ_0 are material constants. In particular, σ_0 is the yield stress in tension and n can vary in the range $1\le n\le\infty$. Equation (1) reduces to Tresca's yield criterion if n=1 or $n\to\infty$ and to Mises' yield criterion if n=2. With no loss of generality it has been assumed that

$$\sigma_1\ge\sigma_2\ge\sigma_3\,. \tag{2}$$

The associated flow rule reads

$$\begin{gathered}\xi_1=\lambda\left[(\sigma_1-\sigma_2)^{n-1}+(\sigma_1-\sigma_3)^{n-1}\right],\quad \xi_2=\lambda\left[(\sigma_2-\sigma_3)^{n-1}-(\sigma_1-\sigma_2)^{n-1}\right],\\ \xi_3=-\lambda\left[(\sigma_2-\sigma_3)^{n-1}+(\sigma_1-\sigma_3)^{n-1}\right]\end{gathered} \tag{3}$$

where ξ_1, ξ_2 and ξ_3 are the principal strain rates and λ is a non-negative multiplier. The equation of incompressibility is a consequence of Eq. (3). Eqs. (1) and (3) should be supplemented with the equilibrium equations. In the case under consideration, the latter reduce to

$$\frac{\partial\sigma_h}{\partial r}+\frac{\partial\tau_{rr}}{\partial r}+\frac{\partial\tau_{rz}}{\partial z}+\frac{\tau_{rr}-\tau_{\theta\theta}}{r}=0,\quad \frac{\partial\tau_{rz}}{\partial r}+\frac{\partial\sigma_h}{\partial z}+\frac{\partial\tau_{zz}}{\partial z}+\frac{\tau_{rz}}{r}=0\,. \tag{4}$$

Here σ_h is the hydrostatic stress and τ_{rr}, $\tau_{\theta\theta}$, τ_{zz}, and τ_{rz} are the deviatoric stress components in the cylindrical coordinate system. The other deviatoric stress components vanish.

The boundary conditions imposed on the shear stress are

$$\tau_{rz}=0 \tag{5}$$

for $r=b$ and

$$\tau_{rz}=k \tag{6}$$

for $r=a$. Here k is the shear yield stress. The boundary condition (6) expresses the maximum friction law. Therefore, according to the general theory (Alexandrov and Richmond, 2001) the velocity field is singular in the vicinity of the surface $r=a$. It follows from Eqs. (1) and (3) that

$$k=\frac{\sigma_0}{\sqrt[n]{1+2^{n-1}}}\,. \tag{7}$$

The exact stress boundary conditions at the ends of the tube are replaced with an approximate condition of the form

$$\int_a^b \sigma_{zz}\big|_{z=L} rdr = 0\,. \tag{8}$$

Here σ_{zz} is the axial stress, $\sigma_{zz}=\sigma_h+\tau_{zz}$. The condition (8) is approximately equivalent to the condition that $\sigma_{zz}=0$ at $z=L$. The exact boundary condition $\sigma_{rz}=0$ at $z=L$ is ignored.

The boundary conditions imposed on the radial velocity, u_r, are

$$u_r = 0 \tag{9}$$

for $r=a$ and

$$u_r = -U \tag{10}$$

for $r=b$. The exact velocity boundary condition at the plane of symmetry is replaced with an approximate condition of the form

$$\int_a^b u_z\big|_{z=0} rdr = 0\,. \tag{11}$$

Here u_z is the axial velocity. The average pressure p (Fig. 1) can be found from the solution by means of

$$p = -\frac{1}{L}\int_0^L \sigma_{rr}\big|_{r=b} dz\,. \tag{12}$$

Here σ_{rr} is the radial stress, $\sigma_{rr}=\sigma_h+\tau_{rr}$.

In the case under consideration the strain rate intensity factor, D, is defined by the following equation (Alexandrov and Richmond 2001)

$$\xi_{eq} = \frac{D}{\sqrt{r-a}} + o\left(\frac{1}{\sqrt{r-a}}\right) \tag{13}$$

as $r \to a$. Here ξ_{eq} is the equivalent strain rate defined as

$$\xi_{eq} = \sqrt{\frac{2}{3}\left(\xi_1^2+\xi_2^2+\xi_3^2\right)}\,. \tag{14}$$

It is also possible to introduce the plastic work rate intensity factor, ω, by (Alexandrov and Mustafa 2014)

$$W = \frac{\omega}{\sqrt{r-a}} + o\left(\frac{1}{\sqrt{r-a}}\right) \tag{15}$$

as $r \to a$. Here W is the plastic work rate defined as

$$W = \sigma_1\xi_1 + \sigma_2\xi_2 + \sigma_3\xi_3\,. \tag{16}$$

3. Solution

The solution of Spencer (1965) suggests that the radial velocity should be of the form

$$u_r = -Ug(r) \tag{17}$$

where $g(r)$ is an arbitrary function of r. Using Eq. (17) the boundary conditions (9) and (10) can be transformed to

$$g = 0 \tag{18}$$

for $r=a$ and

$$g = 1 \tag{19}$$

for $r=b$. It follows from Eq. (17) that the radial and circumferential strain rates are

$$\xi_{rr} = \frac{\partial u_r}{\partial r} = -U\frac{dg}{dr}, \quad \xi_{\theta\theta} = \frac{u_r}{r} = -U\frac{g}{r}. \tag{20}$$

The axial strain rate is found from the incompressibility equation $\xi_{zz}=-\xi_{rr}-\xi_{\theta\theta}$. Substituting Eq. (20) into this equation leads to

$$\xi_{zz} = \frac{\partial u_z}{\partial z} = U\left(\frac{dg}{dr} + \frac{g}{r}\right). \tag{21}$$

Integrating this equation gives

$$\frac{u_z}{U} = \left(\frac{dg}{dr} + \frac{g}{r}\right)z + f(r) \tag{22}$$

where $f(r)$ is an arbitrary function of r. Using Eqs. (17) and (22) the shear strain rate in the cylindrical system of coordinates is determined as

$$\xi_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) = \frac{U}{2}\left[z\frac{d}{dr}\left(\frac{d(rg)}{rdr}\right) + \frac{df}{dr}\right]. \tag{23}$$

Assume that

$$\sigma_{\theta\theta} = \sigma_2 \tag{24}$$

where $\sigma_{\theta\theta}=\sigma_h+\tau_{\theta\theta}$ is the circumferential stress. This assumption should be verified a posteriori. Let ψ be the angle the σ_1 principal stress direction makes with the direction of r. Then

$$\tan 2\psi = \frac{2\tau_{rz}}{\tau_{rr} - \tau_{zz}}. \tag{25}$$

The model under consideration is coaxial (i.e., the principal stress and strain rate directions coincide). Therefore

$$\tan 2\psi = \frac{2\xi_{rz}}{\xi_{rr} - \xi_{zz}}. \tag{26}$$

Substituting Eqs. (20)-(22) into Eq. (26) results in

$$\tan 2\psi = -\left[\frac{d}{dr}\left(\frac{d\left(rg\right)}{rdr}\right)z + \frac{df}{dr}\right]\left(2\frac{dg}{dr} + \frac{g}{r}\right)^{-1}. \tag{27}$$

The solution of Spencer (1965) suggests that ψ is independent of z. It is possible if and only if $d\left(rg\right)/dr = 2rA_1$ where A_1 is constant. The general solution of this equation is

$$g = A_1 r + A_2 r^{-1} \tag{28}$$

where A_2 is a constant of integration. Eqs. (18), (19) and (28) combine to give

$$A_1 = \frac{b}{\left(b^2 - a^2\right)}, \quad A_2 = -\frac{ba^2}{\left(b^2 - a^2\right)}. \tag{29}$$

Substituting Eq. (29) into Eq. (28) leads to

$$g = \frac{b}{\left(b^2 - a^2\right)}\left(r - \frac{a^2}{r}\right). \tag{30}$$

Using this equation it is possible to transform Eq. (27) to

$$\tan 2\psi = \frac{r^2\left(a^2 - b^2\right)}{b\left(3r^2 + a^2\right)}\frac{df}{dr}. \tag{31}$$

Eqs. (25) and (31) combine to give

$$\frac{2\tau_{rz}}{\tau_{rr} - \tau_{zz}} = \frac{r^2\left(a^2 - b^2\right)}{b\left(3r^2 + a^2\right)}\frac{df}{dr}. \tag{32}$$

Substituting Eq. (30) into Eqs. (20), (21) and (23) leads to

$$\xi_{rr} = -\frac{Ub}{\left(b^2 - a^2\right)}\left(1 + \frac{a^2}{r^2}\right), \quad \xi_{\theta\theta} = -\frac{Ub}{\left(b^2 - a^2\right)}\left(1 - \frac{a^2}{r^2}\right), \quad \xi_{zz} = \frac{2Ub}{\left(b^2 - a^2\right)}, \quad \xi_{rz} = \frac{U}{2}\frac{df}{dr}. \tag{33}$$

The principal stresses σ_1 and σ_3 are related to the (r,z) stress components according to the equations

$$\sigma_{rr} = \sigma + q\cos 2\psi, \quad \sigma_{zz} = \sigma - q\cos 2\psi, \quad \tau_{rz} = q\sin 2\psi \tag{34}$$

where

$$\sigma = \frac{\sigma_1 + \sigma_3}{2}, \quad q = \frac{\sigma_1 - \sigma_3}{2} > 0. \tag{35}$$

Analogously

$$\xi_{rr} = \frac{\left(\xi_1 + \xi_3\right)}{2} + \frac{\left(\xi_1 - \xi_3\right)}{2}\cos 2\psi, \quad \xi_{zz} = \frac{\left(\xi_1 + \xi_3\right)}{2} - \frac{\left(\xi_1 - \xi_3\right)}{2}\cos 2\psi, \quad \xi_{rz} = \frac{\left(\xi_1 - \xi_3\right)}{2}\sin 2\psi. \tag{36}$$

The direction of flow dictates that $\tau_{rz}\geq 0$ (Fig. 1). Moreover, it is reasonable to assume that $\sigma_{zz}>\sigma_{rr}$. Then, it follows from Eqs. (34) and (35) that

$$\frac{\pi}{4}\leq\psi\leq\frac{\pi}{2}. \tag{37}$$

It follows from Eq. (24) that $\xi_2=\xi_{\theta\theta}$ and then from Eq. (36) that

$$\frac{\xi_{rr}-\xi_{zz}}{\xi_{\theta\theta}}=\frac{\xi_{rr}-\xi_{zz}}{\xi_2}=\frac{\left(\xi_1-\xi_3\right)}{\xi_2}\cos 2\psi. \tag{38}$$

Substituting Eq. (3) into Eq. (38) and using Eqs. (24) and (33) result in

$$\frac{\left(3r^2+a^2\right)}{\left(r^2-a^2\right)}=\frac{\left[\left(\sigma+q-\sigma_{\theta\theta}\right)^{n-1}+2\left(2q\right)^{n-1}+\left(\sigma_{\theta\theta}+q-\sigma\right)^{n-1}\right]\cos 2\psi}{\left[\left(\sigma_{\theta\theta}-\sigma+q\right)^{n-1}-\left(\sigma-\sigma_{\theta\theta}+q\right)^{n-1}\right]}. \tag{39}$$

Eqs. (1), (24) and (35) combine to give

$$\left[\frac{\left(\sigma+q-\sigma_{\theta\theta}\right)^n+\left(\sigma_{\theta\theta}-\sigma+q\right)^n+\left(2q\right)^n}{2}\right]^{1/n}=\sigma_0. \tag{40}$$

Assuming that q is independent of z and substituting Eq. (34) into Eq. (4) lead to

$$\frac{\partial\sigma}{\partial r}+\frac{d\left(q\cos 2\psi\right)}{dr}+\frac{\sigma-\sigma_{\theta\theta}+q\cos 2\psi}{r}=0,\quad \frac{\partial\sigma}{\partial z}+\frac{d\left(q\sin 2\psi\right)}{dr}+\frac{q\sin 2\psi}{r}=0. \tag{41}$$

These equations are compatible if

$$\frac{\sigma}{\sigma_0}=B_0\left(r\right)+Bz \qquad \text{and} \qquad \frac{\sigma-\sigma_{\theta\theta}}{\sigma_0}=\mu\left(r\right) \tag{42}$$

where B is constant whereas $B_0(r)$ and $\mu(r)$ are arbitrary functions of r. The second equation in Eq. (42) is also compatible with both Eqs. (39) and (40). Substituting Eq. (42) into Eq. (41) yields

$$\sigma_0\frac{dB_0}{dr}+\frac{d\left(q\cos 2\psi\right)}{dr}+\frac{\sigma_0\mu\left(r\right)+q\cos 2\psi}{r}=0,\quad \frac{d\left(q\sin 2\psi\right)}{dr}+\frac{q\sin 2\psi}{r}=-B\sigma_0. \tag{43}$$

Integrating the second equation in Eq. (43) gives

$$\frac{q\sin 2\psi}{\sigma_0}=-\frac{Br}{2}+\frac{B_1}{r} \tag{44}$$

where B_1 is a constant of integration. Taking into account Eqs. (1), (7), (34), (35) and (37) the boundary conditions (5) and (6) can be rewritten as

$$\psi=\frac{\pi}{2} \tag{45}$$

for $r=b$ and

$$\psi = \frac{\pi}{4} \tag{46}$$

for $r=a$. Equations (44) and (45) combine to give $B_1=Bb^2/2$. Then, Eq. (44) becomes

$$\frac{q\sin 2\psi}{\sigma_0} = -\frac{B}{2}\left(r - \frac{b^2}{r}\right). \tag{47}$$

Moreover, $q=k$ at $r=a$. Therefore, it follows from Eqs. (7), (46) and (47) that

$$B = \frac{2a}{\sqrt[n]{1+2^{n-1}\left(b^2-a^2\right)}}. \tag{48}$$

Substituting Eqs. (42) and (47) into Eqs. (39) and (40) yields

$$\frac{\left(3r^2+a^2\right)}{\left(r^2-a^2\right)} = \frac{\left\{\begin{array}{l}\left[2r\mu\sin 2\psi + B\left(b^2-r^2\right)\right]^{n-1} + 2^n B^{n-1}\left(b^2-r^2\right)^{n-1} + \\ \left[B\left(b^2-r^2\right) - 2r\mu\sin 2\psi\right]^{n-1}\end{array}\right\}\cos 2\psi}{\left[B\left(b^2-r^2\right)-2r\mu\sin 2\psi\right]^{n-1} - \left[2r\mu\sin 2\psi + B\left(b^2-r^2\right)\right]^{n-1}} \tag{49}$$

and

$$\begin{gathered}\left[2r\mu\sin 2\psi + B\left(b^2-r^2\right)\right]^n + \left[B\left(b^2-r^2\right)-2r\mu\sin 2\psi\right]^n + \\ +2^n B^n\left(b^2-r^2\right)^n = 2^{n+1} r^n \sin^n 2\psi.\end{gathered} \tag{50}$$

Eqs. (49) and (50) in which B should be eliminated by means of Eq. (48) should be solved numerically to find ψ and μ as functions of r. However, it will be seen in the next section that the strain rate and plastic work rate intensity factors can be found without having this numerical solution.

Once Eqs. (49) and (50) have been solved, the dependence of q on r can be readily determined from Eqs. (47) and (48). Therefore, Eqs. (34) and (42) give σ_{zz} as a function of r and z. This solution also involves $B_0(r)$. This function should be determined by integrating Eq. (43)[1]. The solution so found contains a new constant of integration. Eq. (8) can be used to determine this constant. Then, p can be found from Eq. (12). In order to find the velocity field, ψ should be eliminated in Eq. (31) by means of the solution of Eqs. (49) and (50). Then, the function f can be found by integration with respect to r. As a result, the right hand side of Eq. (22) will involve an arbitrary constant. This constant should be found from Eq. (11). Finally, it is necessary to verify Eq. (24). The circumferential stress is determined from Eq. (42) and the solution of Eqs. (49) and (50). Since the other principal stresses have been already found, it is straightforward to verify Eq. (24).

4. Strain rate and plastic work rate intensity factors

Eq. (14) can be rewritten as

$$\xi_{eq} = \sqrt{\frac{2}{3}\left(\xi_{rr}^2 + \xi_{\theta\theta}^2 + \xi_{zz}^2 + 2\xi_{rz}^2\right)}\,. \tag{51}$$

Since the normal strain rates in the cylindrical coordinate system are bounded in the vicinity of the maximum friction surface $r=a$, Eqs. (13) and (51) combine to give

$$\xi_{rz} = \frac{\sqrt{3}}{2}\frac{D}{\sqrt{r-a}} + o\left(\frac{1}{\sqrt{r-a}}\right) \tag{52}$$

as $r \to a$. Analogously, Eqs. (15) and (16) result in

$$\xi_{rz}\tau_{rz} = \frac{\omega}{2\sqrt{r-a}} + o\left(\frac{1}{\sqrt{r-a}}\right) \tag{53}$$

as $r \to a$. Using Eqs. (26) and (33) the shear strain rate in the cylindrical coordinate system is represented as

$$\xi_{rz} = -\frac{Ub}{2\left(b^2 - a^2\right)}\left(3 + \frac{a^2}{r^2}\right)\tan 2\psi. \tag{54}$$

It is seen from this equation that the asymptotic behaviour of ξ_{rz} in the vicinity of the surface $r=a$ is completely controlled by the asymptotic behaviour of the function $\psi\,(r)$. In the vicinity of the maximum friction surface $r=a$ Eq. (54) can be represented as

$$\xi_{rz} = \frac{Ub}{\left(b^2 - a^2\right)}\left(\psi - \frac{\pi}{4}\right)^{-1} + o\left[\left(\psi - \frac{\pi}{4}\right)^{-1}\right] \tag{55}$$

as $\psi \to \pi/4$. Comparing Eqs. (52) and (55) shows that

$$\psi = \frac{\pi}{4} + \psi_1\sqrt{r-a} + o\left(\sqrt{r-a}\right) \tag{56}$$

as $r \to a$. Here ψ_1 is constant. Since $\mu=0$ at $r=a$, it follows from the binomial theorem that

$$\begin{aligned}
\left[B\left(b^2 - r^2\right) + 2r\mu\sin 2\psi\right]^{n-1} &= B^{n-1}\left(b^2 - r^2\right)^{n-1} + 2r\mu\left(n-1\right)B^{n-2}\left(b^2 - r^2\right)^{n-2}\sin 2\psi + o\left(\mu\right),\\
\left[B\left(b^2 - r^2\right) - 2r\mu\sin 2\psi\right]^{n-1} &= B^{n-1}\left(b^2 - r^2\right)^{n-1} - 2r\mu\left(n-1\right)B^{n-2}\left(b^2 - r^2\right)^{n-2}\sin 2\psi + o\left(\mu\right)
\end{aligned} \tag{57}$$

as $\mu \to 0$. Moreover

$$\cos 2\psi = -2\left(\psi - \frac{\pi}{4}\right) + o\left(\psi - \frac{\pi}{4}\right) \tag{58}$$

as $\psi \to \pi/4$. Substituting Eqs. (56)-(58) into Eq. (49) leads to

$$\mu = \frac{B\left(2^n + 2\right)\left(b^2 - a^2\right)\psi_1}{4\left(n-1\right)a^2}\left(r-a\right)^{3/2} + o\left[\left(r-a\right)^{3/2}\right] \tag{59}$$

as $r \to a$. Substituting Eqs. (56) and (59) into Eq. (50), equating coefficients of like powers of $r - a$ and using Eq. (48) yield

$$\psi_1 = \sqrt{\frac{b^2 + a^2}{2a\left(b^2 - a^2\right)}}. \tag{60}$$

Then, it follows from Eqs. (52), (55), (56), and (60) that

$$D = \frac{2\sqrt{2a}Ub}{\sqrt{3}\sqrt{b^4 - a^4}}. \tag{61}$$

It is seen from this equation that the strain rate intensity factor is independent of n. In particular, Eq. (61) coincides with the expression for the strain rate intensity factor found in Alexandrov (2009) for Tresca's yield criterion. Substituting Eqs. (7), (52), and (61) into Eq. (53) gives

$$\omega = \frac{2\sqrt{2}\sigma_0 Ub\sqrt{a}}{\sqrt[n]{1 + 2^{n-1}}\sqrt{b^4 - a^4}}.$$

It is seen from this equation that the plastic work rate intensity factor depends on n and attains its minimum value at $n \approx 2.767$.

5. Conclusions

A semi-analytical solution for compression of an axisymmetric plastic tube on a rigid fibre has been found. It has been assumed that the material of the tube obeys the yield criterion proposed in Hosford (1972) and its associated flow rule. The objective of the present paper is to reveal the effect of the yield criterion on the strain rate and plastic work rate intensity factors. The yield criterion proposed in Hosford (1972) is quite general and includes widely used the von Mises and Tresca yield criteria as particular cases. The strain rate and plastic work rate intensity factors have been derived from the general solution by means of asymptotic analysis. It has been shown that the strain rate intensity factor is independent of the yield criterion. However, the plastic work rate intensity factor depends on the yield criterion and this dependence is not monotonic with respect to the parameter n involved in the yield criterion (see Eq. (1)). In particular, the plastic work rate intensity factor attains its minimum value at $n \approx 2.767$.

Acknowledgments

The research described has been supported by the grants RFRB-16-51-540001 and VAST.HTQT.Nga.02/16-17.

References

Alexandrov, S. (2009), "The strain rate intensity factor and its applications: a review", *Mater. Sci. Forum*, **623**, 1-20.

Alexandrov, S.E. and Goldstein, R.V. (2015), "On constructing constitutive equations in metal thin layer near friction surfaces in material forming processes", *Doklady Physics*, **60**(1), 39-41.

Alexandrov, S. and Jeng, Y.R. (2013), "Singular rigid/plastic solutions in anisotropic plasticity under plane strain conditions", *Cont. Mech. Therm.*, **25**(5), 685-689.

Alexandrov, S., Jeng, Y.R. and Hwang, Y.M. (2015), "Generation of a fine grain layer in the vicinity of frictional interfaces in direct extrusion of AZ31 alloy", *ASME J. Manuf. Sci. Eng*., **137**(5), Paper 051003.

Alexandrov, S. and Lyamina, E. (2006), "Prediction of fracture in the vicinity of fiction surfaces in metal forming processes", *J. Appl. Mech. Techn. Phys.*, **47**(5), 757-761.

Alexandrov, S. and Mustafa, Y. (2014), "The strain rate intensity factor in the plane strain compression of thin anisotropic metal strip", *Meccanica*, **49**(12), 2901-2906.

Alexandrov, S. and Richmond, O. (2001), "Singular plastic flow fields near surfaces of maximum friction stress", *Int. J. Nonlin. Mech*., **36**(1), 1-11.

Choi, Y. (2010), "Influence of a white layer on the performance of hard machined surfaces in rolling contact", *Proceedings of the Institution of Mechanical Engineers Part B: Journal of Engineering Manufacture*, **224**(8), 1207-1215.

Griffiths, B.J. (1987), "Mechanisms of white layer generation with reference to machining and deformation processes", *ASME J. Tribology*, **109**(3), 525-530.

Griffiths, B.J. and Furze, D.C. (1987), "Tribological advantages of white layers produced by machining", *ASME J. Tribology*, **109**(2), 338-342.

Hosford, W.F. (1972), "A generalized isotropic yield criterion", *Tran. ASME J. Appl. Mech.*, **39**, 607-609.

Kajino, S. and Asakawa, M. (2006), "Effect of "additional shear strain layer" on tensile strength and microstructure of fine drawn wire", *J. Mater. Proc. Technol*., **177**, 704-708.

Murai, T., Matsuoka, S., Miyamoto, S. and Oki, Y. (2003), "Effects of extrusion conditions on microstructure and mechanical properties of AZ31B magnesium alloy extrusions", *J. Mater. Proc. Technol*., **141**, 207-212.

Shield, R.T. (1955), "Plastic flow in a converging conical channel", *J. Mech. Phys. Solid.*, **3**, 246-258.

Spencer, A.J.M. (1965), "A theory of the failure of ductile materials reinforced by elastic fibres", *Int. J. Mech. Sci*., **7**, 197-209.

Trunina, T.A. and Kokovkhin, E.A. (2008), "Formation of a finely dispersed structure in steel surface layers under combined processing using hydraulic pressing", *J. Mach. Manuf. Reliab*., **37**(2), 160-162.

Warren, A.W. and Guo, Y.B. (2005), "Numerical investigation on the effects of machining-induced white layer during rolling contact", *Tribology Tran.*, **48**(3), 436-441.